Asymmetries in e^+e^- collisions from the $\mathrm{Sp}(6)_L \times \mathrm{U}(1)_Y$ family model

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We examine the forward-backward and left-right asymmetries of lepton pairs produced in e^+e^- collisions as probes of new physics resulting from the $\mathrm{Sp}(6)_L \times \mathrm{U}(1)_Y$ family model. The asymmetries are found to be generation dependent and can be used to easily distinguish the $\mathrm{Sp}(6)_L \times \mathrm{U}(1)_Y$ model.

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The standard model [1] of electroweak interactions based on the $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ group has demonstrated remarkable success in describing neutral- and charged-current processes and in determining the mass of the W and Z gauge bosons. However, the family repetition of quarks and leptons strongly suggests that the standard model needs to be extended. Instead of an $\mathrm{SU}(2)_L$ interaction which connects the doublets in one generation, it seems natural to have a larger flavor gauge group that can interchange fermions in different generations. The six left-handed quarks (leptons) form three doublets under the $\mathrm{SU}(2)_L$ flavor group. It is desirable to include them in a six-dimensional representation **6** of a simple flavor gauge group. It was shown [2] that there is a unique extension of $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ into the anomaly-free $\mathrm{Sp}(6)_L \times \mathrm{U}(1)_Y$. Under Sp(6), the left-handed fermions (quarks or leptons) transform like **6** while the right-handed ones are all singlets. This extension is natural since $\mathrm{SU}(2) \cong \mathrm{Sp}(2)$. A doublet of $\mathrm{Sp}(2)_L[\mathrm{SU}(2)]$ is thus readily generalized into the anomaly-free **6** of Sp(6), for three generations. Sp(6) can be naturally broken into $[\mathrm{SU}(2)]^3 = \mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2 \times \mathrm{SU}(2)_3$, where $\mathrm{SU}(2)_i$ operates on the ith generation exclusively. Thus, the standard $\mathrm{SU}(2)_L$ is to be identified with the diagonal SU(2) subgroup of $[\mathrm{SU}(2)]^3$. In terms of the $\mathrm{SU}(2)_i$ gauge boson $\mathbf{A}_i$, the $\mathrm{SU}(2)_L$ gauge bosons are given by $\mathbf{A} = (1/\sqrt{3})(\mathbf{A}_1 + \mathbf{A}_2 + \mathbf{A}_3)$. Of the other orthogonal combinations of $\mathbf{A}_i$, it was found that $\mathbf{A}' = (1/\sqrt{6})(\mathbf{A}_1 + \mathbf{A}_2 - 2\mathbf{A}_3)$ has a mass scale in the TeV range [3].

There have been a number of phenomenological studies [4] on the $\mathrm{Sp}(6)_L \times \mathrm{U}(1)_Y$ model. In the $\mathrm{Sp}(6)_L \times \mathrm{U}(1)_Y$ model the gauge boson Z' couples equally to the first and second generations, but differently to the third. This universality violation gives rise to distinctive observable features. In this work, we would like to investigate the effects of the generation-dependent couplings of Z' on leptonic asymmetries in high-energy e^+e^- collisions.

Several articles have dealt with the effects of an additional neutral gauge boson in e^+e^- collisions [5]. In general, with the additional gauge boson Z', the neutral-current Lagrangian is generalized to contain an additional term:

$$-\mathcal{L}_{\mathrm{NC}} = eJ^{\mu}_{\mathrm{em}} A_\mu + g_Z J^{\mu}_{Z} Z_\mu + g_{Z'} J^{\mu}_{Z'} Z'_\mu , \quad (1)$$

where $g_{Z'} = \sqrt{(1-x_W)/2}\, g_Z = g/\sqrt{2}$, $x_W = \sin^2\theta_W$, and $g = e/\sin\theta_W$. In this paper we use $x_W = 0.23$. The neutral currents J_Z and $J_{Z'}$ are given by

$$J^{\mu}_{Z} = \frac{1}{2} \sum_f \bar{\psi}_f \gamma^\mu (g^f_V + g^f_A \gamma_5) \psi_f , \quad (2)$$

$$J^{\mu}_{Z'} = \frac{1}{2} \sum_f \bar{\psi}_f \gamma^\mu (g'^f_V + g'^f_A \gamma_5) \psi_f , \quad (3)$$

where $g^f_V = (T_{3_L} - 2x_W Q)_f$, $g^f_A = (T_{3_L})_f$, and $g'^f_V = g'^f_A = (T_{3_L})_f$ for the first two generations and $g'^f_V = g'^f_A = -2(T_{3_L})_f$ for the third. Here $(T_{3_L})_f$, and Q_f are the third component of the weak isospin and electric charge of the fermion f, respectively. Let ϕ denote the mixing angle between Z and Z'; then the physical (mass eigenstate) gauge bosons, denoted by Z_1 and Z_2, are given as linear combinations of Z and Z':

$$\begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} Z \\ Z' \end{bmatrix} , \quad (4)$$

and the neutral-current Lagrangian reads

$$-\mathcal{L}_{\mathrm{NC}} = g_Z \sum_{i=1}^{2} \left[\sum_f \bar{\psi}_f \gamma_\mu (g^f_{V_i} + g^f_{A_i} \gamma_5) \psi_f \right] Z_i , \quad (5)$$

where $g^f_{V_i}$ and $g^f_{A_i}$ are the vector and axial-vector couplings of fermion f to physical gauge boson Z_i, respectively, and they are given by

$$g^f_{V_1, A_1} = \frac{1}{2} \left[g^f_{V,A} \cos\phi + \frac{g_{Z'}}{g_Z} g'^f_{V,A} \sin\phi \right] , \quad (6)$$

$$g^f_{V_2, A_2} = \frac{1}{2} \left[-g^f_{V,A} \sin\phi + \frac{g_{Z'}}{g_Z} g'^f_{V,A} \cos\phi \right] . \quad (7)$$

The change in the fermion couplings given by Eqs. (6) and (7) will affect measurements in e^+e^- collisions. Among the quantities that are sensitive to this change are the forward-backward and the left-right asymmetries. The neutral-current Lagrangian in the mass eigenstates basis of the gauge bosons can be used to determine the cross section $\sigma(e^+e^- \to f\bar{f})$. The integrated forward-backward asymmetry is defined as

$$A_{\rm FB}=\frac{\int_0^1(d\sigma/dz)dz-\int_{-1}^0(d\sigma/dz)dz}{\int_0^1(d\sigma/dz)dz+\int_{-1}^0(d\sigma/dz)dz}\,, \tag{8}$$

where $z=\cos\theta$ and θ is the angle between the outgoing fermion and the incident electron. The left-right asymmetry is defined by

$$A_{\rm LR}=\frac{\sigma_L-\sigma_R}{\sigma_L+\sigma_R}\,, \tag{9}$$

where σ_L (σ_R) is the cross section for scattering of a left- (right-) handed electron on an unpolarized positron. With the couplings given by Eqs. (6) and (7), the general expressions for the forward-backward and left-right asymmetries are written explicitly as

$$A_{\rm FB}=\frac{3}{4D}\left\{2\sum_{j=1}^{2}g^e_{A_j}g^f_{A_j}\,{\rm Re}\Delta_j+\sum_{j,k=1}^{2}(g^e_{V_j}g^e_{A_k}+g^e_{A_j}g^e_{V_k})\times(g^f_{V_j}g^f_{A_k}+g^f_{A_j}g^f_{V_k})\,{\rm Re}(\Delta_j\Delta_k^*)\right\}, \tag{10}$$

$$A_{\rm LR}=\frac{1}{D}\left\{2\sum_{j=1}^{2}g^f_{V_j}g^e_{A_j}\,{\rm Re}\Delta_j+\sum_{j,k=1}^{2}2g^e_{V_j}g^e_{A_k}\times(g^f_{V_j}g^f_{V_k}+g^f_{A_j}g^f_{A_k})\,{\rm Re}(\Delta_j\Delta_k^*)\right\}, \tag{11}$$

where the superscript f refers to the final-state lepton and

$$D=1+2\sum_{j=1}^{2}g^e_{V_j}g^f_{V_j}\,{\rm Re}\Delta_j+\sum_{j,k=1}^{2}(g^e_{V_j}g^e_{V_k}+g^e_{A_j}g^e_{A_k})\times(g^f_{V_j}g^f_{V_k}+g^f_{A_j}g^f_{A_k})\,{\rm Re}(\Delta_j\Delta_k^*)\,, \tag{12}$$

where

$$\Delta_j=\frac{s}{x_W(1-x_W)[(s-M^2_{Z_j})+iM_{Z_j}\Gamma_{Z_j}]}\,, \tag{13}$$

here M_{Z_j} and Γ_{Z_j} are the mass and total width of gauge boson Z_j, respectively. The total width Γ_{Z_j} is defined by

$$\Gamma(Z_j\to{\rm all})=\sum_f\Gamma(Z_j\to f\bar f)=\sum_f\tfrac{2}{3}N_fM_{Z_j}\left[\frac{GM^2_{Z_1}}{\sqrt{2}\pi}\right][(g^f_{V_j})^2+(g^f_{A_j})^2]\,, \tag{14}$$

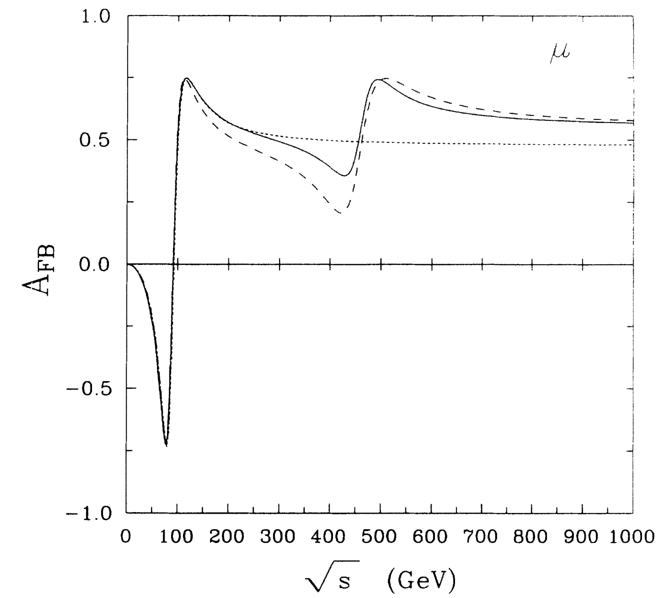

FIG. 1. The forward-backward asymmetry $A_{\rm FB}$ for $e^+e^-\to\mu^+\mu^-$ as a function of $\sqrt{s}$ for $M_{Z_2}=500$ GeV and $\phi=0.05$ (solid curve) and $\phi=-0.05$ (dashed curve). The dotted curve is the standard model predictions.

where N_f is a color factor ($N_f=3$ for quarks and $N_f=1$ for leptons).

The effects of the presence of Z' on top of the Z_1 resonance were studied [6] where deviations from the standard model predictions were expected. However, it is possible that there is not much mixing between Z and Z'. In this case no such deviations will show up on the Z_1 resonance. But, as we will show, pronounced effects can show up off the Z_1 resonance regardless of the value of the mixing angle. The mixing angle is bounded from

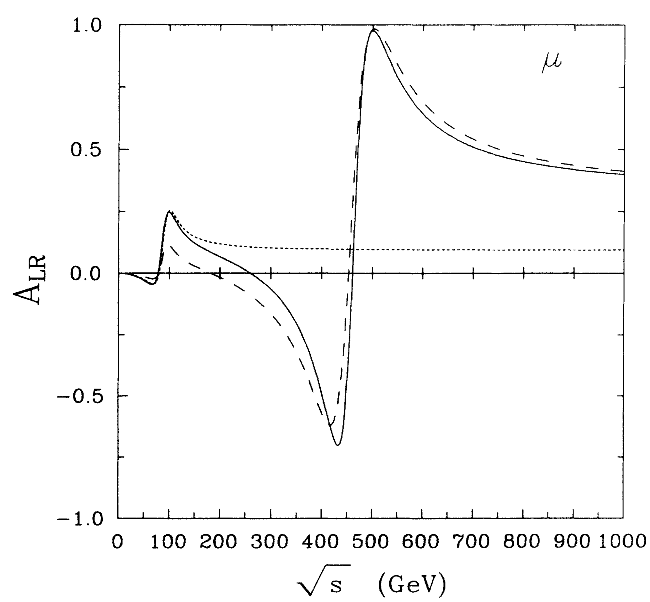

FIG. 2. The left-right asymmetry $A_{\rm LR}$ for $e^+e^-\to\mu^+\mu^-$ as a function of $\sqrt{s}$ for the same mass and mixing angles considered in Fig. 1. The dotted curve is the standard model predictions.

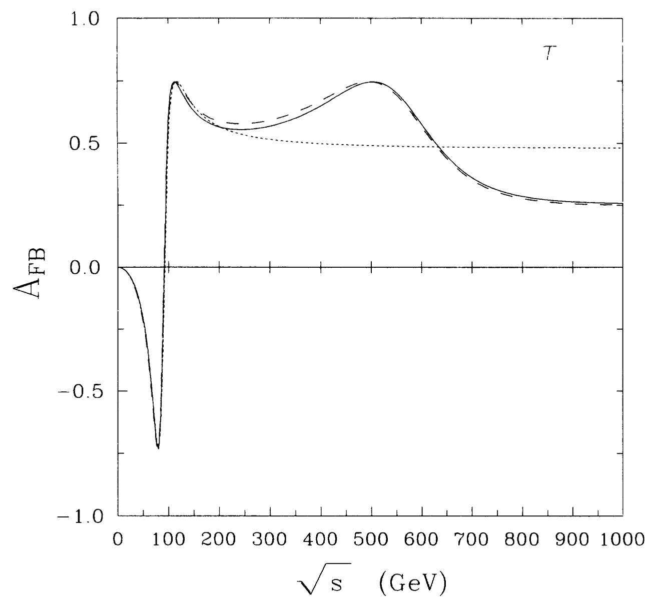

FIG. 3. The forward-backward asymmetry A_{FB} for $e^+e^- \to \tau^+\tau^-$ as a function of $\sqrt{s}$ for the same mass and mixing angles considered in Fig. 1. The dotted curve is the standard model predictions.

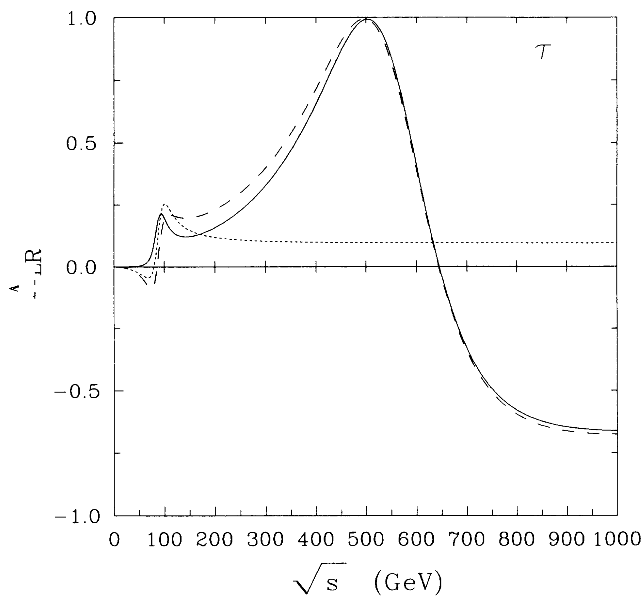

FIG. 4. The left-right asymmetry A_{LR} for $e^+e^- \to \tau^+\tau^-$ as a function of $\sqrt{s}$ for the same mass and mixing angles considered in Fig. 1. The dotted curve is the standard model predictions.

above since it is generally believed to be of the order of $M_Z/M_{Z'}$. $M_{Z'}$ is at least ≥ 300 GeV from the direct search limit of the Collider Detector at Fermilab (CDF) [7]. In this work we will consider $|\phi| \leq 0.05$.

Now we turn to our results. In Fig. 1, we consider $M_{Z_2} = 500$ GeV and we present the expected forward-backward asymmetry for the process $e^+e^- \to \mu^+\mu^-$ as a function of $\sqrt{s}$ for $\phi = -0.05$ and 0.05. For comparison, we also present the forward-backward asymmetry predicted by the standard model. We find a distinctive modification of the standard model predictions featured in the existence of a dip due to cancellation among different contributions. The location of the dip is about 15% below the Z_2 threshold. The dip is followed by a peak on the Z_2. Figure 2 shows the left-right asymmetry as a function of $\sqrt{s}$ for the same process. The effect of the presence of Z' on the left-right asymmetry is found to be even more pronounced where a sharp dip exists about 15% below the Z_2 threshold followed by a sharp peak on the Z_2.

Since the couplings of the τ lepton are different as compared to those of the muon, a different behavior is expected for the $\tau^+\tau^-$ final state. In Fig. 3, we present the forward-backward asymmetry for the process $e^+e^- \to \tau^+\tau^-$ where a broad peak instead of a dip shows up at about 15% below the Z_2 threshold. The left-right asymmetry for the same process is shown in Fig. 4 where a large peak shows up on the Z_2 followed by a large dip well below the standard model predictions.

The deviations for A_{FB} and A_{LR} from those of the standard model are typically of the order of 10–20 %, when one reaches energies $\gtrsim 200$ GeV. From the known accuracies achieved at the CERN e^+e^- collider LEP I and SLAC, these effects should be measurable at LEP II. If future e^+e^- colliders with energies in the range 500 GeV – 1 TeV become a reality, even bigger effects are expected and they cannot escape detection. We are thus hopeful that our results will be put to experimental test in the future.

In conclusion, we have examined the effects of the presence of an extra neutral gauge boson suggested by the $Sp(6)_L \times U(1)_Y$ family mode on leptonic asymmetries in high-energy e^+e^- collisions. We find that there are pronounced deviations in the forward-backward and left-right asymmetries from the standard model predictions. The opposite behavior of the $\mu^+\mu^-$ and $\tau^+\tau^-$ final states which results from their different couplings provided by the universality violation inherent in the $Sp(6)_L \times U(1)_Y$ model is a clear signal for the existence of an additional neutral gauge boson predicted by the model.

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