

Proton decay and related processes models with gauged baryon number

Palash B. Pal

Center for Particle Physics, University of Texas, Austin, Texas 78712

Utpal Sarkar

Theory Group, Physical Research Laboratory, Ahmedabad 380009, India

(Received 3 August 1993)

In unification models based on $SU(15)$ or $SU(16)$, baryon number is part of the gauge symmetry, broken spontaneously. In such models, we discuss various scenarios of important baryon-number-violating processes such as proton decay and neutron-antineutron oscillation. Our analysis depends on the effective operator method, and covers many variations of symmetry breaking, including different intermediate groups and different Higgs-boson content. We discuss processes mediated by gauge bosons and Higgs bosons in parallel. We show how accidental global or discrete symmetries present in the full gauge invariant Lagrangian restrict baryon-number-violating processes in these models. In all cases we find that baryon-number-violating interactions are sufficiently suppressed to allow grand unification at energies much lower than the usual 10^{16} GeV.

PACS number(s): 12.10.Dm, 11.30.Ly, 11.30.Qc, 13.30.-a

I. INTRODUCTION

In all gauge theories reasonably verified by experiments, fermions transform as the fundamental representations of the non-Abelian gauge groups. Quarks transform as the fundamental representation of the color group $SU(3)$, left-handed fermions are fundamental representations of the electroweak $SU(2)$. Inspired by this, it is intriguing to consider the idea that all fermions transform like the fundamental representation of the grand unified gauge group. This leads to grand unified models based on the maximal symmetry group [1] for each generation, $SU(16)$, where the fermions all appear in the fundamental multiplet:

$$\Psi_L \equiv \left(u_r u_b u_y \ d_r d_b d_y \ \widehat{d}_r \widehat{d}_b \widehat{d}_y \ \widehat{u}_r \widehat{u}_b \widehat{u}_y \ \nu_e e^- e^+ \widehat{\nu}_e \right)_L. \quad (1.1)$$

The indices r, b, y are three colors, and carets denote antiparticles for any fermion field ψ :

$$\widehat{\psi} = C\gamma_0^T \psi^*, \quad (1.2)$$

where $C^{-1}\gamma_\mu C = -\gamma_\mu^T$. Thus, for example, $\widehat{\nu}_{eL}$ is the antiparticle of the right-handed neutrino ν_{eR} , assuming that it exists. The same pattern is repeated for other generations. Mirror fermions are needed to cancel the anomalies. One important feature of this model is that both baryon number (B) and lepton number (L), which are known symmetries of low energy physics, appear as gauge symmetries at high energy. In fact, this was one of the main motivations of Pati, Salam, and Strathdee who first introduced such models [1].

A new variant of these models has received some attention lately, where the gauge group is $SU(15)$ [2,3]. The difference with $SU(16)$ is that the right-handed neutrinos, which are not confirmed experimentally, are assumed not

to exist, so that there are only 15 left-chiral fermionic fields per generation. Baryon number is still part of the gauge symmetry although lepton number is not.

The interesting point about these models is that their characteristics are very different from the standard unification models based on the gauge groups $SU(5)$, $SO(10)$, E_6 , etc. For example, it has been shown that renormalization group analysis of certain symmetry breaking chains of these models yield low unification scales [2-5], as low as 10^8 GeV in some cases. All the known chains with such low unification scale have the property that they all break the unified group in such a way that at intermediate scales, quarks and leptons transform under separate subgroups of the gauge group. Because of low unification scales, these models do not suffer from the cosmological monopole problem [6], in sharp difference with $SU(5)$ models. Important and interesting constraints on rare processes can be put in these models [4,7]. Although many of these points were first made with the $SU(15)$ gauge group, it is now known that there are symmetry breaking chains of the original $SU(16)$ gauge group as well which show these characteristics [8-10].

One crucial question arises now. How can a low unification scale be consistent with known bounds on proton lifetime? Of course, it is easy to see that gauge interactions do not violate B in the unbroken phase. This is another important difference with $SU(5)$, $SO(10)$, or E_6 models. With a limited number of Higgs multiplets, it was argued that baryon number symmetry (B) is not violated [3] even after symmetry breaking has taken place. Subsequently, it was emphasized [4,11] that since B is part of the gauge symmetry of these models, it must be broken spontaneously in order to avoid a massless gauge boson corresponding to an unbroken B symmetry, and therefore the Higgs sector must be expanded.

Once this was pointed out, various scenarios of proton

decay were considered by different authors [12,13,5,8]. Particularly powerful is the method of effective operators, which will be explained in Sec. II. A simple dimensional analysis performed with these effective operators [13,8] shows that proton decay amplitude in these models are suppressed by as many as the fifth power of the grand unification mass, as opposed to the second power in the case of standard unification models. For this reason, the low unification scale is consistent with the known bounds on the proton lifetime.

However, this analysis was performed only with proton decay mediated by Higgs bosons, in a handful of scenarios. But there is another kind of contribution to the proton decay amplitude in these models. It is true that in the unbroken theory, each gauge boson carries a well-defined baryon number and therefore cannot mediate B -violating processes [2,3]. However, once the gauge group is spontaneously broken, gauge bosons with different baryon numbers can mix with one another and therefore the mass eigenstates of gauge bosons are not, in general, eigenstates of baryon number. They can therefore mediate baryon-number-violating processes. Although such contributions were discussed in some detail [14,15] in some early papers on the SU(16) model, the possibility of low unification scale was not realized at that time. In the context of low energy unification, it was discussed briefly only at the tree level in a very specific scenario [5].

In this paper our focus is threefold. First, we discuss not only proton decay which is a $|\delta B| = 1$ process, but also neutron-antineutron oscillation which is a $|\delta B| = 2$ process to see whether both are consistent with low energy unification. Of course, even when we will be explicitly talking about “proton decay,” the comments can easily be translated to the baryon-number-violating decays of the neutron as well. Second, we include both gauge-boson-mediated and Higgs-boson-mediated processes in our analysis. Third, we point out accidental global symmetries in the full gauge invariant Lagrangians of the models which seriously constrain possible baryon-number-violating processes. For example, in one case we find that there are no operators involving four fermions which can give baryon number violation even after baryon number symmetry is spontaneously violated. In such cases we extend the analysis of proton decay to operators involving six fermions, which has not been done before.

II. GENERAL CONSIDERATIONS

From the symmetries of the standard model alone, one can argue that the dimension of any proton decay operators must be six or higher [16,17], whereas for $|\Delta B| = 2$ processes it is at least nine [18]. Thus, we should be looking at nonrenormalizable operators generated by the theory. In general, these operators can involve both ordinary and mirror fermions since the physical up quark, for example, can be a superposition of the ordinary and the mirror quark fields. However, for the sake of simplicity, we will assume that the mirrors do not mix with ordinary fermions. This can be attained naturally if we im-

pose a discrete symmetry, $\Psi^{(M)} \rightarrow -\Psi^{(M)}$ where $\Psi^{(M)}$ stands for the mirror fields. This is the most popular discrete symmetry considered in the context of mirror fermions. In the presence of this symmetry the model ceases to remain vectorial and hence the fermion masses are protected. Hence the survival hypothesis is applicable to these theories. The mirrors can be heavier than the ordinary fermions if their Yukawa couplings are consistently larger. In that case, they will not figure in any of the low energy processes we will be discussing. We will also assume that there are no hitherto unknown bosons lighter than the nucleon mass. Low energy operators involving nucleons should then involve ordinary fermionic fields only.

To analyze these operators we adopt the procedure used in Refs. [19,13], where one first constructs effective operators which are invariant under the gauge group. For baryon-number-violating processes at low energy, the full gauge invariant operators will in general also contain some scalar fields so that, when these scalars develop vacuum expectation values (VEV's), one obtains operators involving the fermionic fields only. If the VEV's are baryon number violating, the fermion field operator generated after putting the VEV's would violate baryon number. This is the main difference with operator analysis for proton decay performed in the context of SU(5) or SO(10) grand-unified models [16,17], where the operators have to obey only the symmetries of the standard model since the unbroken grand-unified model does not conserve baryon number. Here, the unbroken operators must obey the symmetries of the unified model, which of course is much larger than that of the standard model. This requirement severely restricts the type of baryon-number-violating operators that one can construct.

The above discussion is applicable equally for any baryon-number-violating process induced by Higgs-boson and gauge-boson exchanges. We now consider proton decay in particular. Here, one needs an operator where the number of fermionic fields is at least four [16], and in the most part we will discuss operators where the number is in fact four, except in Sec. IIIB where we will find that such operators are forbidden for proton decay.

For Higgs-boson-mediated processes, the relevant bilinears are $(\Psi_L)^T C \Psi_L$, where C is the conjugation matrix for fermions. For gauge-boson-mediated processes, the relevant bilinears are $\bar{\Psi}_L \gamma_\lambda \Psi_L$. The important difference is that the first Ψ appears with a complex conjugation here. If we put in the gauge indices, this will mean a lower index for the first Ψ and an upper index for the second one. Thus, the gauge invariants constructed will have a quite different nature than the ones for the Higgs-boson-mediated processes.

The discussion so far implies that, for processes mediated by gauge bosons, the 4-fermionic operators must appear in the effective operator in the combination

$$\mathcal{K} [(\bar{\Psi}_L)_i \gamma^\lambda (\Psi_L)^j] [(\bar{\Psi}_L)_k \gamma_\lambda (\Psi_L)^l], \quad (2.1)$$

where i, j, k, l denote gauge indices, and we have suppressed the generation indices. On the other hand,

for Higgs-boson-mediated processes, the combinations should be

$$\mathcal{K} [(\Psi_L)^i C(\Psi_L)^j] [(\Psi_L)^k C(\Psi_L)^l]. \quad (2.2)$$

One can think of other operators such as $[(\widehat{\Psi}_R)_i C(\widehat{\Psi}_R)_j] [(\widehat{\Psi}_R)_k C(\widehat{\Psi}_R)_l]$ or $[(\Psi_L)^i C(\Psi_L)^j] [(\widehat{\Psi}_R)_k C(\widehat{\Psi}_R)_l]$, where $\widehat{\Psi}_R$ is the multiplet which contains the antiparticles of the fields in Ψ_L , but these are either just Hermitian conjugates of the operators in Eqs. (2.1) and (2.2), or can be Fierz transformed to them. So, we need not discuss them separately.

Our goal is to find gauge invariant operators which can give rise to the four-fermion operators of Eqs. (2.1) and (2.2) after symmetry breaking. This discussion involves the Higgs content of the model and the precise way in which baryon number is violated, and therefore has to be done separately for SU(15) and SU(16). This will be done in the ensuing sections. Also, as we said before, we will encounter specific models where 4-fermion operators are inconsistent with the symmetries of the model. For such cases, we describe here the general case where the fermionic part of the operator has $2n$ number of fields. Let us denote such an operator symbolically as $\mathcal{K}_{(2n)} \psi^{2n}$. The coefficient $\mathcal{K}_{(2n)}$ has a mass dimension $4 - 3n$. Thus, neglecting the masses of all decay products, a simple dimensional analysis will give

$$\tau_p^{-1} \approx m_p^{6n-7} \mathcal{K}_{(2n)}^2. \quad (2.3)$$

Then, the known limits on the proton lifetime,¹

$$\tau_p > 3 \times 10^{32} \text{ yr}, \quad (2.4)$$

implies

$$\mathcal{K}_{(2n)} \lesssim 10^{-32} m_p^{4-3n}. \quad (2.5)$$

In the specific case where $n = 2$ (which is the most frequent case so that we will omit the subscript of \mathcal{K} in this case) we obtain

$$\mathcal{K} \lesssim 10^{-32} \text{ GeV}^{-2}. \quad (2.6)$$

In standard unification models such as SU(5) or SO(10), $\mathcal{K} \simeq g^2 M_G^{-2}$, so that one needs $M_G/g > 10^{16}$ GeV in order for the models to be phenomenologically viable. In the models that we consider, we will see that \mathcal{K} is further suppressed by ratios of different mass scales, and that is why smaller unification scales will be consistent with phenomenology.

III. SCENARIOS OF BARYON NUMBER VIOLATION IN SU(15) MODELS

Various scenarios of baryon number violation has been discussed in the literature [12,13,5] in the context of the

SU(15) gauge group. We present some such chains later in this section. In all these chains, for all symmetry breakings above the weak scale, we use Higgs bosons either in completely antisymmetric representations, or in the adjoint representation or representations which can be obtained by taking tensor products of two adjoint representations. This is done for the sake of economy and definiteness. We also assume that, unless mentioned otherwise, the only Higgs boson multiplets present in the model are the ones which have VEV's.

At the weak scale, however, we make an exception. Here, unless otherwise specified, we assume that the symmetry breaking is performed by a *symmetric* rank-2 multiplet S . This is motivated phenomenologically. If we use the antisymmetric tensor to be the only field to couple to fermions, the fermion mass matrices would be antisymmetric. For three generations, this will imply that one mass eigenvalue is zero and the other two equal for particles of any given charge. This is very unrealistic, so we will not consider this possibility further.

A. Baryon number violated by an antisymmetric rank-3 multiplet

1. Symmetries of the model

For the SU(15) model, Pal [13] introduced the most economic Higgs boson spectrum that leads to a breaking pattern with “unified” intermediate stages. The multiplets necessary for this purpose are the antisymmetric rank-3 multiplet $\Phi^{[ijk]}$, the adjoint T^i_j , the symmetric rank-2 multiplet S^{ij} which gives fermion masses, and an additional one, $H_{[kl]}^{ij}$, which will be called the antisymmetric biadjoint since it appears in the tensor product of two adjoint representations and both the upper and the lower indices are antisymmetrized. Here and henceforth, the square and curly brackets denote antisymmetrization and symmetrization of indices.

In Fig. 1 we show the complete chain of symmetry breaking, where the numbers n denote a factor SU(n) in the gauge group if $n > 1$, and a U(1) factor if $n = 1$. Thus, at the highest stage, the multiplet $\Phi^{[ijk]}$ develops a VEV $\langle \Phi^{\nu_e e^- e^+} \rangle$, which breaks the unification group SU(15) down to $SU(12)_q \times SU(3)_\ell$, where the subscripts indicate which fermions transform nontrivially under the subgroup— q stands for quarks and antiquarks, ℓ for leptons and antileptons. This VEV also breaks lepton number by 1 unit. At the next stage, $SU(12)_q$ breaks to $SU(6)_{qL} \times SU(6)_{qR} \times U(1)_B$. This can be performed by a VEV in the adjoint representation, as shown in the figure and explained in the figure caption. At the scale $M_{\delta qL}$, the $SU(6)_{qL}$ breaks to its maximal subgroup $SU(3) \otimes SU(2)$, under which the fundamental of SU(6) transforms like (3,2). In SU(6), the lowest dimensional multiplet which has the component whose VEV can induce this breaking is the 189-dimensional antisymmetric biadjoint. Naturally, it is contained in the antisymmetric biadjoint representation of the SU(15). At the next stage, $SU(6)_{qR}$ breaks to $SU(3)_{uR} \times SU(3)_{dR} \times U(1)_{qRA}$.

¹The precise bounds depend on the specific decay mode. For rough estimates we use the same bound for all modes.

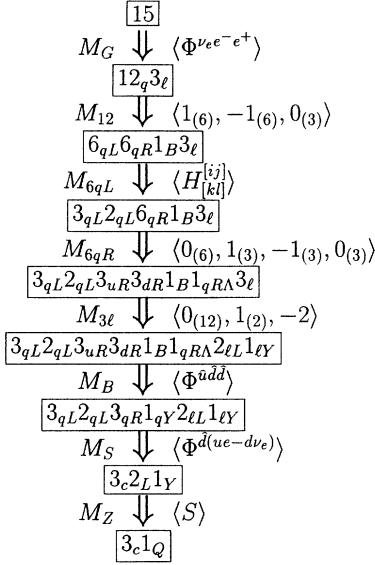


FIG. 1. SU(15) symmetry breaking where baryon number is broken by the VEV of an antisymmetric rank-3 multiplet. If one considers the adjoint Higgs multiplet T as a traceless matrix, its VEV's are diagonal and the notation $1_{(6)}$, e.g., stands for six consecutive entries of unity. In the multiplet Φ , the symbol $\langle \hat{d}ue \rangle$, e.g., stands for the VEV of the color singlet combination of the components with one index having the quantum numbers of \hat{d} , another of u , and another of e .

The SU(3) factors here operate nontrivially only on the \hat{u}_L and \hat{d}_L components, respectively, whereas the U(1) quantum numbers are defined to be +1 for the up-type quarks and -1 for the down-type ones. Baryon number is broken at the next stage, where $SU(3)_{uR} \times SU(3)_{dR}$ also is broken to the diagonal SU(3) subgroup which is called $SU(3)_{qR}$. At this stage, the gauge group that appears is the square of the standard model gauge group, where the quarks and leptons transform under different SU(2) and different U(1) factors. This has been discussed under the name “ununified” model by some authors [20]. On the other hand, the right and left chirality of quarks transform under different color groups, which has been discussed in the literature under the name “chiral color” [15,21]. At the next stage, the standard model gauge group appears, which is why this scale is called M_S .

It should be understood that some variations of this chain are obviously possible. For example, the scale M_{6qL} can be lower than M_{6qR} or even $M_{3\ell}$. On the other hand, some scales can merge, so that the standard model is reached in less number of steps. These will not essentially change the conclusions of the subsequent discussions and hence will not be discussed separately. Similar comments apply for other chains which we will discuss later in the paper.

For this and various other scenarios that we are going to discuss, we find that the full gauge invariant Lagrangian involving the specified fields often contains some accidental global or discrete symmetries which commute with the gauge symmetry. These restrict the type of potentially baryon-number-violating operators. To see such symmetries in the present case let us write the full La-

grangian in the suggestive manner

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}' . \quad (3.1)$$

Here, \mathcal{L}_0 is the part which is invariant under independent phase rotations of all complex multiplets present in the model. These would include, e.g., all gauge interactions, scalar interactions involving only the adjoint Higgs multiplet and the antisymmetric biadjoint. There will also be some terms involving other multiplets, e.g., terms such as $\Phi^{ijk}\Phi_{ijk}$ or $S^{ij}S_{jk}S^{kl}S_{li}$. Obviously, symmetry of \mathcal{L}_0 is much larger than the gauge symmetry. However, \mathcal{L}' contains other terms which are allowed by the gauge symmetry. In the present case, the Yukawa couplings are the only terms which fall in this class. Thus,

$$\mathcal{L}' = \mathcal{Y}(\Psi_L)^k (\Psi_L)^l S_{kl} + \text{H.c.} , \quad (3.2)$$

where the generational indices on Ψ_L and \mathcal{Y} have been omitted. However, it is easy to see that, even with this term, the full gauge invariant Lagrangian has the following accidental global charges which are conserved:

Multiplet	Ψ^k	S^{kl}	Φ^{klm}
Q_1	1	2	0
Q_2	0	0	1

(3.3)

Notice that the global phase of the multiplet Φ is indeed a global symmetry of the Lagrangian [19]. In addition, there is another one, which has been labeled as Q_1 .

Consider now a generic effective operator of the form

$$(\Psi)^{2f} S^{n_S} \Phi^{n_\Phi} , \quad (3.4)$$

which stands for $2f$ number of fermionic fields with upper indices, n_S number of the multiplet S with upper indices, etc. Each of these numbers can be positive, negative (if the relevant multiplet contributes a net number of lower indices), or zero. Thus, for example, the operator $\Phi^{ijk}\Phi_{ijk}$ will have $n_\Phi = 0$ since Φ^{ijk} contributes $n_\Phi = 1$ but Φ_{ijk} contributes $n_\Phi = -1$. On the other hand, $\Phi^{ijk}\Phi^{lmn}S_{il}S_{jm}S_{kn}$ has $n_\Phi = 2$, $n_S = -3$.

Conservation of the charges Q_1 and Q_2 tells us that, in Eq. (3.4),

$$f + n_S = 0 , \quad (3.5)$$

$$n_\Phi = 0 . \quad (3.6)$$

Any effective operator generated by the theory must then obey these two conditions, and we will discuss some such operators below. Note that both these conditions remain unaffected if the operator in Eq. (3.4) contains the adjoint or the multiplet $H_{[kl]}^{[ij]}$, since they contribute equally to the number of upper and lower indices, and since they are neutral under the global symmetries of Eq. (3.3).

One general characteristic of baryon number violation in this model can be immediately noted. As we said earlier, in order to obtain purely fermionic operators, we need to replace the scalar fields in Eq. (3.4) by their VEV's. From Fig. 1 we note that Φ has three types of VEV's. One of these gives $\delta B = -1$, and each of the other two give $\delta L = 1$. Let us say that in the operator

with fermionic fields only, there are n_{Φ_B} VEV's of the first type, and n_{Φ_L} VEV's of the second kind. Since the purely fermionic operator comes from the gauge invariant operator of Eq. (3.4), they must obey Eq. (3.6), which implies $n_{\Phi_B} + n_{\Phi_L} = 0$. The total violation of B and L in the purely fermionic operator can now be written as

$$\delta B = -n_{\Phi_B}, \quad \delta L = n_{\Phi_L} = -n_{\Phi_B}, \quad (3.7)$$

so that

$$\delta(B - L) = 0. \quad (3.8)$$

This immediately tells us that *there is no n - \hat{n} oscillations in the model.*

One comment needs to be made here. Lepton number is not part of the gauge symmetry in $SU(15)$. However, it is well defined for all components of Ψ . This can be used to assign lepton number to the gauge bosons and Higgs bosons. In other words, for any multiplet $\phi^{ij\dots}$, we can count a lepton number $+1$ for each occurrence of the indices 13 or 14, and -1 for each occurrence for the index 15. Since lower indices are complex conjugates, they will have just the opposite assignments. It is then easy to see that lepton number conservation is assured in the Lagrangian by gauge invariance, and therefore must be violated spontaneously. This is one characteristic of this particular version of $SU(15)$ models which is not shared if we break baryon number by higher rank multiplets, as we will see later.

2. Proton decay operators

Since proton decay requires both baryon number and lepton number violation, and since both these violations come through VEV's of different components of Φ , we must need at least two factors of Φ in the gauge invariant operator. Of course, one of them must come with upper indices and the other with lower indices in order that Eq. (3.6) is satisfied.

a. Gauge boson mediated: For gauge boson mediated proton decay for which $f = 0$ in the generic operator of Eq. (3.4), we obtain $n_S = 0$ from Eq. (3.5). Indeed, there is one operator which is consistent with all these numbers:

$$\mathcal{O}_1 = [(\overline{\Psi}_L)_i \gamma^\lambda (\Psi_L)^j] [(\overline{\Psi}_L)_k \gamma_\lambda (\Psi_L)^l] \Phi^{ikr} \Phi_{jlr}. \quad (3.9)$$

In Fig. 2 we have shown a tree-level diagram which gives rise to a particular component of this operator. This is the component with $\{ikr\} \equiv \{\hat{u}\hat{d}\hat{d}\}$. Since the other VEV must have one index contracted with this one, and it has to be lepton number violating, we have to use either $\{jlr\} = \{\hat{d}\hat{u}e\}$ or $\{\hat{d}\hat{d}\nu_e\}$. These two possibilities are shown in Figs. 2(a) and 2(b), respectively. Once the VEV's are put in, 4-fermion operators result, whose co-

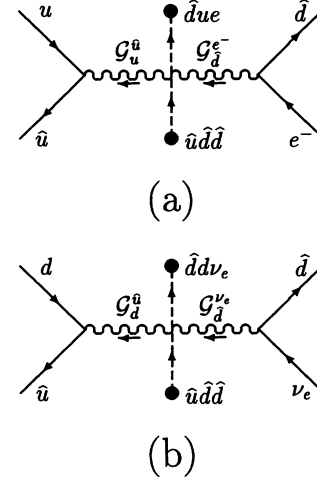


FIG. 2. Tree-level diagram giving rise to the operator \mathcal{O}_1 of Eq. (3.9). All the indices should be considered as upper ones, except the ones for gauge bosons \mathcal{G} where upper and lower indices have been shown explicitly.

efficient can be easily computed by looking at these diagrams:

$$\mathcal{K}_1 \simeq \frac{g^2 M_B M_S}{M_{12}^2 M_G^2}. \quad (3.10)$$

Here, the factor g^2 is just the gauge coupling constant coming from two vertices with fermions. M_B/g and M_S/g give the VEV's, and the four-boson vertex gives a factor g^2 . The denominator comes from the propagators of the gauge bosons. In both diagrams, the gauge boson coming out of the left vertex has the quantum numbers of a diquark. Such gauge bosons belong to the coset space $SU(12)_q/[SU(6)_{qL} \times SU(6)_{qR}]$, and acquire masses of order of $SU(12)_q$ -breaking scale, M_{12} . The other gauge boson which couples a quark to a lepton belongs to the coset space $SU(15)/SU(12)_q \times SU(3)_l$, and therefore has mass at the unification scale M_G .

Proton lifetime bounds now imply, from Eq. (2.6), the constraints

$$M_G^2 \frac{M_{12}^2}{g^2 M_B M_S} > 10^{32} \text{ GeV}^2. \quad (3.11)$$

Notice that M_B and M_S are by definition smaller than M_{12} , and $g < 1$. Thus this condition can be satisfied with a low grand unification scale.

We now discuss the proton decay modes obtained from the diagrams of Fig. 2. In the figure we have suppressed all generation indices of the fermions. We now notice that there is a property of the operator of Eq. (3.9) which forbids all fermions to be of the same generation. This is because

$$\begin{aligned}
[(\overline{\Psi}_L)_i \gamma^\lambda (\Psi_L)^j] [(\overline{\Psi}_L)_k \gamma_\lambda (\Psi_L)^l] &= - [(\overline{\Psi}_L)_i \gamma^\lambda (\Psi_L)^j] [(\widehat{\Psi}_R)^l \gamma_\lambda (\widehat{\Psi}_R)_k] \\
&= 2 [(\overline{\Psi}_L)_i (\widehat{\Psi}_R)_k] [(\widehat{\Psi}_R)^l (\Psi_L)^j] \\
&= 2 [(\widehat{\Psi}_R)_i C (\widehat{\Psi}_R)_k] [(\Psi_L)^l C (\Psi_L)^j] .
\end{aligned} \tag{3.12}$$

Here, the first step is obtained by the definition of $\widehat{\Psi}$, and the next one is obtained by Fierz transformation. Since the matrix C is antisymmetric, the last form shows that the spinor indices of Ψ^i and Ψ^k are antisymmetric in Eq. (3.9). Because the gauge indices are contracted with Φ^{ikr} which is antisymmetric, the gauge indices i and k are also antisymmetric. Therefore, in order to satisfy the Fermi principle, they must be antisymmetric in their (unshown) generation indices. The same comment can be made about Ψ^j and Ψ^l . Thus, disregarding charm quark fields which will be kinematically forbidden in proton decay, we obtain that the 4-fermion operator generated by Fig. 2(a) is

$$\mathcal{K}_1 [\widehat{u}_L \gamma_\lambda u_L] [\widehat{s}_L \gamma^\lambda \mu_L] . \tag{3.13}$$

Similarly, Fig. 2(b) generates

$$\mathcal{K}_1 [\widehat{u}_L \gamma_\lambda d_L] [\widehat{s}_L \gamma^\lambda \nu_{\mu L}] . \tag{3.14}$$

The first one predicts the decay mode

$$p \rightarrow \mu^+ K^0 , \tag{3.15}$$

whereas the second one gives

$$p \rightarrow \widehat{\nu}_\mu K^+ . \tag{3.16}$$

Notice that these are unusual decay modes which are suppressed in unification models such as SU(5) or SO(10), although they occur in their supersymmetric versions. In the present case, it is predicted in the absence of supersymmetry because of the Fermi symmetry between all particles in a generation.

b. Higgs boson mediated: In this case we should put $f = 2$ in Eq. (3.4), as discussed in Sec. II. The constraint of Eq. (3.5) now implies $n_S = -2$. Operators of this type were discussed earlier by one of us [13]. Here is one example:

$$\mathcal{O}_2 = [(\Psi_L)^i C (\Psi_L)^j] [(\Psi_L)^k C (\Psi_L)^l] \Phi_{ikr} \Phi^{pqr} S_{ip} S_{jq} . \tag{3.17}$$

Since the effective operator now involves two occurrences of the field S whose VEV's are of order M_W , any contribution coming from this operator must have a suppression factor of $(M_W/M_G)^2$. For example, the tree diagram of Fig. 3 gives a contribution

$$\mathcal{K}_2 \sim \left(\frac{m_f}{M_W} \right)^2 \frac{\lambda_{S\Phi}^2 M_S M_B M_W^2}{M_G^6} . \tag{3.18}$$

Here, the quantity m_f is the mass of a typical fermion, and comes from the Yukawa couplings. The quartic

scalar couplings are denoted by λ_{SS} and $\lambda_{S\Phi}$ in obvious notation, and we have assumed that all the virtual colored scalars in this diagram have masses of order M_G , the largest scale in the model. Then notice that

$$\frac{\mathcal{K}_2}{\mathcal{K}_1} = \lambda_{SS} \lambda_{S\Phi} \left(\frac{m_f}{M_G} \right)^2 \left(\frac{M_{12}}{M_G} \right)^2 . \tag{3.19}$$

Even if $\lambda_{SS}, \lambda_{S\Phi} \sim 1$ which is the limit allowed by perturbative procedure, we obtain $\mathcal{K}_2 \ll \mathcal{K}_1$ since $M_{12} \leq M_G$ by definition and $m_f \ll M_G$.

B. Baryon number violated by an antisymmetric rank-4 multiplet

1. Symmetries of the model

The rank-4 antisymmetric representation Δ was introduced by Brahmachari, Sarkar, Mann, and Steele (BSMS) [5]. A possible chain of symmetry breaking has been shown in Fig. 4. Notice that the symmetry breaking above the scale M_B is performed by the same VEV's as in Fig. 1. At the scale M_B , the still unbroken gauge group $3_{qL} 2_{qL} 3_{uR} 3_{dR} 1_B 1_{qRA} 2_{\ell L} 1_{\ell Y}$ is broken by the VEV of the rank-4 multiplet, $\Delta^{\widehat{u}\widehat{u}\widehat{d}e^+}$. The 3_{uR} and the 3_{dR} subgroups combine to give the diagonal subgroup which we call 3_{qR} . Also, out of the three U(1) factors, one combination breaks, leaving two unbroken ones, one of which is the hypercharge of the standard model, and the other is called $U(1)_F$, whose generator is proportional to $\text{diag}(9_{(6)}, -22_{(3)}, 4_{(3)}, -7_{(2)}, 14)$.

Notice that the VEV $\Delta^{\widehat{u}\widehat{u}\widehat{d}e^+}$ breaks both baryon number and lepton number by -1 . Since this is the only baryon number violating VEV in this scheme, it must appear in the gauge invariant operator giving rise to proton decay. Moreover, it must appear an odd number of

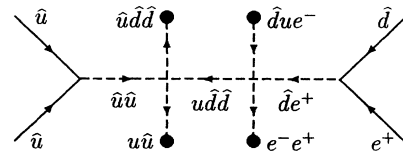


FIG. 3. Proton decay mediated by Higgs bosons, giving rise to the operator in Eq. (3.17). The notation about indices has been explained in Fig. 2. One can similarly contemplate a diagram where the VEV of the component $\Phi^{\widehat{d}d\nu e}$ appears instead of $\Phi^{\widehat{u}\widehat{u}\widehat{d}e^+}$.

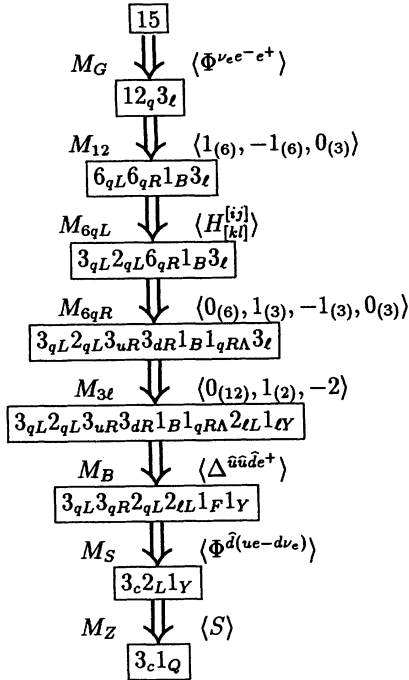


FIG. 4. SU(15) symmetry breaking where baryon number is broken by the VEV of an antisymmetric rank-4 multiplet. The notation for VEV's has been explained in Fig. 1.

times. Thus, it is obvious that if the model contains a discrete symmetry

$$\Delta \rightarrow -\Delta \quad (3.20)$$

with all other fields invariant, one cannot generate any term that violates baryon number by an odd integer. The proton will then be absolutely stable. This comment applies irrespective of whether proton decay is mediated by gauge boson or Higgs boson exchange.

Even if such a symmetry is not imposed on the Lagrangian, analysis of the full gauge invariant Lagrangian reveals accidental global symmetries as discussed in Sec. III A 1, restricting the type of potentially baryon-number-violating operators. To see this we use the notation of Eq. (3.1) and note that here

$$\mathcal{L}' = \mathcal{Y}(\Psi_L)^k (\Psi_L)^l S_{kl} + \lambda \Delta_{klmn} S_{pr} \Phi^{klp} \Phi^{mnr} + \lambda' [\Delta \Delta \Delta \Phi]_\epsilon + \text{H.c.}, \quad (3.21)$$

where in the last term, the indices (not shown) are all upper indices which are contracted by an antisymmetric ϵ tensor having 15 indices, which is indicated by the square brackets with a subscript ϵ . The first term is the Yukawa coupling term.

It is easy to see that, even with the presence of the above terms, there is a global U(1) symmetry of the Lagrangian under which the quantum numbers of various multiplets are

Multiplet	Ψ^i	S^{ij}	Φ^{ijk}	Δ^{ijkl}	
Charge	1	2	$\frac{6}{7}$	$-\frac{2}{7}$	(3.22)

Consider now a generic effective operator of the form

$$(\Psi)^{2f} S^{n_S} \Phi^{n_\Phi} \Delta^{n_\Delta}, \quad (3.23)$$

in the notation used in Eq. (3.4). The global symmetry of Eq. (3.22) implies

$$2f + 2n_S + \frac{6}{7}n_\Phi - \frac{2}{7}n_\Delta = 0. \quad (3.24)$$

On the other hand, all the indices should be contracted, which means that the total number of upper indices should either be zero or be divisible by 15 (so that they can be contracted by ϵ symbols). For the model of Sec. III A, this condition is already contained in Eqs. (3.5) and (3.6). Here, it produces an independent condition²

$$2f + 2n_S + 3n_\Phi + 4n_\Delta = 15N, \quad (3.25)$$

where N is an integer, denoting the number of times a vertex involving the ϵ symbol appears in the diagram giving rise to the operator of Eq. (3.23). As noted in Sec. III A 1, both conditions remain unaffected if the operator in Eq. (3.23) contains the adjoint or the antisymmetric biadjoint.

The solution of Eqs. (3.24) and (3.25) can be written as

$$f + n_S = n_\Delta - 3N, \quad n_\Phi = 7N - 2n_\Delta. \quad (3.26)$$

Let us now check what the above solution means for the violation of baryon and lepton numbers. Baryon number, as noted before, is part of the gauge symmetry and is broken only spontaneously through the VEV of Δ . Thus, clearly,

$$\delta B = -n_\Delta. \quad (3.27)$$

On the other hand, lepton number violation comes from three different sources: each VEV of Δ (with upper indices) gives $\delta L = -1$; each VEV of Φ (with upper indices) induces $\delta L = 1$; each occurrence of a term with an ϵ symbol will have 15 upper indices which are all different, contributing to an explicit violation $\delta L = 1$ in the unbroken Lagrangian. Taking all these contributions we can write

$$\delta L = -n_\Delta + n_\Phi + N. \quad (3.28)$$

Using Eqs. (3.26)–(3.28) we therefore finally obtain

$$\delta(3B - L) = -8N, \quad (3.29)$$

which is the selection rule for this model. Immediately, it tells us that in this model, *there cannot be any neutron-antineutron oscillations*.

²This condition is necessary, but not sufficient, since it does not take into account the fact that the indices to be contracted by the ϵ symbols have to be antisymmetric.

2. Proton decay operators

Specializing to the simplest case when $N = 0$, Eq. (3.29) tells us that $3B - L$ is conserved, which means that there will be three leptons in the final state for proton decay. This cannot occur with four fermionic fields only, since three of these fields must be quark-antiquark fields in order to obtain a $\delta B = -1$ operator. For other values of N , one needs an even higher number of lepton-antilepton pairs in the final state, which cannot be accommodated in a 4-fermion operator for the same reason. Thus, we conclude that in this model there is no proton decay operator with four fermionic field operators. The result is true for operators mediated by gauge or Higgs bosons.

The lowest dimensional operators will thus have six fermionic fields. They can have $f = 3$ where all the indices are upper. Alternatively, they may have $f = 1$ where two of the fields have lower indices, but the other four have upper ones. Of course, one can similarly have $f = -3$ and $f = -1$.

Among these possibilities, $f = 1$ can yield a solution to Eq. (3.26) with the smallest number of scalar fields, given by

$$f = 1, \quad n_S = 0, \quad n_\Phi = -2, \quad n_\Delta = 1. \quad (3.30)$$

An operator of this type is

$$\mathcal{O} = [(\overline{\Psi}_L)_i \gamma_\lambda (\Psi_L)^j] [(\overline{\Psi}_L)_k \gamma^\lambda (\Psi_L)^l] \times [(\Psi_L)^p C (\Psi_L)^r] \Delta^{ikab} \Phi_{ajr} \Phi_{plb}. \quad (3.31)$$

We show in Fig. 5 how this operator can arise at the tree level. The amplitude of the purely fermionic operators can be easily determined. Assuming the scalar interaction couplings to be of order unity, we obtain

$$\mathcal{K}_{(6)} \sim \frac{g M_G M_B M_S}{M_{12}^2 M_G^2 M_{\hat{d}ue^-e^+}^2 M_{e\nu}^2}, \quad (3.32)$$

where the last two factors in the denominator represent the masses of the internal Higgs boson lines. Of these, the former one is a colored boson, whose mass is expected to be of order M_G . But the latter one is uncolored, whose mass we keep as an unknown. Experimental bounds, however, tell us that, being a charged scalar, its mass cannot be much less than 100 GeV. Thus, if this opera-

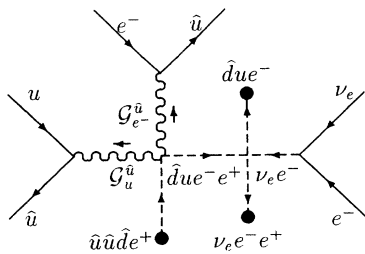


FIG. 5. Tree-level diagram giving rise to the operator \mathcal{O} of Eq. (3.31). The notation about indices has been explained in Fig. 2. One can similarly contemplate a diagram where the VEV of the component $\hat{\Phi}^{\hat{d}d\nu e^-}$ appears instead of $\hat{\Phi}^{\hat{d}ue^-}$.

tor contributes to proton decay, using Eq. (2.5), we can rewrite Eq. (3.32) as

$$M_G^3 > 10^{28} \text{ GeV}^3 \left(\frac{g M_B M_S}{M_{12}^2} \right) \left(\frac{100 \text{ GeV}}{M_{e\nu}} \right)^2. \quad (3.33)$$

Since $M_{12} > M_B, M_S$ by definition and $g < 1$, this bound can be satisfied for any unification scale larger than about 10^9 GeV.

However, there is a subtle reason why this operator cannot contribute to proton decay. In order to accommodate baryon number violation, the indices on the fields Δ must be $\hat{u}\hat{u}\hat{d}e^+$ in any permutation. Now, these indices contract either with the indices of Φ , or those of $\overline{\Psi}$. But Φ does not have any VEV which contains the index \hat{u} . Thus, both the indices on $\overline{\Psi}$ have to be \hat{u} indices. However, as argued in connection with Eq. (3.12), the fields $(\overline{\Psi}_L)_i$ and $(\overline{\Psi}_L)_k$ must come from different generations. Therefore, one of them must be the charm quark and therefore proton decay is kinematically forbidden from this operator.

To get out of this impasse one can use a slightly modified operator:

$$\mathcal{O}' = [(\overline{\Psi}_L)_i \gamma_\lambda (\Psi_L)^j] [(\overline{\Psi}_L)_k \gamma^\lambda (\Psi_L)^l] \times [(\Psi_L)^p C (\Psi_L)^r] T^i_m \Delta^{mkab} \Phi_{ajr} \Phi_{plb}. \quad (3.34)$$

This is still an operator of the type of Eq. (3.30), but now the gauge indices i and k are not antisymmetric, and therefore $(\overline{\Psi}_L)_i$ and $(\overline{\Psi}_L)_k$ can refer to fields from the same generation. A diagram for this operator can be obtained from Fig. 5 by attaching an adjoint Higgs boson to any line which carries at least one $SU(12)_q$ index. This will provide further suppression to the 4-fermion operators since the extra propagator is expected to have a mass M_G , but the largest VEV available for the adjoint multiplet is at the scale M_{12} . The quark level transition induced by this operator is $ue^-e^- \nu \rightarrow \hat{u}\hat{u}$, which implies a decay mode

$$p \rightarrow \pi^- e^+ e^+ \hat{\nu}_e. \quad (3.35)$$

C. Baryon number violated by an antisymmetric rank-5 multiplet

In an early paper, Frampton and Kephart [12] discussed baryon number violation by the VEV of an antisymmetric rank-5 multiplet $J^{[ijklm]}$. Although less economical than the ones discussed above, we include this possibility for the sake of completeness. Figure 6 gives a chain involving this rank-5 multiplet. The VEV that breaks $U(1)_B$ has the gauge transformation properties of $\hat{d}\hat{d}\hat{d}\nu_e e^-$, i.e, it has $B = -1, L = 2$. Notice also that since this VEV does not involve both \hat{u} - and \hat{d} -type indices, it cannot break $3_{uR} 3_{dR}$ part of the symmetry to 3_{qR} , as is done in the models described earlier. Therefore, this symmetry breaking is performed by a VEV in the antisymmetric biadjoint $H_{[kl]}^{[ij]}$. This multiplet certainly has a component which is the antisymmetric bi-

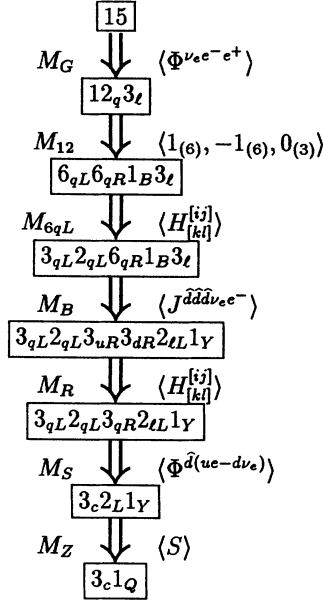


FIG. 6. SU(15) symmetry breaking where baryon number is broken by the VEV of an antisymmetric rank-5 multiplet. The notation for VEV's has been explained in Fig. 1. The two different VEV's of the multiplet H have been described in the text.

adjoint of the subgroup 6_{qR} and singlet under the rest. This part is a 189-dimensional representation of SU(6) which has a component that transforms like $(8, 8, 0)$ under its subgroup $3_{uR} 3_{dR} 1_{qRA}$. A VEV here would perform the desired symmetry breaking. On the other hand, one now does not need the adjoint to break the 6_{qR} subgroup, since the baryon number violating VEV itself performs the job. In fact, the VEV $\langle J^{\tilde{d}\tilde{d}\nu_e e^-} \rangle$ also breaks the leptonic subgroup 3_l to 2_{lL} , and the leptonic and quark hypercharges combine to the total hypercharge of the standard model.

In this case, using the notation introduced earlier, we obtain

$$\mathcal{L}' = \mathcal{Y}(\Psi_L)^k (\Psi_L)^l S_{kl} + \mu [JJJ]_e + \text{H.c.} \quad (3.36)$$

Obviously, the entire Lagrangian respects the discrete symmetry

$$J \rightarrow e^{2\pi i/3} J \quad (3.37)$$

with all other fields neutral. This is a Z_3 symmetry. The number of J fields in any effective operator arising in this model must then be a multiple of 3. Since baryon number violation comes from the VEV of J only, in purely fermionic operators we will have

$$|\delta B| = 3N \quad (3.38)$$

for some integer N . Therefore, *neither proton decay nor neutron-antineutron oscillation is possible in this model.*³ Notice that this conclusion is reached only from the accidental symmetries present in the full gauge invariant Lagrangian.

D. Introduction of antisymmetric Yukawa couplings

So far, we have assumed that the only Higgs bosons which can couple to fermions belong to the symmetric rank-2 multiplet $S^{[ij]}$. The situation changes if, in addition there is also the multiplet $A^{[ij]}$ which couples antisymmetrically. In this case, some of the symmetries described in the above sections may be broken explicitly and hence more baryon-number-violating processes may be allowed.

For the model of Sec. III A, such is not the case. We still have the condition in Eq. (3.6), which leads to $B - L$ conservation. However, in the model of Sec. III B, there is an important change. This is because, with the introduction of the multiplet A , there are the following new terms which are allowed in \mathcal{L}' :

$$\mathcal{L}'_A = \mathcal{Y}_A (\Psi_L)^k (\Psi_L)^l A_{kl} + \mu A_{ij} A_{kl} \Delta^{ijkl} + \text{H.c.} \quad (3.39)$$

There is now no way that one can assign a quantum number of A which keeps the symmetry of Eq. (3.22). Thus, one can have the operator

$$\mathcal{O} = [(\Psi_L)^i C (\Psi_L)^j] [(\Psi_L)^k C (\Psi_L)^l] \Delta^{ijkl}. \quad (3.40)$$

In Fig. 7 we show how this can be generated through the interactions appearing in Eq. (3.39). In the figure we suppressed the generation indices. Turning to Eq. (3.40) we see that since the gauge group indices i and j appear in antisymmetric combination in Δ^{ijkl} , and since the matrix C is antisymmetric, the generation indices for the two fermionic fields in the first bilinear must be antisymmetric in order to maintain Fermi symmetry. The same can be said about the fermionic fields in the other bilinear. Thus, the quark level operator coming from Fig. 7 is $[\hat{u}C\mu^+][\hat{u}C\hat{s}]$, which gives rise to a proton decay mode

$$p \rightarrow \mu^+ K^0. \quad (3.41)$$

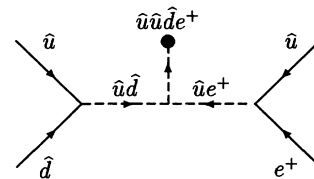


FIG. 7. Higgs-boson-mediated diagram giving rise to the operator of Eq. (3.40).

³Indeed, the baryon-number-violating diagram given by Frampton and Kephart [12] for this model has $\delta B = 3$, as was noted by one of us earlier [13].

The amplitude for the 4-fermion operator is given by

$$\mathcal{K} \sim \mathcal{Y}_A^2 \frac{\mu M_B}{M_G^4}, \quad (3.42)$$

assuming, once again, that the colored Higgs bosons have masses of order M_G . The quantity \mathcal{Y}_A in this formula stands symbolically for two factors of the Yukawa coupling with the multiplet A . Since the antisymmetric Yukawa couplings, if any, are expected to be smaller than the symmetric ones, and since $M_B < M_G$ by definition, this again shows the suppression of proton decay rate.

For the model of Sec. III C, the changes are more dramatic. Here, the extra terms can appear in \mathcal{L}' due to the introduction of A are given by

$$\begin{aligned} \mathcal{L}'_A = & \mathcal{Y}_A (\Psi_L)^k (\Psi_L)^l A_{kl} + \lambda_{JJ\Phi A} [JJ\Phi A]_e \\ & + \mu' J^{ijklm} \Phi_{klm} A_{ij} + \text{H.c.} \end{aligned} \quad (3.43)$$

There still is a Z_3 symmetry in the full Lagrangian, defined as

$$\frac{\text{Multiplet } \Psi^k \ S^{kl} \ A^{kl} \ \Phi^{klm} \ J^{ijklm}}{Z_3 \text{ charge } \ 1 \ 2 \ 2 \ 2 \ 1} \quad (3.44)$$

Consider now a generic effective operator of the form

$$(\Psi)^{2f} S^{n_S} A^{n_A} \Phi^{n_\Phi} J^{n_J} \quad (3.45)$$

in the notation used before. Using the Z_3 symmetry and the requirement that all indices must be contracted, we obtain the conditions

$$2f + 2(n_S + n_A) + 2n_\Phi + n_J = 3N, \quad (3.46)$$

$$2f + 2(n_S + n_A) + 3n_\Phi + 5n_J = 15N', \quad (3.47)$$

where N and N' are both integers. Thus,

$$n_\Phi = 15N' - 3N - 4n_J \quad (3.48)$$

$$2f + 2(n_S + n_A) = 9N - 30N' + 7n_J.$$

Following arguments similar to those in Sec. III B, we now obtain

$$\delta B = -n_J, \quad (3.49)$$

$$\delta L = 2n_J + n_\Phi + N', \quad (3.50)$$

so that, using Eq. (3.48), we obtain

$$\delta(2B - L) = 3N - 16N', \quad (3.51)$$

which is the selection rule in this case.

For proton decay which requires $n_J = 1$, notice that the integer N must be odd because of its definition in Eq. (3.46). The solution of Eq. (3.48) involving minimum number of scalar fields is now given by $N = -1$, $N' = 0$, i.e.,

$$n_\Phi = -1, \quad f + n_S + n_A = -1. \quad (3.52)$$

Using Eqs. (3.49) and (3.50), it is now easy to see that in this case, proton decay operators will satisfy the selection rule $\delta(B + L) = 0$.

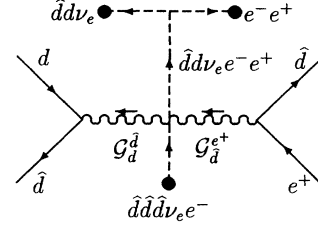


FIG. 8. Gauge-boson-mediated diagram giving rise to the operator of Eq. (3.53).

An example of a gauge-boson-mediated diagram of proton decay is provided in Fig. 8, which has $f = 0$ and $n_A = -1$. The operator here has the form

$$\begin{aligned} \mathcal{O}_1 = & [(\overline{\Psi}_L)_i \gamma^\lambda (\Psi_L)^j] \\ & \times [(\overline{\Psi}_L)_k \gamma_\lambda (\Psi_L)^l] J^{ikmnp} \Phi_{jmn} A_{lp}, \end{aligned} \quad (3.53)$$

and Fig. 8 shows how it can be generated at the tree level. Because of the Fierz transformation property shown in Eq. (3.12), the generation indices of Ψ^i and Ψ^k must be different here. But the same cannot be said about Ψ^j and Ψ^l since their gauge indices are not antisymmetric. Thus, the quark-level transition obtained from Fig. 8 is $d e^+ \rightarrow \widehat{d}$. This implies a proton decay mode

$$p \rightarrow \pi^+ K^+ e^-, \quad (3.54)$$

which conserves $B + L$, as argued before on general grounds. The coefficient of the 4-fermion operator is given by

$$\mathcal{K}_1 \simeq \frac{\mu' M_B M_S M_W}{M_{12}^2 M_G^4}, \quad (3.55)$$

assuming that the colored scalar internal line has a mass of order M_G . Once again, since M_B , M_S , and M_W are each smaller than either M_{12} or M_G by definition, a low unification scale is allowed.

A Higgs boson mediated diagram, with $f = -2$ and $n_A = 1$, was given in Ref. [13]. The operator responsible for this is

$$\mathcal{O}_2 = [(\Psi_L)^i C (\Psi_L)^j] [(\Psi_L)^k C (\Psi_L)^l] J_{ijkqr} \Phi^{pqr} A_{lp}. \quad (3.56)$$

It apparently looks like it has the same VEV's as the operator in Eq. (3.53). But this need not be the case, as seen from Fig. 9. Here, one can use the VEV of Φ

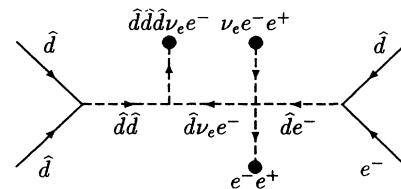


FIG. 9. Higgs-boson-mediated diagram giving rise to the operator of Eq. (3.56).

which occurs at the scale M_G . Thus, we obtain, for the strength of the 4-fermion operator,

$$\kappa_2 \sim \left(\frac{m_f}{M_W} \right)^2 \frac{\mu' M_B M_W}{M_G^2}. \quad (3.57)$$

Depending on the magnitude of the scales M_S and M_{12} , this may or may not dominate over the gauge-boson-mediated decay. The decay mode is the same as that given in Eq. (3.54) since Ψ^i and Ψ^j have to belong to different generations in the operator of Eq. (3.56).

It must also be noticed that, unlike the previous models, neutron-antineutron oscillations are not ruled out in this model. However, it is very suppressed. This can be seen from Eq. (3.51), where we can put $\delta B = 2$ and $\delta L = 0$ as is necessary for neutron-antineutron oscillations. The simplest solution for this situation is obtained when $N = -4$, $N' = -1$, which means

$$n_J = -2, \quad n_\Phi = 5, \quad f + n_S + n_A = -10. \quad (3.58)$$

Obviously, it is a very high dimensional operator, so we will ignore it.

IV. SCENARIOS OF BARYON NUMBER VIOLATION IN SU(16) MODELS

It might seem that SU(16) scenarios of baryon number violation should look similar to the SU(15) ones, since the groups are not all that different. There are, however, some important differences, which should carefully be taken into account. The first is that baryon number violation occurs spontaneously and therefore is sensitive to the choice of the Higgs sector, as amply demonstrated in Sec. III. Being a larger group, SU(16) in general requires more VEV's to break it down to the standard model gauge group, which affect the operator analysis. Second, baryon number processes such as proton decay involves lepton number violation as well, and the nature of the latter is very different in the groups SU(15) and SU(16). The reason is that in SU(16) lepton number is part of the gauge symmetry and can be violated only spontaneously. This is a difference from the SU(15) models where the Lagrangian can violate lepton number. Third, the symmetric rank-2 multiplet of SU(16), unlike its SU(15) version, can have a lepton number violating VEV $S^{\hat{\nu}\nu}$, which does not violate the symmetries of the standard model. This VEV can give neutrinos a Majorana mass at the tree level. To keep our discussion simple we will neglect this VEV.

A. Baryon number violated by an antisymmetric rank-4 multiplet

1. Symmetries of the model

Breaking SU(16) down to $SU(12)_q \times SU(3)_\ell$ requires the VEV of an antisymmetric rank-3 multiplet. Similarly, breaking SU(16) down to $SU(12)_q \times SU(4)_\ell$ requires the VEV of an antisymmetric rank-4 multiplet Δ . Therefore,

it is reasonable to try to see if there are suitable VEV's in the multiplet Δ , the adjoint T^i_j and the biadjoint $H_{[kl]}^{[ij]}$ which can break the grand unification symmetry down to the symmetry of the standard model. In Fig. 10 we show how it can be done. At the weak scale, the symmetry is broken by the rank-2 symmetric multiplet $S^{\{ij\}}$, as before.

For this model we find

$$\mathcal{L}' = \mathcal{Y}(\Psi_L)^i (\Psi_L)^j S_{ij} + \lambda' [\Delta \Delta \Delta \Delta]_\epsilon + \text{H.c.} \quad (4.1)$$

It is easy to see that it has an accidental global $U(1) \times Z_4$ symmetry, under which the charges of various multiplets are

Multiplet	Ψ^i	S^{ij}	Δ^{ijkl}
U(1) charge	1	2	0
Z_4 charge	0	0	1

(4.2)

Considering now a generic effective operator of the form

$$(\Psi)^{2f} S^{n_S} \Delta^{n_\Delta}, \quad (4.3)$$

the requirements of the $U(1) \times Z_4$ symmetry and of the contraction of all indices give the conditions

$$2f + 2n_S = 0, \quad (4.4)$$

$$n_\Delta = 4N, \quad (4.5)$$

for some integer N . Notice that both baryon number and lepton number violation come from only one VEV, viz., $\langle \Delta^{\hat{a}\hat{d}\hat{b}\hat{\nu}} \rangle$. This VEV gives $\delta B = -1$, $\delta L = -1$. Thus, in this model,

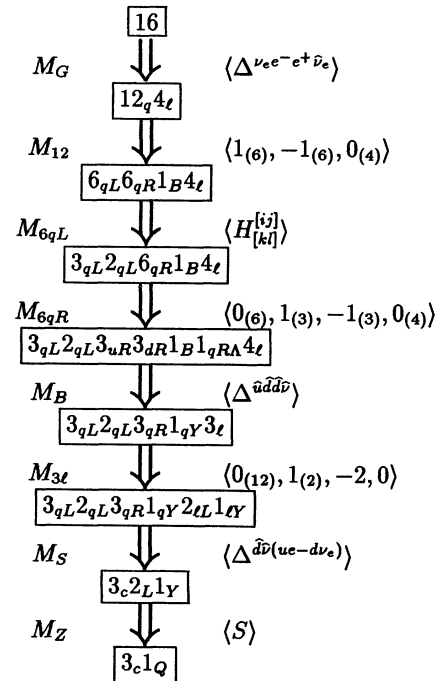


FIG. 10. SU(16) symmetry breaking where baryon number is broken by the VEV of an antisymmetric rank-4 multiplet. The notation for VEV's has been explained in Fig. 1.

$$\delta(B - L) = 0. \quad (4.6)$$

Once again, *neutron-antineutron oscillation is not possible in this model*. The possibilities of proton decay are discussed below.

2. Proton decay operators

Obviously, the simplest solution to Eqs. (4.4) and (4.5) are given by

$$f = n_S = n_\Delta = 0. \quad (4.7)$$

An operator of this type is

$$\mathcal{O}_1 = [(\bar{\Psi}_L)_i \gamma^\lambda (\Psi_L)^j] [(\bar{\Psi}_L)_k \gamma_\lambda (\Psi_L)^l] \Delta^{ikpq} \Delta_{pqjl}. \quad (4.8)$$

This can give gauge-boson-mediated proton decay, as shown in Fig. 11. Notice that this diagram is very similar to Fig. 2. The analysis is also the same, leading to the constraint in Eq. (3.11). In fact, one can also show that Higgs-boson-mediated diagrams, having $n_S = 2$, will be suppressed in this model, as shown in Eq. (3.19).

B. Baryon number violated by an antisymmetric rank-3 multiplet

Deshpande, Keith, and Pal [8] advocated a model where baryon number symmetry is violated by a VEV of an antisymmetric rank-3 multiplet as in Sec. III A. In Fig. 12 we show this chain with some slight modifications which helps eliminate a fundamental Higgs multiplet which was used by them.

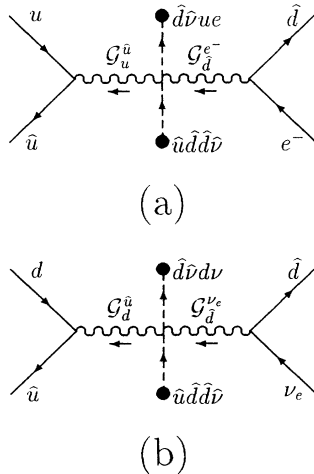


FIG. 11. Tree-level diagram giving rise to the operator \mathcal{O}_1 of Eq. (4.8).

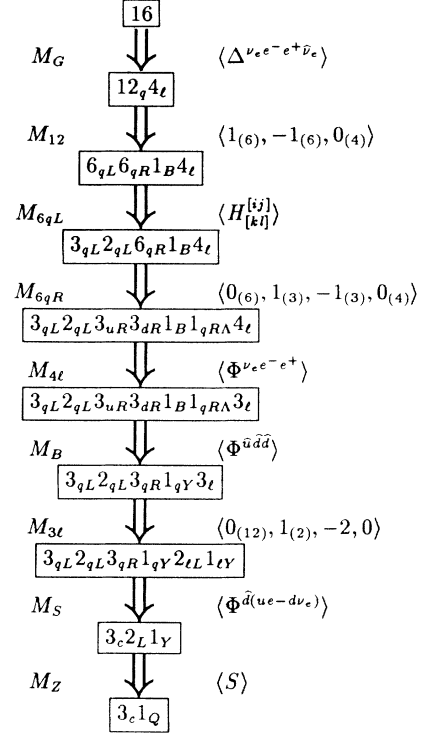


FIG. 12. SU(16) symmetry breaking where baryon number is broken by the VEV of an antisymmetric rank-3 multiplet. The notation for VEV's has been explained in Fig. 1.

With the introduction of the multiplet Φ , there is one more term in \mathcal{L}' :

$$\mathcal{L}' = \mathcal{Y}(\Psi_L)^i (\Psi_L)^j S_{ij} + \lambda \Delta_{klmn} S_{pr} \Phi^{klp} \Phi^{mnr} + \lambda' [\Delta \Delta \Delta \Delta]_\epsilon + \text{H.c.} \quad (4.9)$$

However, we now have an accidental $U(1) \times Z_8$ symmetry, with the charge assignments

Multiplet	Ψ^i	S^{ij}	Φ^{ijk}	Δ^{ijkl}
U(1) charge	1	2	1	0
Z_8 charge	0	0	1	2

(4.10)

So now, the generic effective operator of the form

$$(\Psi)^{2f} S^{n_S} \Phi^{n_\Phi} \Delta^{n_\Delta} \quad (4.11)$$

is subject to the constraints

$$2f + 2n_S + n_\Phi = 0, \quad (4.12)$$

$$n_\Phi + 2n_\Delta = 8N, \quad (4.13)$$

for some integer N . The first of these equations now implies that n_Φ must be even. The simplest solution to these conditions is given by all n 's being zero, which gives the operator of Eq. (3.9), and the phenomenological conclusions are the same as in Sec. III A.

V. CONCLUSIONS

We have analyzed a variety of symmetry breaking chains within the gauge groups SU(15) and SU(16) in

which the grand unified gauge group breaks to “unified” subgroups under which quarks and leptons have separate symmetries. As mentioned in the Introduction, such chains are interesting because some of them are known to predict low unification scales, sometimes as low as 10^8 GeV. Our analysis shows that low unification scale is not phenomenologically ruled out in these models because proton decay operators are very suppressed. The amount of suppression, of course, depends on the Higgs boson sector of the models and therefore varies from one model to another. We have also shown that in these models, since operator analysis can be performed on the full gauge invariant operators, and since such operators have a large Fermi symmetry, the proton cannot decay into nonstrange hadrons. Such modes are preferred in supersymmetric SU(5) or SO(10) models, but here we obtain this conclusion without any supersymmetry in our models. In fact, inclusion of supersymmetry in SU(15) or SU(16) models typically make the unification scales large [22]. In that case, with all the suppression mentioned in this paper, proton decay should be unobservably slow.

One remarkable result that comes out from our anal-

ysis is that, most of these models contain accidental global or discrete symmetries. Of course, these symmetries depend on the Higgs boson contents of the model, much like the $B - L$ conservation in the simplest SU(5) unification model. Such symmetries provide selection rules to baryon-number-violating processes. For example, neutron-antineutron oscillation is strictly forbidden in most of the models, as we pointed out. Of course, one can always further complicate the models, using more Higgs boson multiplets than are necessary for breaking the symmetries. The presence of these multiplets will explicitly break some or all of the accidental symmetries that we discovered, and therefore will allow more baryon number violating processes. We provided examples of this by introducing, in Sec. III D, the antisymmetric rank-2 tensor which couples to fermions. Another example could be a multiplet $X^{\{ijk\}[lmn]}$, which exists in the symmetric part of the tensor product of two antisymmetric rank-3 multiplets. Once this multiplet is introduced, one can show that neutron-antineutron oscillations become allowed in most cases through the operator

$$[(\Psi_L)^i C(\Psi_L)^j][(\Psi_L)^k C(\Psi_L)^l][(\Psi_L)^m C(\Psi_L)^n] X^{\{ijk\}[lmn]}. \quad (5.1)$$

Our analysis, however, deals mostly with “minimal” models in the sense that we do not introduce any Higgs boson multiplets which are not necessary for symmetry breaking.

Our results can also be used to look for other baryon- and lepton-number-violating processes in these models. For example, using Eqs. (3.8), (3.29), and (4.6), we can conclude that neutrinos cannot have any Majorana mass in the SU(15) models of Secs. III A and III B, as well as in the SU(16) model of Sec. IV A. For the first of these models, this result was derived by earlier authors [19], but for the other models, the result is new. This and other new results can be readily derived from the accidental

symmetries that we have discovered in this article for various models of interest.

ACKNOWLEDGMENTS

The work of P.B.P. was supported by the Department of Energy of the United States. U.S. would like to thank Professor Patrick J. O’Donnell for arranging his visit to the University of Toronto and the NSERC of Canada for financial support. We thank Daniel Wyler for reading the manuscript carefully and making insightful comments.

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- [1] J. C. Pati, A. Salam, and J. Strathdee, *Nuovo Cimento* **26A**, 72 (1975); *Nucl. Phys.* **B185**, 445 (1981).
 - [2] S. L. Adler, *Phys. Lett. B* **225**, 143 (1989).
 - [3] P. H. Frampton and B-H. Lee, *Phys. Rev. Lett.* **64**, 619 (1990).
 - [4] P. B. Pal, *Phys. Rev. D* **43**, 236 (1991).
 - [5] B. Brahmachari, U. Sarkar, R. B. Mann, and T. G. Steele, *Phys. Rev. D* **45**, 2467 (1992).
 - [6] P. B. Pal, *Phys. Rev. D* **44**, R1366 (1991).
 - [7] P. B. Pal, *Phys. Lett. B* **311**, 153 (1993).
 - [8] N. G. Deshpande, E. Keith, and P. B. Pal, *Phys. Rev. D* **47**, 2897 (1993).
 - [9] B. Brahmachari, *Phys. Rev. D* **48**, 1266 (1993).
 - [10] L. Lavoura, *Phys. Rev. D* **49**, 575 (1994).
 - [11] U. Sarkar, R. B. Mann, and T. G. Steele, Report No. PRL-TH-90-18, 1990 (unpublished).
 - [12] P. H. Frampton and T. W. Kephart, *Phys. Rev. D* **42**, 3892 (1990).
 - [13] P. B. Pal, *Phys. Rev. D* **45**, 2566 (1992).
 - [14] R. N. Mohapatra and M. Popović, *Phys. Rev. D* **25**, 3012 (1982).
 - [15] A. Raychaudhuri and U. Sarkar, *Phys. Rev. D* **26**, 3212 (1982).
 - [16] S. Weinberg, *Phys. Rev. Lett.* **43**, 1566 (1979).
 - [17] F. Wilczek and A. Zee, *Phys. Rev. Lett.* **43**, 1571 (1979).
 - [18] S. Weinberg, *Phys. Rev. D* **22**, 1694 (1980).
 - [19] N. G. Deshpande, P. B. Pal, and H. C. Yang, *Phys. Rev. D* **44**, 3702 (1991).
 - [20] S. Rajpoot, *Mod. Phys. Lett.* **1**, 645 (1986); H. Georgi, E. Jenkins, and E. H. Simmons, *Phys. Rev. Lett.* **62**, 2789 (1989); **63**, 1540 (E) (1989); D. Choudhury, *Mod. Phys. Lett. A* **6**, 1185 (1991).
 - [21] P. H. Frampton and S. L. Glashow, *Phys. Lett. B* **190**, 157 (1987); *Phys. Rev. Lett.* **58**, 2168 (1987).
 - [22] B. Brahmachari and U. Sarkar, *Phys. Lett. B* **303**, 260 (1993).