Experimental and theoretical implications of new sequential leptons

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If new sequential leptons $E^{\pm}$ and N^0 exist, the CERN LEP bound implies $m_E,\ m_N > M_Z/2$. The heaviness of the neutral lepton breaks away from the pattern of the first three generations. The minimal model is to have 4 left-handed lepton doublets and 4 right-handed charged lepton singlets, but only one right-handed neutral lepton singlet. Since in general the third and fourth generations should mix, and since $|m_N - m_E|$ should not be too large, neither E nor N would be stable, and both tend to decay via the Cabibbo suppressed $E \to \nu_\tau$ or $N \to \tau$ charged currents. This leads to the interesting signature of like-sign W pair production via $E^+N \to \bar{\nu}_\tau \tau^- W^+ W^+$ at the SSC and CERN LHC. The popular seesaw mechanism cannot plausibly accommodate the near masslessness of the light neutrinos and the heaviness of N^0 simultaneously. The representation structure poses a difficulty to the traditional approach of SO(10)-based grand unified theories. The discovery of such new heavy leptons would thus have interesting implications.

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I. INTRODUCTION

Neutrino counting on the Z^0 resonance gives [1], at present,

$$N_\nu = 2.99 \pm 0.04. \tag{1}$$

Although the τ neutrino ν_τ still needs to be established as distinct from ν_e and ν_μ, there is no denying that there exists three and only three light neutrino species. It is tempting to go one step further and state that there are *only* three generations of fermions in nature. However, leaving prejudices aside, a direct search for sequential leptons at the CERN e^+e^- collider LEP yields the limits [1]

$$m_E,\ m_N \gtrsim M_Z/2, \tag{2}$$

where we denote the new sequential charged and neutral leptons as E^- and N^0, respectively. New sequential leptons are permitted, but they have to be very heavy.

New sequential fermions would join the already rather heavy top quark as "heavy" fermions (F), where Yukawa couplings λ_F are of order the gauge couplings. In contrast, except for the top quark, known fermions can be viewed as "light," almost chiral fermions f with $\lambda_f \simeq 0$. Since $\lambda_F \gg 1$ is unreasonable, the generational hierarchy has to stop, which is rather interesting. What is perhaps more striking, however, is the deviation in the neutral lepton sector, that is, $m_N \gtrsim M_Z/2$ [Eq. (2)], which is in strong contrast to the apparent $m_\nu \cong 0$ for the first three generations.

A simple extension that could accommodate this was recently pointed out by King [2]. One adds a *single* right-handed neutral lepton singlet N_R to four sequential generations of the standard type. If Majorana masses are forbidden, one automatically has three strictly massless neutrinos just like the usual implementation of the three-generation standard model (SM). We shall explore the extension of King and compare with other possible ways of accommodating Eq. (2). After working out the parametrization of the model, we show that E, N could have sizable mixings with τ and ν_τ (assuming negligible mixings with the first two generations), and would therefore likely undergo rapid decay. The heaviness of E and N and the expectation that E-N splitting is not too large influence their decay properties, leading to interesting implications for future search strategies. When right-handed Majorana masses are added, various modifications of the traditional seesaw mechanism [3] tend to run into difficulties. If the fourth generation has one extra N_R singlet compared to the first three generations, one would have problems grouping the fermions into standard grand unified theory (GUT) multiplets.

II. PARAMETRIZATION OF THE MODEL

The quark sector is composed of four standard generations; hence, we shall not comment further on it, except recalling the fact that b' may have unusual decay properties [4]. Following King [2], in addition to the four left-handed lepton doublets $\ell'_{iL} = (\nu'_{iL},\ e'_{iL})$ and four right-handed charge -1 leptons e'_{iR}, we add just one right-handed (by convention) gauge singlet lepton N_R. We shall call this situation "3 + 1" generations.

We are interested in the couplings of fourth-generation leptons. Our approach is slightly different from that of Refs. [2, 5]. Assuming one Higgs boson doublet, upon symmetry breaking, one has the lepton mass terms

$$\bar{\nu}'_{iL}\, m_i\, N_R + \bar{e}'_{iL}\, m_{ij}\, e'_{jR}, \tag{3}$$

where we have assumed in addition that a Majorana mass for N_R is forbidden (achieved, e.g., by assigning some unbroken, perhaps discrete, charge). Note that m_i can be chosen to be real. The charged leptons can be diagonalized as usual by a biunitary transform from the gauge basis $e'_{L,\ R}$ to the mass basis $e_{L,\ R}$. We define $e_i \equiv (e_k,\ E)$ for both left- and right-handed components,

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where $e_k = e, \mu, \tau$ for $k = 1, 2, 3$. The left-handed Dirac partner of N_R is trivially chosen to be

$$\bar{\nu}'_{iL} m_i \equiv \bar{N}_L m_N, \tag{4}$$

where $m_N^2 = m_1^2 + m_2^2 + m_3^2 + m_4^2$, and there is a single massive neutral lepton N with Dirac mass

$$m_N \bar{N}_L N_R. \tag{5}$$

The remaining three (left-handed) neutrinos remain strictly massless, much like in SM. Because of this threefold degeneracy, one can arbitrarily redefine them without changing the physics. We denote $\nu_{iL} \equiv (\nu^0_{kL},\ N_L)$, $k = 1$–3.

The neutral current remains diagonal, and the standard ZNN and ZEE couplings lead to the bound of Eq. (2). The leptonic charged current is (suppressing Dirac matrices)

$$\bar{\nu}'_{iL} e'_{iL} = \bar{\nu}_{iL} V_{ij} e_{jL}, \tag{6}$$

where

$$V^{(3+1)} = \begin{pmatrix} U^{(3)} & 0 \\ 0 & 1 \end{pmatrix} K^{(3+1)}. \tag{7}$$

In Eq. (7), $U^{(3)}$ is a 3×3 unitary matrix, the zeros stand for three component column or row matrices, and $K^{(3+1)}$ is the 4×4 Kobayashi-Maskawa (KM) fermion mixing matrix [6].

Since the left-handed leptons are standard repetitions, we have chosen to use the standard procedure to arrive first at the KM matrix K, which possesses six rotation angles and three CP-violating phases. The degeneracy of the three massless neutrinos gives rise to the freedom of an arbitrary unitary matrix $U^{(3)}$. We would like to see to what extent the latter matrix reduces further the physical number of parameters in V. One starts with $3^2 = 9$ parameters in the matrix U. Three phases are absorbed by the unobservable phases of the three massless neutrino fields. The six remaining parameters consist of three angles and three phases. One immediately sees that V has *just three* mixing angles and no phases. These physical angles describe the mixing of N_L with e_L, μ_L, and τ_L, respectively.

One can easily generalize to $n + 1$ generations (had there been n light neutrino species). The $(n+1)\times(n+1)$ KM matrix K would have $n^2 = n(n + 1)/2$ (angle) + $n(n - 1)/2$ (phase) parameters, while U would have $n^2 - n = n(n - 1)/2$ (angle) + $n(n - 1)/2$ (phase) parameters. All phase parameters are removed and one is left with n rotation angles, precisely the number needed to describe the mixing between the heavy neutral lepton and the n light charged leptons. Similarly, one can easily generalize to $3 + m$ generations, e.g., if m new sequential generations exist and one just adds m new right-handed neutral lepton fields. Following similar arguments, there should be $m(6 + m - 1)/2$ mixing angles, describing $3m$ angles between heavy and light and $m(m - 1)/2$ angles among the heavies, and $(m-1)(6+m-2)/2$ phases, which can be similarly decomposed. For the case of $m = 2$, there should be seven angles and three phases in the lepton sector, compared to the KM prescription of ten angles and six phases in the quark sector. CP-violation effects may then occur in the lepton sector in processes that involve the two new generations. Further generalizations to $n + m$ generations is straightforward.

Returning to the $3 + 1$ case, it is useful to have an explicit parametrization of the three physical mixing angles. We have to make a suitable choice of basis in the massless neutrino sector. Recall that with three standard generations, i.e., when K is the 3×3 KM matrix, U is chosen such that V is the unit matrix. That is, the neutrinos are *defined* to carry the label of the associated charged lepton, and lepton number is separately conserved. Since the neutrinos are physically degenerate, one takes the same unitary transform that diagonalizes the charged lepton sector. It is clearly advisable to stay close to this convention, since the three mixing angles in the $3 + 1$ case just describes mixing between N_L and the three light charged letpons. We therefore choose to build up V by three such rotations [7], between N_L-e_L, N_L-μ_L, and N_L-τ_L, where the rotations are denoted as s_e, s_μ, and s_τ, respectively. Thus, the lepton charged current is defined as

$$\left(\bar{\nu}_{eL}, \bar{\nu}_{\mu L}, \bar{\nu}_{\tau L}, \bar{N}_L\right) \begin{pmatrix} c_e & -s_e s_\mu & -s_e c_\mu s_\tau & s_e c_\mu c_\tau \\ 0 & c_\mu & -s_\mu s_\tau & s_\mu c_\tau \\ 0 & 0 & c_\tau & s_\tau \\ -s_e & -c_e s_\mu & -c_e c_\mu s_\tau & c_e c_\mu c_\tau \end{pmatrix} \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \\ E_L \end{pmatrix}. \tag{8}$$

III. PHENOMENOLOGY

The choice of zeros in Eq. (8) is quite arbitrary, and we have adopted the convention that $\bar{\nu}_{\mu L} e_L$, $\bar{\nu}_{\tau L} e_L$, and $\bar{\nu}_{\tau L} \mu_L$ are absent. Note, however, that lepton numbers are separately violated. The physical observable (when massless neutrino states are involved) is always the product $\left(\sum_{k=1}^{3} V_{ki} V_{kj}\right)^2$. For example, in the two-neutrino experiment [8], the ratio of number of electrons produced versus muons should be $s_e^2 c_e^2 s_\mu^2/(c_\mu^2 + s_e^2 s_\mu^2)^2$. The expected smallness of s_e and s_μ, of course, makes this effectively unmeasurable. Note that, if this experiment could be repeated at high energy, the τ to μ ratio would be of order $s_\mu^2 s_\tau^2$.

We would like to explore the constraints on this model, and, in particular, the expected properties of the new leptons E and N [2, 5].

A. Low energy constraints

As remarked, clearly the N_L-e_L and N_L-μ_L mixing angles s_e and s_μ should be rather small. The best constraint is expected to be $\mu \to e\gamma$ and $\mu \to e$ conversion on nuclei. The former gives [9] $s_e^2 c_e^2 s_\mu^2 < 7 \times 10^{-6}$, while the latter is expected [10] to give the more stringent bound

$$s_e^2 c_e^2 s_\mu^2 < 10^{-8}. \tag{9}$$

Although these are not separate bounds on s_e and s_μ, they do suggest that s_e and s_μ are extremely small, and in any case these two angles are rather hard to test separately. In the following we shall assume that both s_e and s_μ are negligibly small.

Based on observed patterns in the three-generation quark sector, the largest mixing angle is in fact expected to be the N_L-τ angle s_τ. Sizable s_τ values have been suggested as an explanation for the "τ decay puzzle" [10]. The latter, however, has largely evaporated with new m_τ measurements from the BES Collaboration [11] and new τ lifetime measurements. Nevertheless, it is easy to see that N_L-τ mixing can still be of Cabibbo strength; that is,

$$s_\tau \lesssim 0.2 \tag{10}$$

is still permitted by present data. From the slightly more theoretical standpoint, even if one assumes no N_L-ν_τ mixing [$\nu'_{4L} \equiv N_L$ in Eq. (4)], the usual rule of thumb from quark mixing patterns [12] leads to τ_L-E_L mixing of order $\sqrt{m_\tau/m_E}$, which ranges from 0.2 to 0.08 for $M_Z/2 \lesssim m_E < 300$ GeV, quite similar to Eq. (10). Continued improvements on τ decay studies would remain the best indirect searching ground for the existence of new neutral leptons.

B. Decay properties of E and N

What is more exciting is, of course, the direct production and detection of E or N. Most work in the past [13] tends to assume $m_N \ll m_E$, while some recent work has focused on the case when E and N do not mix with light generations [14]. The former is certainly no longer justified. For the latter, we have seen that, although it is reasonable to assume that s_e and s_μ are vanishingly small, s_τ can still be quite sizable. We shall consider the charged current involving E and N as effectively described by

$$(\bar{\nu}_{\tau L}, \bar{N}_L) \begin{pmatrix} c_\tau & s_\tau \\ -s_\tau & c_\tau \end{pmatrix} \begin{pmatrix} \tau_L \\ E_L \end{pmatrix}. \tag{11}$$

Since there is no reason to believe that s_τ is vanishingly small in this model, whether E or N is the heavier one, they would necessarily undergo rapid decay because of their heaviness [Eq. (2)]. Their decay rate is typically $s_\tau^2 \times \Gamma_t$ or higher, where Γ_t is the top quark decay rate assuming $m_t \sim m_N$ or m_E, even when E and N are degenerate. Thus, the possibility of having stable charged or neutral leptons, of relevance for cosmological considerations [14] and for study at colliders [15], seems rather improbable.

Let us consider the case where $m_N > m_E \gtrsim M_Z/2$. The E would decay via $E \to \nu_\tau W^{(*)}$, while there are two decay chains for N, $N \to EW^{(*)}$, or $\tau W^{(*)}$. Since $m_t > M_W$, the W boson, whether real or virtual, would decay further via $W^{(*)} \to e\bar{\nu}_e,\ \mu\bar{\nu}_\mu,\ \tau\bar{\nu}_\tau,\ \bar{u}d,\ \bar{c}s$. Ignoring m_τ and other "light" fermion masses, the decay rates are

$$\Gamma(N \to EW^{(*)}) = 9 \times c_\tau^2 \frac{G_F^2 m_N^5}{192\pi^3} f\left(\frac{m_N^2}{M_W^2}, \frac{m_E^2}{m_N^2}, \frac{\Gamma_W^2}{M_W^2}\right), \tag{12}$$

$$\Gamma(N \to \tau W^{(*)}) = 9 \times s_\tau^2 \frac{G_F^2 m_N^5}{192\pi^3} f\left(\frac{m_N^2}{M_W^2}, 0, \frac{\Gamma_W^2}{M_W^2}\right), \tag{13}$$

where the function $f(\rho, \mu, \gamma)$, accounting for decays via both real and virtual W bosons, can be found in [16]. The $E \to \nu_\tau W^{(*)}$ rate is identical to Eq. (13) with m_N replaced by m_E. Naively, one would have expected the "Cabibbo-favored" $N \to E$ chain to be the dominant N decay channel. However, when m_E is close to m_N, this chain has rather limited phase space and may suffer from W propagator effects. There are also reasons such as ρ parameter limits that suggest N-E splitting should not be too large, or else it would affect the global fit of present day electroweak precision tests [1]. In contrast, the $N \to \tau$ sequence, though suffering from "Cabibbo suppression" through the factor of s_τ^2 (in rate), it does not suffer from phase space. Thus, it is not impossible that the "Cabibbo-suppressed" $N \to \tau$ process could in fact be dominant over the "Cabibbo-favored" $N \to E$ process. This effect is displayed in Figs. 1 and 2.

With an eye towards the two major regions of experimental study in the future, Figs. 1 and 2 are for the mass ranges $m_{E,N} \in (50,\ 100)$ GeV and (100, 300) GeV, respectively. The dashed curves are for the $N \to \tau$ process. The solid curves are for the $N \to E$ process for $m_E = 50, 60, 70, 80, 90$ GeV for Fig. 1, and $m_E = 100, 150, 200, 250$ GeV for Fig. 2. For the latter set, we switch to dotdash lines for $m_N > m_E + M_W$. We illustrate with $s_\tau = 0.2$. With this s_τ value, we see

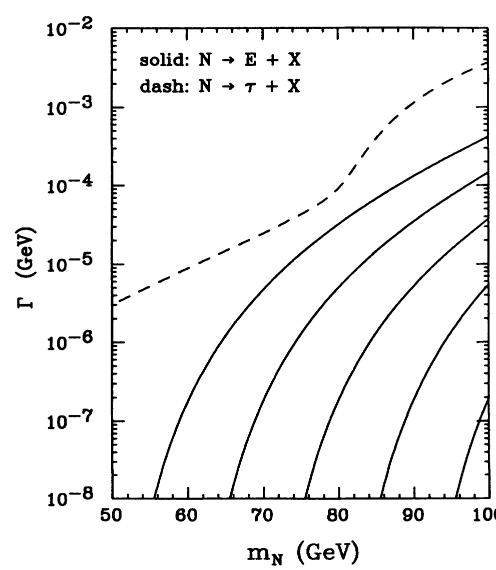

FIG. 1. Decay rate for $N \to EW^{(*)}$ (solid line) and $\tau W^{(*)}$ (dashed line) with $s_\tau = 0.2$. The solid curves correspond to $m_E = 50, 60, 70, 80$, and 90 GeV.

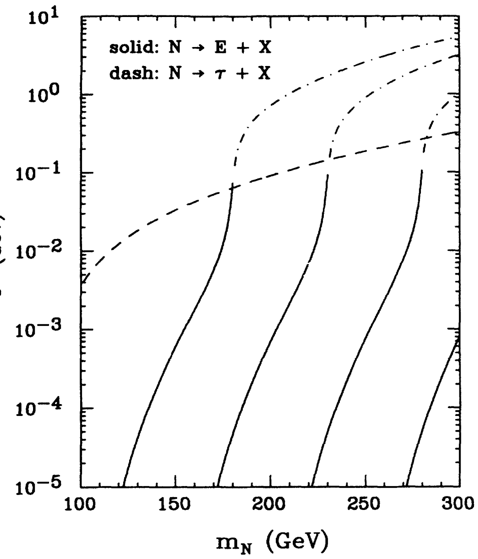

FIG. 2. Same as Fig. 1 except the solid curves correspond to $m_E = 100, 150, 200, 250$ GeV. For $m_N > m_E + M_W$, the curves for $N \to EW^{(*)}$ switch to dot-dashed.

from Fig. 1 that $N \to \tau + W^*$ is typically orders of magnitude higher than $N \to EW^*$. This would largely hold even if s_τ is much smaller than 0.2, where one can simply scale down the dashed curve. For the heavier mass case, $N \to \tau$ also dominates over $N \to E$ for the plausible mass range $m_N - m_E \lesssim M_W$, beyond which the $N \to EW$ rate turns on sharply. The eminence of $N \to \tau W^{(*)}$ has interesting implications on search strategies.

In case $m_E > m_N$, everything above holds true upon making the interchange of $N \leftrightarrow E$ and $\tau \leftrightarrow \nu_\tau$.

C. Search strategies

In case m_N or $m_E < M_W$, one could search for $W \to \tau N$ or $E\nu$ [2]. One could also indirectly check for sizable s_τ by studying e-μ-τ universality since $W \to \tau\nu$ would be suppressed by $1 - s_\tau^2$ [2]. This test would demand rather high statistics and low systematic background.

In the following, we focus on the direct production of E and/or N pairs. We shall always discuss the case $m_N > m_E$, as the opposite can be easily reached by the interchange mentioned eariler.

The mass range 50 GeV $< m_E$, $m_N <$ 100 GeV would be of immediate interest at LEP II as soon as it turns on. The e^+e^- collider environment is rather clean such that there should be no difficulty in finding E^+E^- and $N\bar{N}$ pair production, although one suffers from low event rates. The E^+E^- pair results in the signature $\nu_\tau\bar{\nu}_\tau W^{(*)+}W^{(*)-}$, which is distinctive enough for m_E below m_W. Running at $\sqrt{s} < 160$ GeV would reveal the existence of such charged leptons [17]. However, for $m_E > M_W$, the signature becomes $\nu_\tau\bar{\nu}_\tau W^+W^-$, and one is swamped by direct $e^+e^- \to W^+W^-$ background that is typically 10 times larger [17]. It is not clear whether the extra missing energy and the difference in WW angular distributions would be sufficient to suppress background at LEP II energies.

The purpose of Fig. 1 is to show that, with present knowledge that s_τ could be as large as 0.2, the neutral lepton N would dominantly decay via the Cabibbo-suppressed $N \to \tau W^{(*)}$ mode, rather than the Cabibbo-favored $N \to EW^*$ mode. This is even more true when $m_N - m_E$ is small; i.e., if E is found at LEP II first, then the heavier it is, the lesser the likelihood that N would be discovered via the Cabibbo-favored $N \to E$ channel, even with s_τ much smaller than 0.2. Hence, the discovery channel for N is most likely $N\bar{N} \to \tau^+\tau^- W^{(*)+}W^{(*)-}$. Of course, if m_E is close to 50 GeV, and s_τ is smaller than 0.2, it is possible to have $N \to EW^*$ as the dominant N decay mode. In this case N could be discovered via the decay sequences $N\bar{N} \to E^-W^{*+}E^+W^{*-} \to \nu_\tau\bar{\nu}_\tau W^{(*)+}W^{(*)-}W^{*+}W^{*-}$, or $N\bar{N} \to \tau^-W^{*+}E^+W^{(*)-} \to \tau^-\bar{\nu}_\tau W^{(*)+}W^{(*)-}W^{*+}$, depending on the strength of s_τ.

Note that, unless the mixing angle s_τ is much smaller than 0.001–0.0001, both N and E should have sufficiently short lifetime such that they would decay in the detector. Of course, if $E^\pm$ is sufficiently long lived, it could show up as minimally ionizing charged tracks. Note also that in case $m_E > m_N$ and M_W, if s_τ is sufficiently small and $m_E - m_N$ is suitably large, the heavy E may be discovered via $E \to NW^* \to \tau W^{*+}W^{*-}$ i.e., in $e^+e^- \to \tau^+\tau^-W^{*+}W^{*-}W^{*+}W^{*-}$ above W^+W^- threshold.

For m_E, $m_N > 100$ GeV, one would either need a high energy e^+e^- linear collider which should be able to cover the full mass range m_E, $m_N \lesssim \sqrt{s}/2$, or one would have to resort to hadronic supercolliders such as the Superconducting Super Collider (SSC) or CERN Large Hadron Collider (LHC). It is usually claimed that heavy leptons that decay via on-shell W bosons suffer from large vector boson pair production backgrounds and would be difficult to detect at hadron supercolliders [13, 18] (for a counterview, see, however, Ref. [19]). However, in these earlier studies, it is usually assumed that the fourth neutral lepton is massless. This is clearly no longer the case, and it is of interest to see if the conclusions can be evaded.

The Drell-Yan production mechanism (via virtual γ, Z, or W bosons) yields E^+E^-, $N\bar{N}$, and $E^-\bar{N}$ (or E^+N) pairs, with cross sections at the SSC ($\sqrt{s}$ = 40 TeV) ranging from 10 to 0.1 pb as m_N and m_E range from 100 to 300 GeV [13]. The background problem lies with $pp \to W^+W^- + X$ production, which is of order 200 pb and completely swamps the $E^+E^- \to \nu_\tau\bar{\nu}_\tau W^+W^-$ process. For the $N\bar{N}$ and $E\bar{N}$ modes, one needs to know the $N \to E$ and $N \to \tau$ branching ratios.

It is seen from Fig. 2 that, with $s_\tau \simeq 0.2$ and $m_N - m_E < M_W$, the Cabibbo-suppressed $N \to \tau^-W^+$ mode dominates (quite often by orders of magnitude) over the Cabibbo-favored $N \to E^-W^{*+}$ mode. Thus, for the $N\bar{N}$ production process, the detection final state is $\tau^+\tau^-W^+W^-$. Although the signal is one to three orders of magnitude smaller than the W^+W^- pair production background, if the additional high p_T, isolated $\tau^+\tau^-$ pair can be utilized, perhaps one could still separate the signal. Detection in the $\tau^+\tau^- + 4$ *jets* mode may in fact allow reconstruction of m_N using kinematic tricks [20].

What is more exciting is the $E\bar{N}$ mode. In our scenario that $N \to \tau$ decay is likely to dominate over $N \to E$, we find that E^+N decays into the final state $\bar{\nu}_\tau \tau^- W^+ W^+$. That is, the Drell-Yan production of E^+N or $E^-\bar{N}$ leads to *like-sign* W boson pairs plus isolated $\tau\nu$. The corresponding electroweak background in this mass range is of order 1 pb or less [21]. Thus, the $\nu\tau^\mp W^\pm W^\pm$ signature should allow E and N to be simultaneously discovered at the SSC (similar conclusions should hold for the LHC). However, the additional neutrino makes the reconstruction of m_E and m_N rather difficult.

As the $N \to EW^{(*)}$ branching ratio is raised, which could come about if s_τ is considerably smaller than 0.2, or if $m_N - m_E > M_W$, the signal switches to the more complicated $E^+N \to \nu_\tau \bar{\nu}_\tau W^+ W^- W^{(*)+}$, and $\bar{N}N \to \nu_\tau \tau^\mp W^\pm W^\pm W^{(*)\mp}$ or $\nu_\tau \bar{\nu}_\tau W^+ W^- W^{(*)+} W^{(*)-}$. That is, one may have triple or quadruple (up to two being virtual) W boson production with additional associated handles like $\nu\bar{\nu}$ or $\nu\tau^\pm$. In the corresponding case of $m_E > m_N$, the final state E^+N or $E^-\bar{N} \to \tau^+\tau^- W^+ W^- W^{(*)+}$ may allow for m_N and m_E reconstruction.

We conclude that the SSC and LHC should be able to discover N and E with masses above 100 GeV, especially via E^+N or $E^-\bar{N} \to \nu\tau^\mp W^\pm W^\pm$. However, further detailed studies are needed to confirm this.

IV. THEORETICAL IMPLICATIONS

It is foreseen that new limits on E and N would first come from the onset of LEP II. This should allow the full exploration of m_N, m_E in the mass range up to the beam energy, of order 90–100 GeV. Beyond this, one would need either a high energy linear e^+e^- collider, or one would have to study the signatures discussed above at the SSC or LHC. The Tevatron is not a good place for heavy lepton search. It is of interest to ask, if new sequential leptons are found, what would be the meaning of such discoveries? The implications turn out to be rather interesting and should add to the impetus for conducting heavy lepton searches.

A. Problems with seesaw mechanism

The most salient feature of discovering new sequential leptons is the departure from previous patterns in first three generations: The new neutral lepton must be rather heavy. This would pose as a serious challenge to the usual seesaw mechanism for explaining the near masslessness of known neutrinos.

In the standard seesaw mechanism [3], one introduces right-handed neutrinos for *each* neutrino species. Since these extra fields are gauge singlets, it is possible that they carry a lepton-number-violating Majorana-type neutrino mass, denoted generically as M_R. Assuming that the Dirac-type neutrino mass is of order the corresponding charged lepton mass $m_{\ell^\pm}$, if $M_R \gg m_\ell$, then the left-handed neutrino effectively developes a Majorana mass of order $m_{\nu_L} \simeq m_\ell^2/M_R$.

Although M_R can be quite arbitrary, and there are many tailor-made models constructed for rather specific purposes [22], the most popular and most natural setting for discussing the seesaw mechanism is within GUT theories [3], especially GUT theories where right-handed neutrinos are incorporated in multiplets together with other fermions. Not only does the M_R scale get independently motivated, it also seems [23] to provide the best particle physics explanation for the solar neutrino problem via the Mikheyev-Smirnov-Wolfenstein (MSW) effect [24].

With three seemingly massless neutrinos, the seesaw mechanism provides a rich playground for neutrino physics [22]. However, if a new sequential neutral lepton N is discovered, one would have to reassess the utility of the mechanism. Our (King) model is quite extreme in that we forbid a Majorana mass for N_R, and of course, we have no right-handed neutrino fields for the first three families. Thus, we cannot accommodate the seesaw mechanism. Is it possible to construct models where the heaviness of N and the lightness of ν_k, $k = 1$–3 are incorporated within the framework of the seesaw mechanism? Let us list the known options.

(1) Weak-scale seesaw. Motivated by dynamical symmetry breaking ideas with $\bar{t}t$ condensation, where one faces the problem of too heavy a top quark, it was found desirable to introduce fourth-generation fermions. Hill and Paschos [25] proposed that M_R is perhaps of order 100 GeV. In this way, assuming m_E is of similar order of magnitude, small neutrino masses and Eq. (2) can both be satisfied, but at a price. With the seesaw mechanism retained, the model is rather precarious since *all* the neutrino masses lie *just* at the border of present limits [25]. It is hard to believe that we are just at the juncture in time such that the model is viable, although it certainly makes the model interesting in terms of immediate experimental checks. However, the main merit of the seesaw mechanism, the MSW explanation for the solar neutrino problem, is lost, since the three light neutrinos are too heavy.

(2) Singular seesaw. Several groups [27] have noticed that, with n neutrino species, the right-handed neutrino mass M_R can be viewed as a $n \times n$ matrix. There is no strong reason that this matrix should be necessarily rank n, and det M_R could vanish, hence the name "singular" seesaw. This notion gained some recent popularity because of the 17 keV neutrino problem. The latter problem, however, has by now evaporated [26]. Applying the idea to the present case, one envisions the standard type of seesaw for the first three neutrinos, but for the fourth neutrino, m_{4R} "accidentally" has a vanishing eigenvalue solution; hence, the resulting "neutrino" is not necessarily very light.

There are at least two serious problems with this picture. First of all, a high degree of tuning is needed to maintain the smallness of m_ν for the first three generations and satisfy Eq. (2) for the fourth "neutrino." This is especially so if one wants to invoke the MSW mechanism to explain the solar neutrino problem, that is, when $M_{kR} \sim M_{\rm GUT}$ for $k = 1$–3, while $m_{4R} \sim m_E$. This is reminiscent of the gauge hierarchy problem. In the particular application to the fourth-generation case, Fukugita and Yanagida [27] had to construct a global SU(4) family model that is broken in a complicated way by multi-Higgs fields at some high scale. Second, it seems artificial to have larger M_R for otherwise lighter ($k = 1$–3) generations, while smaller M_R for otherwise heavier ($k \geq 4$) ones.

(3) Radiative seesaw. Babu and Ma [28, 29] have constructed a model similar to our $3+1$ model, but allowing for Majorana masses for N_R. In this model, Majorana masses are specifically given to N_R, and one of the left-handed neutrinos acquires a seesaw mass [that has to satisfy Eq. (2)]. The chiral symmetry of the three originally massless left-handed neutrinos is then broken, and they acquire radiative Majorana masses through the two-loop two-W graph. The idea is interesting and may be explored further. Although it may be difficult to conceive a realistic working model, it can be viewed as an extension of our $3 + 1$ model. Again there is only one right-handed neutrino singlet.

B. Problems with grand unification

If the $3 + 1$ model is realized in nature, it seems that, as a corollary to the problem with the standard seesaw mechanism, we would have to rethink our strategies regarding unifying particle interactions.

The problem lies with the deviation from monotonous repetition of representation structure. In most GUT theories, the fermion generations are not unified, but rather, each generation serves as one copy of the multiplet(s) structure of the GUT group. The original GUT proposal, SU(5) [30] puts each generation into a **10** plus a $\bar{\mathbf{5}}$, with the possible inclusion of ν_R as a gauge singlet. This model is a direct generalization of the standard model, and therefore could straightforwardly accommodate $3+1$ generations. The original SU(5) GUT, however, is ruled out by experiment [1].

Beyond SU(5), it is customary to put each fermion generation into one single multiplet; e.g., the **16** of SO(10) has the SU(5) decomposition of $\mathbf{10}\oplus\bar{\mathbf{5}} \oplus \mathbf{1}$, and therefore necessarily requires a right-handed neutrino [SU(3)×SU(2)×U(1) singlet] for each generation. Originally [3], this came hand in hand with the seesaw mechanism and convinced many that right-handed neutrinos exist, that the left-handed neutrino is extremely light, and one had the extra bonus of providing a basis for explaining the solar neutrino problem, as mentioned earlier. Any GUT theory that puts each generation in one single multiplet would face difficulty if generations repeat, but not entirely sequentially, like in the $3 + 1$ case discussed here. In this sense, it may be of interest to put more emphasis on SU(5)-based GUT models.

V. CONCLUSION

The existence of three seemingly massless neutrinos while the fourth neutral lepton is very heavy clearly breaks from the traditional pattern. The discovery of new sequential leptons would provide great impetus for us to reconsider traditional thinking in regards neutrinos, GUT theories, and in particular, the question of fermion flavor. We have explored the $3 + 1$ model of King, where one adds just one right-handed neutral lepton singlet N_R to four generations of sequential fermions. We elucidate the mixing properties and demonstrate that there are only three rotational angles in the lepton charged current, and no CP-violating phases. Such phases start to appear if there are more heavy generations with associated heavy neutral leptons. In the $3 + 1$ case, charged current mixing is expected to be mostly in the $\nu_{\tau L}$-N_L and τ_L-E_L sector, described by one single mixing angle s_τ which could be as large as the Cabibbo angle. Both E and N should be quite unstable, even for the lighter of the two, because of the possible decay chain $E \to \nu_\tau$ and $N \to \tau$. The mass range $M_Z/2 < m_{N,E} < 100$ GeV can be explored soon at LEP II and also via W decays. If the mixing angle s_τ is not too small and if N-E splitting is not too large ($|m_N - m_E| \lesssim M_W$), it is expected that the Cabibbo-suppressed decays $E \to \nu_\tau$ and $N \to \tau$ are the dominant ones for both E and N, whether E or N is lighter. This leads to the distinctive like-sign W boson pair production signal $\nu\tau^\mp W^\pm W^\pm$ via E^+N or $E^-\bar{N}$ Drell-Yan production at the SSC or LHC. If they are found, the traditional seesaw mechanism, and SO(10)-based GUT theories, would be at jeopardy, and one may face a serious challenge with the solar neutrino problem.

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