Unquenching the ρ meson

Derek B. Leinweber* and Thomas D. Cohen
*Department of Physics and Center for Theoretical Physics, University of Maryland,
College Park, Maryland 20742*
(Received 9 July 1993)

Two-pion-induced self-energy contributions to the ρ-meson mass are examined in relation to the quenched approximation of QCD, where the physics associated with two-pion intermediate states has been excluded from vector-isovector correlation functions. Corrections to quenched QCD calculations of the ρ-meson mass are estimated to be small at the order of a few percent of the ρ-meson mass. The two-pion contributions display nonanalytic behavior as a function of the pion mass as the two-pion cut is encountered. The implications of this nonanalytic behavior in extrapolations of full QCD calculations are also discussed. We note that for full QCD, the error made in making a linear extrapolation of the ρ mass, neglecting nonanalytic behavior, increases as one approaches the two-pion cut.

I. INTRODUCTION

The lattice regularized approach to quantum field theory provides the best forum for the examination of the fundamental nonperturbative aspects of QCD. In the low momentum transfer regime, it is the only approach which in the foreseeable future holds reasonable promise of confirming or rejecting the validity of QCD as the underlying theory of the strong interactions.

Recently, it has become possible to perform quenched QCD calculations in which all systematic uncertainties are quantitatively estimated. Thus if the effects of quenching can be understood, the validity of QCD may be tested in the nonperturbative regime. Of particular note is the recent determination of the QCD coupling constant from the $1S - 1P$ mass splitting of charmonium [1]. In this case one believes that the effects of quenching may be estimated with minimal model dependence. Corrections to the utilization of the quenched approximation of QCD have been estimated and are currently the dominant source of uncertainty in the final predictions [2].

In this paper we will continue efforts along this line through the examination of systematic uncertainties in hadron mass spectrum calculations. In particular, the importance of the two-pion induced self-energy contribution to the ρ-meson mass is evaluated in relation to the quenched approximation of QCD and to full QCD. In the quenched approximation, the physics associated with two-pion intermediate states has been excluded in the numerical simulations.

This investigation is motivated by recent results from the GF11 group [3] for the low-lying hadron mass spectrum in the quenched approximation of QCD. Their analysis is the first to systematically extrapolate QCD calculations to physical quark mass, zero lattice spacing, and infinite volume. Their predictions display an impressive agreement with experiment. Of eight hadron mass ratios, six agree within one standard deviation and the remaining two ratios agree within 1.6σ. However, since the quenched approximation leaves out so much important physics, one might question whether these results are actually *too* good [4].

In the perturbative regime, many of the effects of not including disconnected quark loops when preparing an ensemble of gauge configurations may be accounted for in a simple renormalization of the strong coupling constant. However, one also anticipates nonperturbative effects in making the quenched approximation. Unlike a global renormalization of the coupling constant, these effects are expected to be channel specific. For example, the quenched approximation of QCD leaves out the physics associated with the decay of the ρ meson to two pions. This physics must be accounted for and added to the quenched results prior to comparing with experimental data. Moreover, the calculated hadron masses are extrapolated as a function of the pion mass squared to the point at which the pion mass vanishes using linear extrapolation functions. Such an approach neglects nonlinear and indeed, nonanalytic behavior in the continuum extrapolation function. For example, in the case of the ρ-meson mass, one expects nonanalyticity associated with the onset of the two-pion cut.

A priori one does not know the relative importance of two-pion intermediate states of the ρ meson in describing the ρ mass. The substantial width of the ρ meson at 151.5 ± 1.2 MeV indicates its coupling to pions is not small and correspondingly these dynamics may have significant influence on the ρ-meson mass.

Geiger and Isgur were the first to study the possible importance of the two-pion-induced self-energy of

---
*Present address: Department of Physics, The Ohio State University, 174 West 18th Avenue, Columbus, OH 43210-1106.

the ρ meson in relation to lattice QCD calculations [5]. Their results are based on a string-breaking quark model and predict large corrections to smooth extrapolations of the ρ-meson mass at approximately 70 MeV. These authors were motivated by the long-standing problem of QCD predictions for the N/ρ-mass ratio being too large. Their hope was the two-pion-induced self-energy correction would sufficiently raise the ρ mass to solve this problem. However, we now understand that both the finite lattice spacing and the finite volume of the lattice act to push up this mass ratio. Current estimates [3] for this ratio corrected to the infinite volume, continuum limit are 1.22 ± 0.11 in excellent agreement with the experimental value of 1.22.

Geiger and Isgur [5] advocate using the nonlinearities in the ρ mass as a function of the quark mass to correct for the linear extrapolation of lattice results. We wish to stress that such a procedure is only sensible for the extrapolation of full QCD calculations. As we shall argue, the entire two-pion-induced self-energy is absent in the quenched approximation and should be added on to quenched QCD results prior to comparison with experiment. We note that for the model of Ref. [5], quenched lattice calculations of the ρ-meson mass would be *reduced* by 160 MeV instead of increased by 70 MeV. This would further exacerbate the "m_N/m_ρ problem" discussed in their paper, rather that curing it.

In the quenched approximation, the ρ meson cannot decay to two light pseudoscalars. As discussed in Ref. [6] it is not possible to generate intermediate states of the ρ meson in the quenched approximation in which one has two isovector pseudoscalars. One might worry about the presence of the light isoscalar pseudoscalar η' which fails to obtain its heavy mass in the quenched approximation [6–9]. However, decay of the ρ meson to a $\pi \eta'$ is forbidden by G parity and decay to two η' mesons is of course forbidden by the isospin invariance of the strong interactions. Hence the physics associated with two-pion intermediate states of the ρ should simply be added onto the results extracted from calculations of quenched QCD, provided the lattice spacing is defined by physical observables which do not have a similar dependence on pion decays.

Current calculations of full QCD typically employ quark masses which place $2m_\pi > m_\rho$. As a result, the functional form of the extrapolation function should account for the nonanalytic behavior in the ρ-meson mass as the two-pion cut is encountered. On the lattice, the spectral density does not have a cut but rather a series of poles at the points satisfying

$$\sqrt{s} = 2 \left(m_\pi^2 + p_n^2 \right)^{1/2} , \qquad (1)$$

where p_n are the discrete momenta allowed in the finite volume of the lattice. Obviously, to fully account for the two-pion-induced self-energy of the ρ in quark mass extrapolations, one must first extrapolate the lattice results to zero lattice spacing and infinite volume prior to extrapolating the quark masses to physical values [10].

In evaluating the integrals describing the coupling of pions to the ρ one must take into account the q^2 depen-

dence of the ρ to two π coupling constant $g_{\rho\pi\pi}$ reflecting the internal structure of these mesons. While this q^2 dependence was extracted from a string-breaking quark model in Ref. [5], we elect to take a more agnostic approach and consider different methods of cutting off the integral. In particular we investigate a sharp θ-function cutoff and a dipole cutoff. The underlying reason for our agnosticism is the belief that models of the sort underlying Ref. [5] are not likely to correctly describe the structure of pseudo Goldstone bosons such as the pion, as they do not incorporate chiral symmetry. While there are models of the $\rho\pi\pi$ vertex which incorporate chiral symmetry and chiral symmetry breaking [13–15], these models are based on particular dynamical assumptions. Accordingly, it is difficult to assess the reliability of such models. Instead, we consider a range of possibilities for the vertex and to simplify this task, we consider convenient phenomenological forms.

One other paper addressing this issue [16] sidesteps the problems surrounding the q^2 dependence of $g_{\rho\pi\pi}$ by fixing $g_{\rho\pi\pi}$ to a constant and making two subtractions of the divergent integral at $q^2 = 0$. These subtractions are absorbed into a mass and wave function renormalization. However, this approach excludes any analysis of ρ-meson mass extrapolations as the subtraction terms themselves have an unknown m_π dependence which has been lost in the renormalization procedure. Moreover, contributions from virtual two-pion states have been absorbed into the bare lattice parameters which is inconsistent with the dynamics contained in the quenched approximation.

The outline of this paper is as follows. In Sec. II the model used in examining the two-pion-induced self-energy is outlined. Two methods for regulating the divergent self-energy are explored. In Sec. III the relevance of the self-energy corrections to quenched QCD simulations is discussed. Section IV addresses the quark mass extrapolation of full QCD calculations and the importance of nonlinear behavior in the ρ-meson mass. Finally, the implications of this investigation are summarized in Sec. V.

II. THE SELF-ENERGY

In modeling the two-pion-induced self-energy of the ρ meson, $\Sigma_{\rho\pi\pi}$, the standard $\rho\pi\pi$ interaction motivated by low-energy current algebra is used. The effective Lagrange interaction has the form [17]

$$\mathcal{L}_{\text{int}} = -i g_{\rho\pi\pi} \rho^\mu \left(\pi \overset{\leftrightarrow}{\partial_\mu} \pi \right) + g_{\rho\pi\pi}^2 \pi^2 \rho^2 . \qquad (2)$$

The pions are further assumed to interact exclusively through the ρ channel as summarized in the Schwinger-Dyson equation for the ρ propagator:

$$G_{\mu\nu} = G_{\mu\nu}^0 + G_{\mu\sigma}^0 \Sigma^{\sigma\tau} G_{\tau\nu} , \qquad (3)$$

where

$$G_{\mu\nu}^0 = \frac{-i}{q^2 - M_0^2 + i\epsilon} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) , \qquad (4)$$

in Landau gauge, and M_0 is the bare ρ-meson mass. The self-energy $\Sigma_{\rho\pi\pi}$ is defined through the solution

$$G_{\mu\nu} = \frac{-i}{q^2 - M_0^2 - \Sigma_{\rho\pi\pi} + i\epsilon} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right), \qquad (5)$$

where

$$\Sigma^{\sigma\tau} \equiv \Sigma_{\rho\pi\pi} \left(g^{\sigma\tau} - \frac{q^\sigma q^\tau}{q^2} \right). \qquad (6)$$

$\Sigma^{\sigma\tau}$ is given by the standard one loop integrals

$$-i\Sigma^{\sigma\tau} = \int \frac{d^4k}{(2\pi)^4} g_{\rho\pi\pi}^2 \left\{ \frac{(q^\sigma - 2k^\sigma)(q^\tau - 2k^\tau)}{\left[(q-k)^2 - m_\pi^2 + i\epsilon\right]\left[k^2 - m_\pi^2 + i\epsilon\right]} - \frac{2g_{\mu\nu}}{k^2 - m_\pi^2 + i\epsilon} \right\}. \qquad (7)$$

Physically, the integral is convergent due to the momentum dependence of $g_{\rho\pi\pi}$. However, this momentum dependence is unknown. In light of this uncertainty, it is reasonable to parametrize the momentum dependence in terms of some given functional form with an adjustable parameter controlling how the function falls off as a function of momentum transfer. We consider two regulation prescriptions. In one, we assume a monopole form for $g_{\rho\pi\pi}$, and for comparison, we also consider a sharp θ-function cutoff.

The simplest fashion for introducing a covariant cutoff function is through the use of a dispersion relation. The second term of (7) is q independent and serves only to subtract the quadratic divergence of the first term in maintaining current conservation. As a result, we write a dispersion relation for $\Sigma(q^2)$ with one subtraction at $q^2 = 0$:

$$\Sigma(q^2) \equiv \frac{1}{\pi} \int_0^\infty ds \frac{q^2}{s} \frac{\mathrm{Im}\,\Sigma(s)}{s - q^2}. \qquad (8)$$

Of course, the imaginary part of $\Sigma_{\rho\pi\pi}$ may be easily determined using any number of techniques for rendering the integral of (7) finite. The imaginary part is

$$\mathrm{Im}\,\Sigma_{\rho\pi\pi}(q^2) = \frac{g_{\rho\pi\pi}^2}{48\pi} q^2 \left(1 - \frac{4m_\pi^2}{q^2} \right)^{3/2} \theta\left(q^2 - 4m_\pi^2\right). \qquad (9)$$

The value of $g_{\rho\pi\pi}$ at $q^2 = m_\rho^2$ is fixed by equating the imaginary parts of

$$M_0^2 + \Sigma_{\rho\pi\pi} \equiv \left(m_\rho + \frac{i\Gamma}{2} \right)^2, \qquad (10)$$

at $q^2 = m_\rho^2$. The physical values [18] $m_\rho = 768.1$ MeV and $\Gamma = 151.5$ MeV fix $g_{\rho\pi\pi}$ at ~ 6.0.

A. θ-function cutoff

To illustrate the physics associated with the real part of the self-energy we first consider the integral of (8) cut off covariantly by a sharp θ function at $s = \Lambda^2$. The functional form is

$$\mathrm{Re}\,\Sigma_{\rho\pi\pi} = \frac{g_{\rho\pi\pi}^2}{48\pi^2} q^2 \left\{ \ln\left(\frac{1 - \sigma_\Lambda}{1 + \sigma_\Lambda} \right) + \frac{8m_\pi^2}{q^2} \sigma_\Lambda \right.$$

$$\left. -\sigma_q^3 \ln\left(\frac{\sigma_\Lambda - \sigma_q}{\sigma_\Lambda + \sigma_q} \right) \right\}, \qquad (11)$$

where

$$\sigma_q = \left(1 - \frac{4m_\pi^2}{q^2} \right)^{1/2} \text{ and } \sigma_\Lambda = \left(1 - \frac{4m_\pi^2}{\Lambda^2} \right)^{1/2}. \qquad (12)$$

Figure 1 illustrates the real part of $\Sigma_{\rho\pi\pi}$ evaluated at $q^2 = m_\rho^2$ for a variety of cutoffs ranging from slightly above m_ρ^2 to 4 GeV^2. For small cutoffs, most of the strength in the integral lies below the ρ mass and consequently the ρ mass is pushed up due to the mixing with pion states. Of course, this behavior is completely consistent with that anticipated by simple quantum mechanical arguments. For larger cutoffs the strength above the ρ mass acts to reduce the ρ mass.
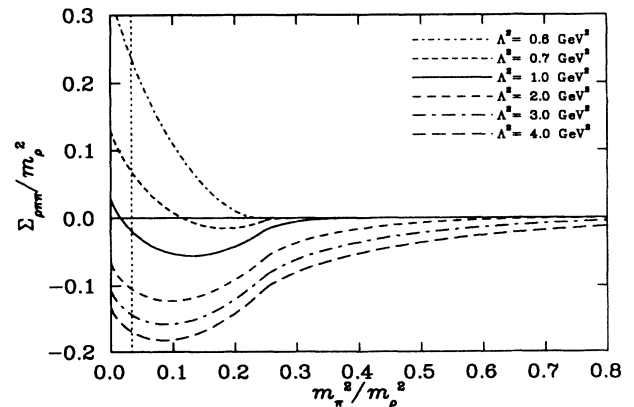

FIG. 1. The real part of $\Sigma_{\rho\pi\pi}$ from (11) evaluated at $q^2 = m_\rho^2$ for a variety of cutoffs, at $s = \Lambda^2$. In this and the following figures, the finely dashed vertical line marks the position of the physical point. For small cutoffs, most of the strength in the integral lies below the ρ mass and as a result the ρ mass is pushed up.

B. Dipole cutoff

While the θ function is useful as an illustrative tool, it suffers from being physically artificial and the results can be very sensitive to the value of the cutoff, as illustrated in Fig. 1. In an attempt to better represent the q^2 dependence of $g_{\rho\pi\pi}$ a monopole form for each vertex is introduced and the dispersion relation of (8) is evaluated with

$$g_{\rho\pi\pi}^2 \to g_{\rho\pi\pi}^2 \left(\frac{q^2 + \Lambda^2}{s + \Lambda^2} \right)^2 . \tag{13}$$

This form maintains the normalization of $g_{\rho\pi\pi}$ defined at the physical ρ mass and renders the integral of (8) finite.

$$\text{Re } \Sigma_{\rho\pi\pi} = \frac{g_{\rho\pi\pi}^2}{48\,\pi^2}\, q^2 \left\{ \left(1 + \frac{8\,m_\pi^2}{q^2} + \frac{12\,m_\pi^2}{\Lambda^2}\right) \left(1 + \frac{q^2}{\Lambda^2}\right) \right.$$
$$\left. + \left(1 + \frac{10\,m_\pi^2}{\Lambda^2} + \frac{6\,m_\pi^2 q^2}{\Lambda^4}\right) \beta_\Lambda \ln\left(\frac{\beta_\Lambda - 1}{\beta_\Lambda + 1}\right) - \sigma_q^3 \ln\left(\frac{1 - \sigma_q}{1 + \sigma_q}\right) \right\}, \tag{14}$$

where

$$\sigma_q = \left(1 - \frac{4\,m_\pi^2}{q^2}\right)^{1/2} \quad \text{and} \quad \beta_\Lambda = \left(1 + \frac{4\,m_\pi^2}{\Lambda^2}\right)^{1/2} . \tag{15}$$
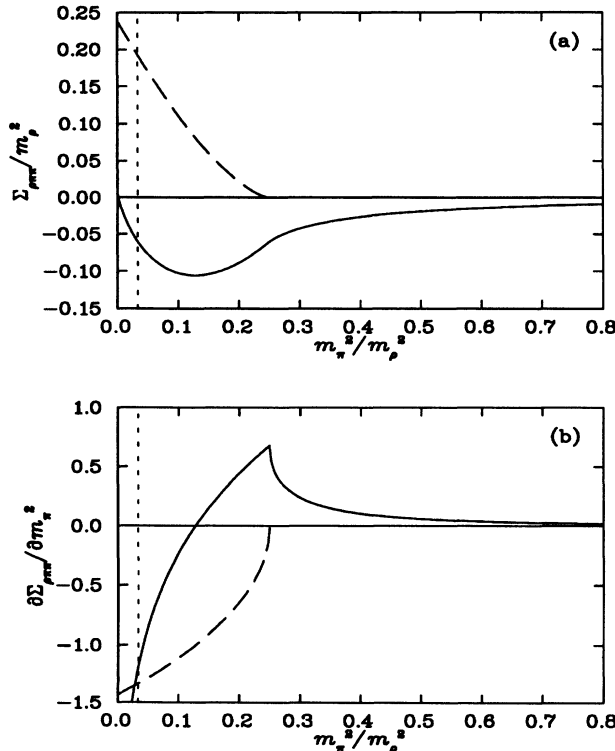
The imaginary part is recovered as in (9).

FIG. 2. The two-pion-induced self-energy (a) and its derivative with respect to m_π^2 (b) for a regulator mass of $\Lambda^2 = 2$ GeV^2. Both real (solid line) and imaginary (dashed line) parts are illustrated. The derivative clearly displays the nonanalytic behavior encountered at $m_\rho = 2m_\pi$.

Of course, this approach is not without a few unphysical side effects. The most obvious problem is the introduction of spurious poles in the spacelike region for the s dependence of $g_{\rho\pi\pi}$. However, the dispersion integral only samples the timelike region and the presence of these unphysical poles should not affect the results. One could consider other functional forms. However, the effective physical value for the regulator mass, Λ, is itself unknown. Our aim is to estimate the importance of the two-pion-induced self-energy relative to the ρ mass, as opposed to attempting to evaluate the actual correction. For this reason we view a consideration of the dipole regulator to be adequate.

Evaluation of the dispersion relation of (8) with (13) leads to the following functional form for the real part of $\Sigma_{\rho\pi\pi}$:

Figure 2 illustrates the real part of the self-energy and its derivative with respect to m_π^2 at $\Lambda^2 = 2$ GeV^2. The derivative clearly displays the nonanalytic behavior encountered at $m_\rho = 2m_\pi$. The second derivative is discontinuous at $m_\rho = 2m_\pi$ and is infinite from above. The imaginary part of the self-energy [19] is also illustrated in Fig. 2.

A comparison with Fig. 1 indicates that at $\Lambda^2 = 2$ GeV^2 the results are not too sensitive to the manner in which the integral is regulated. Figure 3 illustrates the real part of the self-energy for the same values of Λ^2 used in Fig. 1. The sensitivity of the results to the value of Λ is greatly reduced and all curves display the same qualitative behavior. In the limit of $\Lambda \to \infty$ both (11) and (14) reduce to

$$\text{Re } \Sigma_{\rho\pi\pi} = -\frac{g_{\rho\pi\pi}^2}{48\,\pi^2}\, q^2 \ln\left(\frac{\Lambda^2}{q^2}\right) . \tag{16}$$
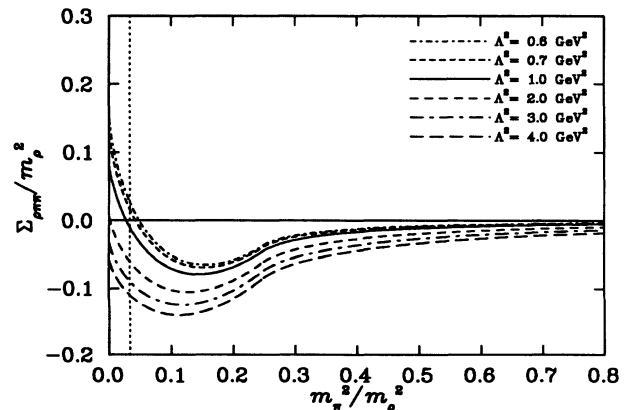

FIG. 3. The real part of the self-energy for dipole regulator masses Λ taking the same values used in Fig. 1. The sensitivity of the results to the value of Λ is greatly reduced.

III. APPLICATION TO QUENCHED QCD

The effects of quenching QCD may be categorized as perturbative or global effects and nonperturbative or channel specific effects. As discussed in the Introduction, the effects in the perturbative regime may be accounted for through a simple renormalization of the coupling constant. However, the effects in the nonperturbative regime will, of course, be channel specific. The ρ-meson channel might be particularly vulnerable to the nonperturbative effects of quenching QCD due to the fact that the ρ is unstable in full QCD and stable in the quenched approximation. As we shall see however, our estimated "unquenching" corrections turn out to be rather small.

Figure 4 displays ρ and squared pion masses for the three lightest quark masses used in the quenched QCD analysis of the GF11 group [3] for the $32 \times 30 \times 32 \times 40$ lattice at $\beta = 6.17$ for 219 configurations. The dashed line illustrates the linear relationship assumed in extrapolating the hadron masses to the critical point where the pion mass vanishes.

Provided the lattice spacing is defined by physical observables which do not have a similar dependence on pion decays, the physics associated with two-pion intermediate states of the ρ should simply be added onto the results extracted from calculations of quenched QCD. The correction is particular to the ρ meson and is unlikely to be accounted for if the lattice spacing is fixed by the nucleon mass for example.

The solid line of Fig. 4 illustrates the addition of the self-energy correction to the linear extrapolation where the regulator mass $\Lambda^2 = 1$ GeV^2 has been selected [20]. This choice of Λ is physically motivated and indicates the correction to the ρ mass at the physical point is neg-
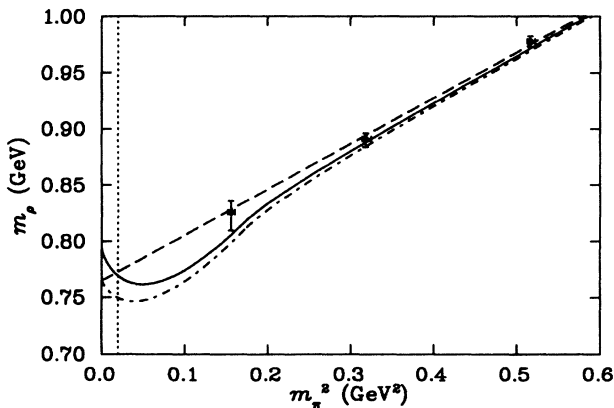

FIG. 4. ρ and squared pion masses for the three lightest quark masses used in the quenched QCD analysis of the GF11 group [3] on their largest lattice. The dashed line illustrates the linear relationship assumed in extrapolating the hadron masses to the critical point. The solid curve displays the self-energy correction for a regulator mass of $\Lambda^2 = 1$ GeV^2 where the correction to the ρ mass at the physical point is negligible. The dot-dash curve corresponds to $\Lambda^2 = 2$ GeV^2. A lattice scale parameter of 2.73 GeV has been applied to the otherwise dimensionless lattice results.

ligible. The dot-dashed curve illustrates the correction when $\Lambda^2 = 2$ GeV^2.

Figure 3 indicates that $\Sigma_{\rho\pi\pi}$ is less than 6% of the squared ρ mass for $\Lambda^2 < 2$ GeV^2. This corresponds to a less than 3% correction to the ρ mass itself. The conclusion that may be drawn from this analysis, which differs from previous considerations of these issues, is that the predictions of the GF11 group [3] are not "too good to be true." The channel specific nonperturbative correction to the ρ-meson mass may actually be rather small.

The magnitude of the corrections estimated here is much smaller than that anticipated in the analysis of Ref. [5] for any reasonable choice of Λ. Geiger and Isgur predict an unquenching correction to the ρ mass of approximately -160 MeV in contrast with our prediction of a 0 to 25 MeV reduction of the ρ mass.

IV. FULL QCD CALCULATIONS

In full QCD simulations, the two-pion-induced self-energy corrections are, of course, already included. However, the continuum predictions derived here will differ from those anticipated on the lattice [16] largely due to the discretization of the momenta on the lattice and the lattice regularization itself. This renders the divergent integral of (7) to a finite sum over a few two-pion states. In fact, if one hopes to recover the continuum physics, it will be necessary to first correct the hadron masses determined at unphysical quark masses to the continuum, infinite volume limit prior to extrapolating to physical quark masses.

Figure 4 suggests that it may be extremely difficult to see the effects of virtual two-pion intermediate states in full QCD. This is in accord with the recent findings of Ref. [21] where an attempt to observe the effects of virtual pion states in ρ correlation functions failed.

Ultimately, full QCD calculations will reach the point where linearly extrapolating the ρ mass to physical quark masses ignoring nonanalytic effects will introduce a relevant systematic error. To this end we present Fig. 5 which illustrates the relative amount which should be added onto the ρ mass extracted from a linear extrapolation of full QCD data corrected to the continuum, infinite volume limit [22]. We remind the reader that these effects are being calculated using the dipole form for the cutoff which we have more or less arbitrarily chosen. Thus the curves are illustrative only. The x-axis indicates the point at which the derivative is determined for the linear extrapolation. Values for Λ^2 are as in Figs. 1 and 3. For example, if the effective physical value for Λ^2 is 1 GeV^2 and the derivative is determined at $m_\pi^2 \simeq 0.25$ GeV^2, the ρ mass extracted from a linear extrapolation should be augmented by approximately 25 MeV prior to comparing with experiment.

The size of the corrections to linear extrapolations illustrated in Fig. 5 are generally smaller than the 70 MeV addition predicted by the Geiger and Isgur. However, it is possible to recover a correction to the ρ-meson mass extrapolation similar in magnitude to that of the Geiger-Isgur analysis, provided one linearly extrapolates from
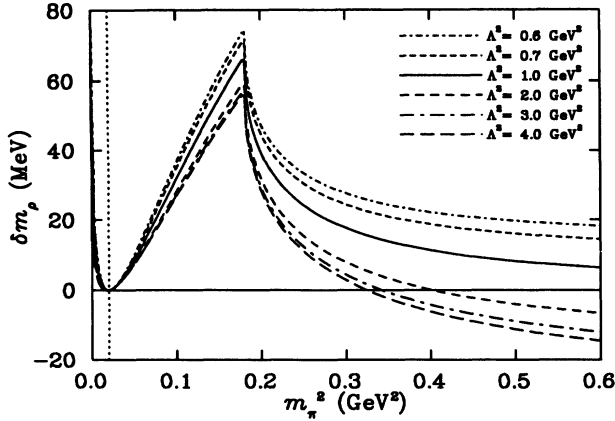
FIG. 5. The amount to be added to the ρ mass extracted from a linear extrapolation of full QCD data. The x axis indicates the point at which the derivative is determined for the linear extrapolation. Values for Λ^2 are as in Figs. 1 and 3.

the onset of the cut as in their investigation. Of course, it is not practical to extrapolate from the onset of the cut in actual lattice calculations. In fact, Fig. 5 indicates that this is the worst possible place to attempt an extrapolation to physical quark masses. The derivative is more likely to be averaged from a number of quark masses corresponding to squared pion masses of 0.2 to 0.6 GeV^2. Hence the corrections are expected to be the order of 10–20 MeV and possibly negligible if the effective physical value for Λ^2 is 2 GeV^2 or more.

An important point to mention here is that a simple calculation of the ρ-meson mass in full QCD will not circumvent the problems associated with its decay to pions. In fact, as one works hard to drive down the quark mass, linear extrapolations to the physical point will increasingly underestimate the ρ mass. Determination of the last few percent of the physical ρ-meson mass requires additional information describing the q^2 dependence of

$\rho\pi\pi$ interactions such that a suitable extrapolation function may be identified. Fortunately, these corrections are small and may be neglected until systematic uncertainties associated with the finite volume and finite spacings of the lattice are understood and eliminated.

V. SUMMARY

We have calculated the two-pion-induced self-energy correction to the ρ-meson mass in a manner that allows an estimation of the correction to quenched QCD calculations and an analysis of extrapolations of full QCD results.

The analysis of full QCD extrapolations indicates linear extrapolations of typical ρ-meson masses extracted from lattice correlation functions will underestimate the physical ρ mass by 10–20 MeV. An important point to draw from the analysis is that as the quark masses become lighter, linear extrapolations to the physical point will increasingly underestimate the ρ-meson mass.

We estimate the corrections to quenched calculations to be the order of a few percent and quite possibly negligible (0 to -25 MeV). These results lend credence to the success of quenched QCD in describing the physical low-lying hadron mass spectrum.

ACKNOWLEDGMENTS

[1] A. El-Khadra, G. Hockney, A. Kronfeld, and P. Mackenzie, Phys. Rev. Lett. 69, 729 (1992).

[2] Ultimately, these uncertainties will be eliminated by directly calculating the observables in full QCD calculations. However, explorations of the parameter space for full QCD are in the early stages. A thorough understanding and elimination of lattice systematics in full QCD remains a focus of current investigations.

[3] F. Butler et al., Phys. Rev. Lett. 70, 2849 (1993).

[4] S. R. Sharpe, in The Fermilab Meeting, Proceedings of the Annual Meeting of the Division of Particles and Fields of the APS, Batavia, Illinois, 1992, edited by C. Albright et al. (World Scientific, Singapore, 1993), p. 247.

[5] P. Geiger and N. Isgur, Phys. Rev. D 41, 1595 (1990).

[6] T. D. Cohen and D. B. Leinweber, Comments Nucl. Part. Phys. 21, 137 (1993).

[7] C. Bernard and M. Golterman, Phys. Rev. D 46, 853 (1992).

[8] S. R. Sharpe, Phys. Rev. D 46, 3146 (1992).

[9] Only the first two terms of the geometric series of pseudoscalar-meson and glueball states (which can generate the η' mass when geometrically summed) survive in the quenched approximation.

[10] In principle it is possible to calculate resonance properties on the lattice [11,12] by exploiting the relation between the infinite volume scattering matrix and the two-particle energy spectrum in a periodic box. However, implementation of this approach is currently impracticable for the ρ meson and is likely to remain so for some time.

[11] M. Lüscher, Nucl. Phys. B354, 531 (1991).

[12] M. Lüscher, Nucl. Phys. B364, 237 (1991).

[13] J. Praschifka, C. Roberts, and R. Cahill, Int. J. Mod. Phys. A 2, 1797 (1987).

[14] C. Roberts, R. Cahill, and J. Praschifka, Int. J. Mod. Phys. A 4, 719 (1989).

[15] L. Hollenburg, C. Roberts, and B. McKellar, Phys. Rev. C 46, 2057 (1992).

[16] T. A. DeGrand, Phys. Rev. D 43, 2296 (1991).

[17] M. Herrmann, B. Friman, and W. Norenberg, Z. Phys. A **343**, 119 (1992); GSI Report No. GSI-93-12 [Nucl. Phys. A (to be published)].

[18] Particle Data Group, K. Hikasa *et al.*, Phys. Rev. D **45**, S1 (1990).

[19] In Euclidean space where lattice calculations are performed, the existence of a Minkowski-space imaginary part of the ρ-meson self-energy is reflected in the spectral density $\rho(s)$, in which the δ function strength at the stable ρ mass is spread out for a resonance. The time evolution of a lattice two-point correlation function is altered

according to $G_2(t) = \int_{2m_\pi}^{\infty} \rho(s) \exp(-st) \, ds$.

[20] Ideally, the lattice results should be corrected to the infinite volume, continuum limit prior to adding the self-energy correction.

[21] C. Bernard *et al.*, Phys. Rev. D **48**, 4419 (1993).

[22] In preparing Fig. 5 we have accounted for the quark mass (or m_π) dependence of the ρ-meson mass. For illustrative purposes we set $m_\rho = m_0 + 0.5 \, (\text{GeV}^{-1}) \, m_\pi^2$ and fixed m_0 to reproduce the physical ρ mass at the physical pion mass.