Constituent quark model analysis of weak mesonic decays of charm baryons

Taruni Uppal and R. C. Verma
Department of Physics, Panjab University, Chandigarh, India 160014

M. P. Khanna*
International Centre for Theoretical Physics, Trieste, Italy 1-34100
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In view of recent experimental trends we investigate the weak nonleptonic decays of charm baryons within the framework of the constituent quark model. Branching ratios and asymmetry parameters for all $\frac{1}{2}^+ \to \frac{1}{2}^+ + 0^-$ charm-changing modes are calculated with appropriate QCD corrections. The effect of flavor dependence on the scale is found to be quite significant.

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I. INTRODUCTION

The discovery of the charm particles began a new era in particle physics. Ever since, charm hadrons have been under an active probe, but data constraints directed most of the theoretical efforts to the understanding of the weak decays of charm mesons. The advent of the B factories and a change in the experimental trend has now brought charmed baryons under active investigation [1–3], with results encouraging enough to warrant a detailed theoretical analysis. Moreover, the large event samples of B-meson decays will allow an accurate and extensive study of all charm baryonic decays in the near future. The detailed knowledge of these decay properties is essential as, in addition to explaining the charm sector, it will form the core to the quality of information that can be extracted from $b \to c$ physics.

Charmed hadrons can decay into numerous channels, yet the data on the exclusive modes are very limited [1]. There have been many recent theoretical attempts to study charm baryons in the weak mesonic modes [4–9]. To study the two-body decays exclusively, it would be ideal to have a reliable and direct theoretical evaluation of the three-body matrix elements. However, a direct calculation of $\langle BM|H_W|B_c\rangle$ at the quark level involves some uncertainties. Strong interaction interference effects between different processes, which are prevalent among these modes, cast a shadow on the exact contribution of each process. Final-state interactions (FSI's) among the hadrons may further complicate the situation as in the charm meson decays [10]. The $B \to B'P$ weak modes have traditionally been studied through the standard current algebra approach using the soft pion theorems [11]. It has been shown for quite some time that though this approach successfully reproduces separately the s- and p-wave amplitudes of the hyperons, and their relative sign, it fails to predict their relative magnitudes [12]. To have better agreement between theory and experiment, the importance of including the factorizable contributions, which vanish in the soft pion limit, has been recognized [13].

Generally the spectator diagram is considered to be the significant decay mechanism for the charm meson decays. However, for baryons other contributing processes such as the W- exchange mechanism are also possible [14–16]. Unlike the meson weak decays, this W-exchange mechanism in baryons is neither helicity nor color suppressed since there may exist a spin-0 two-quark system inside the baryon. In fact, the contribution from this process has been found to be proportional to $|\psi(0)|^2$, thereby making it more significant for the heavy baryon weak decays [15]. Experimentally also the lifetime differences among D^0, D^+, Λ_c^+, Ξ_c^+, and Ξ_c^0 are indicative of the presence of W-exchange mechanism. The signal [17] for $\Lambda_c^+ \to \Delta^{++}K^-$, and recent measurements by CLEO [1] on exclusive modes such as $\Xi_c^0 \to \Omega^- K^+$ and $\Lambda_c^+ \to \Xi^0 K^+$, which can occur most likely via a W-exchange diagram, lend credence to this interpretation.

The $B_c \to B + \pi/K$ decay has been recently analyzed by several authors in the framework of the conventional soft-meson technique with inclusion of factorizable terms [6–9]. In the quark model language, the factorization contributions are the same as the spectator processes, while the baryon pole terms and the equal-time commutator (ETC) term involve the W-exchange diagrams. Theoretical prejudices indicate domination of factorization contributions in the charm sector [10]. This may be responsible for the general theoretical predictions of the branching ratios being larger than the experimental results, in particular, $\Lambda_c \to \Lambda\pi^+$. However, modes such as $\Lambda_c^+ \to \Sigma^0\pi^+$ can occur only through a baryon pole, and experimental observation of

$$B(\Lambda_c^+ \to \Sigma^0\pi^+) \approx B(\Lambda_c^+ \to \Lambda\pi^+)$$

indicates the importance of the nonspectator processes in the charm baryonic decays. Potentially important modifications can arise for these decays from the baryon

*Permanent address: Department of Physics, Panjab University, Chandigarh, India 160014.

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pole contribution, if the effect of SU(4) breaking is included in the scale $|\psi(0)|^2$ [16] and the strong-coupling constants [18].

In this paper, we analyze the weak nonleptonic decays $(\frac{1}{2}^+ \to \frac{1}{2}^+ + 0^-)$ of the $C=1$ charm baryons in the current algebra framework with inclusion of factorization terms. Section II describes the relevant weak Hamiltonian and various contributing processes. We employ the constituent quark model to evaluate the weak Hamiltonian matrix elements and the form factors appearing in the decay amplitudes. The effects of short-distance quantum chromodynamical (QCD) modifications are present in the Hamiltonian. We also calculate the SU(4) broken strong-coupling constants. Section III deals with the discussion of the results obtained and the inconsistency between theory and experiment. In Sec. IV we study the effect of flavor dependence on the scale of the weak matrix element, and hence on the branching ratios and asymmetry parameters for these decays. In this study we find that both factorization and pole and/or ETC terms are equally important, though one may dominate over the other, depending on the decay channel. In addition to the Cabibbo-enhanced modes, we have studied the charm-changing weak nonleptonic decays of $C=1$ baryons for the Cabibbo-suppressed and doubly suppressed modes. We end with the summary and conclusions in Sec. V.

II. GENERAL FRAMEWORK

A. Weak Hamiltonian

The relevant effective weak Hamiltonian including QCD short-distance effects for the charm-changing decays is as follows.

Cabibbo enhanced: $\Delta C = \Delta S = -1$,

$$\frac{G_F \cos^2\theta_C}{\sqrt{2}}[c_1(\bar{s}c)(\bar{u}d)+c_2(\bar{u}c)(\bar{s}d)] ;$$

Cabibbo suppressed: $\Delta C = -1$, $\Delta S = 0$,

$$\frac{G_F \cos\theta_C \sin\theta_C}{\sqrt{2}}\{c_1[(\bar{s}c)(\bar{u}s)-(\bar{d}c)(\bar{u}d)] + c_2[(\bar{u}c)(\bar{s}s)-(\bar{u}c)(\bar{d}d)]\} ;$$

Cabibbo doubly suppressed: $\Delta C = -\Delta S = -1$,

$$\frac{-G_F \sin^2\theta_C}{\sqrt{2}}[c_1(\bar{d}c)(\bar{u}s)+c_2(\bar{u}c)(\bar{d}s)] , \tag{1}$$

where the abbreviation $\bar{q}q = \bar{q}\gamma_\mu(1-\gamma_5)q$. $c_1 = \frac{1}{2}(c_+ + c_-)$ and $c_2 = \frac{1}{2}(c_+ - c_-)$ represent combinations of the QCD coefficients c_- and c_+. In the leading log approximation these are given by

$$c_\pm(\mu) = \left[\frac{\alpha_s(\mu^2)}{\alpha_s(m_W^2)}\right]^{d_\pm/2b} \tag{2}$$

with $d_- = -2d_+ = 8$ and $b = 11 - \frac{2}{3}N_f$, N_f being the number of flavors, μ the mass scale and α_s is the strong fine-structure constant. The precise value of these QCD coefficients is difficult to assign, depending as they do on the mass scale and $\Lambda_{\rm QCD}$. c_1 and c_2 are fixed from $D \to K\pi$ data [19] to be $c_1 = 1.2$, $c_2 = -0.5$.

The amplitude for the process $B \to B'P$ is defined by

$$\langle B_f, P_k | H_W^{\rm eff} | B_i \rangle = \iota \bar{u}_{B_f}(A - B\gamma_5)u_{B_i} . \tag{3}$$

B_i and B_f are ground state $\frac{1}{2}^+$ baryons, A and B are the respective s-wave and p-wave amplitudes. The main quantities of experimental interest are the decay rate

$$\Gamma(B_i \to B_f + P_k) = \frac{p_c}{8\pi m_i^2}\{[(m_i+m_f^2)-m_k^2]|A|^2 + [(m_i-m_f^2)-m_k^2]|B|^2\} \tag{4}$$

and the asymmetry parameter

$$\alpha = \frac{2x\,{\rm Re}(A^*B)}{|A|^2+x^2|B|^2} , \tag{5}$$

where $\mathbf{p}_c$ is the center-of-mass three-momentum in the rest frame of B_i, and $x = p_c/(E_f+m_f)$.

B. Weak decay amplitudes

Among the $J^P = \frac{1}{2}^+$ charmed baryons comprising the **20′** multiplet of SU(4), only the members of the SU(3) submultiplets **3***, **3**, and Ω_c^0 of **6** decay weakly. The remaining decay strongly or radiatively to **3***. We have studied the weak nonleptonic decays of $C=1$ charm baryons of **3*** and Ω_c^0. To evaluate the decay rate and asymmetry parameters all we require is the estimation of the total parity-violating (PV) and parity-conserving (PC) amplitudes A and B, respectively.

Using standard current algebra techniques, the evaluation of the $B \to B'P$ involves relating the three-hadron amplitude $\langle BP|H_W|B_c\rangle$ to the baryon-baryon transition matrix element $\langle B|H_W|B_c\rangle$ through the PCAC (partial conservation of axial-vector current) hypothesis [11]. Adding the factorization term gives the general form

$$\langle B_f P_k | H_W | B_i \rangle = \frac{1}{f_P}\langle B_f | [Q_k^5, H_W] | B_i \rangle + M_{\rm pole} + M_{\rm fac} . \tag{6}$$

We discuss the contribution of each of these terms in the context of PC and PV amplitudes. The first term corresponds to the equal-time current commutator, which is essentially the matrix element of H_W between the two $\frac{1}{2}^+$ baryon states:

$$\langle B_f | H_W^{\rm eff} | B_i \rangle = \bar{u}_f(a_{if} + b_{if}\gamma_5)u_i . \tag{7}$$

It is well known that the PV matrix element $b_{\rm if}$ vanishes for the hyperons since $\langle B_f | H_W^{\rm PV} | B_i \rangle = 0$ in the SU(3) limit. In the case of the charm decays, in analogy with hyperons, it has been shown that $b_{if} \ll a_{if}$, and in the nonrelativistic limit only the PC term survives. Thus the ETC term contributes only to the s-wave amplitude:

$$A^{\rm ETC} = \frac{1}{f_k}\langle B_f | [Q_k^5, H_W^{\rm PV}] | B_i \rangle = \frac{1}{f_k}\langle B_k | [Q_k, H_W^{\rm PC}] | B_i \rangle . \tag{8}$$

Q_k and Q_k^5 are the vector and axial-vector charges, respectively. The p waves are then described by the $\frac{1}{2}^+$ pole contribution. The baryon-pole terms arising from s and u channels contribute only to the PC amplitude, and are given by

$$B^{\text{pole}}=\frac{g_{lfk}a_{il}}{m_i-m_l}\frac{m_i+m_f}{m_l+m_f}+\frac{g_{il'k}}{m_f-m_l'}\frac{m_i+m_f}{m_i+m_l'}\ , \tag{9}$$

where g_{ijk} are the strong baryon-meson coupling constants, and l, l' are the intermediate baryon states corresponding to the respective channels. The term M_{pole} is actually a modified pole term and contains the contribution from the surface term, the soft-meson Born-term contraction and the baryon-pole term [11], combined in a well-defined way.

The third term is the factorization term obtained by inserting vacuum intermediate states, which reduces it to a product of two current matrix elements that vanish in the soft-meson limit. Factorization may be viewed as a correction to the current algebra term, as it is directly proportional to the meson four-momenta. In this approach the quark currents of the weak Hamiltonian are considered as interpolating hadron fields, directly generating a $q\bar{q}$ state. The separable combination of $B_i \to B_f P_k$ is given by

$$\langle P_k | A_\mu | 0 \rangle \langle B_f | V^\mu - A^\mu | B_i \rangle\ , \tag{10}$$

where

$$\langle P_k | A_\mu | 0 \rangle = \iota f_k q_\mu\ , \tag{11}$$

where $q_\mu=(p_i-p_f)_\mu$, and f_k is the decay constant of the emitted pseudoscalar meson P. The baryon-baryon transition matrix element is defined in terms of the invariant vector and axial-vector form factors:

$$\begin{aligned}\langle B_f(p_f)| V_\mu | B_i(p_i)\rangle &= \bar{u}_f(p_f)(f_1\gamma_\mu - f_2\iota\sigma_{\mu\nu}q^\nu + f_3 q_\mu)u_i(p_i)\ ,\\ \langle B_f(p_f)| A_\mu | B_i(p_i)\rangle &= \bar{u}_f(p_f)(g_1\gamma_\mu\gamma_5 - g_2\iota\sigma_{\mu\nu}q^\nu\gamma_5 + g_3 q_\mu\gamma_5)u_i(p_i)\ .\end{aligned} \tag{12}$$

f_i and g_i denote the vector and axial-vector form factors and are functions of q^2. Up to first order of parametrization, the factorization PV and PC amplitudes are

$$\begin{aligned}A^{\text{fac}} &= -\frac{G_F}{\sqrt{2}}F_C f_k c_k (m_i - m_f) f_1^{B_i,B_f}(m_k^2)\ ,\\ B^{\text{fac}} &= \frac{G_F}{\sqrt{2}}F_C f_k c_k (m_i + m_f) g_1^{B_i,B_f}(m_k^2)\ ,\end{aligned} \tag{13}$$

where F_C denotes Cabibbo factors and c_k is the QCD coefficient equal to $c_1(c_2)$ depending on the emitted meson. The total amplitude is given by

$$\begin{aligned}A &= A^{\text{ETC}} + A^{\text{fac}}\ ,\\ B &= B^{\text{pole}} + B^{\text{fac}}\ .\end{aligned} \tag{14}$$

We use the constituent quark model to evaluate the weak matrix element a_{ij} for the baryon-baryon transition and the form factors $f_i(q^2)$ and $g_i(q^2)$ appearing in the weak decay amplitudes. The SU(4) broken strong-coupling constants g_{ik} are evaluated using a symmetry-breaking ansatz and the Coleman-Glashow null result [18]. We briefly review the calculation of each of these terms.

1. Baryon matrix element of $\langle B_f | H_W^{\text{eff}} | B_i \rangle$

In recent studies the weak amplitudes have been calculated by many authors [5,6] using the symmetry arguments and a simple $c \to s$ substitution; e.g., $a_{\Lambda_c^+\Sigma^+}$ can be obtained from $a_{\Sigma^+ p}$ through the relation

$$\langle \Sigma^+ | H_W^{\text{PC}} | \Lambda_c^+ \rangle = \frac{1}{\sqrt{6}}\cot\theta_C \langle p | H_W^{\text{PC}} | \Sigma^+ \rangle\ . \tag{15}$$

However, the flavor symmetry SU(4) is badly broken and this relation ignores the difference in QCD enhancements at different mass scales. A direct method of estimating a_{ij} is to use the constituent quark model which describes both s and p waves adequately. Following the analysis of Riaazuddin and Fayyazuddin [15] in the hyperon sector, we reduce the charm-changing Hamiltonian for the Cabibbo enhanced mode in the nonrelativistic limit to give

$$H_W^{\text{PC}} = c_-(m_c)(s^+cu^+d - s^+\sigma c\cdot u^+\sigma d)\delta^3(\mathbf{r})\ . \tag{16}$$

The presence of c_- in the overall scale may be understood by noticing that the portion of the Hamiltonian corresponding to c_+ is symmetric in color indices and hence does not contribute. The enhancement due to hard gluon exchanges in the charm sector $c_-(m_c)=1.7$ is lower than that in the hyperon sector with $c_-(m_s)=2.2$, and will affect the naive relation (15). Effects of long-distance QCD reflected in the bound-state wave functions have generally been ignored. We will discuss these in a later section.

2. Form factors

The conventional method of evaluating the form factors using SU(3) symmetry or at $\mathbf{q}=0$ is found to give very large values [5], e.g., $f_1^{\Lambda_c^+\Lambda}(0)\approx 0.95$ and $g_1^{\Lambda_c^+\Lambda}(0)\approx 0.86$. If these are first evaluated at maximum q^2 and then extrapolated to $q^2=0$ assuming a dipole q^2 dependence, they are reduced by nearly half. This has been done by Pérez-Marcial *et al.* [20] using a Briet frame where the emitted baryons have momenta $p_i=-p_f=q/2$; $q_0=E_1-E_2$. Moreover, at maximum momentum transfer $q^2=(m_i-m_f)^2$ the quark model wave functions are found to simulate the compound hadron states in the best possible manner. Following the method of Pérez-Marcial *et al. we evaluate* all required form factors in the constituent quark model. Unlike the bag model estimate, the constituent quark model is free from numerous parameters. Form factors at $q^2=\Delta M^2$ are given by

$$f_1=\left[1-\frac{\Delta M^2}{4M_1M_2}\right]\alpha_1-\frac{\Delta M^2}{4M_1M_2}\alpha_2\ ,$$
$$g_1=\left[1-\frac{\Delta M^2}{16M_{12}}\right]\alpha_2\ , \tag{17}$$

where $\Delta M=M_1-M_2$, $M_{12}=M_1+M_2$, and α_1 and α_2 are the combinations of the Clebsch-Gordan coefficients. We go from $q^2=\Delta M^2$ to $q^2=0$ using the dipole q^2 dependence:

$$f_i(q^2)=\frac{f_i(0)}{(1-q^2/m_{V_i}^2)^2}\ ,\quad g_i(q^2)=\frac{g_i(0)}{(1-q^2/m_{A_i}^2)^2}\ . \tag{18}$$

3. *Strong-coupling constant*

In many attempts [5,6,8], SU(4) symmetric coupling constants have been used. Since flavor symmetries are badly broken, it is quite erroneous to use SU(4) symmetric values in the charm baryons. Extending an earlier analysis [18] we estimate the broken strong couplings employing the Coleman-Glashow null result [18] from tadpole-type symmetry breaking. The baryon-meson strong couplings are then given by

$$g_{BB'P}=\frac{M_B+M_{B'}}{2M_N}\frac{1}{\alpha_P}g_{BB'P}^{\mathrm{sym}} \tag{19}$$

where $g_{BB'P}^{\mathrm{sym}}$ is the SU(4) predicted value. $\alpha_P=1$ for π,K and $\alpha_P=\delta M_c/\delta M_s$ for D mesons.

III. DISCUSSION OF RESULTS

Having determined all the ingredients of the weak decay amplitudes, we can now compute the decay amplitudes using Eq. (14). Experimentally measured [1] masses of the charm baryons have been used. The branching ratios and asymmetry parameters are given in columns 2 and 3 of Tables I, II, and III corresponding to the Cabibbo-enhanced, suppressed, and doubly suppressed decay modes.

A. Cabibbo-enhanced modes

The weak charm baryonic nonleptonic decays which proceed through a $(cd)\to(su)$ transition are enhanced by the Cabibbo factor $\cos^2\theta_C$. In this mode the decays involving the emission of a π^+ or a $\bar{K}^0$ get contributions from the spectator diagrams and are seen to be significantly larger than the other modes arising through the pole and ETC contributions. The branching ratio for the π^+ modes is greater than the $\bar{K}^0$ modes due to color enhancement. However, in a typical π^+ emitting decay, $\Lambda_c^+\to\Sigma^0\pi^+$, the factorization term vanishes due to the Clebsch-Gordan coefficient, and hence it proceeds only through nonspectator contributions. Among the decays arising through purely pole and ETC diagrams, the branching ratios decrease in the order of $\pi^0>\eta>\eta'>K^-$ emitting modes. The pole contributions are obviously significant in channels where the factorization term cannot appear, and an accurate experimental observation of these decays can clearly determine the relative strength of the pole diagrams in charm baryon decays.

Experimentally [1], only three branching ratios and one asymmetry have been measured:

TABLE I. Branching ratios and asymmetry parameters for Cabibbo-enhanced modes.

Process	Without $\lvert\Psi(0)\rvert^2$ scale variation Branching ratio (%)	Asymmetry	With $\lvert\Psi(0)\rvert^2$ scale variation Branching ratio (%)	Asymmetry
$\Delta C=\Delta S=-1$				
$\Lambda_c^+\to p+\bar{K}^0$	1.25	−0.99	2.34	−0.99
$\Lambda_c^+\to\Lambda+\pi^+$	2.15	−0.87	2.33	−0.85
$\Lambda_c^+\to\Sigma^0+\pi^+$	0.55	−0.32	2.43	−0.32
$\Lambda_c^+\to\Sigma^++\pi^0$	0.55	−0.32	2.43	−0.32
$\Lambda_c^+\to\Xi^0+K^+$	0.05	0.00	0.23	0.00
$\Lambda_c^+\to\Sigma^++\eta$	0.22	−0.94	0.99	−0.99
$\Lambda_c^+\to\Sigma^++\eta'$	0.05	0.68	0.20	0.68
$\Xi_c'^+\to\Sigma^++\bar{K}^0$	0.45	−0.43	0.38	−0.25
$\Xi_c'^+\to\Xi^0+\pi^+$	4.67	−0.73	1.55	0.25
$\Xi_c'^0\to\Lambda+\bar{K}^0$	0.27	−0.92	0.77	−0.80
$\Xi_c'^0\to\Sigma^++K^-$	0.04	0.00	0.19	0.00
$\Xi_c'^0\to\Sigma^0+\bar{K}^0$	0.10	−0.24	0.11	0.87
$\Xi_c'^0\to\Xi^-+\pi^+$	2.30	−0.99	3.49	−0.99
$\Xi_c'^0\to\Xi^0+\pi^0$	0.31	−0.80	1.38	−0.80
$\Xi_c'^0\to\Xi^0+\eta$	0.07	0.13	0.32	0.13
$\Xi_c'^0\to\Xi^0+\eta'$	0.03	0.75	0.11	0.75
$\Omega_c^0\to\Xi^-+\bar{K}^0$	1.27	0.36	12.40	0.50

$$B(\Lambda_c^+ \to p\bar{K}^0) = (1.6 \pm 0.4)\% ,$$
$$B(\Lambda_c^+ \to \Lambda\pi^+) = (0.58 \pm 0.16)\% ,$$
$$\alpha(\Lambda_c^+ \to \Lambda\pi^+) = (-0.96 \pm 0.42) ,$$
$$B(\Lambda_c^+ \to \Sigma^0\pi^+) = (0.55 \pm 0.26)\% .$$

(1) The agreement of branching ratio predictions for the decay $B(\Lambda_c^+ \to p\bar{K}^0) = 1.25\%$ is good within experimental errors, but lies on the lower side. We find its asymmetry $\alpha(\Lambda_c^+ \to p\bar{K}^0) = -0.99$.

(2) Theoretically determined $\alpha(\Lambda_c^+ \to \Lambda\pi^+) = -0.87$ is in good agreement with the experimental measurement [3]

$$\alpha(\Lambda_c^+ \to \Lambda\pi^+) = (-0.96 \pm 0.42) .$$

However, the branching ratio $B(\Lambda_c^+ \to \Lambda\pi^+) = 2.15\%$ is found to be greater than the expected value. It is noticeable that in most of the theoretical efforts this branching ratio has been predicted on the higher side, sometimes by an order of magnitude or even more. This may indicate the need of new physics. Final-state interactions, which are well known to substantially alter the decay predictions in charm mesons as well as hyperons, may provide a clue here. On the other hand, the experimental information itself is not clean. The predictions of the branching ratios for all the three measured modes is made with respect to the branching ratio

$$B(\Lambda_c^+ \to pK^-\pi^+) = (3.2 \pm 0.8)\% .$$

This value, quoted by the Particle Data Group [1], is in fact an average of the three measurements $(4.3 \pm 1.0)\%$, $(4.1 \pm 2.4)\%$, and $(2.2 \pm 0.8)\%$, of which the lowest one was measured way back in 1980. The other two recent values are consistent mutually as well as with the latest measurement [3]

TABLE II. Branching ratios and asymmetry parameters for Cabibbo singly suppressed modes.

	Without $\lvert\Psi(0)\rvert^2$ scale variation		With $\lvert\Psi(0)\rvert^2$ scale variation	
Process	Branching ratio (%)	Asymmetry	Branching ratio (%)	Asymmetry
$\Delta C = -1, \Delta S = 0$				
$\Lambda_c^+ \to p + \pi^0$	0.01	0.82	0.02	0.85
$\Lambda_c^+ \to n + \pi^+$	0.08	−0.13	0.09	0.67
$\Lambda_c^+ \to \Lambda + K^+$	0.12	−0.99	0.09	−0.99
$\Lambda_c^+ \to \Sigma^+ + K^0$	0.04	−0.80	0.08	−0.80
$\Lambda_c^+ \to \Sigma^0 + K^+$	0.02	−0.80	0.08	−0.80
$\Lambda_c^+ \to p + \eta$	0.03	−1.00	0.03	−0.79
$\Lambda_c^+ \to p + \eta'$	0.004	0.87	0.02	0.87
$\Xi_c'^+ \to p + \bar{K}^0$	0.14	−0.93	0.61	−0.94
$\Xi_c'^+ \to \Lambda + \pi^+$	0.02	0.26	0.22	−0.79
$\Xi_c'^+ \to \Xi^0 + K^+$	0.61	−0.86	0.45	−0.01
$\Xi_c'^+ \to \Sigma^+ + \pi^0$	0.07	−0.81	0.17	−0.35
$\Xi_c'^+ \to \Sigma^0 + \pi^+$	0.27	−1.00	0.39	−0.86
$\Xi_c'^+ \to \Sigma^+ + \eta$	0.06	−0.74	0.14	−0.08
$\Xi_c'^+ \to \Sigma^+ + \eta'$	0.01	0.83	0.05	0.83
$\Xi_c'^0 \to p + K^-$	0.002	0.00	0.01	0.00
$\Xi_c'^0 \to n + \bar{K}^0$	0.01	−0.50	0.06	−0.50
$\Xi_c'^0 \to \Lambda + \pi^0$	0.004	−0.84	0.03	−0.94
$\Xi_c'^0 \to \Sigma^+ + \pi^-$	0.00	0.00	0.01	0.00
$\Xi_c'^0 \to \Sigma^0 + \pi^0$	0.01	−0.99	0.02	−1.00
$\Xi_c'^0 \to \Sigma^- + \pi^+$	0.10	−0.99	0.15	−0.99
$\Xi_c'^0 \to \Xi^- + K^+$	0.15	−0.99	0.20	−0.99
$\Xi_c'^0 \to \Xi^0 + K^0$	0.01	−0.48	0.06	−0.48
$\Xi_c'^0 \to \Lambda + \eta$	0.01	−0.99	0.02	−1.00
$\Xi_c'^0 \to \Lambda + \eta'$	0.003	0.86	0.01	0.86
$\Xi_c'^0 \to \Sigma^0 + \eta$	0.01	−0.74	0.002	−0.16
$\Xi_c'^0 \to \Sigma^0 + \eta'$	0.001	0.83	0.004	0.83
$\Omega_c^0 \to \Lambda + \bar{K}^0$	0.59	0.32	2.62	0.32
$\Omega_c^0 \to \Sigma^+ K^-$	0.10	0.00	0.46	0.00
$\Omega_c^0 \to \Sigma^+ \bar{K}^0$	0.05	0.00	0.23	0.00
$\Omega_c^0 \to \Xi^- + \pi^+$	1.02	0.03	2.90	−0.20
$\Omega_c^0 \to \Xi^0 + \pi^0$	0.31	−0.23	1.12	−0.36
$\Omega_c^0 \to \Xi^0 + \eta$	0.02	−0.36	0.13	−0.96
$\Omega_c^0 \to \Xi^0 + \eta'$	0.09	−0.38	0.39	−0.38

$$B(\Lambda_c^+ \to pK^-\pi^+) = (4.0 \pm 0.8 \pm 0.3)\% .$$

Therefore, it seems more appropriate to take their average value

$$B(\Lambda_c^+ \to pK^-\pi^+) = (4.1 \pm 0.9)\% ,$$

which in turn yields

$$B(\Lambda_c^+ \to p\bar{K}^0) = (2.01 \pm 0.38)\% ,$$

$$B(\Lambda_c^+ \to \Lambda\pi^+) = (0.75 \pm 0.41)\% , \quad (20)$$

$$B(\Lambda_c^+ \to \Sigma^0\pi^+) = (0.70 \pm 0.58)\% ,$$

bringing theoretical predictions closer to experiment. Some authors [8,9] have included $\frac{1}{2}^-$ poles to lower the branching ratio of $B(\Lambda_c^+ \to \Lambda\pi^+)$, but these adversely affect the $B(\Lambda_c^+ \to p\bar{K}^0)$. Relevance of $\frac{1}{2}^-$ poles may come from data on $\Lambda_c^+ \to \Xi^0 K^+$ which occurs through the pole contributions only.

(3) The decay $B(\Lambda_c^+ \to \Sigma^0\pi^+) = 0.55\%$ is found consistent within experimental errors, though on the lower side. Its asymmetry is found to be $\alpha(\Lambda_c^+ \to \Sigma^0\pi^+) = -0.32$.

For the remaining decays we predict the following:

(4) The branching ratio

$$B(\Lambda_c^+ \to \Sigma^+\pi^0) = B(\Lambda_c^+ \to \Sigma^0\pi^+) ,$$

which matches with the expectations from naive isospin arguments. The asymmetry is also the same for both channels.

(5) The branching ratio $B(\Lambda_c^+ \to \Xi^0 K^+) = 0.05\%$. This decay is the cleanest of all Λ_c^+ decays as it has only a small p-wave contribution to its decay amplitude and a null asymmetry.

(6) Among the $\Xi_c'^+$ decays, there are only two possible modes. Both channels get contributions from the factorization, pole, and ETC terms, yet the decay $(\Xi_c'^+ \to \Xi^0\pi^+)$ dominates over $(\Xi_c'^+ \to \Sigma^+\bar{K}^0)$ by an order of magnitude:

$$B(\Xi_c'^+ \to \Xi^0\pi^+)/B(\Xi_c'^+ \to \Sigma^+\bar{K}^0) = 12.5 .$$

(7) The $\Xi_c'^0$ decays also exhibit similar trends. The dominant mode is $B(\Xi_c'^0 \to \Xi^-\pi^+) = 2.30\%$, which is in fact the largest of all the Cabibbo-enhanced modes. Its decay rate is about 2.5 times greater than that of $(\Lambda_c^+ \to \Lambda\pi^+)$, and can be expected to be measured soon.

TABLE III. Branching ratios and asymmetry parameters for Cabibbo doubly suppressed modes.

Process	Without $\lvert\Psi(0)\rvert^2$ scale variation Branching ratio (10^{-4})	Asymmetry (10^{-4})	With $\lvert\Psi(0)\rvert^2$ scale variation Branching ratio	Asymmetry
$\Delta C = -\Delta S = -1$				
$\Lambda_c^+ \to p + K^0$	0.04	0.20	0.05	0.25
$\Lambda_c^+ \to n + K^+$	0.51	−0.57	0.26	−0.02
$\Xi_c'^+ \to p + \pi^0$	0.03	0.00	0.15	0.00
$\Xi_c'^+ \to n + \pi^+$	0.07	0.00	0.30	0.00
$\Xi_c'^+ \to \Lambda + K^+$	0.26	0.31	0.44	0.94
$\Xi_c'^+ \to \Sigma^+ + K^0$	1.15	−0.99	2.03	−0.96
$\Xi_c'^+ \to \Sigma^0 + K^+$	2.28	−0.98	3.04	−0.99
$\Xi_c'^+ \to p + \eta$	0.56	−0.62	2.62	−0.62
$\Xi_c'^+ \to p + \eta'$	0.07	0.90	0.30	0.90
$\Xi_c'^0 \to p + \pi^-$	0.01	0.00	0.04	0.00
$\Xi_c'^0 \to n + \pi^0$	0.005	0.00	0.02	0.00
$\Xi_c'^0 \to \Lambda + K^0$	0.01	0.80	0.08	0.04
$\Xi_c'^0 \to \Sigma^0 + K^0$	0.08	−0.99	0.15	−0.96
$\Xi_c'^0 \to \Sigma^- + K^+$	0.66	−0.98	0.88	−0.99
$\Xi_c'^0 \to n + \eta$	0.08	−0.62	0.36	0.60
$\Xi_c'^0 \to n + \eta'$	0.01	0.90	0.04	0.90
$\Omega_c^0 \to p + K^-$	0.07	0.00	0.32	0.00
$\Omega_c^0 \to n + \bar{K}^0$	0.08	0.00	0.33	0.00
$\Omega_c^0 \to \Lambda + \pi^0$	Decay forbidden			
$\Omega_c^0 \to \Sigma^+ + \pi^-$	0.48	0.00	2.12	0.00
$\Omega_c^0 \to \Sigma^0 + \pi^0$	0.48	0.00	2.12	0.00
$\Omega_c^0 \to \Sigma^- + \pi^+$	0.48	0.00	2.10	0.00
$\Omega_c^0 \to \Xi^- + K^+$	5.51	0.64	12.44	0.59
$\Omega_c^0 \to \Xi^0 + K^0$	2.58	0.58	7.78	0.55
$\Omega_c^0 \to \Lambda + \eta$	3.14	0.23	13.83	0.23
$\Omega_c^0 \to \Lambda + \eta'$	0.32	−0.42	1.41	−0.42
$\Omega_c^0 \to \Sigma^0 + \eta$	Decay forbidden			
$\Omega_c^0 \to \Sigma^0 + \eta'$	Decay forbidden			

(8) Next in order of magnitude are the branching ratios of ($\Xi_c'^0 \to \Lambda \bar{K}^0 / \Xi^0 \pi^0$) modes. In these the $\bar{K}^0$ emitting mode gets contributions from both spectator and nonspectator processes, while the decay involving a π^0 comes only from the pole contributions.

(9) The decay ($\Xi_c'^0 \to \Sigma^+ K^-$) like ($\Lambda_c^+ \to \Xi^0 K^+$) has only a p-wave contribution. These decays are among the good candidates to test theoretical models.

(10) For the sextet particle Ω_c^0, only one ($\Omega_c^0 \to \Xi^0 \bar{K}^0$) channel is allowed, which acquires an amplitude from both pole and factorization terms:

$$B(\Omega_c^0 \to \Xi^0 \bar{K}^0) = 1.27\% \ .$$

(11) We have neglected $\eta - \eta'$ mixing in this study. The branching ratios for the η, η' decays are lower by a factor of 5–10 as compared to the other dominant modes.

B. Cabibbo-suppressed modes

The Cabibbo singly suppressed decays are of the order of 5% of the enhanced modes. The decays in which there exist both pole and factorization contributions are the π^+ or K^+ emitting modes, and are larger than the others by an order of magnitude. Factorization is also present in the decays emitting π^0, η, and η', but these are lower than the π^+, K^+ decays, due to the lower QCD factor c_2 and the Clebsch coefficients. In the modes arising through only nonspectator diagrams, the decays involving $\bar{K}^0$, K^0 are seen to be greater than those emitting the charged particles K^-, π^-.

Salient features of the Cabibbo singly suppressed decays, particularly the dominant modes, are summarized below:

(1) Λ_c^+: $B(\Lambda_c^+ \to \Lambda K^+) = 0.12\%$,

$B(\Lambda_c^+ \to n \pi^+) = 0.08\%$;

(2) $\Xi_c'^+$: $B(\Xi_c'^+ \to \Xi^0 K^+) = 0.61\%$,

$B(\Xi_c'^+ \to \Sigma^0 \pi^+) = 0.27\%$,

$B(\Xi_c'^+ \to p \bar{K}^0) = 0.14\%$;

(3) $\Xi_c'^0$: $B(\Xi_c'^0 \to \Xi^- K^+) = 0.15\%$,

$B(\Xi_c'^0 \to \Sigma^- \pi^+) = 0.10\%$;

(4) Ω_c^0: $B(\Omega_c^0 \to \Xi^- \pi^+) = 1.02\%$,

$B(\Omega_c^0 \to \Lambda^0 \bar{K}^0) = 0.59\%$.

It is interesting to note that the suppression due to the Cabibbo factor in the Ω_c^0 decays is canceled by the QCD enhancement factor c_1, making the π^+ emitting decay comparable to the Cabibbo-enhanced mode. This mode then can be considered as a viable candidate for experimental observations.

The Cabibbo doubly suppressed modes also exhibit a similar behavior. These are lower as expected, to be about 0.1% of the enhanced modes. Decays including factorization contributions are larger and found in the K^0, K^+ emitting modes. The largest among these are the branching ratio for

$$B(\Xi_c'^0 \to \Sigma^- K^+) = 0.01\% \ ,$$

$$B(\Omega_c^0 \to \Xi^- K^+) = 0.06\% \ ,$$

$$B(\Omega_c^0 \to \Xi^0 K^0) = 0.03\% \ ,$$

$$B(\Omega_c^0 \to \Lambda \eta) = 0.03\% \ .$$

IV. EFFECT OF $|\psi(0)|^2$ VARIATION

Though the data on the charm baryon decays is meager, it is still unexplained. In the above section, we have discussed the branching ratios of the individual decay modes of Λ_c^+, whose experimental numbers are sensitive to the choice of $B(\Lambda_c^+ \to p K^- \pi^+)$. A comparison of decay ratios is expected to be more accurate. However, there exists a discrepancy even between the experimental and theoretical predictions of their ratios; see, e.g., the following:

(i) The experimentally measured ratio [1],

$$\frac{\Gamma(\Lambda_c^+ \to \Lambda \pi^+)}{\Gamma(\Lambda_c^+ \to p \bar{K}^0)} = (0.41 \pm 0.09)\% \tag{21}$$

is in stark contrast to the naive expectations from QCD coefficients $(c_1/c_2)^2 \approx 6$, on the basis of color enhancement. This ratio has been theoretically estimated to be as high as 13 in some of the earlier attempts [5].

(ii) An earlier measurement by Albrecht *et al.* [3] and a recent measurement by CLEO [2], are both consistent with unity, for the ratio

$$\frac{B(\Lambda_c^+ \to \Sigma^0 \pi^+)}{B(\Lambda_c^+ \to \Lambda \pi^+)} = \frac{(2.0 \pm 0.7 \pm 0.4)}{(2.2 \pm 0.3 \pm 0.4)}$$

$$(1.0 \pm 0.2 \pm 0.1) \ , \tag{22}$$

whereas the theoretical predictions lies in the range 0.3–0.5.

We find that these inconsistencies appearing in the ratios may be due to the lack of a proper treatment of the pole contributions. The above experimental ratios indicate the significance of nonspectator processes, and Eq. (22) clearly shows that the pole contribution is comparable to the factorization contribution, if not larger.

In the preceding analysis, though we have accounted for the different enhancement of hard-gluon QCD effects in the strange and charm sectors, we have assumed the $|\psi(0)|^2$ scale for the charm baryons to be the same as that of the hyperons, i.e.,

$$\langle \psi_\Lambda | \delta^3(r) | \psi_{\Lambda_c^+} \rangle \approx \langle \psi_p | \delta^3(r) | \psi_{\Sigma^+} \rangle \ . \tag{23}$$

However, since $|\psi(0)|^2$ is a dimensional quantity, it may be incorrect to ignore its variation with flavor. Evidence to corroborate this has been found in the quark model [16,21] as well as in lattice calculations [22]. The flavor dependence reflected in the scale factor corresponding to the spatial matrix element is due to the long-distance QCD effects. Evaluation of $|\psi(0)|^2$ is as yet uncertain for baryons and more complicated, because, unlike the mesons, these are three-body systems. In fact the charm baryons may provide a good and perhaps even dramatic way of testing the flavor dependence of the confinement forces. The absence of an exact dynamical theory of low-energy interactions between quarks limits our evaluation of $|\psi(0)|^2$ from first principles. However, a naive estimate for the scale parameter may be obtained using the hyperfine splitting

$$\Delta E_{\rm HFS}=\frac{4\pi\alpha_s}{9m_1m_2}|\psi(0)|^2\langle\sigma_1\cdot\sigma_2\rangle\ , \tag{24}$$

which leads to

$$\frac{\Sigma_c-\Lambda_c}{\Sigma-\Lambda}=\frac{|\psi(0)|_c^2}{|\psi(0)|_s^2}\frac{\alpha_s(m_c)}{\alpha_s(m_s)}\frac{m_c-m_u}{m_s-m_u}\frac{m_s}{m_c}\ . \tag{25}$$

For a choice of $\alpha_s(m_c)/\alpha_s(m_s)\approx 0.53$ we get

$$|\psi(0)|_c^2/|\psi(0)|_s^2\approx 2.1\ . \tag{26}$$

We discuss the implications of the variation in the spatial wave function overlap on the branching ratios and the asymmetry parameters. The effective enhancement due to this dependence is manifest in the pole and the ETC contributions. An immediate consequence of this would be to increase the branching ratios for purely nonspectator processes such as $\Lambda_c^+\to\Sigma^0\pi^+$ and $\Lambda_c^+\to\Xi^0K^+$ by a factor of about 4, making them comparable to the other modes, while leaving their asymmetry unchanged. Decays also involving the factorization are not so straightforward and the effect of scale variation could provide some insight into the decay processes. The branching ratios and asymmetry parameters calculated with inclusion of scale variation, are given in columns 4 and 5 of Tables I, II, and III corresponding to the Cabibbo-enhanced, suppressed, and doubly suppressed decay modes. We will discuss some of their significant features here.

A. Cabibbo-enhanced modes

(1) Inclusion of flavor dependence in these decay modes reduces the ratio

$$\frac{B(\Lambda_c^+\to\Lambda\pi^+)}{B(\Lambda_c^+\to p\bar{K}^0)}=0.99$$

in the required direction, and enhances

$$\frac{B(\Lambda_c^+\to\Sigma^0\pi^+)}{B(\Lambda_c^+\to\Lambda\pi^+)}=1.04$$

in excellent agreement with experimental measurement ($1.0\pm0.2\pm0.1$) [1].

(2) The $B(\Lambda_c^+\to p\bar{K}^0)$ is nearly doubled to 2.34% in better agreement with experiment, and can also account for the lack of expected color suppression in the experimental prediction of the ratio $\Gamma(\Lambda\pi^+)/\Gamma(p\bar{K}^0)$.

(3) Effect of scale variation leaves the $B(\Lambda_c^+\to\Lambda\pi^+)=2.33\%$ almost unaffected. This is because the factorization term in the PC amplitude is larger than the pole due to the QCD enhancement factor c_1, and the ETC term is absent in this channel.

(4) The branching ratios for $B(\Lambda_c^+\to\Sigma^0\pi^+)$ and $(\Lambda_c^+\to\Sigma^+\pi^0)$ are both enhanced by a factor of 4.4 to about 2.4%, which is larger than the experimental number for $(\Lambda_c^+\to\Sigma^0\pi^+)$.

(5) The decay mode $\Xi_c'^+\to\Xi^0\pi^+$ presents an interesting study. Because of the destructive interference between the pole and factorizable contributions, the effect of $|\Psi(0)|^2$ variation is to decrease the decay rate from 0.82 to 0.27 and to alter the asymmetry parameter from -0.73 to $+0.25$. A measurement of $\Xi_c'^+\to\Xi^0\pi^+$ will provide a good test for variation of $|\Psi(0)|^2$.

(6) The decay $\Xi_c'^+\to\Sigma^+\bar{K}^0$ also shows a slight decrease in the branching ratio from 0.45% to 0.38%, and the asymmetry is seen to be halved from -0.43 to -0.25.

(7) The decay rates for $\Gamma(\Xi_c^{0\prime}\to\Xi^-\pi^+)$, $\Gamma(\Xi_c^{0\prime}\to\Lambda\bar{K}^0)$, and $\Gamma(\Xi_c^{0\prime}\to\Xi^0\pi^0)$ are enhanced by factors of about 2, 3, and 4, respectively, making them comparable in magnitude to Λ_c^+ decays. The branching ratio for $\Xi_c'^0\to\Sigma^0\bar{K}^0$ remains unaffected but the asymmetry is modified from -0.24 to $+0.87$.

(8) The decay $\Omega_c^0\to\Xi^-\bar{K}^0$ shows by far the largest increase in the decay rate, by a factor of 10, making it comparable to the Λ_c^+ decays and should be easier to detect.

B. Cabibbo-suppressed modes

In the singly suppressed $\Delta C=-1$ and $\Delta S=0$ modes, the decays not involving factorization show the expected order of enhancement by a factor of 4.4 in their decay rates. Among the other decays, some show marked changes in their asymmetries as well as their decay rates. The more prominent ones now raised to the order of the other decays are

(1) Λ_c^+: $B(\Lambda_c^+\to\Sigma^+K^0)=0.08\%$,
$B(\Lambda_c^+\to\Sigma^0K^+)=0.08\%$;

(2) $\Xi_c'^+$: $B(\Xi_c'^+\to p\bar{K}^0)=0.61\%$,
$B(\Xi_c'^+\to\Lambda\pi^+)=0.22\%$,

whose branching ratio is enhanced nearly 11 times and the asymmetry parameter changes from 0.26 to -0.79. For $B(\Xi_c'^+\to\Xi^0K^+)=0.45\%$ both the branching ratio and asymmetry decrease:

(3) $\Xi_c'^0$: $B(\Xi_c'^0\to n\bar{K}^0)=0.06\%$,
$B(\Xi_c'^0\to\Xi^0K^0)=0.06\%$;

(4) Ω_c^+: $B(\Omega_c^0\to\Xi^0\pi^0)=1.12\%$,
$B(\Omega_c^0\to\Sigma^+K^-)=0.46\%$.

V. SUMMARY AND CONCLUSION

We have performed a constituent quark model analysis of the exclusive two-body charm baryon nonleptonic decays for the Cabibbo-enhanced, -suppressed, and doubly suppressed modes. In this work we restrict ourselves to $B_c\to B+P(0^-)$ decays of $C=1$ charm baryons. We relate the matrix element $\langle B',P|H_W|B\rangle$ to $\langle B'|H_W|B\rangle$, employing the standard current algebra framework, and include the spectator contribution, which vanish in the soft-meson limit. Contrary to the conventional calculation of form factors at $q^2=0$, we evaluate the form factors f_i and g_i in the constituent quark model using the approach of Pérez-Marcial *et al.* [20]. Unlike other authors we explicitly calculate the relevant form factors for each decay rather than relating them through symmetry principles. The weak matrix elements are evaluated through the expansion of the W-exchange Hamiltonian, with the flavor dependence inherent through short-distance QCD corrections to the Hamiltonian. We also include SU(4)-breaking effects in the evaluation of the strong-coupling constants, using the Coleman-Glashow null result.

It has generally been observed that the factorization contribution dominates over the pole and commutator contributions in a naive estimate. However, the picture is unable to explain even the limited data. For instance, factorization does not generate decays such as $\Lambda_c^+ \to \Sigma^0\pi^+$, which has been measured comparable to the decay $\Lambda_c^+ \to \Lambda\pi^+$, and hence is a definite indication of significant nonspectator processes. This discrepancy motivated us to explore the effects of mass dependence on the weak scale. We find that the inclusion of flavor dependence on the scale $|\Psi(0)|^2$ can in fact raise the pole and ETC contributions to the same order as the factorization terms, if not more. This then effectively explains the experimentally observed lack of color suppression in the ratio

$$B(\Lambda_c^+ \to \Lambda\pi^+)/B(\Lambda_c^+ \to p\bar{K}^0) ,$$

and also justifies the near equality between the measured branching ratios

$$B(\Lambda_c^+ \to \Lambda\pi^+) \approx B(\Lambda_c^+ \to \Sigma^0\pi^+) .$$

We have presented the salient features of other decay channels in the Cabibbo-enhanced and Cabibbo-suppressed modes.

Note added in proof. A recent measurement [23] of the asymmetry

$$\alpha = (\Lambda_c^+ \to \Sigma^+\pi^0) = -0.43 \pm 0.23$$

is in good agreement with our calculated value.

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