Decays of the B_c meson

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The decays of the B_c meson are calculated systematically by introducing a suitable approach. The obtained results are discussed and compared with those obtained by other approaches.

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I. INTRODUCTION

According to the rule of naming a hadron, the meson B_c denotes the ground state of the bound system of heavy quarks, a heavy quark c and an antiquark $\bar{b}$. In addition to the well-studied bound systems of $(b\bar{b})$ and $(c\bar{c})$, the system and its antiparticles probably are the only ones of the double heavy quark systems which can form bound states, i.e., mesons, before one of its constituents has decayed, because the other possible double heavy quark systems must contain one top quark at least, and the top will decay with so great a possibility that it has no time to form a bound state with another heavy antiquark, if the top mass m_t is larger than 120–140 GeV, as indicated by indirect analyses. The reason is that the top quark's lifetime decreases rapidly as its mass is increasing, especially when greater than m_W+m_b. Thus the meson B_c is the only potentially possible double heavy one carrying known flavors and should be discovered in the near future [1–3]. Because of the fact that the B_c meson carries flavor explicitly, not like the mesons of $c\bar{c}$ and $b\bar{b}$, there is no gluon or photon annihilation via strong interaction or electromagnetic interaction. It can decay only via weak interaction, so it has a very long lifetime. Thus it will offer ideal new samples to study the weak decay mechanism of heavy flavors. They are even better than what we have had for certain purposes of the study. In fact, the study of the B_c meson is becoming one of the currently more interesting topics, especially since the experimental studies of the B_c meson will be accessible soon, as pointed out in Refs. [1–3].

As for the weak decays of hadrons, the short-distance effects responsible for the decays, i.e., the quark weak decay interaction and its QCD corrections, are relatively well known owing to the achievement of the standard model of electroweak interactions and perturbative QCD; however, the long-distance effects responsible for the hadronization from quarks to experimentally measurable hadrons are of a nonperturbative nature, and still remain obscure in several aspects. The situation in general may be summarized as follows: Many uncertainties have not been clarified in calculating the decays of light hadrons, due to the entanglement of the long-distance effects and the short-distance effects. The long-distance effects in the decays are hard to manage satisfactorily, especially for the energetic nonleptonic decays. Recently, the heavy quark effective theory (HQET) has achieved great success in describing the heavy meson $(Q\bar{q})$ or baryon (Qqq) decays due to the fact that an $SU(2)\times SU(2)$ spin-flavor symmetry in the limit when the heavy quark mass is approaching infinity is newly recognized. Up to now the next-order QCD, $1/m_Q$, and even higher-order corrections have been performed already in the HQET [4–9]. Being different from those of the mesons $(Q\bar{q})$, the meson B_c consists of two heavy quarks; hence whether or not the HQET is suitable at least needs to be examined. However, in this paper we will adopt a different approach from that of the HQET to the problem.

On the other hand, it is known that the QCD-motivated nonrelativistic heavy quark potential model has achieved great success in describing the $c\bar{c}$ and $b\bar{b}$ systems [10–15]. According to the QCD-inspired potential model, for the present system $c\bar{b}$, the difference from the above two is about the reduced mass only; i.e., its reduced mass is just between theirs and will have the same potential as them. Therefore to describe the relative motion of the two constituents of the B_c meson, the nonrelativistic approximation should be expected to work well and the potential framework should be suitable, provided we take the difference of the reduced masses mentioned above into account. The binding energy and the wave function of the B_c meson can be predicted well in this framework by the flavor-independent potential in which the parameters have been fixed totally by $c\bar{c}$ and $b\bar{b}$ spectra and decays. Therefore when calculating the decays of the B_c meson, we may use the obtained wave function reliably at the concerned accuracy level; hence, with its help the hadronization related to the B_c meson in the decays is relatively easy to deal with. One will see that our approach is to calculate the decays with the help of the well-established potential model as much as possible.

In the potential model we describe the relative motion of the two heavy components in the meson's center mass system (c.m.s.), so the wave functions are of this system too. However, for the decays, especially those with a large recoil in momentum, one cannot find a system in which the initial meson and the concerned produced

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meson in the decay are both at rest in the meantime; i.e., there is no common c.m.s. for them; hence to apply the wave function obtained by the potential model to the present problem is not straightforward. To overcome this difficulty and as the first step, we start with the Bethe-Salpeter (BS) equation to depict the meson as a bound state, and then take the so-called instantaneous nonrelativistic approximation but in a "covariant" form. Under the approximation, the relation between the BS wave function and the Schrödinger one of the potential model can be established in a covariant form. The second step of our approach is with the help of the Mandelstam formalism [24], to write down the decay matrix (the weak current matrix in fact) properly and then to make an instantaneous nonrelativistic approximation similarly for the whole matrix element, that the BS wave function(s) appearing in the formula turns out to be related to the Schrödinger one(s) in a similar manner as that established by the first step automatically. Therefore we have a proper calculation of the matrix elements. We should note here that our approach is to make the instantaneous nonrelativistic approximation in a general frame not only for the wave functions but also for the matrix element as whole; that is different from those approaches that attribute the problem to having only one meson wave function in the related moving frame while the wave function is obtained by a simple Lorentz boost of the Schrödinger one at rest. We will present our approach in detail and illustrate its reliability in Sec. III and the Appendix.

In respect to the short-distance effects, the effective Hamiltonian including the QCD corrections [16–18] for the weak decays is adopted and we will focus our attention on the main decay channels in the paper. Moreover as most references are based on the $1/N$ expansion consideration, the spectator mechanism dominance as well as the others to single out the long-distance effects from the short-distance ones are assumed for the nonleptonic decays also.

The authors of Ref. [19] have calculated the B_c decays by using the BSW (Bauer, Stech, and Wirbel) model [20] and the IGSW [21] (Isgur, Grinstein, Scora, Wise) model, respectively. The authors of Ref. [22] formulated the B_c decays under the HQET framework. However, we apply the approach as outlined above to calculate the decays of the B_c meson systematically, and, to understand our approach, we make comparisons of the obtained results with the IGSW's precisely and at the end we will point out the differences among them. We hope the experiments will test the different approaches soon.

This paper is organized as follows. In addition to the Introduction, in Sec. II, the inclusive decays and the lifetime for the meson are estimated. In Sec. III, the formalism of the form factors for the decays is presented in general. In Sec. IV, the exclusive semileptonic decays of the meson are evaluated, and the numerical results are presented in tables. Section V is parallel to the previous section, but is devoted to the nonleptonic decays. Section VI is devoted to discussions. Finally, the formalism for calculating the weak current matrix elements under the so-called covariant instantaneous approximation is collected in detail in the Appendix.

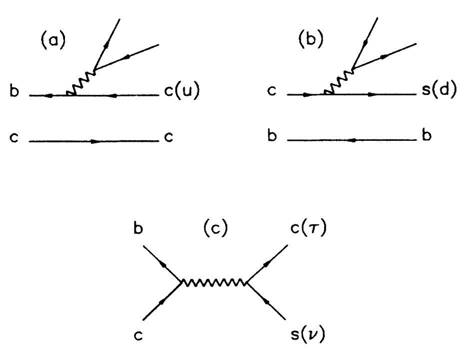

FIG. 1. Feynman diagrams for inclusive B_c decays. (a) $\bar{b}$ decays with c spectator; (b) c decays with $\bar{b}$ spectator; (c) $\bar{b}$ and c annihilate.

II. INCLUSIVE DECAYS AND THE LIFETIME

According to the known mechanisms for the weak decays of heavy mesons, the B_c meson may decay mainly via three categories of subprocesses: i.e., (1) the $\bar{b}$ component decays with c being a spectator; (2) the c decays but with $\bar{b}$ being a spectator, (3) the two components annihilate weakly (see Fig. 1).

The first one is similar to that in the case of B^+ (or B^0) meson decays; the $\bar{b}$ quark decays into either the semileptonic modes $\bar{b}\to\bar{c}(\bar{u})+l\nu_l(l=e,\mu,\tau)$ or the nonleptonic modes $\bar{b}\to\bar{c}(\bar{u})+u\bar{d}'(c\bar{s}')$ (d' and s' denote the eigenstates of the down and strange quarks of the weak interaction). Generally speaking, as for the B_c meson, being different from the B^0 meson decays, there is a destructive interference in the mode of $\bar{b}\to\bar{c}+(c\bar{s}')$ because of the identity quarks of c appearing in the final state (one is from the decay, the other just is the spectator), the same as in the case of D^+, and it will lead to a partial width that is slightly smaller than that of the B^0 meson. Nevertheless, one may approximately ignore this effect for the moment.[1]

The second one is similar to those in the cases of D^0 (D_s) decays: either the semileptonic decays $\bar{c}\to\bar{s}'+l\nu_l(l=e^+,\mu^+)$ or the nonleptonic decays $c\to s'+u+\bar{d}'$. The partial width due to the decay of the component c, should be close to that of the D^0 (or D_s) because there is no destructive interference here at all. However, since the mass of the spectator m_b is larger than the mass of the decay quark m_c, the phase space of the hadronic final state is comparatively tightly constrained so that the partial width due to the decays of the component c quark should be smaller than that of a c quark inside a D meson. Lusignoli and Masetti [19] took

[1]In fact, as for the B^+ decays there is a similar factor in the decay mode $\bar{b}\to\bar{c}+(u\bar{d}')$. However, the experiments indicate $\tau_{B^+}\sim\tau_{B^0}$; i.e., the effect is not significant. It is not surprising, because here there are two Cabibbo favorable decay channels $\bar{b}\to\bar{c}+(u\bar{d}')$ and $\bar{b}\to\bar{c}+(c\bar{s}')$ instead of one as in the case of the D^+ meson, so the effect is quite diluted. Thus we would not expect the effect being great in the B_c meson case either.

a factor 0.6 to depict it by some arguments. Another approach to take the effect into account is to sum up all of the widths of the exclusive main processes and a different factor from one is acquired too. Here for the same reason as in Ref. [19], in our estimation the factor 0.6 of the partial width to that of the D^0 (D_s) meson is adopted also. One will see that this value is consistent with a summation of the main exclusive modes calculated in the following sections of this paper. It should be emphasized here that this is still an open problem to investigate further.

The third one, weak annihilation, has a not tiny contribution to the total width. It is because in the present case the annihilation via a virtual W boson may create such a final state which contains a heavy lepton τ or quark c, that the helicity suppression is not very effective. The inclusive partial width for the annihilation can be easily calculated by

$$\Gamma(B_c \to f_1 \bar{f}_2) = C\frac{G_F^2}{8\pi}|V_{bc}|^2 f_{B_c}^2 M_{B_c} m_1^2 (1 - m_1^2/M_{B_c}^2)^2 , \tag{1}$$

where the constant $C=3$ (1) for creating a quark pair (a lepton pair), m_1 is the mass of the comparatively massive fermion created in the final state, while the other one is ignorable in the present case. In fact, there are two annihilation channels which are Cabibbo favorable and helicity suppression does not affect very much: $B_c \to c+\bar{s}'$ (whose contribution is about 5% in total width) and $B_c \to \tau^+ + \nu_\tau$ (less than 2%), while the other channels such as $B_c \to u+\bar{d}'$, $\mu^+\nu_\mu$, and $e^+ + \nu_e$ may be ignorable, due to a very strong helicity suppression from a very small m_1^2, though Cabibbo favorable. The decay constant f_{B_c} is defined by

$$\langle 0|\bar{b}\gamma_\mu\gamma_5 c|B_c^+(p)\rangle = if_{B_c}p_\mu . \tag{2}$$

From numerical calculations by a typical potential with $\Lambda_{\overline{MS}}=200$ MeV, where $\overline{\text{MS}}$ denotes the modified minimal subtraction scheme, and in the convention $f_\pi=135$ MeV, it reads

$$f_{B_c}=480 \text{ MeV} . \tag{3}$$

This value is expected to have an accuracy within 20% from the fact that the potential [15] gives a very successful prediction to the leptonic width of the $c\bar{c}$ and $b\bar{b}$ systems within the accuracy.

In summary, the total width of the B_c meson may be estimated approximately by the equation

$$\frac{1}{\tau_{B_c}} = \frac{1}{\tau_B} + \frac{0.6}{\tau_{D^0}} + \Gamma_{\text{anni}} . \tag{4}$$

It follows that

$$\tau_{B_c} = 4.0 \times 10^{-13} \text{ sec} , \tag{5}$$

where the values of the measurements on τ_{B^0} ($\tau_{B^\pm}$) and τ_{D^0} have been put into Eq. (4).

III. FORM-FACTOR FORMALISM

To calculate the exclusive weak decays of the B_c meson, one needs to evaluate the hadronic matrix elements, i.e., the weak current operator sandwiched between the initial state of the B_c meson and the concerned hadronic final state. In this section, we restrict ourselves to evaluating them in the simplest cases, i.e., only those decays in their final state having one hadron for semileptonic ones, but two hadrons for nonleptonic ones. In these cases, one may attribute the problem to evaluating a matrix element of the weak current operator sandwiched by two single-hadron states (for nonleptonic decays it is due to the factorization assumption of calculating the decay amplitude).

With the notation of a weak charged current $J_\mu = V_\mu - A_\mu$, where V_μ, A_μ are the vector and the axial-vector currents, respectively, the matrix elements are related to the form factors [21,23] as

$$\begin{aligned} \langle P(p')|V_\mu|B_c(p)\rangle &= f_+(p+p')_\mu + f_-(p-p')_\mu , \\ \langle V(p',\epsilon^*)|V_\mu|B_c(p)\rangle &= ig\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}(p+p')^\rho \\ &\quad \times (p-p')^\sigma , \\ \langle V(p',\epsilon^*)|A_\mu|B_c(p)\rangle &= f\epsilon_\mu^* + a_+(\epsilon^*\cdot p)(p+p')_\mu \\ &\quad + a_-(\epsilon^*\cdot p)(p-p')_\mu , \end{aligned} \tag{6}$$

where p,p' are the momenta of the B_c and the outgoing hadron, respectively, P and V denote the pseudoscalar and the vector mesons, respectively, and ϵ is the polarization vector of the vector meson. The form factors are functions of the Lorentz invariant variable $r^2 \equiv (p-p')^2$.

So far there are two kinds of approaches to calculate these form factors. One of them is the BSW model [20], in which the authors calculated the form factors at the maximum recoil $r^2=0$ by means of the wave functions defined at the light cone system under the quark model framework, and then extrapolated the result to all values of r^2 by assuming the form factors dominated by a proper pole of the nearest ones. The other is the IGSW model [21]. The authors of Ref. [21] calculated the form factors by using the wave functions of the quark model ("mock meson"), which treats the hadrons as a nonrelativistic object. As argued by the authors, the approach is exactly valid in the limit of weak binding and at the point of zero recoil. However, in the cases with a large recoil, there is a question whether the formulation is valid. For instance, for the decay $B_c \to J/\psi + \rho$, which we are considering, although the initial state B_c and the final state J/ψ both are of weak binding, the recoil of the decay is not small.

To calculate the weak current matrix element with a comparatively large recoil, one not only needs to know the appropriate forms of the meson's wave function in its rest frame and in a moving one but also the relations of the matrix element to the wave functions.

Recently, great progress has been achieved in understanding weak decays of the mesons containing one light and one heavy quark due to the work of Isgur and coworkers [4–6]. One of the applicable conditions of their formalism is that the mass of the light quark should be smaller than $\Lambda_{\rm QCD}$ and much smaller than that of the heavy one. However, for the B_c meson, both the $\bar{b}$ and c are heavy quarks so both may be considered nonrelativistic, and their masses are compatible; thus, the formalism [4–6] may not be very appropriate; at least a careful examination and considerable modification can be expected. Furthermore, even though their approach is valid for the B_c meson, to establish a link between the universal Isgur-Wise function $\xi(v\cdot v')$ and the nonrelativistic wave function of the heavy meson obtained by the potential model would be also very interesting.

To overcome the difficulty due to the two constituents being very heavy, we introduce a "new" approach. We start with the Mandelstam formalism [24] and then apply a generalized instantaneous nonrelativistic approximation as a whole to it and the BS wave functions(s) appearing in it, i.e., to make the approximation in a "covariant" form by introducing some Lorentz invariant variables so as to establish not only the connection between the Lorentz covariant BS wave function and the one obtained by the potential model for the heavy quark system properly, but also the dependence of the matrix element on the wave functions as well. The approach is very complicated, because in the case with a large recoil, to calculate the weak current matrix elements we cannot put the initial and the final mesons into one frame in which both are at rest.

Now let us proceed to write down the matrix element to describe the approach explicitly. It is known that the BS equation of a fermion-antifermion bound state takes the form

$$(\not{p}_1-m_1)\chi_p(q)(\not{p}_2+m_2)=i\int\frac{d^4k}{(2\pi)^4}V(p,k,q)\chi_p(k)\ , \tag{7}$$

where p_1 and p_2 are the momenta of the constituent particles 1 and 2, respectively. They can be expressed in terms of the total and the relative momenta p and q as

$$p_1=\alpha_1 p+q\ ,\quad \alpha_1=\frac{m_1}{m_1+m_2}\ ;$$
$$p_2=\alpha_2 p-q\ ,\quad \alpha_2=\frac{m_2}{m_1+m_2}\ . \tag{8}$$

$V(p,k,q)$ is the interaction kernel. It is well known that the BS wave function $\chi_p(q)$ satisfies the normalization condition

$$\int\frac{d^4q}{(2\pi)^4}\frac{d^4q'}{(2\pi)^4}\mathrm{tr}\left\{\bar{\chi}_p(q)\frac{\partial}{\partial p_0}[S_1^{-1}(p_1)S_2^{-1}(p_2)\delta^4(q-q')+V(p,q,q')]\chi_p(q')\right\}=2ip_0\ . \tag{9}$$

According to the mechanism shown in Fig. 2 and the Mandelstam formalism [24], the weak current matrix element involving only one hadron in the initial and the final states, respectively, may be expressed in terms of the BS wave functions:

$$l^\mu(r)=i\int\frac{d^4q}{(2\pi)^4}\mathrm{tr}[\bar{\chi}_{p'}(q')\Gamma_1^\mu\chi_p(q)(\not{p}_2+m_2)]\ , \tag{10}$$

where $\chi_p(q),\chi_{p'}(q')$ are the BS wave functions of the initial state and the final state with the total momenta p,p' and the relative momenta q,q', respectively; p_1,m_1, $p_1'm_1'$, and p_2,m_2 are the momenta and the masses of the decay quark, the final one, and the spectator, respectively; Γ_1^μ is the weak interaction vertex and to the lowest order, Γ_1^μ has the form of $\gamma_\mu(1-\gamma_5)$ for the charged current.

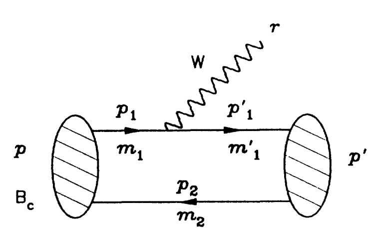

FIG. 2. Feynman diagram corresponding to the weak current matrix element sandwiched by the B_c meson state as the initial state and a single-particle state of the concerned final state.

As pointed out above, the BS wave function $\chi_p(q)$ of the heavy quark pair system can be evaluated by solving the corresponding Schrödinger equation with the help of the nonrelativistic instantaneous approximation and the potential model. Usually, it is convenient to make the nonrelativistic instantaneous approximation for the BS equation in the rest frame of the concerned bound state. Namely when the kernel at the rest frame has a simple form

$$V(p,k,q)\sim V(|\mathbf{k}-\mathbf{q}|)\ , \tag{11}$$

the integration over the q_0 component for the BS equation Eq. (7) can be easily carried through. As a result, the BS equation Eq. (7) is deduced into a three-dimensional equation, i.e., the Schrödinger equation in the momentum space for the system. However, as pointed out above, because of the nonzero recoil effects in the decays, the decaying meson in the initial state and the concerned meson in the final state cannot be put into a frame in which both of them are at rest. Therefore, it is necessary now to construct the matrix element in the Mandelstam formalism Eq. (10) and the BS wave function under a generalized instantaneous approximation in a Lorentz covariant form. To pursue this purpose, we need to divide the relative momentum q into two parts, $q_{p\parallel}$ and $q_{p\perp}$, a parallel part and an orthogonal one to p, respectively, i.e.,

$$q^\mu=q^\mu_{p\parallel}+q^\mu_{p\perp}\ , \tag{12}$$

where $q^\mu_{p\parallel}\equiv(p\cdot q/M_p^2)p^\mu$; $q^\mu_{p\perp}\equiv q^\mu-q^\mu_{p\parallel}$. Correspondingly, we have two Lorentz invariant variables:

$$q_p=\frac{p\cdot q}{M_p}\ ,\quad q_{pT}=\sqrt{q_p^2-q^2}=\sqrt{-q_{p\perp}^2}\ . \tag{13}$$

In the rest frame of the meson, i.e., $\mathbf{p}=\mathbf{0}$, they turn back to the usual component q_0 and $|\mathbf{q}|$, respectively. In terms of these variables, the covariant form of the wave function can be obtained.

Now the volume element of the relative momentum k can be written in an invariant form:

$$d^4k=dk_p k_{pT}^2 dk_{pT} ds\, d\phi\ , \tag{14}$$

where ϕ is the azimuthal angle, $s=(k_pq_p-k\cdot q)/k_{pT}q_{pT}$. The interaction kernel can be denoted as

$$V(|\mathbf{k}-\mathbf{q}|)=V(k_{p\perp},s,q_{p\perp})\ , \tag{15}$$

which is independent of ϕ, k_p, and q_p.

Defining

$$\varphi_p(q_{p\perp}^{\mu})\equiv i\int\frac{dq_p}{2\pi}\chi_p(q_{p\parallel}^{\mu},q_{p\perp}^{\mu})\ ,$$
$$\eta(q_{p\perp}^{\mu})\equiv\int\frac{k_{pT}^2dk_{pT}ds}{(2\pi)^2}V(k_{p\perp},s,q_{p\perp})\varphi_p(k_{p\perp}^{\mu})\ , \tag{16}$$

the BS equation can be rewritten as

$$\chi_p(q_{p\parallel},q_{p\perp})=S_1(p_1)\eta(q_{p\perp})S_2(p_2)\ , \tag{17}$$

where $S_1(p_1)$ and $S_2(p_2)$ are the propagators of the free particles and they can be decomposed as

$$S_i(p_i)=\frac{\Lambda_{ip}^{+}(q_{p\perp})}{J(i)q_p+\alpha_iM-\omega_{ip}+i\epsilon}+\frac{\Lambda_{ip}^{-}(q_{p\perp})}{J(i)q_p+\alpha_iM+\omega_{ip}-i\epsilon}\ , \tag{18}$$

with

$$\omega_{ip}=\sqrt{m_i^2+q_{pT}^2}\ ,$$
$$\Lambda_{ip}^{\pm}(q_{p\perp})=\frac{1}{2\omega_{ip}}\left[\frac{\not p}{M}\omega_{ip}\pm J(i)(m_i+\not q_{p\perp})\right]\ , \tag{19}$$

$i=1,2$ and $J(i)=(-1)^{i+1}$.

Here $\Lambda_{ip}^{\pm}(q_{p\perp})$ satisfies the relations

$$\Lambda_{ip}^{+}(q_{p\perp})+\Lambda_{ip}^{-}(q_{p\perp})=\frac{\not p}{M}\ ,$$
$$\Lambda_{ip}^{\pm}(q_{p\perp})\frac{\not p}{M}\Lambda_{ip}^{\pm}(q_{p\perp})=\Lambda_{ip}^{\pm}(q_{p\perp})\ , \tag{20}$$
$$\Lambda_{ip}^{\pm}(q_{p\perp})\frac{\not p}{M}\Lambda_{ip}^{\mp}(q_{p\perp})=0\ .$$

Thus, $\Lambda_{ip}^{\pm}(q_{p\perp})$ may be referred to as p-projection operators (p is the momentum of the bound state) while in the rest frame they correspond to the energy projection operators.

If defining $\varphi_p^{\pm\pm}(q_{p\perp})$ as

$$\varphi_p^{\pm\pm}(q_{p\perp})\equiv\Lambda_{1p}^{\pm}(q_{p\perp})\frac{\not p}{M}\varphi_p(q_{p\perp})\frac{\not p}{M}\Lambda_{2p}^{\mp C}(q_{p\perp})\ , \tag{21}$$

where the upper index C denotes the charge conjugation. In our notation,

$$\Lambda_{2p}^{\pm C}(q_{p\perp})\equiv\Lambda_{2p}^{\mp}(q_{p\perp})\ .$$

Integrating over q_p on both sides of Eq. (17), we obtain

$$(M-\omega_{1p}-\omega_{2p})\varphi_p^{++}(q_{p\perp})=\Lambda_{1p}^{+}(q_{p\perp})\eta_p(q_{p\perp})\times\Lambda_{2p}^{-c}(q_{p\perp})\ ,$$
$$(M+\omega_{1p}+\omega_{2p})\varphi_p^{--}(q_{p\perp})=\Lambda_{1p}^{-}(q_{p\perp})\eta_p(q_{p\perp})\times\Lambda_{2p}^{+c}(q_{p\perp})\ , \tag{22}$$
$$\varphi_p^{+-}(q_{p\perp})=\varphi_p^{-+}(q_{p\perp})=0\ .$$

The normalization condition reads, in covariant form,

$$\int\frac{q_{pT}^2dq_{pT}}{2\pi^2}\mathrm{tr}\left[\bar\varphi_p^{++}(q_{p\perp})\frac{\not p}{M}\varphi_p^{++}(q_{p\perp})\frac{\not p}{M}-\bar\varphi\phi_p^{--}(q_{p\perp})\frac{\not p}{M}\varphi_p^{--}(q_{p\perp})\frac{\not p}{M}\right]=2M\ . \tag{23}$$

Now let us introduce two auxiliary three-momenta $\tilde{\mathbf{p}}_1$ and $\tilde{\mathbf{p}}_2$ for the following usage:

$$\tilde{\mathbf{p}}_1\equiv\frac{\omega_{1p}}{M}\mathbf{p}+\mathbf{q}_{p\perp}\ ,\quad \tilde{\mathbf{p}}_2\equiv\frac{\omega_{2p}}{M}\mathbf{p}-\mathbf{q}_{p\perp}\ . \tag{24}$$

In the case of weak binding, the wave function can be constructed approximately as

$$\varphi_p^{\lambda++}(q_{p\perp})=\sum_{ss'}\frac{1}{\sqrt{4\omega_{1p}\omega_{2p}}}u_s(\tilde{\mathbf{p}}_1)\bar v_{s'}(\tilde{\mathbf{p}}_2)\phi_p^{+}(q_{pT})\chi_{ss'}^{\lambda}\ ,$$
$$\varphi_p^{\lambda--}(q_{p\perp})=\sum_{ss'}\frac{1}{\sqrt{4\omega_{1p}\omega_{2p}}}v_s(\tilde{\mathbf{p}}_1)\bar u_{s'}(\tilde{\mathbf{p}}_2)\phi_p^{-}(q_{pT})\chi_{ss'}^{\lambda}\ , \tag{25}$$

where $u_s(\tilde{\mathbf{p}}_i)$, $v_{s'}(\tilde{\mathbf{p}}_i)$ $(i=1,2)$ are the Dirac spinors of free particles with masses m_i; $\chi_{ss'}^{\lambda}$ is the Clebsch-Gordan coefficients that make s' and s couple to λ; and $\phi^{\pm}(q_{pT})$ is the scalar part of the wave function.

In the case of weak binding, the $\varphi_p^{\lambda--}(q_{p\perp})$ is a small component and can be ignored. In fact, if the kernel is of scalar and/or vector, the $\varphi_p^{\lambda--}(q_{p\perp})$ is of the order of $(v/c)^4$ to $\varphi_p^{\lambda++}(q_{p\perp})$ [25]. Furthermore, if we ignore the components proportional to the $q_{p\perp}$ in the spinor structure due to the nonrelativistic nature, $\varphi_p^{\lambda++}(q_{p\perp})$ can be simplified:

$$\varphi_p^{\lambda++}(q_{p\perp})=\frac{\not p+M}{2\sqrt{2}M}(\alpha\gamma_5+\beta\not\epsilon)\phi(q_{pT})\ , \tag{26}$$

where $\alpha=1$, $\beta=0$ for an ${}^1S_0$ state and $\alpha=0$, $\beta=1$ for a ${}^3S_1$ state, while the "radius" wave function $\phi(q_{pT})$ satisfies the Schrödinger equation

$$\frac{q_{pT}^2}{2\mu}\phi(q_{pT})+\int\frac{k_{pT}^2dk_{pT}ds}{(2\pi)^2}V(s,k_{p\perp},q_{p\perp})\phi(k_{pT})=E\phi(q_{pT})\ , \tag{27}$$

with the reduced mass $\mu=m_1m_2/(m_1+m_2)$ of the system.

Thus, we have established the relation between the covariant form of the BS wave function and the solution of the Schrödinger equation with the nonrelativistic instantaneous approximation.

Now we proceed to establish the relation between the matrix elements and the obtained wave functions. Integrating over the q_p component of Eq. (10) with Eqs. (17) and (18) and certain reasonable approximations, we will have (the detail is presented in the Appendix)

$$l_\mu(r) \simeq \int \frac{q_{pT}^2 dq_{pT} ds}{(2\pi)^2} \mathrm{tr}\left[\bar{\varphi}_{p'}^{++}(q'_{p'\perp})\Gamma_\mu \varphi_p^{++}(q_{p\perp})\frac{\not{p}}{M}\right] \times \frac{\omega'_{2p'}}{\omega_{2p}} , \tag{28}$$

where

$$\omega_{2p} = \sqrt{\mathbf{q}^2 + m_2^2} , \quad \omega_{2p'} = \frac{E'\omega_{2p} + \mathbf{r}\cdot\mathbf{q}}{M'} , \quad q'_{p'T} = \sqrt{\omega_{2p}'^2 - m_2^2} . \tag{29}$$

We should note here that based on the adopted extra approximations (see the Appendix), Eq. (28) is valid only with not too large recoils, i.e., $\gamma \equiv |r|/M' \leq 1$; however, most of the B_c decays, e.g., our concerning processes $B_c \to J/\psi + X$, $B_c \to B_s + X$, satisfy the condition. Using the wave functions in the form of Eq. (25) for both of the initial state and the final state, it follows that

$$l_\mu(r) = \int \frac{q_{pT}^2 dq_{pT} ds}{(2\pi)^2} [\bar{u}_l(\tilde{\mathbf{p}}'_1)\Gamma_\mu u_m(\tilde{\mathbf{p}}_1)]\bar{\varphi}_{p'}^{+}(q'_{p'T}) \times \varphi_p^{+}(q_{pT})\chi_{ls}^{\lambda}\chi_{s'm}^{\lambda'} \left[\frac{\omega'_{2p'}}{4\omega_{2p}\omega_p\omega'_{1p'}}\right]^{1/2} , \tag{30}$$

where

$$\omega'_{1p'} = \sqrt{q_{p'T}'^2 + m_1'^2} , \quad p_1 = (\omega_1, \mathbf{q}) , \quad p'_1 = \frac{\omega_{1p}'' + \omega_{2p}''}{M'}p' - \frac{\omega_{1p} + \omega_{2p}}{M}p + p_1 , \tag{31}$$

and as for the spectator (the antifermion with momentum p_2 in Fig. 2), the normalization condition

$$\bar{v}_{s'}(p_2)\frac{\not{p}}{M}v_{l'}(p_2) = 2\omega_2\delta_{s'l'} \tag{32}$$

has been used.

There is some arbitrariness for the choice of the directions of the spins of the quarks. However, it is convenient to take them orthogonal to the p and p' because the spin in this direction remains unchanged throughout the Lorentz boost along p' directions.

In the Appendix of Ref. [1], a covariant formalism to calculate the creation of a pair of fermion-antifermion has been derived in the spirit of the helicity amplitude [26]. A similar formalism can be obtained for a fermion scattered by a virtual $W^\pm$ boson; i.e., the amplitudes with possible spin directions read as

$$M_{1,2}^\mu = L_+ \tfrac{1}{2}\mathrm{tr}\left[(\not{p}'_1 + m'_1)\frac{1 \pm \gamma_5 \not{k}_1}{2}(\not{p}_1 + m_1)\Gamma^\mu\right] , \quad M_{3,4}^\mu = L_- \tfrac{1}{2}\mathrm{tr}\left[(\not{p}'_1 + m'_1)\gamma_5\frac{1 \pm \gamma_5 \not{k}_1}{2}(\not{p}_1 + m_1)\Gamma^\mu\right] , \tag{33}$$

where

$$L_\pm = [\tfrac{1}{2}(p_1 \cdot p'_1 \pm m_1 m'_1)]^{-1/2} , \tag{34}$$

and k_1 is the spacelike vector which is orthogonal to the initial momentum p_1 and the final one p'_1. It is easy to see that the first equation of Eq. (33) describes the spin-nonflip amplitude while the second one describes a spin-flip one. Both fermions are fully polarized along the $\pm k_1$ directions. It should be noted here that in these formulas the relative phases of the spinors among those states with different polarizations have been fixed.

Let us construct the spinor wave functions into a definite spin. For an $^1S_0$ state

$$\chi_{ss'} = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) . \tag{35}$$

For an $^3S_1$ state, the spin structures corresponding to three possible independent polarizations are

$$\chi_{ss'}^{k_1} = \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow) , \quad \chi_{ss'}^{k_2} = \frac{1}{\sqrt{2}}(\uparrow\uparrow + \downarrow\downarrow) , \quad \chi_{ss'}^{k_3} = \frac{i}{\sqrt{2}}(\uparrow\uparrow - \downarrow\downarrow) , \tag{36}$$

where k_1, k_2, and k_3 denote three polarization directions which are orthogonal to each other.

Thus, for a transition $P \to P' + X$, the amplitude reads

$$M_0^\mu = L_+ \tfrac{1}{2}\mathrm{tr}[(\not{p}'_1 + m'_2)\gamma_5\gamma_5(\not{p}_1 + m_1)\Gamma^\mu] , \tag{37}$$

and, for a transition $P \to V + X$, the corresponding amplitudes read

$$M_1^\mu = L_+ \tfrac{1}{2}\mathrm{tr}[(\not{p}'_1 + m'_1)\gamma_5\not{k}_1(\not{p}_1 + m_1)\Gamma^\mu] , \quad M_2^\mu = L_- \tfrac{1}{2}\mathrm{tr}[(\not{p}'_1 + m'_1)\not{k}_1(\not{p}_1 + m_1)\Gamma^\mu] , \quad M_3^\mu = L_- \tfrac{1}{2}\mathrm{tr}[(\not{p}'_1 + m'_1)\gamma_5(\not{p}_1 + m_1)\Gamma^\mu] , \tag{38}$$

where M_1, M_2, and M_3 correspond to those with various polarizations of the final vector meson. In fact, if we choose

$$k_2^\mu = -i\tfrac{1}{2}L_+L_-\epsilon^{\mu\nu\rho\sigma}p'_{1\nu}k_{1\rho}p_{1\sigma} , \quad k_3^\mu = \frac{L_+L_-}{2M'}[(p'\cdot p'_1)p_1^\mu - (p'\cdot p_1)p_1'^\mu] \tag{39}$$

as the polarizations of M_2 and M_3, the amplitudes of the $P \to V + X$ transition can be written down in a compact form:

$$M^\mu = L_+ \tfrac{1}{2}\mathrm{tr}[(\not{p}'_1 + m'_1)\gamma_5\not{\epsilon}'(\not{p}_1 + m_1)\Gamma^\mu] , \tag{40}$$

where

$$\epsilon'^{\mu}=\epsilon^{\mu}-\frac{C(\epsilon\cdot p)}{p^2-(p\cdot p')^2/M'^2}p^{\mu}_{p'\perp}\ ,$$
$$C=\frac{1}{L_+\left[\frac{(p'\cdot p_1)(p'\cdot p_1')}{M'^2}-\frac{1}{L_-^2}\right]^{1/2}}-1\ , \qquad (41)$$
$$p_{p'\perp}=p-\frac{(p\cdot p')}{M'^2}p'\ .$$

After a straightforward calculation, the form factor formalism is deduced:

$$f_{\pm}=\xi\left[\frac{1}{M}\left[1-\frac{m_2}{m_1'}\right]\pm\frac{\omega_1'+\omega_2'}{M'm_1'}\right],$$
$$g=\xi\frac{\omega_1'+\omega_2'}{MM'm_1'}\ ,$$
$$f=\xi\left[\frac{(p\cdot p_1')}{Mm_1'}+1\right], \qquad (42)$$
$$a_{\pm}=\xi\left[\frac{2m_2}{M^2m_1'}+\delta\mp\left[\frac{\omega_1'+\omega_2'}{MM'm_1'}+\frac{(p\cdot p')}{M'^2}\delta\right]\right],$$

where

$$\delta=-\frac{C[1+(p\cdot p_1')/Mm_1']}{p^2-(p\cdot p')^2/M'^2}\ . \qquad (43)$$

In the case of the zero recoil vicinity ($\mathbf{r}\to\mathbf{0}$),

$$\delta\to-\frac{m_2}{M^2m_1'}\ , \qquad (44)$$

while the "common" factor is written in the frame of the initial meson at rest ($\mathbf{p}=\mathbf{0}$):

$$\xi=\left[\frac{2\omega_2'm_1^2m_1'^2}{[(p_1\cdot p_1')+m_1m_1']\omega_1\omega_1'\omega_2}\right]^{1/2}\times\int\frac{d^3\mathbf{q}}{(2\pi)^3}\phi'^{*}_{p'}(q'_{p'T})\cdot\phi_p(|\mathbf{q}|)\ , \qquad (45)$$

where $\phi'^{*}_{p'}(q'_{p'T})$ and $\phi_p(|\mathbf{q}|)$ correspond to the radius parts of the wave functions of the meson in the initial state and the one in the final state, respectively. Being in a covariant form one and a rest one in the c.m.s. of the initial meson, they may be obtained by solving the Schrödinger equation Eq. (27) and its specific one (that in its center mass system), respectively, as long as the QCD-inspired potential is rewritten in the corresponding form in the equation. We should note the following. (i) Of the approximations, the "covariant" instantaneous one for the matrix element is essential, and we think it is reliable for the purposes throughout this paper due to the fact that, as for the BS equation, the approximation is proven in many cases for weak binding systems. (ii) The wave functions obtained in the present way are more reliable than those in other ways because the adopted potential is proven to work well for heavy quark systems in the potential model, although the "common" function ξ attributed to the overlap integration is not very sensitive to the specific radius wave functions, as there is a normalization condition controlling it; i.e., the overlap integration of the wave functions is approaching the normalization when the recoil is approaching zero and the reduced mass of the final state is approaching that of the initial one in the meantime. (iii) When carrying through the trace for γ matrices in Eqs. (37) and (40) so as to reach to the form factors, the contributions from $(\vec{\gamma}\cdot\mathbf{q})$ in terms $\not p_1'$ and $\not p_1$ have been ignored safely in the considered accuracy, because they are small in the case of weak binding, and when carrying through the integration Eq. (45) of the integrand, being of ground states, all terms proportional to $\mathbf{q}$ in odd power will vanish (only even power terms contribute).

IV. EXCLUSIVE SEMILEPTONIC DECAYS

For the B_c meson, there exist two types of semileptonic decays: i.e., $\bar{b}$ decays (the c quark inside the meson as a spectator) and c decays ($\bar{b}$ quark as a spectator).

Obviously, only the decay modes $\bar{b}\to\bar{c}\bar{l}\nu_l$ ($\bar{l}=e^+,\mu^+,\tau^+$) and $c\to s\bar{l}\nu_l$ ($\bar{l}=e^+,\mu^+$) are Cabibbo favored comparatively.

Here we will adopt the formalism presented in the previous section to calculate the exclusive semileptonic decays.

Following Refs. [21,23], the decay matrix element for $B_c\to Xl\nu_l(\bar{l}\nu)$ can be written as

$$T=\frac{G_F}{\sqrt{2}}V_{ij}\bar{u}_l\gamma_\mu(1-\gamma_5)v_\nu\langle X(p',\epsilon^*)|J^{\mu}_{ij}|B_c(p)\rangle\ , \qquad (46)$$

where V_{ij} is the Cabibbo-Kobayaski-Maskawa (CKM) matrix element and J^{μ}_{ij} is the charged hadronic current.

The hadronic tensor which is defined by

$$h_{\mu\nu}\equiv\sum_s\langle B_c(p)|J^{\dagger}_{\nu}|X(p',s)\rangle\langle X(p',s)|J_\mu|B_c(p)\rangle\ , \qquad (47)$$

based on Lorentz covariance analysis, can be written as

$$\begin{aligned}h_{\mu\nu}=&-\alpha g_{\mu\nu}+\beta_{++}(p+p')_\mu(p+p')_\nu\\&+\beta_{+-}(p+p')_\mu(r)_\nu+\beta_{-+}(r)_\mu(p+p')_\nu\\&+\beta_{--}(r)_\mu(r)_\nu+i\gamma\epsilon_{\mu\nu\rho\sigma}(p+p')^\rho(r)^\sigma\ . \end{aligned}\qquad (48)$$

By a straightforward calculation, the differential decay rate is obtained:

$$\frac{d^2\Gamma}{dx\,dy}=|V_{ij}|^2\frac{G_F^2M^5}{32\pi^3}\left\{\alpha\frac{y}{M^2}+2\beta_{++}\left[2x\left[1-\frac{M'^2}{M^2}+y\right]-4x^2-y\right]-\gamma y\left[1-\frac{M'^2}{M^2}-4x+y\right]\right\}, \qquad (49)$$

where $x\equiv E_l/M_{B_c}$ and $y\equiv t/M_{B_c}^2=(p-p')^2/M_{B_c}^2$.

The coefficient functions α,β_{++},γ's, ... can be expressed in terms of the form factors. For instance, for the decay $B_c\to Pe\nu_e$ (P denotes a pseudoscalar meson) we have

$$\alpha=\gamma=0\ ,\quad \beta_{++}=f_+^2\ , \qquad (50)$$

TABLE I. Exclusive semileptonic decay width (in 10^{-6} eV) for various modes calculated by our model.

$B_c \to \eta_c + e^+\bar{\nu}_e$	14.2
$B_c \to J/\psi + e^+\bar{\nu}_e$	34.4
$B_c \to D^0 + e^+\bar{\nu}_e$	0.094
$B_c \to D^{0*} + e^+\bar{\nu}_e$	0.269
$B_c \to \eta_c' + e^+\bar{\nu}_e$	0.727
$B_c \to \psi(2S) + e^+\bar{\nu}_e$	1.45
$B_c \to B_s + e^+\bar{\nu}_e$	26.6
$B_c \to B_s^* + e^+\bar{\nu}_e$	44.0
$B_c \to B^0 + e^+\bar{\nu}_e$	2.30
$B_c \to B^{0*} + e^+\bar{\nu}_e$	3.32

but for the decay $B_c \to V e \nu_e$ (V denotes a vector meson), the situation is complicated a little because of the polarization of the vector meson. Corresponding to the polarization of the vector meson, the hadronic tensor is better to be decomposed into a longitudinal part (L) and a transverse part (T), i.e., $h_{\mu\nu} = h_{\mu\nu}^{(L)} + h_{\mu\nu}^{(T)}$. After doing so we have

$$\begin{aligned}
&\alpha^{(L)} = \gamma^{(L)} = 0 \ , \\
&\alpha^{(T)} = f^2 + 4M^2 g^2 \mathbf{p}'^2 \ , \\
&\gamma^{(T)} = 2gf \ , \\
&\beta_{++}^{(L)} = \frac{M^2}{16\mathbf{p}'^2 M'^2}\left[1 - y - \frac{M'^2}{M^2}\right]^2 f^2 \\
&\qquad + \frac{M^2}{2M'^2}\left[1 - y - \frac{M'^2}{M^2}\right] f a_+ + \frac{M^2}{M'^2}\mathbf{p}'^2 a_+^2 \ , \\
&\beta_{++}^{(T)} = \left[\frac{1}{4M'^2} - \frac{M^2}{16\mathbf{p}'^2 M'^2}\left[1 - y - \frac{M'^2}{M^2}\right]^2\right] f^2 \\
&\qquad - M^2 g^2 y \ .
\end{aligned} \tag{51}$$

By calculating out all values of the form factors first, putting them into the formula Eq. (49) for the differential decay rates, and then integrating out the differential ones, the concerned semileptonic decay rates are calculated out finally. We list the results in Table I. For the $B_c \to J/\psi + l + \nu$ process the estimate happens to close to that of the IGSW model [19] due to the fact that the authors of Ref. [19] adopt a factor $\kappa = 0.7$ to take the recoil effects into account in their approach when calculating the form factors. However, some deviation from theirs for the $B_c \to B_s + l + \nu$ decays is remarkable; i.e., ours are larger than theirs [19]. We should note here that we cannot expect that the prediction on $B_c \to B_s + l + \nu$ decay by our approach is as reliable as that on $B_c \to J/\psi + l + \nu$, since the s quark inside the B_s meson is not so heavy that one cannot expect the nonrelativistic approximation is very appropriate.

V. EXCLUSIVE NONLEPTONIC DECAYS

There are three types of the nonleptonic decays for the B_c mesons: i.e., $\bar{b}$ decays with the c inside the meson being as a spectator, or alternatively, c decays with the $\bar{b}$ quark as a spectator, and the constituents c and $\bar{b}$ annihilate. Those exclusive nonleptonic decays, involving only two hadrons in the final states and dominated by the spectator mechanism, may be estimated comparatively well based on the factorization assumption and the approach on the weak current matrix elements described in Sec. III. The short-distance corrections of the strong interaction for the weak nonleptonic decays can be taken into account by perturbative QCD and the renormalization-group equation (RGE) techniques [16,17]. As a result, the effective and Cabibbo favored Hamiltonian for the b quark decay and the c quark decay can be written, respectively, as

$$\begin{aligned}
H_{\text{eff}}^b &= \frac{G_F}{\sqrt{2}} V_{cb}[c_1^b(\mu_b) Q_1^{cb} + c_2^b(\mu_b) Q_2^{cb}] + \text{H.c.} \ , \\
H_{\text{eff}}^c &= \frac{G_F}{\sqrt{2}} V_{cs}[c_1^c(\mu_c) Q_1^{cs} + c_2^c(\mu_c) Q_2^{cs}] + \text{H.c.} \ ,
\end{aligned} \tag{52}$$

where $c_i(\mu)$ are the Wilson coefficients, Q_1^{ij} and Q_2^{ij} are the local four-quark operators,

$$\begin{aligned}
Q_1^{bc} &= [(\bar{d}'u)_{V-A} + (\bar{s}'c)_{V-A}](\bar{c}b)_{V-A} \ , \\
Q_2^{bc} &= (\bar{c}c)_{V-A}(\bar{s}'b)_{V-A} + (\bar{c}u)_{V-A}(\bar{d}'b)_{V-A} \ , \\
Q_1^{cs} &= (\bar{c}s)_{V-A}(\bar{d}'u)_{V-A} \ , \\
Q_2^{cs} &= (\bar{d}'s)_{V-A}(\bar{c}u)_{V-A} \ ,
\end{aligned} \tag{53}$$

where d' and s' denote weak eigenstates of the down and strange quarks, respectively; $(\bar{q}'q)_{V-A} \equiv \bar{q}'\gamma_\mu(1-\gamma_5)q$, etc. Note that those terms from operator mixing due to penguin diagrams are ignored in Eq. (52) for the following reasons: (i) in our concerning processes their contributions are comparatively small; (ii) in order to compare our approach with others easily we had better ignore them as done by the others [19–20]. In addition, we should also note that we will ignore the annihilation for the concerning decays in the following estimates due to similar reasons as the penguin's above, i.e., smallness and as done in Refs. [19–20]. The smallness of the annihilation may be understood: first its contribution is about less than 5% in whole by estimating the inclusive processes with the help of Eqs. (1)–(5); second the decay product of the annihilation is two energetic jets; however, according to the theories of fragmentation and the experiences of the experiments about jets and nonleptonic decays of τ lepton, the two jets are more likely to make fragments into multiparticle modes rather than those of two-particle final states as concerned here.

The Wilson coefficients may be calculated by means of RGEs and the matching conditions of renormalization [17]. The operators $Q_\pm = Q_1 \pm Q_2$, being linear combinations of the operators Q_1 and Q_2, have diagonal anomalous dimensions [16,17]. In the c decay case having the renormalization parameter $\mu > m_c$, the anomalous dimensions read as [16]

$$\gamma_- = -2\gamma_+ = -\frac{2\alpha_s}{\pi} \ . \tag{54}$$

To solve the RGE, the coefficients $c_\pm^c(\mu)$ and at $m_b > \mu > m_c$, are

$$c_+^c(\mu)=\left[\frac{\alpha_s(M_W)}{\alpha_s(m_b)}\right]^{6/23}\left[\frac{\alpha_s(m_b)}{\alpha_s(\mu)}\right]^{6/25},$$
$$c_-^c(\mu)=[c_+^c(\mu)]^{-2}\ . \qquad (55)$$

In the b decay case, the evaluation of the Wilson coefficient $c_+^b(\mu)$ is different from that in the c decay case [18]. For the region $\mu\geq m_b$, the anomalous dimensions are just the same as that in the c decay case, while for the region $m_c<\mu<m_b$, due to the fact that the b quark behaves as a static color source, the anomalous dimensions should be

$$\gamma_-=4\gamma_+=-\frac{2\alpha_s}{\pi}\ . \qquad (56)$$

It leads to the evaluation of the Wilson coefficients as follows:

$$c_+^b(\mu)=\left[\frac{\alpha_s(M_W)}{\alpha_s(m_b)}\right]^{6/23}\left[\frac{\alpha_s(m_b)}{\alpha_s(\mu)}\right]^{-3/25},$$
$$c_-^b(\mu)=\left[\frac{\alpha_s(M_W)}{\alpha_s(m_b)}\right]^{-12/23}\left[\frac{\alpha_s(m_b)}{\alpha_s(\mu)}\right]^{-12/25}. \qquad (57)$$

There exist some ambiguities for the choice of the parameter μ. We would take $\mu=m_c$ for the c decays and $\mu=m_b$ for the b decays. Setting $\Lambda_{QCD}=250$ MeV, one obtains the numerical results of the Wilson coefficients:

$$c_1(m_c)=1.26\ ,\quad c_2(m_c)=-0.51\ ,$$
$$c_1(m_b)=1.12\ ,\quad c_2(m_b)=-0.26\ . \qquad (58)$$

The next step is to calculate the hadronic matrix elements, i.e., the effective Hamiltonian so that the four-fermion operators Q_1,Q_2 appearing in the effective Hamiltonian are sandwiched by the considered initial and final states. To calculate them, the so-called factorization ansatz cannot be avoided [29]. For instance, for the decay $B_c\to J/\psi\pi$, the amplitude can be expressed as

$$A=\frac{G_F}{\sqrt{2}}V_{cb}V_{ud}^*a_1\langle\pi^+|(\bar{u}d)_A|0\rangle\langle J/\psi|(\bar{b}c)_V|B_c\rangle\ , \qquad (59)$$

where

$$a_1=c_1(\mu)+\zeta c_2(\mu)\ . \qquad (60)$$

The second term $\zeta c_2(\mu)$ comes from the factorizable color singlet of the Fierz-reordered operator Q_2, and the equation

$$\zeta=\frac{1}{N_c}$$

is obtained, based on simply counting for the color index. However, the experimental data of D meson decays indicate that $\zeta\approx 0$ [30]; thus, as done in Refs. [18–20], we also take $\zeta=0$.

According to the factorization properties and the property of the operators Q_i, the nonleptonic two-body decays of the B_c meson by the "spectator" mechanism can be divided into three classes. The first, the produced mesons, can be generated directly by the operator Q_1, such as $B_c\to J/\psi\pi^+$, as discussed above. The second, the mesons, can be generated directly by the operator Q_2, such as the decay $B_c\to D^+D^0$. In this case, the corresponding factor a_2, a similar factor to the a_1 [as in Eq. (60)] for the first class, can be written as

$$a_2=c_2(\mu)+\zeta c_1(\mu)\ . \qquad (61)$$

The third, the mesons, can be generated directly by both operators Q_1 and Q_2, for instance, the decay mode $B_c\to J/\psi+D_s$ is one of this class. Of the last class there is an interference of the two operators; thus the corresponding factor a_3 reads as

$$a_3=a_1+ka_2\ . \qquad (62)$$

where k is a constant depending on the modes concerned.

To calculate the decays, the decay constants of the mesons are taken as follows:

TABLE II. Exclusive two-body nonleptonic decay rates (in 10^{-6} eV) with c spectator. For the modes including $c\bar{c}$ state, only η_c and J/ψ are contained.

		$a_1=1.26$ $a_2=-0.51$
$B_c\to\eta_c+\pi^+$	$a_1^2 2.07$	3.29
$B_c\to\eta_c+\rho$	$a_1^2 5.48$	8.70
$B_c\to J/\psi+\pi^+$	$a_1^2 1.97$	3.14
$B_c\to J/\psi+\rho$	$a_1^2 5.95$	9.45
$B_c\to\eta_c+K^+$	$a_1^2 0.161$	0.256
$B_c\to\eta_c+K^{+*}$	$a_1^2 0.286$	0.453
$B_c\to J/\psi+K^+$	$a_1^2 0.152$	0.242
$B_c\to J/\psi+K^{+*}$	$a_1^2 0.324$	0.514
$B_c\to D^++D^0$	$a_2^2 0.664$	0.173
$B_c\to D^++D^{0*}$	$a_2^2 0.695$	0.181
$B_c\to D^{+*}+D^0$	$a_2^2 0.653$	0.170
$B_c\to D^{+*}+D^{0*}$	$a_2^2 1.08$	0.281
$B_c\to D_s+D^0$	$a_2^2 0.340\times10^{-1}$	8.85×10^{-3}
$B_c\to D_s+D^{0*}$	$a_2^2 0.354\times10^{-1}$	9.20×10^{-3}
$B_c\to D_s^*+D^0$	$a_2^2 0.334\times10^{-1}$	8.68×10^{-3}
$B_c\to D_s^*+D^{0*}$	$a_2^2 0.564\times10^{-1}$	0.015
$B_c\to\eta_c+D^+$	$(a_1 0.193+a_2 0.440)^2$	3.40×10^{-4}
$B_c\to\eta_c+D^{+*}$	$(a_1 0.181+a_2 0.430)^2$	7.40×10^{-5}
$B_c\to J/\psi+D^+$	$(a_1 0.177+a_2 0.442)^2$	0.382×10^{-6}
$B_c\to\eta_c+D_s$	$(a_1 1.13+a_2 1.98)^2$	0.173
$B_c\to\eta_c+D_s^*$	$(a_1 1.04+a_2 1.90)^2$	0.118
$B_c\to J/\psi+D_s$	$(a_1 1.02+a_2 1.95)^2$	0.085
$B_c\to\eta_c'+\pi^+$	$a_1^2 0.268$	0.426
$B_c\to\eta_c'+\rho$	$a_1^2 0.622$	0.987
$B_c\to\psi(2S)+\pi^+$	$a_1^2 0.251$	0.398
$B_c\to\psi(2S)+\rho$	$a_1^2 0.710$	1.13
$B_c\to\eta_c'+K^+$	$a_1^2 0.020$	0.032
$B_c\to\eta_c'+K^{+*}$	$a_1^2 0.031$	0.049
$B_c\to\psi(2S)+K^+$	$a_1^2 0.018$	0.029
$B_c\to\psi(2S)+K^{+*}$	$a_1^2 0.038$	0.060
$B_c\to\eta_c'+D^+$	$(a_1 0.220+a_2 0.403)^2$	5.06×10^{-3}
$B_c\to\eta_c'+D^{+*}$	$(a_1 0.174+a_2 0.366)^2$	1.10×10^{-3}
$B_c\to\psi(2S)+D^+$	$(a_1 0.174+a_2 0.373)^2$	8.56×10^{-4}
$B_c\to\eta_c'+D_s$	$(a_1 1.31+a_2 1.84)^2$	0.502
$B_c\to\eta_c'+D_s^*$	$(a_1 0.981+a_2 1.58)^2$	0.185
$B_c\to\psi(2S)+D_s$	$(a_1 0.988+a_2 1.62)^2$	0.173

TABLE III. Exclusive two-body nonleptonic decay rates (in 10^{-6} eV) with b spectator.

		$a_1=1.12$ $a_2=-0.26$
$B_c \to B_s+\pi^+$	$a_1^2 58.4$	73.3
$B_c \to B_s+\rho$	$a_1^2 44.8$	56.1
$B_c \to B_s^*+\pi^+$	$a_1^2 51.6$	64.7
$B_c \to B_s^*+\rho$	$a_1^2 150.$	188.
$B_c \to B_s+K^+$	$a_1^2 4.20$	5.27
$B_c \to B_s^*+K^+$	$a_1^2 2.96$	3.72
$B_c \to B^++K^0$	$a_2^2 96.5$	4.25
$B_c \to B^++K^{0*}$	$a_2^2 68.2$	3.01
$B_c \to B^{+*}+K^0$	$a_2^2 73.3$	3.23
$B_c \to B^{+*}+K^{0*}$	$a_2^2 141$	6.23
$B_c \to B^++\phi$	$a_2^2 14.7$	0.650
$B_c \to B^{+*}+\phi$	$a_2^2 10.7$	0.471
$B_c \to B^0+\pi^+$	$a_1^2 3.30$	4.14
$B_c \to B^0+\rho$	$a_1^2 5.97$	7.48
$B_c \to B^{0*}+\pi^+$	$a_1^2 2.90$	3.64
$B_c \to B^{0*}+\rho$	$a_1^2 11.9$	15.0
$B_c \to B^0+K^+$	$a_1^2 0.255$	0.320
$B_c \to B^0+K^{+*}$	$a_1^2 0.180$	0.226
$B_c \to B^{0*}+K^+$	$a_1^2 0.195$	0.244
$B_c \to B^{0*}+K^{+*}$	$a_1^2 0.374$	0.469
$B_c \to B^++\pi^0$	$a_2^2 1.65$	0.0738
$B_c \to B^++\rho$	$a_2^2 2.98$	0.132
$B_c \to B^{+*}+\pi^0$	$a_2^2 1.45$	0.064
$B_c \to B^{+*}+\rho$	$a_2^2 5.96$	0.263

$$f_\pi=132\ \text{MeV}\ ,\quad f_K=161\ \text{MeV}\ ,\quad f_{K^*}=218\ \text{MeV}\ ,$$
$$f_\rho=216\ \text{MeV}\ ,\quad f_{J/\psi}=380\ \text{MeV}\ ,\quad f_{\psi'}=280\ \text{MeV}\ , \tag{63}$$
$$f_D=220\ \text{MeV}\ ,\quad f_{D^*}=220\ \text{MeV}\ ,\quad f_{D_s}=280\ \text{MeV}\ .$$

Thus now, based on the approach described in Sec. II and the spectator mechanism, the exclusive two-body nonleptonic decays of the B_c meson are calculated conveniently. The numerical results are listed in Tables II and III.

VI. DISCUSSIONS

We have proposed an approach to calculate the weak decay matrix elements, as well as the form factors. It is expected that the approach is available as long as the mesons in initial and final states are of weak binding. It is interesting to compare ours with that of the IGSW model [21]. In the IGSW model, the authors calculated the form factors by using the Gaussian-type wave functions, of which the parameters are determined by the variational method. It is easy to see that in the case of weak binding and at zero recoil vicinity, the formalism of our approach is consistent with theirs except for a tiny difference in the formulation for the overlap integration of the wave functions [21,23]. However in the case still of weak binding but with a large recoil, there are two remarkable deviations between these two approaches. One deviation comes from the difference in the spin structure of the wave functions, the other from the arguments in the wave-function integrand. For instance, the function corresponding to the ξ in the IGSW model is $F_3\sqrt{M_{B_c}/M'}$ and reads [21]

$$\xi'_{\rm IGSW}=\left[\frac{2\beta\beta'}{\beta^2+\beta'^2}\right]^{3/2}\exp\left[-\frac{m_2^2}{2\tilde{M}\tilde{M}'}\frac{t_m-t}{\kappa^2(\beta^2+\beta'^2)}\right]\ , \tag{64}$$

where $t_m=(M-M')^2$; $\tilde{M}$ and $\tilde{M}'$ are the masses of the "mock meson" [21]; β and β' are the variational parameters for the initial state and the final state, respectively; m_2 is the mass of the "spectator" and κ is a parameter introduced by hand. In the IGSW model, κ is adjusted to be 0.7 by fitting the π electromagnetism form factor and the authors of Ref. [21] regarded it as relativistic corrections due to a large recoil. However, our approach is different from theirs; i.e., all factors come into the formula automatically. The interesting thing is that occasionally the numerical calculation shows that the function of ξ obtained in our approach is very close to that of the IGSW model with $\kappa=0.7$. To show this fact, we present the corresponding ξ functions of $B_c\to J/\psi+X$ and $B_c\to B_s+X$ in Fig. 3 and Fig. 4, respectively. The dashed line represents the ξ function obtained by the Gaussian wave functions and with the original formalism of Ref. [21] and $\kappa=1$ as well. The dot-dashed line represents that obtained by the wave functions from solving the Schrödinger equation of the potential model with the same formalism. The dotted line represents that of the IGSW model but with $\kappa=0.7$. Note that the parameters $\beta_{B_c}=0.88$ and $\beta_{J/\psi}=0.65$ which were obtained by

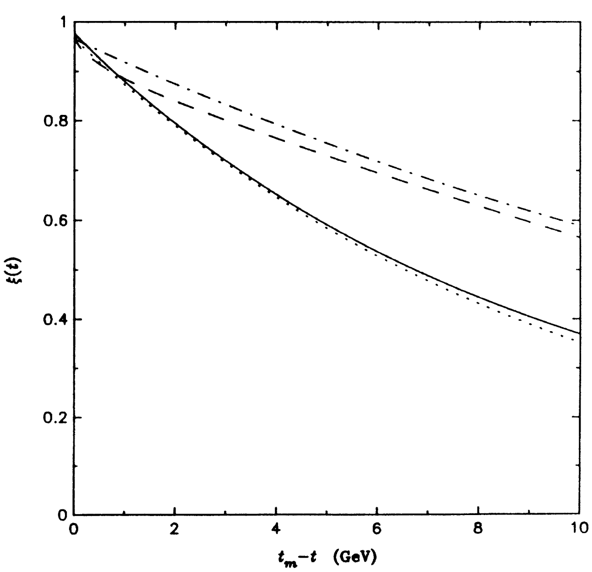

FIG. 3. The ξ function for $B_c\to J/\psi+X$. The dashed line: the ξ function of the IGSW model with $\kappa=1$; the dotted-dashed line: the ξ function obtained by the formalism of Ref. [18] with the wave function solved by potential I [12]; the dotted line: IGSW model with $\kappa=0.7$; the solid line: the ξ function obtained by our approach.

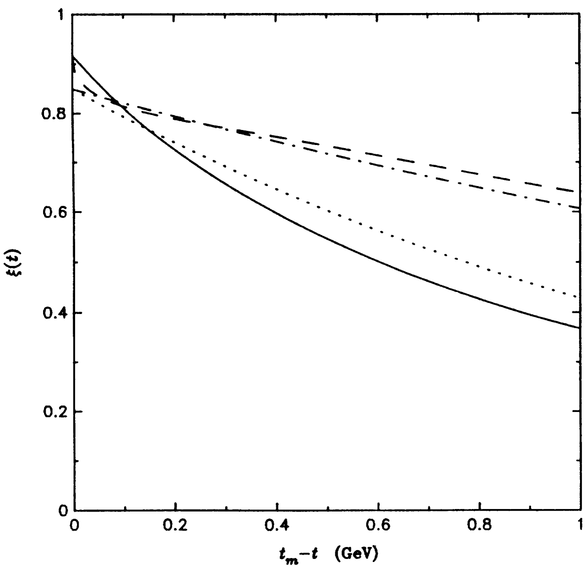

FIG. 4. The ξ function for $B_c \to B_s + X$. The meaning of each type line is the same as that in Fig. 3.

the authors of Ref. [19] are used here for the dotted, dashed, and dot-dashed curves, respectively. The solid line represents the ξ functions achieved by our approach. It is easy to see from Fig. 3 that the result of Eq. (45) is very close to that of the IGSW model with $\kappa = 0.7$. This means that our results involve reasonable effects automatically. In order to have a further comparison we also show the ξ function for $B_c \to B_s + X$ in Fig. 4 and the meaning of each line is the same as in Fig. 3, although it is not expected that our approach is as suitable as that for the formal decays $B_c \to J/\psi + X$, because the quark is not so heavy.

It is also very interesting to see the behavior in the limit when the quark mass is approaching infinity because it will let us see the relation between the form factors obtained here and the universal Isgur-Wise function. At the limit of

$$m_2 \ll m_1, m_1' \quad \text{and} \quad m_1, m_1' \to \infty \ , \tag{65}$$

the formulas of Eqs. (42)–(45) reproduce those of the Isgur-Wise formalism [4–6] for the form factors. In fact, in the limitation of Eq. (65) and from Eq. (31) and Eq. (41), we have

$$p_1 \to m_1 \cdot v \ , \quad p_1' \to m_1' \cdot v' \ , \quad \epsilon' \to \epsilon \ . \tag{66}$$

Hence the Eq. (40) can be rewritten as

$$l_\mu(r) = \xi(v \cdot v') \mathrm{tr}[(1+\not{v}')(\alpha + \beta\gamma_5 \not{\epsilon})(1+\not{v})] \ , \tag{67}$$

where $\xi(v \cdot v')$ is the universal Isgur-Wise function and is expressed now as

$$\xi(v \cdot v') = \frac{\sqrt{2v \cdot v'}}{\sqrt{1+v \cdot v'}} \int \frac{d^2\mathbf{q}}{(2\pi)^3} \phi'^*(q'_{p'T}) \phi(|q|) \ . \tag{68}$$

Thus the Eqs. (67) and (68) reproduce those of the Isgur-Wise formalism in the infinite heavy limit [4,5], so a link between the Isugr-Wise function and the nonrelativistic wave-function overlap integration has been established. The factor of $[0.5(1+v \cdot v')]^{-1/2}$ has been derived by Bjorken [27] by using the Cabibbo-Radicati sum rule [28]. Here in our formalism it automatically appears in Eq. (68).

Here we have applied the approach and obtained formulas to calculate the weak decays of the B_c systematically, thus it is no doubt that when the experimental study of B_c meson has fruitful results our approach will receive serious tests. We would emphasize here that the results for the decay modes $B_c \to J/\psi(\eta_c) + X$, which are the most important channels to reconstructing the B_c meson events in experiments, are more reliable for our approach than others, as in the calculation of the form factors the use of the nonrelativistic wave function is very suitable for the heavy quark bound states appearing in the initial state and in the final state both. Although for the $B_c \to J/\psi(\eta_c) + X$ processes our predictions are very close to those of the IGSW model with $\kappa = 0.7$ [19], there exist some deviations in numerical results for $B_c \to B_s + X$; i.e., our predictions are larger than those of Ref. [19] both for semileptonic decays and nonleptonic decays. The deviations can be understood as follows: although the ξ function for the modes is smaller than those of the IGSW model [19] as shown in Fig. 4, the form factors gain an enhancement from the spinor factor as shown in Eqs. (37) and (40).

Finally we would conclude that we are all at the position that all approaches to the B_c decays remain to be tested in future experiments, and one will learn much about the decay mechanism when they have had thorough experimental tests of the B_c decays; i.e., the experimental study of the B_c meson decays is desired because of the special roles of the meson.

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APPENDIX

In this appendix we derive Eq. (28), the weak current matrix element, by integrating over the q_p (here $\mathbf{P}=0$, $q_p = q_0$) component of Eq. (10) with the method of contour integration.

As discussed in Sec. II, the negative energy parts of the wave functions are very small in the weak binding case so that we can ignore their contributions for the lowest-order approximation. At present this is the case; thus we do so. Putting Eqs. (17) and (18) into Eq. (10), we have

$$l_\mu(r) = \int \frac{d^4q}{(2\pi)^4} \left[\bar{\eta}'_{p'}(q'_{p'\perp}) \frac{\Lambda^+_{1p'}(\pm q'_{p'\perp})}{q'_{p'} + \alpha'_1 M' - \omega_{1'p'} + i\epsilon} \Gamma_{1\mu} \frac{\Lambda^+_{1p}(q_{p\perp})}{q_p + \alpha_1 M + \omega_{1p} + i\epsilon} \eta_p(q_{p\perp}) \frac{\Lambda^+_{2p}(-q_{p\perp})}{q_p - \alpha_2 M + \omega_{2p} - i\epsilon} \right] \ . \tag{A1}$$

In the brackets of the integrand, there are three poles in the complex-q_0 plane at points a_i,

$$\begin{aligned} a_1 &= -\alpha_1 M + \omega_1 - i\epsilon , \\ a_2 &= \alpha_2 M - \omega_2 + i\epsilon , \\ a_1' &= \alpha_2 M - E'\sqrt{(\mathbf{r}+\mathbf{q})^2 + m_1'^2} - i\epsilon , \end{aligned} \tag{A2}$$

and two branch cuts starting at the branch points (ignoring the **q** term):

$$q_0 \simeq m_2 \pm i \frac{m_1'}{\gamma} , \tag{A3}$$

with $\gamma \equiv |\mathbf{r}|/M'$, due to the term $\omega'_{1p'} = \sqrt{q'^2_{p'\perp} + m'^2_1}$.

We perform the integration over the q_0 component on the right-hand side of Eq. (A1) with contour integration, owing to the fact that the branch cuts may be treated approximately, by expanding $\omega'_{1p'}$ as follows:

$$\omega'_{1p'} = \sqrt{q'^2_{p'\perp} + m'^2_1} = m_1' + \frac{q'^2_{p'\perp}}{2m_1'} + \cdots , \tag{A4}$$

in the weak binding limit. According to Cauchy's theorem, the integration of a closing contour on the upper half plane of the complex q_0 for the current matrix element $l_\mu(r)$ is just summing up all the pole's residues. However, as the pole a_2 on the upper half plane is very close to the pole a_1 on the lower half plane in the weak binding limit, in fact, the distance

$$\Delta \equiv a_1 - a_2 \simeq M - m_1 - m_2 + \frac{\mathbf{q}^2}{2\mu_1} \tag{A5}$$

is small; the value of the integration is dominated by the residue of the pole a_2 only (a' is not important).[2] Therefore, approximately, we obtain that

$$\begin{aligned} l_\mu(r) = \int \frac{d^3\mathbf{q}}{(2\pi)^3} \bar{\eta}'_{p'}(q'_{p'\perp}) \\ \times \frac{\Lambda^+_{1p'}(q'_{p'\perp})\Gamma_{1\mu}\Lambda^+_{2p}(-q_{p\perp})}{(q'_{p'} + \alpha_1' M' - \omega'_{1p'})(M - \omega_{1p} - \omega_{2p})} \\ \times \eta_p(q_{p\perp})\Lambda^+_{1p}(q_{p\perp}) , \end{aligned} \tag{A6}$$

where $q'_{p'}$, $\omega'_{1p'}$, $\omega'_{2p'}$, ω_{1p}, ω_{2p} are as expressed in Eqs. (13), (19), (29), and (31) in the text.

Finally, the relations

$$\begin{aligned} q'_{p'} + \alpha_1' M' - \omega'_{1p'} &= M' - \omega'_{1p'} - \omega'_{2p'} , \\ \Lambda^{c+}_{2p}(-q_{p\perp}) &= \frac{\omega'_{2p'}}{\omega_{2p}} \Lambda^{c+}_{2p'}(-q'_{p'\perp})\gamma_0 \Lambda^{c+}_{2p}(-q_{p\perp}) \end{aligned} \tag{A7}$$

are easily proved, and with Eqs. (A6) and (22) the required equation Eq. (28) in the text is obtained.

Based on Eq. (10) and with careful estimates on factors of the matrix element, the weak current sandwiched by two weak bound states at each step, it is safe to say in a not too accurate sense, the approximation taken here is quite valid for a not extremely large recoil, i.e., for $\gamma < 1$, that is satisfied in our concerned cases such as $B_c \to J/\psi(\eta_c) + X$ and $B_c \to B_s + X$, etc., the approximation is valid.

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[2] In fact, in principle, some poles (even cuts) may be induced into the matrix element through $\bar{\eta}'_{p'}(q'_{p'\perp})$, however, for a similar reason as here, they would not contribute substantially to the final results in all the cases of weak binding.

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