Decays of B and D mesons

Fayyazuddin
Physics Department, King Abdulaziz University, P.O. Box 9028 Jeddah 21413, Saudi Arabia

Riazuddin
Physics Department, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia
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The semileptonic decays of the type $B\to\pi l\nu$ are discussed. The form factor describing this decay is calculated and is compared with the experimental data on $D\to Kl\nu$ decay. The $D(J^+)\to D^*(D)\pi$ decays are also discussed. The parameters in the form factor for the semileptonic decays are correlated with $D^*\to D\pi$ and $D_0\to D\pi$ decays. We also derive the relation $f_{B_0}/f_B=m_{B_0}/m_B$.

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I. INTRODUCTION

In the heavy quark limit, one has the heavy quark effective field theory (HQEFT) [1] which simplifies the dynamics involving hadrons with one heavy quark. In particular, the heavy quark spin is decoupled. As a consequence of this, we have spin multiplets such as $[D(0^-),D^*(1^-)]$, $[D_2^*(2^+),D_1(1^+)]$, and $[D_1^*(1^+),D_0(0^+)]$. The degeneracy between two P-wave multiplets is broken by the spin-orbit coupling term involving the spin and orbital angular momentum of the light quark [2]. The degeneracy between two members of each multiplet is broken by terms such as the Fermi term $\mathbf{S}_q\cdot\mathbf{S}_Q$, the spin-orbit term $(\mathbf{S}_q+\mathbf{S}_Q)\cdot\mathbf{L}$, and the tensor term [2–4]. These terms vanish in the heavy quark limit, i.e., as $M\to\infty$. The structure of the angular momentum part of the wave function of the heavy meson [2] has some interesting consequences regarding radiative [2,5] and hadronic decays of P-wave D mesons [3,6].

The matrix elements of current $i\bar{Q}\gamma_\mu(1+\gamma_5)Q$ $(Q=b,c)$ are of interest in transition $B\to D$. These matrix elements have been extensively studied in HQEFT; we will not be concerned with them. The mixed currents $i\bar{q}\gamma_\mu(1+\gamma_5)Q$ are of interest in transitions such as $B,D\to\pi$, K, etc., in the semileptonic decays of B and D mesons. The form factors involving these decays have also been studied in the last couple of years [7–11]. The hadronic decays of heavy mesons involve currents with light quarks only. In this paper we are interested in the last two areas mentioned above.

II. SEMILEPTONIC DECAYS OF HEAVY MESONS

We are interested in semileptonic decays of the type $\bar{B}^0\to\pi^+l^-\bar{\nu}$. For this purpose we define the matrix elements

$$\sqrt{4p_0p_0'}\,\langle\pi^+(p')|i\bar{u}\gamma_\mu b|\bar{B}^0(p)\rangle=[F_+(t)(p+p')_\mu+F_-(t)(p-p')_\mu]\,, \tag{1}$$

$$\sqrt{4p_0p_0'}\,\langle\pi^+|i\bar{u}\gamma_\mu\gamma_5 b|\bar{B}^{*0}\rangle=i[G_0(t)\epsilon_\mu+G_+(t)\epsilon\cdot p'(p+p')_\mu+G_-(t)\epsilon\cdot p'(p-p')_\mu]\,, \tag{2}$$

where $t=-q^2=-(p-p')^2$.

In order to calculate the form factors, we use the following ingredients: (i) the low-energy theorems of current algebra; (ii) the dispersion relation for the form factors; the subtraction constant in the dispersion relation is fixed by the equal time commutator of the current algebra, i.e., by (i); and (iii) the dispersion relation is saturated with low-lying states. The possible low-lying states are $B(0^-)$, $B^*(1^-)$, $B_0(0^+)$, $B_1^*(1^+)$, $B_1(1^+)$, and $B_2^*(2^+)$. Since $\langle 0|i\bar{u}\gamma_\mu(1+\gamma_5)b|B_2^*\rangle=0$, B_2^* cannot contribute to the dispersion integral. By heavy quark spin symmetry, it follows that $f_{B_1}=0=f_{B_2^*}$. Hence only B^* and B_0 contribute to the dispersion integrals for the form factors $F_+(t)$ and $F_-(t)$. Only B_1^* contributes to the form factors G_0,G_+, and B_1^* and B contribute to G_-.

A similar procedure was followed in Ref. [11]. We extend this work so as to take into account possible contributions of all low-lying states. Our approach is close to that of Ref. [10]; it complements it. Using the technique of Ref. [11], we obtain

$$F_+(t)+F_-(t)=\frac{f_B}{f_\pi}+(t-m_B^2)\times\left[\frac{f_{B^*}g_{B^*B\pi}}{m_B^2(m_{B^*}^2-t)}+\frac{f_{B_0}g_{B_0B\pi}}{2m_B(m_{B_0}^2-t)}\right], \tag{3}$$

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$$F_+(t)-F_-(t)=\frac{2f_{B^*}g_{B^*B\pi}}{m_{B^*}^2-t}-\frac{f_{B_0}g_{B_0B\pi}(m_{B_0}^2-m_B^2)}{2m_B(m_{B_0}^2-t)}\;, \tag{4}$$

$$G_0(t)=-\frac{f_{B_*}}{f_\pi}-(t-m_{B^*})\frac{f_{B_1^*}F_{B_1^*}}{m_{B^*}(m_{B_1^*}^2-t)}\;. \tag{5}$$

We will use Eq. (5) later. In deriving Eqs. (3), (4), and (5), we have used the following definitions (for the particles on their mass shell, $-2p'\cdot q=m_{B_0}^2-m_B^2-m_\pi^2$):

$$\langle \bar B^0(p')|J_\pi^-|\bar B^*(p)\rangle=\frac{1}{\sqrt{4p_0p_0'}}g_{B^*B\pi}(2p'-p)\cdot\epsilon\;, \tag{6}$$

$$\langle \bar B^0(p')|J_\pi^-|B_0^-(p)\rangle=\frac{1}{\sqrt{4p_0p_0'}}i\frac{(-p'\cdot q)}{m_B}g_{B_0B\pi}\;, \tag{7}$$

$$\langle \bar B^{*0}(p')|J_\pi|B_1^{*-}(p)=\frac{1}{\sqrt{4p_0p_0'}}i\frac{(-p'\cdot q)}{m_{B^*}}\times\left[F_{B_1^*}\eta\cdot\epsilon+\frac{4G_{B_1^*}}{m_{B_1^*}^2-m_{B^*}^2}\eta\cdot p'\epsilon\cdot p\right]\;, \tag{8}$$

where η and ϵ are the polarization vectors of B_1^* and B^*, respectively. We use the normalization in which $f_\pi=130$ MeV.

Using $f_{B^*}=(m_Bm_{B^*})^{1/2}f_B$ [1,12], $g_{B^*B\pi}=\lambda_B(m_Bm_{B^*})^{1/2}/f_\pi$ [10,11,12], $f_{B_0}=m_{B_0}/m_Bf_B$, and parametrizing $g_{B_0B\pi}=\lambda_{B_0}(2m_B/f_\pi)$ (see the next section), we obtain

$$\begin{aligned}F_+(t)&=\frac{1}{2}\frac{f_B}{f_\pi}\left[1-\lambda_B\frac{m_{B^*}}{m_B}-\lambda_{B_0}\frac{m_{B_0}}{m_B}+2\lambda_Bm_Bm_{B^*}\left[1+\frac{m_{B^*}^2-m_B^2}{2m_B^2}\right]\frac{1}{m_{B^*}^2-t}\right]\\&=\frac{1}{2}\frac{f_B}{f_\pi}\lambda_B\left[\left[-1+\frac{1}{\lambda_B}-\frac{\lambda_{B_0}}{\lambda_B}\right]-\left[\frac{\delta_B}{m_B}+\frac{\lambda_{B_0}}{\lambda_B}\frac{\delta_{B_0}}{m_B}\right]+2\frac{m_{B^*}^2}{m_{B^*}^2-t}\right]\;,\end{aligned} \tag{9}$$

$$\begin{aligned}F(t)&\equiv\frac{m_B^2F_+(t)+tF_-(t)}{m_B^2}\\&=\frac{1}{2}\frac{f_B}{f_\pi}\lambda_B\left\{\frac{t+m_B^2}{m_B^2}+\frac{t-m_B^2}{m_B^2}\left[-\frac{m_B^*}{m_B}-\frac{m_{B_0}}{m_B}\frac{\lambda_{B_0}}{\lambda_B}+\frac{m_B^*}{m_B}(m_{B^*}^2-m_B^2)\frac{1}{m_{B^*}^2-t}+2\frac{m_{B_0}}{m_B}\frac{\lambda_{B_0}}{\lambda_B}\frac{m_{B_0}^2}{m_{B_0}^2-t}\right]\right\}\;,\end{aligned} \tag{10}$$

where $\delta_B=m_{B^*}-m_B$, $\delta_{B_0}=m_{B_0}-m_B$.

The advantage of the dispersion approach is that $1/m_B$ corrections are automatically incorporated. Moreover, every parameter is related to a physical quantity which is experimentally measurable. Our approach does not use the elaborate machinary of effective Lagrangian.

Neglecting the terms of order δ_B/m_B or δ_{B_0}/m_B, we can write

$$F_+(t)\approx\frac{1}{2}\frac{f_B}{f_\pi}\lambda_B\left[\left[-1+\frac{1}{\lambda_B}-\frac{\lambda_{B_0}}{\lambda_B}\right]+2\frac{m_{B^*}^2}{m_{B^*}^2-t}\right]\;, \tag{11}$$

$$F(t)\approx\frac{1}{2}\frac{f_B}{f_\pi}\lambda_B\left\{\frac{t+m_B^2}{m_B^2}+\frac{t-m_B^2}{m_B^2}\left[\left[-1-\frac{\lambda_{B_0}}{\lambda_B}\right]+2\left[\frac{\lambda_{B_0}}{\lambda_B}\right]\frac{m_{B_0}^2}{m_{B_0}^2-t}\right]\right\}\;. \tag{12}$$

For the transition $D\to\pi$, replace B by D. For the $D\to K$ transition, we get, using $f_{D_s^*}=(m_{D_s^*}m_D)^{1/2}f_D$, $g_{D_s^*DK}=\lambda_{D_s}(m_Dm_{D_s^*})^{1/2}f_K$, $f_{D_{s0}}=(m_{D_{s0}}/m_D)f_D$,

$$F_+(t)=\frac{1}{2}\frac{f_D}{f_K}\lambda_{D_s}\left[\left[-1+\frac{1}{\lambda_{D_s}}-\frac{\lambda_{D_{s0}}}{\lambda_{D_s}}\right]-\left[\frac{\delta_{D_s^*}}{m_D}+\frac{\lambda_{D_{s0}}}{\lambda_{D_s}}\frac{\delta_{D_{s0}}}{m_D}\right]+\frac{2m_{D_s^*}^2}{m_{D_s^*}^2-t}\right]\;, \tag{13a}$$

$$\approx\frac{1}{2}\frac{f_D}{f_K}\lambda_{D_s}\left[\left[-1+\frac{1}{\lambda_{D_s}}-\frac{\lambda_{D_{s0}}}{\lambda_{D_s}}\right]+\frac{2m_{D_s^*}^2}{m_{D_s^*}^2-t}\right]\;. \tag{13b}$$

While for the $B \to \pi$ transition, $1/m_B$ corrections are negligible, this may not be true for the $D \to \pi$ or $D \to K$ transition, especially for the latter. Therefore we use for the $D \to K$ transition Eq. (13a), for the $B \to \pi$ transition Eq. (11), and for the $D \to \pi$ transition Eq. (9). We take $m_{D_0} = 2.19$ GeV [2] and $M_{D_{s0}} - m_{D_0} = 100$ GeV.

The decay distribution for $D \to Kl\nu$ is given by

$$\frac{d\Gamma}{dt} = \frac{G_F^2 |V_{cs}|^2}{24\pi^3} p_K^3 [F_+(t)]^2 \equiv \frac{G_F^2 |V_{cs}|^2}{24\pi^3} \Phi(t) ,$$

where

$$p_K = [(m_D^2 - m_K^2 + t)^2 - 4m_D^2 t]^{1/2} / 2m_D \ . \quad (14)$$

In Ref. 13 it was reported that a good fit to the experimental data for the $D \to K$ transition is obtained by using the form factor

$$F_+(t) = \frac{F_+(0)}{1 - t/M^{*2}} , \quad (15)$$

with $M^* = 1.96 \pm 0.11 + 0.16$ GeV and $F_+(0) = 0.76 \pm 0.02$.

In order to make contact with the experimental data, we note that the present experimental data on $D^* \to D\pi$ decay give $\lambda_D < 1$. In principle, λ_{D_0} can be determined from the decay $D_0 \to D\pi$. We do not have this information. Thus we take λ_{D_0}/λ_D as a free parameter. As our analysis in Sec. III shows, $\lambda_{D_0}/\lambda_D < 1$. Accordingly, in our numerical calculation of $F_+(t)$ we have taken this ratio to be 0.875. This ratio seems to give a good fit to the experimental data for $D \to Kl\nu$ decay. However, $\lambda_{D_0}/\lambda_D = 1$ is not excluded by the data. In any case, we assume $\lambda_D = \lambda_B = \lambda_{D_s} = \lambda$, $\lambda_{B_0} = \lambda_{D_0} = \lambda_0$. As an illustration, we have obtained the form factor $F_+(t)$ from Eqs. (13a), (9), and (11) for $D \to K(\pi)$ and $B \to \pi$ transitions, with the following two sets of parameters: (a) $\lambda = 1$, $\lambda_0/\lambda = 0.875$ and (b) $\lambda = 1/\sqrt{2}$, $\lambda_0/\lambda = 0.875$. We take $f_K/f_\pi = 1.25$.

The form factor $F_+(t)$ as calculated above is used to obtain the decay distribution $\Phi(t) \propto d\Gamma/dt$. The results for $D \to K$ transition are shown in Fig. 1. For comparison, the distribution $\Phi(t)$ obtained with the form factor given in Eq. (15) with $M^* = 1.96$ GeV and $F_+(0) = 0.75$ is also shown in Fig. 1. It is clear from Fig. 1 that the distribution obtained using our form factor with the parameter set (b) is in agreement with that obtained with the form factor given in Eq. (15)

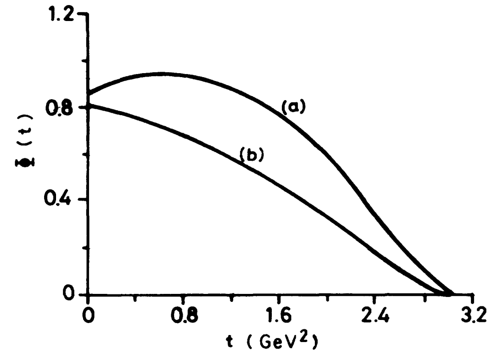

FIG. 2. Distributions $\Phi(t)$ for the decay $B \to \pi l\nu$ for sets (a) and (b).

The decay distribution $\Phi(t)$ for the $D \to \pi$ transition is shown in Fig. 2. In Fig. 3 the distribution $[(t - m_\tau^2)^2/t^2]\Phi(t)$ for the transition $B \to \pi\tau\nu_\tau$ is plotted. Also in this figure, the decay distribution $[(t - m_\tau^2)^2/t^2]\Phi_s(t)$, where

$$\Phi_s(t) = \frac{3}{8} \frac{p_\pi m_\tau^2 (m_B^2 - m_\pi^2)^2}{t m_B^2} [F(t)]^2 , \quad (16)$$

is shown. Note that $F(t)$ is the scalar form factor given in Eq. (12). For comparison with the form factors previously investigated, see Ref. [14].

The following remarks are in order. With $F_+^K(0) = 0.75$, we get $f_D/f_\pi = 2.36$, $F_+^\pi(0) = 1.05$ for set (a); for set (b), we get $f_D/f_\pi = 2.19$, $F_+^\pi(0) = 1.1$. Both values for $F_+^\pi(0)$ are consistent with the experimental value [13] $F_+^\pi(0)/F_+^K(0) = 1.0^{+0.6}_{-0.3}$. For the $B \to \pi$ transition, we get $F_+(0) = 0.8$, and $f_B/f_\pi = 1.4$ and $F_+(0) = 0.7$, $f_B/f_\pi = 1.3$ for sets (a) and (b), respectively. These values follow from the relation $f_B/f_\pi = (\sqrt{m_D/m_B}) f_D/f_\pi = 0.6\, f_D/f_\pi$.

We end this section with the observation that with $\lambda_0/\lambda = 1$, the shape of the curves shown in Figs. 1–3 remains almost the same, but they are shifted upward. For this case, if we use $F_+^K(0) = 0.75$, we get

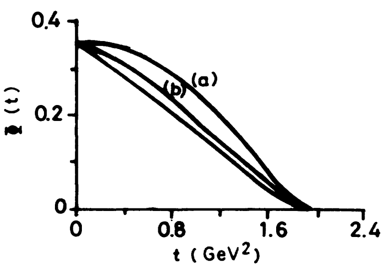

FIG. 1. Distribution $\Phi(t) \propto d\Gamma/dt$ for the decay $D \to Kle\nu$ for sets (a) $\lambda_{D_s} = 1$, $\lambda_{D_{s0}}/\lambda_{D_s} = 0.875$ and (b) $\lambda_{D_s} = 1/\sqrt{2}$, $\lambda_{D_{s0}}/\lambda_{D_s} = 0.875$. Also shown is the distribution obtained by using the phenomenological form factor given in Eq. (15).

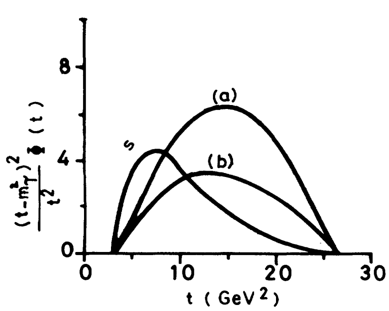

FIG. 3. Distributions $(t - m_\tau^2)^2/t^2 \Phi(t)$ for the decay $B \to \pi\tau\nu_\tau$ for sets (a) and (b). Also shown is the scalar distribution $[(t - m_\tau^2)^2/t^2]\Phi_s(t)$ (marked s) for set (b).

$f_D/f_\pi \approx 2.9$, 2.5 and $F_+^\pi(0)=1.09$, 1.03 for sets (a) and (b), respectively.

III. DERIVATION OF THE RELATION $f_{B_0}/f_B = m_{B_0}/m_B$ AND THE LIMIT ON λ_{B_0}

We first derive the relation $f_{B_0}/f_B = m_{B_0}/m_B$. The heavy quark spin symmetry relates the vector form factors F's with the axial form factors G's [11]:

$$-[(F_+ + F_-)m_B + (F_+ - F_-)E_\pi] = G_0 + G_+ E_\pi^2 - G_- E_\pi^2 . \quad (17)$$

Using Eqs. (3)–(5), we obtain

$$m_B \frac{f_B}{f_\pi} - \frac{t-m_B^2}{t-m_{B_0}^2} f_{B_0} \left[\frac{m_{B_0}^2 + m_B^2}{2m_B^2} \right] g_{B_0 B\pi} = m_B \frac{f_B}{f_\pi} - \frac{t-m_{B^*}^2}{m_{B^*}(t-m_{B_1^*}^2)} f_{B_1^*} F_{B_1^*} . \quad (18)$$

Hence we get

$$\left[\frac{m_{B_0}^2 + m_B^2}{2m_B^2} \right] f_{B_0} g_{B_0 B\pi} = \frac{f_{B_1^*} F_{B_1^*}}{m_{B^*}} , \quad (19)$$

which can also be expressed using $f_{B_1^*} = m_{B_0} f_{B_0}$ as

$$\left[\frac{m_{B_0}^2 + m_B^2}{2m_B^2} \right] g_{B_0 B\pi} = \frac{m_{B_0}}{m_{B^*}} F_{B_1^*} . \quad (20)$$

We shall make use of this relation in the next section. If we consider the matrix elements $\langle \pi^+ | i\bar{u}\gamma_\mu\gamma_5 b | \bar{B}_0^0 \rangle$ and $\langle \pi^+ | i\bar{u}\gamma_\mu b | \bar{B}_1^{*0} \rangle$ and use the same procedure as above, we obtain

$$\frac{m_{B_0}}{m_B} \left[\frac{m_{B_0}^2 + m_B^2}{2m_B^2} \right] f_B g_{B_0 B\pi} = \frac{f_{B_1^*} F_{B_1^*}}{m_B^*} . \quad (21)$$

Comparing Eqs. (19) and (21), we get

$$f_{B_0}/f_B = m_{B_0}/m_B . \quad (22)$$

We now discuss the $B_0 B\pi$ coupling. An estimate of $g_{B_0 B\pi}$ can be obtained from the sum rule considered in Ref. [10], which gives

$$\frac{1}{f_\pi^2} = \frac{g_{B^* B\pi}^2}{m_{B^*}^2} + \frac{g_{B_0 B\pi}^2}{4m_B^2} + \delta^2 , \quad (23)$$

in which δ^2 is the contribution from the higher states in the resonance saturation of the sum rule. Using the previously considered parametrization

$$g_{B^* B\pi} = \frac{\lambda_B m_{B^*}}{f_\pi}, \quad g_{B_0 B\pi} = \lambda_{B_0} \left[\frac{2m_B}{f_\pi} \right] , \quad (24)$$

one obtains

$$\lambda_{B_0} = \sqrt{1-\lambda_B^2-\delta^2} < \sqrt{1-\lambda_B^2} . \quad (25)$$

If we take $\lambda_B = 1/\sqrt{2}$ as previously, then $\lambda_{B_0}/\lambda_B < 1$.

IV. DECAY OF $D(J^+)$, $J=0,1,2$, MESONS TO PION AND D OR D^*

The emission of the pion by $D(J^+)$ would not affect the velocity of the heavy quark. Thus it is the operator $\mathbf{S}_q \cdot \hat{\mathbf{p}}_\pi$ which is relevant for these decays. Taking $\hat{\mathbf{p}}_\pi$ along the z axis, one has to consider the matrix element of $S_{3q}\cos\theta = \sqrt{4\pi/3}\, Y_{10} S_{3}q$ between $|D(J^+)\rangle$ and $|D^*\rangle$ or $|D\rangle$. Hence the helicity amplitude

$$F_\lambda^J \propto \langle D \text{ or } D^*, \lambda | S_3 q Y_{10} | D(J^+), M > \delta_{M\lambda} \quad (26)$$

can be easily calculated by using the wave functions [2]

$$\begin{aligned}
|D_2^*, M\rangle &= \left[\frac{(2+M)(1+M)}{12} \right]^{1/2} Y_{1M-1}\chi_+^{+1} + \left[\frac{4-M^2}{6} \right]^{1/2} Y_{1M}\chi_+^0 + \left[\frac{(2-M)(1+M)}{12} \right]^{1/2} Y_{1M+1}\chi_+^{-1} , \\
|D_1, M\rangle &= -\left[\frac{(2-M)(1+M)}{12} \right]^{1/2} Y_{1M-1}\chi_+^{+1} + \frac{1}{\sqrt{6}} Y_{1M}(M\chi_+^0 + 2\chi_-^0) + \left[\frac{(2+M)(1-M)}{6} \right]^{1/2} Y_{1M+1}\chi_+^{-1} , \\
|D_1^*, M\rangle &= -\left[\frac{(2-M)(1+M)}{6} \right]^{1/2} Y_{1M-1}\chi_+^{+1} + \frac{1}{\sqrt{3}} Y_{1M}(M\chi_+^0 - \chi_-^0) + \left[\frac{(2+M)(1-M)}{6} \right]^{1/2} Y_{1M+1}\chi_+^{-1} , \\
|D_0, 0\rangle &= \frac{1}{\sqrt{3}} (Y_{1-1}\chi_+^{+1} - Y_{10}\chi_+^0 + Y_{11}\chi_+^{-1}) ,
\end{aligned} \quad (27)$$

where

$$\begin{aligned}
\chi_+^{+1} &= |\tfrac{1}{2}\rangle|\tfrac{1}{2}\rangle , \\
\chi_\pm^0 &= \sqrt{\tfrac{1}{2}}(|\tfrac{1}{2}\rangle|-\tfrac{1}{2}\rangle \pm |-\tfrac{1}{2}\rangle|\tfrac{1}{2}\rangle) , \\
\chi_+^{-1} &= |-\tfrac{1}{2}\rangle|-\tfrac{1}{2}\rangle .
\end{aligned} \quad (28)$$

In our conventions, the first spin state corresponds to light quark and the second spin state corresponds to heavy quark. Note that $D_2^*(J=2^+)$ and $D_1(J=1^+)$, $D_1^*(J=1^+)$ and $D_0(J=0^+)$, and $D^*(J=1^-)$ and $D(J=0^-)$ form multiplets of heavy quark spin symmetry. Experimentally, D_1^* and D_0 have not been discovered up until now.

Using Eqs. (27) and (28), it is easy to see that

$$\langle D,0|fS_{3q}Y_{10}|D_2^*,0\rangle \equiv F_0^2=\sqrt{2/3}f\ , \quad (29a)$$

$$\langle D^*,\pm1|fS_{3q}Y_{10}|D_2^*,\pm1\rangle \equiv F_{\pm1}^2=\frac{1}{\sqrt{2}}f\ , \quad (29b)$$

$$\langle D^*,0;\pm1|fS_{3q}Y_{10}|D_1,0;\pm1\rangle \equiv F_0^1;F_{\pm1}^1=\sqrt{2/3}f;-\sqrt{1/6}f\ , \quad (29c)$$

$$\langle D^*,0;\pm1|f'S_{3q}Y_{10}|D_1^*,0;\pm1\rangle \equiv F_0^1;F_{\pm1}^1=-\sqrt{1/3}f';-\sqrt{1/3}f'\ , \quad (30a)$$

$$\langle D,0|f'S_3qY_{10}|D_0,0\rangle \equiv F_0^0=-\sqrt{1/3}f'\ . \quad (30b)$$

Thus we see that

$$\Gamma(D_2^*\to D\pi):\Gamma(D_2^*\to D^*\pi):\Gamma(D_1\to D^*\pi)=\tfrac{2}{15}f^2:\tfrac{1}{5}f^2:\tfrac{1}{3}f^2$$

and $\Gamma(D_1^*\to D^*\pi):\Gamma(D_0\to D\pi)=\frac{1}{3}f'^2:\frac{1}{3}f'^2$. The above results were also obtained in Refs. [3,6].

From Eq. (8), the decay amplitude for the decay $D_1\to D^*\pi$ in terms of the helicity states $\lambda=0$ and $\lambda=1$ in the rest frame of D_1 is given by

$$A_T(\lambda=1)=\frac{m_{D_1}^2-m_{D^*}^2}{2m_{D^*}}F_{D_1}=2m_{D_1}\left[S+\frac{p_0'}{\sqrt{2}m_{D^*}}D\right]\approx 2m_{D_1}\left[S+\frac{D}{\sqrt{2}}\right]\ , \quad (31a)$$

$$A_L(\lambda=0)=\frac{m_{D_1}^2-m_{D^*}^2}{2m_{D^*}}\left[F_{D_1}\frac{p_0'}{m_{D^*}}-\frac{4G_{D_1}}{m_{D_1}^2-m_{D^*}^2}|\mathbf{p}'|^2\frac{m_{D_1}}{m_D^*}\right]=2m_{D_1}\left[S\frac{p_0'}{m_{D^*}}-\sqrt{2}D\right]\approx 2m_{D_1}(S-\sqrt{2}D)\ , \quad (31b)$$

where we have used

$$G_{D_1}=\sqrt{2}m_{D^*}^2\left[1+\frac{p_0'^2}{2m_{D^*}^2}\right]\frac{D}{|\mathbf{p}'|^2}\approx\frac{3}{\sqrt{2}}m_{D^*}^2\frac{D}{|\mathbf{p}_\pi|^2}\ . \quad (32)$$

We have put

$$\frac{p_0'}{m_{D^*}}=\frac{m_{D_1}^2+m_{D^*}^2}{2m_{D_1}m_{D^*}}=1+O\left[\frac{\delta_{D_1}^2}{m_D^2}\right]$$

and $|\mathbf{p}'|=|\mathbf{p}_\pi|$. Here S and D are the S- and D-wave amplitudes for the decay $D_1\to D^*\pi$. They are normalized such that

$$\Gamma(D_1\to D^*\pi)=\frac{1}{2\pi}[|S|^2+|D|^2]|\mathbf{p}_\pi|\ . \quad (33)$$

From Eqs. (29) and (31), we get

$$F_1^1=-\sqrt{1/6}f=\left[S+\frac{D}{\sqrt{2}}\right]\ ,$$
$$F_0^1=\sqrt{2/3}f=(S-\sqrt{2}D)\ . \quad (34)$$

Hence we obtain $S=0$ [i.e., the decay $D_1\to D^*\pi$ is a D-wave decay] and

$$f=-\sqrt{3}D\ . \quad (35)$$

From Eqs. (33), (35), and (29), we obtain

$$\Gamma(D_1\to D^*\pi)=\frac{1}{2\pi}|D|^2|\mathbf{p}_\pi|\ ,$$
$$\Gamma(D_2^*\to D\pi)=\frac{1}{2\pi}\left[\frac{6}{15}\right]|D|^2|\mathbf{p}_\pi|\ , \quad (36)$$
$$\Gamma(D_2^*\to D^*\pi)=\frac{1}{2\pi}\left[\frac{3}{5}\right]|D|^2|\mathbf{p}_\pi|\ .$$

Using Eq. (32) and putting $\sqrt{2/3}G_{D_1}=\lambda_{D_1}m_{D^*}/f_\pi$, we obtain, from Eqs. (36),

$$\Gamma(D_1^0\to D^{*+}\pi^-)=\frac{1}{6\pi}\lambda_{D_1}^2\frac{1}{f_\pi^2m_{D^*}^2}|\mathbf{p}_\pi|^5\approx\lambda_{D_1}^2\ (4.4\ \text{MeV})\ ,$$
$$\Gamma(D_2^*\to D^+\pi^-)=\frac{1}{15\pi}\lambda_{D_1}^2\frac{1}{f_\pi^2m_{D^*}^2}|\mathbf{p}_\pi|^5\approx\lambda_{D_1}^2\ (10\ \text{MeV})\ , \quad (37)$$
$$\Gamma(D_2^*\to D^{*+}\pi^-)=\frac{1}{10\pi}\lambda_{D_1}^2\frac{1}{f_\pi^2m_{D^*}^2}|\mathbf{p}_\pi|^5\approx\lambda_{D_1}^2\ (4\ \text{MeV})\ ,$$

where we have used, for $D_1,D_2^*\to D^*\pi$ and $D_2^*\to D\pi$, $|\mathbf{p}_\pi|=356$, 388, and 504 MeV respectively [15]. From Eqs. (37), we obtain

$$\Gamma(D_2^{*0}\to D^+\pi^-)/\Gamma(D_2^{*0}\to D^{*+}+\pi^-)\approx 2.5$$

to be compared with the experimental value 2.4±0.7 [16]. If we take $\lambda_{D_1}=1$, then we obtain

$$\Gamma(D_1^0\to D^{*+}\pi^-+D^{*0}\pi^0)\approx 7\ \text{MeV}$$

and

$$\Gamma(D_2^{*0}\to D^{*+}\pi^-+D^{*0}\pi^0+D^+\pi^-+D^0\pi^0)\approx 21\ \text{MeV}$$

to be compared with the total decay widths of 20^{+9}_{-5} and 19±7 MeV, respectively [15].

We now come to the decays $D_1^*\to D^*\pi$ and $D_0\to D\pi$. From Eqs. (20) and (31a), we get (replace S and D by S^* and D^* and m_{D_1} by $m_{D_1^*}$)

$$\left[\frac{m_{D_0}^2+m_D^2}{2m_D^2}\right]g_{D_0D\pi}=\frac{m_{D_1}^*}{m_D^*}\left[\frac{4m_D^*m_{D_1}^*}{m_{D_1^*}^2-m_{D^*}^2}\right]\left[S^*+\frac{D^*}{\sqrt{2}}\right]. \quad (38)$$

Since $D_0 \to D\pi$ is an S-wave decay, it follows that $D_1^* \to D^*\pi$ must also be an S-wave decay. Hence we have

$$S^* = \frac{m_{D_1^*}^2 - m_{D^*}^2}{4m_{D_1^*}^2}\left[\frac{m_{D_0}^2 + m_D^2}{2m_D^2}\right] g_{D_0 D\pi} . \quad (39)$$

The above result also follows from Eqs. (30) and (31). From Eq. (33), we get, on using $G_{D_0 D\pi} = \lambda_{D_0} 2m_D / f_\pi$,

$$\Gamma(D_1^{*0} \to D^{*+}\pi^-) \approx \frac{1}{32\pi}\frac{4\lambda_{D_0}^2}{f_\pi^2}(m_{D_1^*} - m_{D^*})^2[(m_{D_1^*} - m_{D^*})^2 - m_\pi^2]^{1/2} . \quad (40)$$

Using the definition (7), we get

$$\Gamma(D_0^0 \to D^+\pi^-) = \frac{1}{32\pi}\frac{4\lambda_{D_0}^2}{f_\pi^2}(m_{D_0} - m_D)^2 \times [(m_{D_0} - m_D)^2 - m_\pi^2]^{1/2} . \quad (41)$$

No experimental data on D_1^* and D_0 are available. One version of the quark model [2] gives $m_{D_1^*} = 2.29$ GeV and $m_{D_0} = 2.19$ GeV. Using these values, we obtain, from Eqs. (40) and (41),

$$\Gamma(D_1^{*0} \to D^{*+}\pi^-) = [\lambda_D(2\lambda_{D_0}/\lambda_D)]^2 \times 11 \text{ MeV} , \quad (42)$$

$$\Gamma(D_0^0 \to D^{*+}\pi^-) = [\lambda_D(2\lambda_{D_0}/\lambda_D)]^2 \times 18 \text{ MeV} . \quad (43)$$

In the discussion of the $D \to \pi$ transition, for the form factor $F_+(t)$, we have used two set of parameters: (a) $\lambda_D = 1$, $\lambda_{D_0}/\lambda_D = 0.875$ and (b) $\lambda_D = 1/\sqrt{2}, \lambda_{D_0}/\lambda_D = 0.875$. Using sets (a) and (b), we get $\Gamma(D_1^{*0} \to D^{*+}\pi^-) \approx 34$ MeV, $\Gamma(D_0^0 \to D^+\pi^-) \approx 55$ MeV and $\Gamma(D_1^{*0} \to D^{*+}\pi^-) \approx 17$ MeV, $\Gamma(D_0^0 \to D^+\pi^-) \approx 27$ MeV, respectively. In future experiments, these predictions can be tested.

Results similar to our results given in Eqs. (37), (40), and (41) were obtained in Ref. [16] using the machinary of the effective Lagrangian. These decays were also studied in Ref. [17] using the trace technique. We have derived these results with simpler techniques.

Finally, we discuss the angular distribution of pions in the decays $D(J^+) \to D^*(D)\pi$. The angular distribution is given by [18]

$$I_{JM}(\theta) = \sum_\lambda |F_\lambda^J||d_{M\lambda}^J|^2 . \quad (44)$$

For $D_0 \to D\pi$, the angular distribution is isotropic. For $D(1^+, M) \to D^*\pi$, we get

$$I_{1M}(\theta) = |F_0^1|^2|d_{M0}^1(\theta)|^2 + |F_1^1|^2[|d_{M1}^1(\theta)|^2 + |d_{M-1}^1(\theta)|^2] . \quad (45)$$

From Eq. (45), we get

$$\begin{aligned} I_{10}(\theta) &= |F_0^1|^2\cos^2\theta + |F_1^1|^2\sin^2\theta , \\ I_{11}(\theta) &= \tfrac{1}{2}[|F_0^1|^2\sin^2\theta + |F_1^1|^2(1+\cos^2\theta)] \\ &= I_{1-1}(\theta) . \end{aligned} \quad (46)$$

For $D_1^* \to D^*\pi$ decay, using Eq. (30), we see that $I_{10}(\theta) = I_{11}(\theta) = I_{1-1}(\theta) = \frac{1}{3}f'^2$; i.e., the angular distribution is isotropic as expected for an S-wave decay. For the $D_1 \to D^*\pi$ decay, using Eq. (29c), we obtain

$$\begin{aligned} I_{10}(\theta) &= \tfrac{1}{6}f^2(1+3\cos^2\theta) , \\ I_{11}(\theta) &= \tfrac{1}{12}f^2(5-3\cos^2\theta) = I_{1-1}(\theta) . \end{aligned} \quad (47)$$

Equations (47) show that the decay is a D-wave decay. However,

$$I_1(\theta) = \tfrac{1}{3}\sum_M I_{1M}(\theta) = \tfrac{1}{3}f^2 ; \quad (48)$$

i.e., the angular distribution $I_1(\theta)$ is isotropic. This is true if D_1 is a pure beam. If this not so, then let f_L be the probability of finding it in an $M=0$ state. In this case, the probability of finding it in an $M=1$ or -1 state is $\frac{1}{2}(1-f_L)$. Hence the angular distribution is given by

$$\begin{aligned} I_1(\theta) &= \tfrac{1}{3}[f_L I_{10}(\theta) + \tfrac{1}{2}(1-f_L)I_{11}(\theta) + \tfrac{1}{2}(1-f_L)I_{1-1}(\theta)] \\ &= \tfrac{1}{3}\{f_L(|F_0^1|^2\cos^2\theta + |F_1^1|^2\sin^2\theta) + (1-f_L)[|F_0^1|^2\sin^2\theta + |F_1^1|^2(1+\cos^2\theta)]\} . \end{aligned} \quad (49)$$

Thus for the $D_1 \to D^*\pi$ decay we obtain

$$I_1(\theta) = \frac{f^2}{3}[\tfrac{1}{6}f_L(1+3\cos^2\theta) + \tfrac{1}{12}(1-f_L)(5-3\cos^2\theta)] ; \quad (50)$$

and for the $D_1^* \to D^*\pi$ decay, we again get isotropic distribution.

For the decay $D_2^* \to D\pi$, using Eqs. (44) and (29), we obtain

$$\begin{aligned} I_{20}(\theta) &= \tfrac{1}{6}f^2(1-3\cos^2\theta)^2 , \\ I_{21}(\theta) &= f^2\cos^2\theta(1-\cos^2\theta) = I_{2-1}(\theta) , \\ I_{22}(\theta) &= \tfrac{1}{4}f^2(1-\cos^2\theta)^2 = I_{2-2}(\theta) . \end{aligned} \quad (51)$$

For the decay $D_2^* \to D^*\pi$, using Eqs. (29b), we obtain

$$\begin{aligned} I_{20}(\theta) &= \tfrac{3}{2}f^2\cos^2\theta(1-\cos^2\theta) , \\ I_{21}(\theta) &= \tfrac{1}{4}f^2[4\cos^4\theta - 3\cos^2\theta + 1] = I_{2-1}(\theta) , \\ I_{22}(\theta) &= \tfrac{1}{4}f^2(1-\cos^4\theta) = I_{2-2}(\theta) . \end{aligned} \quad (52)$$

It may be noted that for D_2^* decays the angular momentum and parity conservation restrict these decays to the D wave. The heavy quark spin symmetry does not give anything new as far as the angular distribution of D_2^* decays is concerned. However, for $D(1^+)$ decays, the general angular distribution is given by Eqs. (46) and (49). However, for these decays, heavy quark spin symmetry gives a definite prediction for $D_1^* \to D^*$ to be isotropic, and for $D_1 \to D^*\pi$ decay, the angular distribution is given in Eqs. (47) and (50). These can be tested in future experiments

To summarize, we have derived the relation $f_{B_0}/f_B = m_{B_0}/m_B$ and have indicated that λ_{B_0}/λ_B is expected to be <1 in the parametrizations $g_{B^*B\pi} = \lambda_B m_B/f_\pi$ $(\lambda_B < 1)$ and $g_{B_0B\pi} = \lambda_{B_0}(2m_B/f_\pi)$, which take into account the possible corrections to the values for $g_{B^*B\pi}$ and $g_{B_0B\pi}$. The parameters λ_B or λ_D and λ_{B_0} or λ_{D_0} determine the form factor $F_+(t)$ for the semileptonic decay of a heavy meson to a light meson. Thus in our approach these decays are correlated with $D^* \to D\pi$ and $D_0 \to D\pi$ decays. We have shown that with $\lambda_{D_s} = \lambda_D = \lambda_B = 1/\sqrt{2}$ and $\lambda_{D_{s0}}/\lambda_{D_s} = 0.875$ or 1, the experimental data on $D \to Kl\nu$ can be fitted with the form factor $F_+(t)$ obtained by us. This fit implies $f_D/f_\pi = 2.2$. For $\lambda_{D_{s_0}}/\lambda_{D_s} = 1$, we get $f_D/f_\pi = 2.5$. The value $\lambda_D = 1/\sqrt{2}$ gives $g_{D^*D\pi} = m_D/\sqrt{2}f_\pi$, i.e., the Kawarabayashi-Suzuki-Riazuddin-Fayyazudin (KSRF) value [19] for the D mesons. The value $\lambda_D = 1/\sqrt{2}$ is favored by the present experimental data on $D^* \to D\pi$ and $D \to Kl\nu$ decays.

The value $\lambda = 1$ gives a pure $V - A$ structure for the heavy meson, i.e., $g_A = 1$. The value $\lambda = 1/\sqrt{2}$ gives a $V - 0.71A$ structure, i.e., $g_A = 0.71$.

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