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Differential distributions in semileptonic decays of heavy flavors in QCD

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A generalization of the operator product expansion is used to find the differential distributions in the inclusive semileptonic weak decays of heavy flavors in QCD. In particular, the double distribution in electron energy and invariant mass of the lepton pair is calculated. We are able to calculate the distributions in an essentially model-independent way as a series in m_Q^{-1} where m_Q is the heavy quark mass. All effects up to m_Q^{-2} are included.

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I. INTRODUCTION

Differential distributions in semileptonic decays of heavy flavors are used for measurements of the Cabbibo-Kobayashi-Maskawa (CKM) matrix elements, key phenomenological parameters of the standard model. To extract the CKM matrix elements from data one needs to disentangle the effects of strong interactions at large distances from the quark-lepton Lagrangian known at short distances.

Up to now essentially two approaches have been applied to describe nonperturbative strong interaction effects in the inclusive weak decays: the naive parton model amended to include the motion of the heavy quark inside the decaying meson [1] and the "exclusive variant" based on summation of different channels, one by one [2]. Both approaches are admittedly model dependent; neither their accuracy nor the connection to the fundamental parameters of QCD are clear *a priori.* Each of them needs an input from a constituent quark model to parametrize nonperturbative effects. The latter play an especially important role in the form of the spectra near the end points.

The need for model-independent QCD-based predictions is apparent. Considerable progress achieved recently in the theory of preasymptotic effects (proportional to powers of $1/m_Q$ where m_Q is the heavy quark mass) allows one to make these predictions.

The theoretical construction presented in this paper is, in a sense, a generalization and combination of the formalisms which are used in deep inelastic scattering and total cross section of e^+e^- annihilation. The expansion parameter in deep inelastic scattering is Q^{-1} where Q is the momentum transfer. In the problem at hand the expansion parameter is m_Q^{-1} or, more exactly, the inverse energy released in the final hadronic state (in the rest frame of the decaying quark).

In classical problems of this type, such as e^+e^- annihilation, there are two alternative ways to get predictions. The first approach having a solid theoretical justification in terms of the operator product expansion (OPE) [3] is based on calculations in the Euclidean domain where one can apply the OPE. Contact with the observable quantities is made through the dispersion relations and in this way predictions for certain integrals are obtained. In the second approach we perform the calculations directly in the Minkowski domain. Although formally this calculation refers to large distances, from the first approach we know that in specific integrals large distance contributions drop out. Therefore results obtained in this way, although not valid literally, should be understood in the sense of duality: Being smeared over some duality interval the theoretical prediction should coincide with the smeared experimental curve. Inclusive weak decays will be treated within the second approach. The averaging mainly refers to the invariant mass of the inclusive hadronic state produced in the decay considered.

If the invariant mass of the final hadronic state is large, this is not a constraint at all since the theory "itself" takes care of the averaging required by duality. In the opposite limit, near a spectral end point, the smearing is not provided for free. The boundary of the distribution corresponds to a low momentum of the quark produced (low momentum of the hadronic final state). At this point the OPE blows up; therefore we do not have any specific prediction for the distributions near the boundary. Nevertheless, the integrals taken over the domain from the kinematical boundary up to a new boundary, defined by the requirement that the OPE be convergent, are predicted. In particular this integration domain should include the resonance range (when m_Q is large a parametrically stronger limitation is imposed by the fact that the heavy quark and meson masses are different). An example of the safe integration is the total decay width where

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the integration domain is maximal.

Although we explicitly work in Minkowski kinematics we always keep in mind the relationship to the Euclidean domain and the corresponding operator product expansion. The first analysis of this type has been outlined in [4] for inclusive heavy flavor decay rates. A general analysis of the semileptonic inclusive spectra along this line is presented in Ref. [5]. In that work it was observed, in particular, that the leading operator and those appearing at next-to-leading order have a gap in dimensions of two units, and, consequently, the $O(m_Q^{-1})$ term should be absent in certain quantities. The analysis presented in [5] was not backed up, however, by concrete calculations of the preasymptotic effects. Recently this formalism has been systematically developed and applied to the nonleptonic decays of heavy flavors [6,7] and the charged-lepton energy spectrum in the semileptonic decays [8] (see also [9]). The present work is a natural continuation of Ref. [8].

We generalize the results of Ref. [8] to find the complete inclusive distributions in the semileptonic decays. The leptonic variables E_e, q^2, and q_0, where E_e is the charged lepton energy and q is the momentum of the lepton pair,[1] are kept fixed which automatically fixes the invariant mass of the inclusive hadronic state. Integrating over q_0 we obtain the double spectral distribution in E_e and q^2.

At the first stage we construct the transition *operator* $T(Q \to X \to Q)$ describing the forward scattering amplitude of the heavy quark Q on a weak current. Our focus is the influence of the "soft" modes (background fields) on the transition operator $T_{\mu\nu}$ which is expressed as an infinite series in the local operators built from gluon and quark fields and bilinear in Q, $\bar{Q}$.

The local operators are ordered according to their dimensions; the coefficient functions contain the corresponding powers of $1/m_Q$ (or $1/E_h$, where E_h is the energy released into the hadronic system). At sufficiently large m_Q or E_h the operators with the lowest dimensions dominate, and the infinite series can be truncated. Generically, we will refer to the power expansion as the $1/m_Q$ expansion, although strictly speaking it is an expansion in $1/E_h$. At the next stage the matrix elements of the relevant operators over the initial heavy hadron H_Q must be evaluated. Unfortunately, in present-day QCD the matrix elements over the hadronic states are not theoretically calculable. In some instances they can be related, through heavy quark symmetries, to measurable quantities [10,2]; in other cases they have to be parametrized. These parameters play the role analogous to the gluon condensate [11]. As a matter of fact, at the level of the leading preasymptotic corrections only two operators are relevant. The matrix element of the first one can be related to the mass splittings of the vector and pseudoscalar heavy mesons. The matrix element of the second one has the meaning of the average square of the spatial momentum of the heavy quark Q in H_Q and the state must be treated as a parameter.

Finally, the observed decay rates and spectra are obtained by taking the discontinuity of the hadronic tensor $\langle H_Q|T_{\mu\nu}|H_Q\rangle$ and convoluting the result with the lepton currents and appropriate kinematic factors.

In this paper we consider the differential distributions in the semileptonic decays at the level of $O(m_Q^{-2})$. The differential distributions are measured experimentally in the B meson decays and will be used for more precise determination of V_{ub}, for example. This was a primary motivation for our investigation. We would like to make it as close to fundamental QCD as possible.

The organization of the paper is as follows. In Sec. II we describe the kinematics and in Sec. III we present the operator product expansion. In Sec. IV we derive the differential distributions. Section V is devoted to the analysis of our distributions and limitations on the their use. Our results are summarized in Sec. VI. The Appendix contains expressions for hadronic invariant functions.

II. KINEMATICAL ANALYSIS

We will consider the inclusive weak decays of the mesons (or baryons) with the open heavy flavor into the lepton pair plus (inclusive) hadronic state

$$H_Q(p_H) \to l(p_l) + \bar{\nu}(p_\nu) + \text{hadrons}.$$

Our final goal is to calculate the differential decay rate

$$\frac{d^3\Gamma}{dE_e dq^2 dq_0}, \tag{1}$$

where E_e is the energy of the emitted electron and $q^\mu = p_l^\mu + p_\nu^\mu$ is the four-momentum of the lepton pair. In order to find the differential distributions we need to know the amplitude of the process, which is given by the expression

$$\mathcal{M} = V_{qQ} \frac{G_F}{\sqrt{2}} \, \bar{e}\,\Gamma_\nu\, \nu \, \langle X|j_\nu|H_Q\rangle. \tag{2}$$

Here V_{qQ} is the corresponding Cabibbo-Kobayashi-Maskawa matrix element, $j_\mu = \bar{q}\Gamma_\mu Q$ is the electroweak currents, and $\Gamma_\mu = \gamma_\mu(1+\gamma_5.)$ (Although our theory is general we will keep in mind the $b \to c$ and $b \to u$ decays, so that $Q = b$ and $q = c$ or u.) The differential distributions we are interested in are given by the modulus squared of the amplitude (2) summed over the final hadronic states.

The modulus squared of the amplitude summed over the final hadronic states can be written as

$$|\mathcal{M}|^2 = |V_{qQ}|^2 G_F^2 M_{H_Q} \, l^{\mu\nu} \, W_{\mu\nu}, \tag{3}$$

where M_{H_Q} is the mass of hadron H_Q, $W_{\mu\nu}$ is the hadronic tensor,

[1] The charged lepton produced will be generically called an "electron" hereafter.

$$W_{\mu\nu} = \sum_X (2\pi)^4 \delta^4(p_{H_Q} - q - p_X) \frac{1}{2M_{H_Q}} \langle H_Q | j^\dagger_\mu(0) | X \rangle \langle X | j_\nu(0) | H_Q \rangle, \quad (4)$$

and $l^{\mu\nu}$ is the lepton tensor:

$$l^{\mu\nu} = 8 \left[(p_e)^\mu (p_\nu)^\nu + (p_e)^\nu (p_\nu)^\mu - g^{\mu\nu}(p_e \cdot p_\nu) + i\epsilon^{\mu\nu\alpha\beta}(p_e)_\alpha (p_\nu)_\beta \right], \quad (5)$$

where $\epsilon^{0123} = 1$. Let us introduce the hadronic structure functions w_i and parametrize the hadronic tensor in the following way:

$$W_{\mu\nu} = -w_1 g_{\mu\nu} + w_2 v_\mu v_\nu + i w_3 \epsilon_{\mu\nu\alpha\beta} v^\alpha q^\beta + w_4 q_\mu q_\nu + w_5 (q_\nu v_\mu + q_\mu v_\nu). \quad (6)$$

Here $q_\mu = (p_e + p_\nu)_\mu$ is the four-momentum of the lepton pair, and $v_\mu = (p_{H_Q})_\mu / M_{H_Q}$ is the four-velocity of the initial *hadron* (not that of the Q quark). Note that we have omitted the structure $q_\mu v_\nu - q_\nu v_\mu$ which cannot appear because of the T invariance. The structure functions w_i depend on two invariant variables $q \cdot v$ and q^2. In the rest frame of H_Q which will be used throughout the paper $q \cdot v = q_0$, and so $w_i = w_i(q_0, q^2)$. The convolution of $W_{\mu\nu}$ with the lepton tensor (5) is given by the expression

$$W_{\mu\nu} l^{\mu\nu} = 4 \{ 2 q^2 w_1 + [4 E_e (q_0 - E_e) - q^2] w_2 + 2 q^2 (2 E_e - q_0) w_3 \}. \quad (7)$$

We see that only three invariant functions are relevant for the processes we are considering in this paper. At this step we encounter the third variable, the electron energy $E_e = p_e \cdot p_{H_Q} / M_{H_Q}$, entering through the leptonic tensor.

Finally the formula for the differential width takes the form

$$\frac{d^3\Gamma}{dE_e\, dq^2 dq_0} = |V_{qQ}|^2 \frac{G_F^2}{64\pi^4} \{ 2q^2 w_1 + [4 E_e (q_0 - E_e) - q^2] w_2 + 2 q^2 (2 E_e - q_0) w_3 \}. \quad (8)$$

This expression concludes the kinematical analysis. Our task is, of course, the calculation of the invariant functions $w_i(q_0, q^2)$. We will proceed to this calculation in the next section.

III. OPERATOR PRODUCT EXPANSION

In this section we will discuss the derivation of the tensor $W_{\mu\nu}$. The operator product expansion (OPE) is similar to that in deep inelastic scattering. It is convenient to introduce the hadronic tensor $h_{\mu\nu}$ (forward scattering amplitude) as

$$h_{\mu\nu} = i \int d^4x e^{-iqx} \frac{1}{2 M_{H_Q}} \langle H_Q | T\{ j^+_\mu(x) j_\nu(0) \} | H_Q \rangle. \quad (9)$$

The absorptive part of this tensor reduces to $W_{\mu\nu}$ discussed above

$$W_{\mu\nu} = (1/i)\, \mathrm{disc}(h_{\mu\nu}). \quad (10)$$

Here $\mathrm{disc}(h_{\mu\nu})$ is the discontinuity of the forward scattering amplitude $h_{\mu\nu}$ on the physical cut in the complex plane of the variable q_0. Of course, $h_{\mu\nu}$ can be expanded into the same set of structures as $W_{\mu\nu}$ [see Eq. (6)]:

$$h_{\mu\nu} = -h_1 g_{\mu\nu} + h_2 v_\mu v_\nu + i h_3 \epsilon_{\mu\nu\alpha\beta} v^\alpha q^\beta + h_4 q_\mu q_\nu + h_5 (q_\nu v_\mu + q_\mu v_\nu), \quad (11)$$

and relation (10) implies that

$$w_i = 2\, \mathrm{Im}\, h_i. \quad (12)$$

Let us remember that $h_{\mu\nu}$ is the matrix element of the transition operator $T_{\mu\nu}$;

$$h_{\mu\nu} = \frac{1}{2 M_{H_Q}} \langle H_Q | T_{\mu\nu} | H_Q \rangle, \quad (13)$$

$$T_{\mu\nu} = i \int d^4x e^{-iqx}\, T\{ j^+_\mu(x) j_\nu(0) \}, \quad (14)$$

and so below we will construct the OPE for the product of currents in Eq. (14). Having in mind the relationship to Euclidean analysis discussed above we will treat our expansion in the same way as a normal Euclidean OPE. In the asymptotic limit $m_Q \to \infty$ the hadronic tensor $h_{\mu\nu}$ is given by the tree graph of Fig. 1. This graph defines the matrix element of the transition operator $T_{\mu\nu}$ over the heavy quark state:

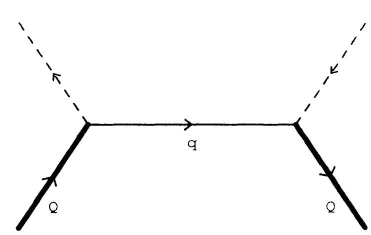

FIG. 1. The tree diagram determining the transition operator $T_{\mu\nu}$ in the leading approximation. The dashed lines correspond to the weak currents, the solid internal line describes the propagation of the quark q, and the bold external lines represent the heavy quark Q.

$$\langle Q|T_{\mu\nu}|Q\rangle = -\bar{u}_Q\Gamma_\mu \frac{1}{\not{P} - \not{q} - m_q}\Gamma_\nu u_Q. \qquad (15)$$

The latter expression represents nothing else but the free quark decay. In the asymptotic regime $m_Q \to \infty$ the interaction of the heavy quark with the gluon and/or light quark medium, as well as its intrinsic motion inside the hadron can be neglected. Then

$$P_\mu = P_{0\,\mu} \equiv m_Q\, v_\mu, \qquad (16)$$

where v_μ is the four-velocity of hadron H_Q .

Equation (15) allows one to immediately write down the operator form in the approximation at hand (only operators bilinear in Q, $\bar{Q}$ are considered; see the discussion of other operators at the end of this section):

$$T_{\mu\nu} = -\bar{Q}\Gamma_\mu \frac{1}{\not{k} - m_q}\Gamma_\nu Q = -\frac{2}{(k^2 - m_q^2)}[g_{\alpha\mu}k_\nu + g_{\alpha\nu}k_\mu - g_{\mu\nu}k_\alpha - i\epsilon_{\mu\nu\alpha\beta}k^\beta]\,\bar{Q}\gamma^\alpha(1+\gamma_5)Q, \qquad (17)$$

where $k = P_0 - q$.

As we see, the two operators $\bar{Q}\gamma_\alpha\bar{Q}$ and $\bar{Q}\gamma_\alpha\gamma_5\bar{Q}$ showed up in the operator expansion at the level considered. Note that the $\bar{Q}\gamma_\alpha\gamma_5\bar{Q}$ term vanishes after averaging over the unpolarized hadronic states.

In this paper the perturbative corrections in α_s are not touched upon at all. As for nonperturbative corrections they appear because of interactions with the soft medium of the light cloud in H_Q. By taking these interactions into account we isolate two types of effects. First, the fast quark q produced does not propagate as a free one, but interacts with the background fields; these corrections will be included explicitly into the OPE coefficients. Second, the heavy quark Q also does not live in the empty space; it is surrounded by the light cloud. In particular, because of this fact the heavy quark momentum does not coincide with $m_Q v_\mu$. This large distance effect will not be calculated explicitly, but implicitly it will be reflected in the H_Q matrix elements of the operators in $T_{\mu\nu}$. This is in full analogy with what is usually done in deep inelastic scattering. The influence of the background field on the transition operator is summarized by the expression

$$T_{\mu\nu} = -\int dx e^{-iqx}\bar{Q}(x)\Gamma_\mu S_q(x,0)\,\Gamma_\nu\, Q(0), \qquad (18)$$

where $S_q(x,0)$ is the propagator of the quark q in an external gluon field A^a_μ. It is convenient to use the Schwinger technique of treating the motion in an external field (for a review of QCD adaptation see, e.g., Ref. [12]). Within that formalism the propagator S_q is presented by the expression

$$S_q(x,0) = \left(x\left|\frac{1}{\not{\mathcal{P}} - m_q}\right|0\right). \qquad (19)$$

Here $\mathcal{P} = \gamma^\mu[p_\mu + A_\mu(X)]$ and $A_\mu = g\,A^a_\mu T^a$ is the gluon field in the matrix representation. Furthermore, the operator of coordinate X_μ and momentum p_μ are introduced [thus the field $A_\mu(X)$ becomes an operator function of X_μ] with the commutation relations

$$[p_\mu, X_\nu] = i\,g_{\mu\nu}, \quad [X_\mu, X_\nu] = 0, \quad [p_\mu, p_\nu] = 0. \qquad (20)$$

The states $|x)$ are the eigenstates of the operator X_μ, $X_\mu|x) = x_\mu|x)$.

Combining Eqs. (18) and (19), we arrive at

$$T_{\mu\nu} = -\int dx e^{-iqx}\left(x\left|\bar{Q}(x)\Gamma_\mu \frac{1}{\not{\mathcal{P}} - m_q}\,\Gamma_\nu\, Q(X)\right|0\right). \qquad (21)$$

As we have discussed above the operator $\mathcal{P}_\mu$ contains a large mechanical part $(P_0)_\mu = m_Q v_\mu$; the deviation from P_0 will be separated explicitly;

$$\mathcal{P}_\mu = (P_0)_\mu + \pi_\mu, \qquad (22)$$

and we will expand in π_μ. In this paper we will limit ourselves to the terms up to $O(\pi^2)$ corresponding to $1/m_Q^2$ corrections. The master formula to perform the expansion is

$$T_{\mu\nu} = -\int dx\left(x\left|\bar{Q}(X)\Gamma_\mu \frac{1}{\not{P}_0 - \not{q} - m_q + \not{\pi}}\,\Gamma_\nu\, Q(X)\right|0\right). \qquad (23)$$

There is a subtle point in the description of the formalism given above. Technically in the computation the $A_\mu(x)$ is assumed to be a c-number background field while in the final expression for local operators it should be understood as a second quantized operator. Since we are not considering any loop corrections, this substitution is justified.

Let us now discuss the set of the operators relevant to the order $O(m_Q^{-2})$. Without loss of generality we can work in the rest frame of the hadron H_Q, i.e., $v_\mu = (1,0,0,0)$. Only those operators will be retained which produce nonvanishing results after being averaged over H_Q. The leading operator, as was discussed above, is

$$\bar{Q}\gamma_0 Q; \qquad (24)$$

its matrix element is fixed by the vector current conservation:

$$\frac{1}{2\,M_{H_Q}}\langle H_Q|\bar{Q}\gamma_0 Q|H_Q\rangle = 1. \qquad (25)$$

Equation (25) is given in the relativistic normalization we are using throughout this paper. In the nonrelativistic normalization there is no need in the factor $1/2M_{H_Q}$ on the left-hand side (LHS).

As has been noted in the Ref. [5] there are no operators of dimension 4 in the problem at hand. The set includes two operators of dimension 5:

$$O_G = \frac{i}{2}\bar{Q}\sigma^{\alpha\beta}G_{\alpha\beta}Q, \quad (26)$$

$$O_\pi = -\bar{Q}\,\mathbf{D}^2 Q = \bar{Q}\,\boldsymbol{\pi}^2 Q, \quad (27)$$

where $\sigma^{\alpha\beta} = \frac{1}{2}(\gamma^\alpha\gamma^\beta - \gamma^\beta\gamma^\alpha)$ and $G_{\alpha\beta} = g\,G^q_{\alpha\beta}T^a$ is the gluon field strength tensor. The classification above takes into account the fact that the quark field Q satisfies the equation of motion. In particular, this classification implies that the operator $\bar{Q}Q$ is not independent but is reducible to three operators (24), (26), and (27):

$$\bar{Q}\,Q = \bar{Q}\,\gamma_0 Q - \frac{1}{2\,m_Q^2}\,\bar{Q}\,\boldsymbol{\pi}^2 Q + \frac{i}{4\,m_Q^2}\,\bar{Q}\sigma^{\alpha\beta}G_{\alpha\beta}Q + O(m_Q^{-4}). \quad (28)$$

To get Eq. (28) we observe that the lower component of Q is related to the upper one in the following way:

$$\frac{1-\gamma_0}{2}\,Q = \frac{1}{2m_Q}\boldsymbol{\pi}\cdot\boldsymbol{\sigma}\frac{1+\gamma_0}{2}\,Q + O(m_Q^{-2}), \quad (29)$$

and the difference between $\bar{Q}Q$ and $\bar{Q}\gamma_0 Q$ is due to the product of the lower components. (Here and below we will stick to the H_Q rest frame.)

A few other useful relations which can be obtained in the same manner and are valid at the level $O(m_Q^{-2})$ are

$$\bar{Q}\boldsymbol{\gamma}\cdot\boldsymbol{\pi}Q = \frac{1}{m_Q}\bar{Q}\left(\boldsymbol{\pi}^2 - \frac{i}{2}\sigma G\right)Q + O(m_Q^{-2}), \quad (30)$$

$$\bar{Q}\boldsymbol{\gamma}\cdot\boldsymbol{\pi}\gamma_0 Q = O(m_Q^{-2}), \quad (31)$$

$$\bar{Q}\pi_0 Q = \frac{1}{2m_Q}\bar{Q}\left(\boldsymbol{\pi}^2 - \frac{i}{2}\sigma G\right)Q + O(m_Q^{-2}). \quad (32)$$

A few comments are in order here concerning the actual technique of constructing the OPE. Since we work in the H_Q rest frame, it is convenient to compute different components of $T_{\mu\nu}$ separately, T_{00}, T_{0i}, T_{i0}, and T_{ij}. The calculation itself is a straightforward although rather tedious procedure of expanding the denominator in Eq. (23) in $\not\pi$ using the properties of the γ matrices, the commutation relation

$$[\pi_\mu, \pi_\nu] = i\,G_{\mu\nu} \quad (33)$$

and Eqs. (30)–(32).

Notice that we must keep the terms of the first order in π_0 and of the second order in $\boldsymbol{\pi}$, since

$$\pi_0\,Q = \frac{(\boldsymbol{\sigma}\cdot\boldsymbol{\pi})^2}{2m_Q}Q + O(m_Q^{-2}). \quad (34)$$

Next, observe that the Green function in the background field can be written as

$$\frac{1}{\not{\mathcal{P}} - \not{q} - m_q} = (\not{\mathcal{P}} - \not{q} + m_q)\frac{1}{(\mathcal{P}-q)^2 + (i/2)\sigma G - m_q^2} \equiv (\not{\mathcal{P}} - \not{q} + m_q)\frac{1}{\Pi}. \quad (35)$$

To transpose $1/\Pi$ with Γ_ν it is convenient to use the identity

$$\frac{1}{\Pi}\Gamma_\nu = \Gamma_\nu\frac{1}{\Pi} + \frac{1}{\Pi}[\Gamma_\nu, \Pi]\frac{1}{\Pi} = \Gamma_\nu\frac{1}{\Pi} + \frac{1}{\Pi}\left[\Gamma_\nu, \frac{i}{2}\sigma G\right]\frac{1}{\Pi}. \quad (36)$$

Acting on Q and using the equations of motion we can now substitute $1/\Pi$ in both terms on the right-hand side by

$$\frac{1}{m_Q^2 - m_q^2 - 2\mathcal{P}q + q^2}, \quad (37)$$

provided that we limit ourselves to terms up to $O(m_Q^{-2})$. The second term in (36) can be simplified even further since here we can additionally neglect π in $\mathcal{P} = P_0 + \pi$.

We split the calculation into three parts: vector×vector, axial×axial, and vector×axial in correspondence with the structure of Γ_μ as a sum of vector and axial vector, $\Gamma_\mu = \gamma_\mu + \gamma_\mu\gamma_5$. The full hadronic tensor $h_{\mu\nu}$ is given then by the expression

$$h_{\mu\nu} = h^{VV}_{\mu\nu} + h^{AA}_{\mu\nu} + h^{AV}_{\mu\nu} + h^{VA}_{\mu\nu} = \frac{1}{2\,M_{H_Q}}\langle H_Q|T^{VV}_{\mu\nu} + T^{AA}_{\mu\nu} + T^{AV}_{\mu\nu} + T^{VA}_{\mu\nu}|H_Q\rangle\,. \quad (38)$$

The complete expressions for the hadronic invariant functions are given in the Appendix. In the order $O(m_Q^{-2})$ they are defined by the matrix elements of operators O_G, O_π given by Eqs.(26), (27):

$$\frac{1}{2\,M_{H_Q}}\left\langle H_Q\middle|\bar{Q}\,\frac{i}{2}\,\sigma_{\mu\nu}G^{\mu\nu}Q\middle|H_Q\right\rangle = \mu_G^2, \quad (39)$$

$$\frac{1}{2\,M_{H_Q}}\langle H_Q|\bar{Q}\,\boldsymbol{\pi}^2 Q|H_Q\rangle = \mu_\pi^2. \quad (40)$$

The parameter μ_G^2 coincides with $m^2_{\sigma H}$ introduced in [15]. For mesonic states it is expressible in terms of the quantity measured experimentally, the hyperfine mass splittings, and it has a zero value for baryonic states of

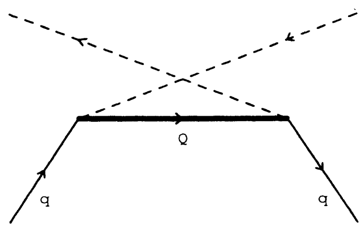

FIG. 2. The tree diagram determining the operator without the heavy quark Q. Now the bold internal line describes the propagation of the heavy quark Q and the solid external lines represent the quark q.

the type of Λ_Q; the parameter μ_π^2 has the meaning of the average square of spatial momentum of the heavy quark Q in the hadronic state H_Q. The two quantities μ_G^2 and μ_π^2 often appear in the combination $\mu_\pi^2 - \mu_G^2$; cf. Eq. (28).

The last comment of this section is about the operators which are not bilinear in $\bar{Q}$, Q fields. The simplest example of appearance of such operators is given by the diagram of Fig. 2 where the heavy quark Q propagates between the current vertices. This diagram is similar to the one of Fig. 1, and the corresponding operator follows from Eq. (17) by the substitution $Q \Rightarrow q$, $m_q \Rightarrow m_Q$, $k_\mu \Rightarrow q_\mu$. The additional term in $T_{\mu\nu}$ has the form

$$\begin{aligned}\Delta T_{\mu\nu} &= -\bar{q}\Gamma_\nu \frac{1}{\not{q} - m_Q}\Gamma_\mu q \\ &= -\frac{2}{(q^2 - m_Q^2)}[g_{\alpha\mu}q_\nu + g_{\alpha\nu}q_\mu - g_{\mu\nu}q_\alpha - i\epsilon_{\mu\nu\alpha\beta}q_\beta] \\ &\quad \times \bar{q}\gamma^\alpha(1+\gamma_5)q. \end{aligned} \tag{41}$$

The matrix element of the operator $\bar{q}\gamma^\alpha q$ over the H_Q state counts the number of quarks q and is not small in general. The operator coefficient given by Eq. (41) is particularly large when $q^2 \to m_Q^2$.

In terms of the intermediate hadronic states in the forward scattering off H_Q this contribution is due to states in the crossing channel containing two Q quarks—the problem was pointed out in Ref. [5]. The cross channel is not related to the weak inclusive decays under consideration. It is reasonable to accept the duality between the operators without heavy quark fields and the cross-channel contributions having nothing to do with heavy flavor decays. We rely on the assumption that we can consistently omit the crossing channel together with operators in $T_{\mu\nu}$ related to this channel.

IV. CALCULATION OF THE DIFFERENTIAL DISTRIBUTIONS

The differential distributions we are interested in are determined by Eq. (8) containing three invariant functions w_1, w_2, and w_3. They are obtained from the results for h_i (see the Appendix) by taking the imaginary parts of the corresponding functions [see Eq. (12)]. The imaginary parts are due to the poles of h_i and are obtained through the relations

$$\mathrm{Im}\frac{1}{z^n} = \pi\,\frac{(-1)^n}{(n-1)!}\,\frac{d^{n-1}}{dz^{n-1}}\,\delta(z), \tag{42}$$

where z is given by

$$z = m_Q^2 - 2\,m_Q\,q_0 + q^2 - m_q^2. \tag{43}$$

We do not present here the expression for the triple differential distribution which can be easily obtained by combining Eqs. (8), (12), (42), and expressions for h_i from the Appendix.

Although the result is derived for the physical quantity $d^3\Gamma/dE_e dq^2 dq_0$, it cannot be directly compared with the experimental data. An obvious signal for this is the presence of the δ function and its derivatives. It is not surprising because we are sitting now right on the mass shell of the q quark. As we discussed in the Introduction our results should be understood in the sense of duality, that is, that the predictions should be smeared over a certain duality interval; At the moment we have no purely theoretical tools to fix the size of the duality interval; therefore we are forced to rely on qualitative arguments and experimental data. For example the duality interval for q_0 can be inferred from the distribution in the invariant mass of the final hadronic states. Our δ functions reflect the resonance structure at low invariant masses. The smearing interval should be chosen in such a way as to cover the entire resonance domain up to the onset of the smooth behavior. Instead of smearing of the distribution one can calculate the average characteristics such as the total width Γ or $\langle M_X^n\rangle$, where M_X is the invariant mass of the final hadronic states. The power corrections we have calculated will enter in a specific way in each particular quantity.

Now let us proceed to the calculation of the double differential distribution $d^2\Gamma/dE_e\,dq^2$. To this end we must integrate over q_0, a rather simple exercise with δ functions. However, if one would perform the integration by merely substituting

$$q_0 \to q_0^* = \frac{m_Q^2 + q^2 - m_q^2}{2\,m_Q}, \tag{44}$$

and taking the derivatives in the case of δ' and δ'', one would get the wrong answer. The point is that the integration domain in q_0 has a boundary from below:

$$q_0 \geq E_e + \frac{q^2}{4\,E_e}, \tag{45}$$

which corresponds to $4\,E_e\,E_\nu \geq q^2$. Therefore one should take into account the fact that q_0 cannot cross the boundary (45). For that we introduce $\theta(q_0 - E_e - q^2/4E_e)$ into the integrand. The occurrence of the θ function is important for the integration of $\delta'(q_0 - q_0^*)$ and $\delta''(q_0 - q_0^*)$ which leads to the appearance of $\delta(q_0^* - E_e - q^2/4E_e)$ and $\delta'(q_0^* - E_e - q^2/4E_e)$ in the double distribution $d^2\Gamma/dq^2\,dE_e$ because of differentiation of the θ function. The final formula for the double differential distribution in the lepton energy E_e and q^2 takes the form

$$\frac{d^2\Gamma}{dx dt} = |V_{qQ}|^2 \frac{G_F^2 m_Q^5}{96\pi^3} x^2 \{ 6(1-t)(1-\rho-x+xt) + G_Q[1-5\rho+2t+10\rho t+10xt-10xt^2 - (-1+6\rho-5\rho^2+x-5\rho x+t-2\rho t+5\rho^2 t+xt+15\rho xt + 5x^2t-2xt^2-10\rho xt^2-10x^2t^2+5x^2t^3)\delta((1-t)(1-x)-\rho)] + K_Q[-3+3\rho+4t-4\rho t-6xt+4xt^2-(1-2\rho+\rho^2-3x+3\rho x-3t+2\rho t+\rho^2 t+11xt-3\rho xt - 3x^2t-6xt^2-2\rho xt^2+2x^2t^2+x^2t^3)\delta((1-t)(1-x)-\rho)+(1-\rho-x+xt)(1-t) \times(1-2\rho+\rho^2-2xt-2\rho xt+x^2t^2)\delta'((1-t)(1-x)-\rho)]\}. \tag{46}$$

Here we have introduced the dimensionless variables

$$x = 2E_e/m_Q, \quad t = q^2/2m_Q E_e, \tag{47}$$

and the parameters

$$\rho = m_q^2/m_Q^2, \quad G_Q = \mu_G^2/m_Q^2, \quad K_Q = \mu_\pi^2/m_Q^2. \tag{48}$$

Let us emphasize that the scale m_Q used in Eq. (47) is the heavy quark mass and does not coincide with M_{H_Q} which is normally used in the experimental distributions.

The fact that the OPE generates corrections only of the order of $O(m_Q^{-2})$ (terms proportional to K_Q and G_Q) is valid for the distributions only if we use m_Q as a scale, i.e., in the variables x, t. Of course one can easily rescale them to M_{H_Q}; then the corrections of the order of $O(m_Q^{-1})$ will show up for trivial kinematical reasons.

We can proceed further and obtain the energy spectrum by integrating over q^2. The range of integration is given by

$$0 \le t \le 1 - \frac{\rho}{1-x}. \tag{49}$$

The result for the energy spectrum coincides with that obtained in [8]. For the sake of completeness we present it here.[2]

$$\frac{d\Gamma}{dx} = |V_{qQ}|^2 \frac{G_F^2 m_Q^5}{192\pi^3}\theta(1-x-\rho)2x^2 \Bigg\{ (1-f)^2(1+2f)(2-x)+(1-f)^3(1-x) + (1-f)\left[(1-f)\left(2+\frac{5}{3}x-2f+\frac{10}{3}fx\right)-\frac{f^2}{\rho}[2x+f(12-12x+5x^2)]\right]G_Q - \left[\frac{5}{3}(1-f)^2(1+2f)x+\frac{f^3}{\rho}(1-f)(10x-8x^2)+\frac{f^4}{\rho^2}(3-4f)(2x^2-x^3)\right]K_Q\Bigg\}, \tag{50}$$

where $f = \rho/(1-x)$. Finally, performing the last integration over x in the domain

$$0 \le x \le 1-\rho, \tag{51}$$

we arrive to the total width coinciding with that in [9]:

$$\Gamma = |V_{qQ}|^2 \frac{G_F^2 m_Q^5}{192\pi^3}[z_0\{1+\tfrac{1}{2}(G_Q-K_Q)\}-2z_1 G_Q], \tag{52}$$

where $z_0 = 1-8\rho+8\rho^3-\rho^4-12\rho^2\ln\rho$ and $z_1=(1-\rho)^4$.

Now let us discuss the characteristic features of the double distribution (46). The most striking one is the presence of the singular terms. The technical reason for occurrence of those terms was that we expanded the denominator of the pole expression (23) in π and σG. Physically this expansion reflects the shifts of the masses of particles due to the nonperturbative effects. As was mentioned above these singularities reflect the structure of the resonance domain and the predictions suitable for comparison with the experimental data require smearing over the corresponding domain. To illustrate the most salient features of our prediction let us concentrate on the physically interesting case of the $b \to u$ transition.

For massless u quark the kinematical region of b quark semileptonic decay is shown in Fig. 3. It has the form of a square with the side equal to 1 in the plane ($x = 2E_e/m_b$, $t = q^2/2m_bE_e$). The right-hand side of the square corresponds to the maximal energy of an electron $E_e = m_b/2$ while the upper side is a maximal energy of a neutrino. In the real B meson decay the kinematical region is certainly wider; if one neglects the pion mass, the region is the square with the side $x_{\max} = t_{\max} = M_B/m_b$. The origin of this window is

[2]Let us draw the reader's attention to the difference of notation: y in [8] is equal to our x.

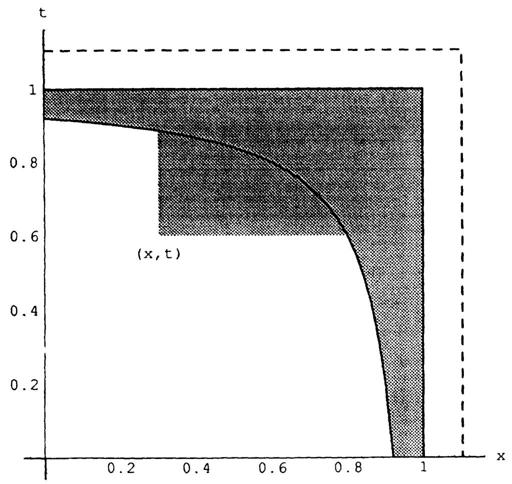

FIG. 3. The kinematical region of the decay for $b \to u$ decays in coordinates $x = 2E_e/m_b$ and $t = q^2/2m_bE_e$. The solid lines are the kinematical boundary for the b quark decay ($x_{\max} = t_{\max} = 1$) and the dashed lines are the boundary for B meson decay ($x_{\max} = t_{\max} = M_B/m_b$). The area of integration for the distribution $P(x,t)$ [see Eq. (55)] is shaded. It includes integration over the resonance domain.

related to the motion of the heavy b quark inside the B meson. In our calculations we account for nonzero momentum of the b quark in the form of an expansion which produced singular δ and δ' terms on the boundary. It is possible to show (see Refs. [8,16]) that the expansion breaks down at distances $\sim (M_B - m_b)/m_b$ near the boundary, and so we need to integrate our distributions over a range of the order of the window between quark and hadron boundaries. It is interesting to note that the distribution spreads off the distances of the order $(M_B - m_b)/m_b$ while the corrections to integrals are only of second order in $1/m_b$.

Another effect we need to account for is the structure of the resonance region near the low end of the hadronic invariant masses. To imitate the effect let us imagine that this region corresponds to the u quark fragmentation into the hadronic states with s (the square of the invariant mass) from $s = 0$ to $s = s_0 = 2$ GeV^2. The curve corresponding to $s = s_0$ in Fig. 3 is given by the equation

$$(1-t)(1-x) = s_0/m_b^2, \tag{53}$$

and the resonance region should be included as a whole into the process of integration; we can predict the integral but not the structure.

V. APPLICATION TO THE ANALYSIS OF THE EXPERIMENTAL DATA

Our theoretical prediction (46) depends on the following parameters: V_{qQ}, m_Q, m_q, K_Q, G_Q. Let us remember that in this paper we do not consider perturbative in α_s corrections (see Ref. [13]), which, of course, should be added. The Cabibbo-Kobayashi-Maskawa matrix element V_{qQ} does not affect the form of the differential distribution; the total semileptonic width is proportional to $|V_{qQ}|^2$. The quark masses enter at the level of the leading approximation while K_Q and G_Q determine $1/m_Q^2$ corrections. It is important that our differential distributions by themselves could be used to fit these parameters. In particular it is a good place to extract the heavy quark mass.

Our purpose here is to give an idea of how important the $1/m_Q^2$ corrections are in the case of charmless B meson decays ($b \to u$ transition). To this end we will use the approximate values for the parameters m_B, K_b, and G_b obtained from other sources. First, we use $m_b \sim 4.8$ GeV as deduced from the QCD sum rule analysis of the Ypsilon system [14], and $m_u = 0$. The parameter G_b can be extracted from the B, B^* mass splitting [15]:

$$G_b = \tfrac{3}{4}[M^2(B^*) - M^2(B)]/m_b^2 \sim 0.017. \tag{54}$$

As a representative value we use for the parameter K_b the value ~ 0.02. A close value was obtained in Ref. [17] from the QCD sum rules. An earlier QCD sum rule result [18] was a factor of 2 higher. Notice that the sensitivity of our results to the value of K_b is essentially less than that to G_b. For example, Eq. (52) for the $b \to u$ transition contains $G_b + \frac{1}{3}K_b$.

In accordance with the discussion at the end of the previous section the comparison with experiment should include the integration of our distribution (46) over the domain which includes the area adjacent to the kinematical boundary. We will choose this area to be given by the resonance domain [see Eq. (53)] with $s_0 = 2$ GeV^2.

Let us introduce the quantity

$$P(x_c,t_c) = \frac{1}{\Gamma_0}\int\int_{A(x_c,t_c)} dx dt\, \frac{d^2\Gamma}{dxdt}, \tag{55}$$

where x_c, t_c is the point in the (x,t) plane sitting not too close to the boundary (outside the resonance range), $\Gamma_0 = |V_{ub}|^2 G_F^2 m_b^5/192\pi^3$, and the area of integration $A(x_c,t_c)$ shown in Fig. 3 as shaded includes the resonance domain plus the domain $x > x_c, t > t_c$. For the experimental distribution the range of integration should be extended to include the window between quark and hadron kinematical boundaries. Notice that in the limit of large m_Q the size of the window $(M_{H_Q} - m_Q)/m_Q$ is parametrically larger then the resonance range s_0/m_Q^2. In the case of the b quark they are numerically close.

The function $P(x,t)$ is plotted as a function of t in Fig. 4 for three values of x equal to 0.3, 0.6, 0.8. The last value of x is close to the border of the resonance region beyond which we cannot make reliable predictions for the distributions considered. The dashed lines in Fig. 4 describe the leading order distributions in t while the solid lines include the QCD corrections we have calculated. As we can see from the curves, the corrections are negative and their relative magnitude is larger near the end points of the spectra.

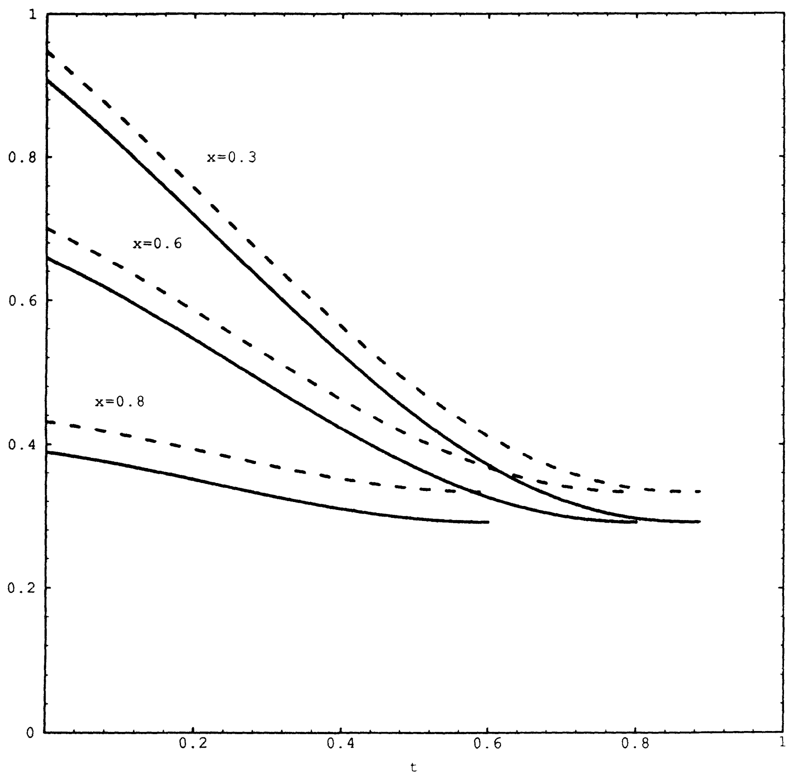

FIG. 4. The integrated distribution $P(x,t)$ [see Eq. (55)] for the case $b \to u$ is plotted as a function of $t = q^2/2m_bE_e$ for few values of $x = 2E_e/m_b$. The dashed lines correspond to the leading order distribution while the solid lines account for nonperturbative corrections [see Eq. (46)]. The lines stop at the border of the resonance region. It follows from the picture that the corrections are negative.

VI. CONCLUSION

Let us now summarize our results. A model, independent approach to nonperturbative effects $(1/m_Q)^n$ is used for calculations of differential distributions. The effects are most pronounced near the end points of the spectra. We discussed how the comparison with experiment should be formulated accounting for the boundary effects. Somewhat disappointing is that we cannot use our results to improve an extraction of V_{ub} by the consideration of q^2 dependence. Indeed, experimentally the signal of $b \to u$ is due to the range of electron energy E_e near the upper end where $b \to c$ is absent. However, as follows from Fig. 3 the distribution in q^2 at such energies is concentrated in the resonance domain, and no model-independent prediction emerges.

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APPENDIX: HADRONIC INVARIANT FUNCTIONS

Here we present the results of calculations of different hadronic invariant functions h_i, introduced by Eqs. (11), (38). The structure functions w_i are simply related to h_i by Eqs. (12) and (42). We use the following notation: $q_0 = q\cdot v$, $\mathbf{q}^2 = q_0^2 - q^2$, and $z = m_Q^2 - 2m_Q q_0 + q^2 - m_q^2$.

For the vector×vector functions we have

$$h_1^{VV} = -\left[(m_Q - m_q - q_0) - (\mu_G^2 - \mu_\pi^2)\frac{1}{2m_Q}\left(\frac{1}{3} + \frac{m_q}{m_Q}\right)\right]\frac{1}{z}$$

$$-\frac{1}{m_Q}\left[\frac{1}{3}\mu_G^2\left[(4m_Q - 3q_0)(m_Q - m_q - q_0) + 2\mathbf{q}^2\right] + \mu_\pi^2\left[q_0(m_Q - m_q - q_0) - \frac{2}{3}\mathbf{q}^2\right]\right]\frac{1}{z^2}$$

$$-\frac{4}{3}\mu_\pi^2\mathbf{q}^2(m_Q - m_q - q_0)\frac{1}{z^3}\,, \tag{A1}$$

$$h_2^{VV} = -\left[2\,m_Q - \frac{5}{3m_Q}\,(\mu_G^2 - \mu_\pi^2)\right]\,\frac{1}{z} - \frac{2}{3}\,[\,2\mu_G^2\,(m_Q - m_q) - 5\,\mu_G^2\,q_0 + 7\,\mu_\pi^2\,q_0]\,\frac{1}{z^2}\; - \frac{8}{3}\,m_Q\,\mu_\pi^2\,\mathbf{q}^2\,\frac{1}{z^3}, \tag{A2}$$

$$h_3^{VV} = 0\,, \tag{A3}$$

$$h_4^{VV} = -\frac{4}{3m_Q}\,(\mu_\pi^2 - \mu_G^2)\,\frac{1}{z^2}\,, \tag{A4}$$

$$h_5^{VV} = \frac{1}{z} - \frac{1}{3}\left[5\,\frac{q_0}{m_Q}(\mu_G^2\, - \mu_\pi^2\,) - 4\,\mu_\pi^2\right]\,\frac{1}{z^2} + \frac{4}{3}\,\mu_\pi^2\,\mathbf{q}^2\,\frac{1}{z^3}\,. \tag{A5}$$

To get the functions h_i^{AA} for axial×axial tensor from h_i^{VV} one should substitute m_q by $(-\,m_q)$ in Eqs. (A1)–(A5). For the axial×vector tensor only one invariant structure survives:

$$h_3^{VA} = \frac{1}{z} + \left[2\,\mu_G^2 + \frac{5}{3}\,(\mu_\pi^2 - \mu_G^2)\,\frac{q_0}{m_Q}\right]\,\frac{1}{z^2}\; + \frac{4}{3}\,\mu_\pi^2\,\mathbf{q}^2\,\frac{1}{z^3}\,. \tag{A6}$$

Summing up we get the result for the full hadronic tensor $h_{\mu\nu}$:

$$h_1 = -\left[2\,(m_Q - q_0) - \frac{1}{3m_Q}\,(\mu_G^2 - \mu_\pi^2)\right]\,\frac{1}{z}$$
$$-\left[\frac{2}{3m_Q}\,\mu_G^2\,(4\,m_Q^2 + 2\,\mathbf{q}^2 - 7\,m_Q\,q_0 + 3\,q_0^2) + \frac{\mu_\pi^2}{2m_Q}\left(4\,q_0\,(m_Q - q_0) - \frac{8}{3}\,\mathbf{q}^2\right)\right]\,\frac{1}{z^2} - \frac{8}{3}\,\mu_\pi^2\,\mathbf{q}^2\,(m_Q - q_0)\,\frac{1}{z^3}\,, \tag{A7}$$

$$h_2 = -\left[4\,m_Q + \frac{10}{3m_Q}\,(\mu_\pi^2 - p\mu_G^2)\right]\,\frac{1}{z} - \left[\frac{28}{3}\,\mu_\pi^2\,q_0 + \mu_G^2\left(\frac{8}{3}\,m_Q - \frac{20}{3}\,q_0\right)\right]\,\frac{1}{z^2}\; - \frac{16}{3}\,\mu_\pi^2\,m_Q\,\mathbf{q}^2\,\frac{1}{z^3}\,, \tag{A8}$$

$$h_3 = -\,2\,\frac{1}{z} - \left[4\,\mu_G^2 + \frac{10}{3}\,(\mu_\pi^2 - \mu_G^2\,)\,\frac{q_0}{m_Q}\right]\,\frac{1}{z^2} - \frac{8}{3}\,\mu_\pi^2\,\mathbf{q}^2\,\frac{1}{z^3}\,, \tag{A9}$$

$$h_4 = -\frac{8}{3m_Q}\,(\mu_\pi^2 - \mu_G^2)\,\frac{1}{z^2}\,, \tag{A10}$$

$$h_5 = 2\,\frac{1}{z} - \frac{2}{3}\left[5\,(\mu_G^2 - \mu_\pi^2)\,\frac{q_0}{m_Q}\; - 4\,\mu_\pi^2\right]\,\frac{1}{z^2} + \frac{8}{3}\,\mu_\pi^2\,\mathbf{q}^2\,\frac{1}{z^3}\,. \tag{A11}$$

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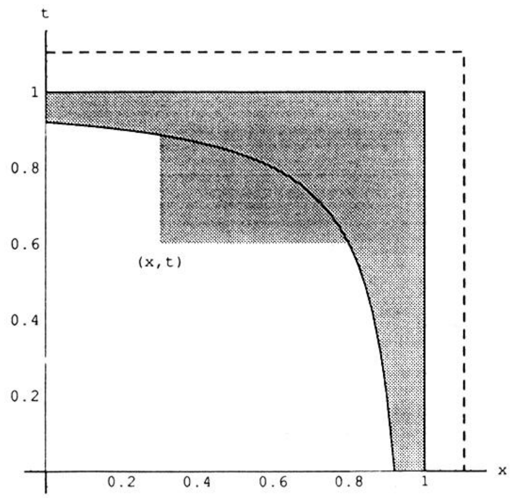

FIG. 3. The kinematical region of the decay for $b \to u$ decays in coordinates $x = 2E_e/m_b$ and $t = q^2/2m_bE_e$. The solid lines are the kinematical boundary for the b quark decay ($x_{\max} = t_{\max} = 1$) and the dashed lines are the boundary for B meson decay ($x_{\max} = t_{\max} = M_B/m_b$). The area of integration for the distribution $P(x,t)$ [see Eq. (55)] is shaded. It includes integration over the resonance domain.