Production, decay, and polarization of excited heavy hadrons

Adam F. Falk* and Michael E. Peskin

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309

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We discuss the production via fragmentation of excited heavy mesons and baryons, and their subsequent decay. In particular, we consider the question of whether a net polarization of the initial heavy quark may be detected, either in a polarization of the final ground state or in anisotropies in the decay products of the excited hadron. The result hinges in part on a nonperturbative parameter which measures the net transverse alignment of the light degrees of freedom in the fragmentation process. We use existing data on charmed mesons to extract this quantity for certain excited mesons. Using this result we estimate the polarization retention of charm and bottom baryons.

I. INTRODUCTION

It is well known that many properties of a hadron containing a single heavy quark Q simplify considerably in the large mass limit $m_Q \to \infty$ [1]. For $m_Q \gg \Lambda_{QCD}$, the light degrees of freedom become insensitive to the mass m_Q, and as far as they are concerned the heavy quark acts simply as a nonrecoiling source of color. Hyperfine effects associated with the heavy-quark chromomagnetic moment also decouple, and a new "heavy-quark, spin-flavor symmetry" emerges. There is now an extensive literature in which this symmetry has been used to make rigorous, model-independent predictions relating heavy hadron spectra, weak matrix elements, and strong decay rates. Corrections to the $m_Q \to \infty$ limit, both radiative and nonperturbative, have been explored in great detail [2].

In this article we will apply the same symmetries to the production of heavy mesons and baryons. In the limit $m_Q \to \infty$ such a process factorizes into short-distance and long-distance pieces. A heavy quark Q is first produced via some high-energy interaction, perhaps as part of a pair $Q\bar{Q}$ with large relative momentum. This process, for example, the decay of a virtual photon or Z boson, is typically calculable in perturbation theory. This perturbative stage is finished in a time short compared to the time scale of the nonperturbative strong interactions. Over a longer time scale, a fragmentation process occurs which eventually forms a physical hadron containing the heavy quark. One might visualize this process as the splitting of a color flux tube which joins the heavy Q to the other colored products of the hard reaction. However one models the fragmentation process, it occurs entirely at length scales of order $1/\Lambda_{QCD}$, and hence involves the redistribution of energies small compared to m_Q. As a result, the velocity of Q remains unchanged once it has

been produced, and its mass and spin, which are determined by the calculable short-distance physics, decouple from the nonperturbative dynamics. The situation here is entirely analogous to that of the much-explored weak decays of heavy hadrons. This analogy has already been exploited in discussions of the production of ground-state pseudoscalar and vector mesons [3].

It is tempting to generalize this philosophy directly to the production of excited heavy mesons and heavy baryons. For these systems, a major issue is the question of the polarization of the heavy state along the axis of fragmentation. We will show that when one computes this polarization, the factorization of heavy- and light-quark physics in the fragmentation process is not quite so straightforward.

Two new ingredients enter the analysis. First, it is often the case that the strict heavy-quark approximation fails for the last stage of fragmentation in systems with c and b quarks. We will present some examples in which light-quark rearrangements, with rates formally independent of m_Q, are slowed by phase-space or angular momentum factors so that they become comparable to the rate, of order $(m_Q)^{-1}$, for processes that flip the heavy-quark spin.

The possibility to transfer angular momentum from the heavy to the light degrees of freedom means that the final heavy-quark polarization will depend on the polarization of the light degrees of freedom created in the fragmentation process. This brings in the second new feature of the analysis. Since fragmentation is a strong interaction process which conserves parity, it cannot select a preferred spin direction along the axis of fragmentation. However, the strong interactions can produce the light degrees of freedom in a way which is anisotropic about this axis, for example, preferring states with longitudinal to those with purely transverse polarization. We will define parameters w_j which characterize the alignment of light degrees of freedom and spin j and show how these affect the polarization of the heavy hadrons and their decay products. The w_j are new parameters of potential importance which provide nontrivial tests of fragmentation models.

Our analysis is organized as follows: In Sec. II, we will

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*On leave from The Johns Hopkins University, Baltimore, MD 21218. Present address: Department of Physics 0319, University of California at San Diego, La Jolla, CA 92039-0319.

give a more detailed discussion of the relative time scales in heavy-quark fragmentation. In Sec. III, we will discuss the polarization of heavy quarks in ground-state heavy mesons D, D^* and B, B^*. This is the simplest case, but we will see that here all polarization information is lost in the fragmentation process. In Sec. IV, we will discuss the polarization of excited heavy mesons. Here we will identify reactions in which light-quark processes are hindered below the heavy-quark spin-flip time. This affects the dependence of the heavy-meson decay distributions on the fragmentation orientation. We will determine the orientation parameter $w_{3/2}$ from data on excited charmed mesons.

In Sec. V, we will discuss the polarization of heavy baryons. The ground-state heavy baryon is the Λ_Q, the bound state of a heavy quark with a light diquark system of spin zero. Using this identification and the $m_Q \to \infty$ limit, Mannel and Schuler [4] and Close, Körner, Phillips, and Summers [5] have argued that Λ_b's produced at the Z^0 resonance should be highly polarized. The second of these groups also pointed out the potential for depolarization when Λ_b's are produced by the decay of excited baryons Σ_b and Σ_b^*. We will discuss this effect quantitatively and show that it potentially leads to significant depolarizations in an interesting pattern. These effects can also be seen in the study of charmed baryons. We will show how these effects are sensitive to the basic parameters governing baryon fragmentation and decay and suggest ways to determine these parameters experimentally.

II. TIME SCALES IN
HEAVY-QUARK FRAGMENTATION

We are concerned in this paper with the dynamics of the spin of a heavy quark produced in a fragmentation process. To begin, we will discuss in this section the various time scales which arise in heavy-quark fragmentation. This will provide a consistent framework for our later analysis.

We always imagine that we begin with a heavy quark which has been ejected at relativistic speed from a hard reaction. We will compute time in the frame of the heavy quark. The axis linking this frame to the center-of-mass frame of the hard process is a preferred direction, which we call the axis of fragmentation. We will take the $\hat{3}$ axis to lie along this line, pointing in the direction of the heavy-quark velocity.

In the rest frame of the heavy quark, the leading operator which couples to the heavy-quark spin is the color magnetic-moment operator, whose coefficient is suppressed by $1/m_Q$. Thus the rate of heavy-quark spin flip is very slow on the scale of Λ_{QCD}. We might imagine the early stages of fragmentation to involve the production of highly excited mesons or baryons containing the heavy quark, which then rapidly eject pions and decay to lighter excited states. Throughout this process, the heavy-quark spin retains its initial orientation. The process continues until we reach a state whose lifetime is comparable to the time required to flip the heavy-quark spin.

This long-lived heavy-quark state is characterized by two angular momenta: $s = \frac{1}{2}$, the heavy-quark spin, and j, the spin of the light degrees of freedom. The combination gives states of total spin $J = j \pm \frac{1}{2}$, which we will call H and H^*. The color magnetic-moment interaction produces a small mass splitting between H and H^*, which we call Δ. This energy splitting Δ can be identified with the rate of heavy-quark spin-flip processes in the (H, H^*) multiplet.

The states of the heavy-quark multiplet can decay either by transitions involving the heavy or light quarks separately or by transitions $H^* \to H$. In the former case, H and H^* have the same decay rate Γ. We will call the rate of the $H^* \to H$ transition γ. This later decay is a QCD or QED magnetic dipole transition. Thus it is suppressed by two powers of $1/m_Q$ from the square of the matrix element and by further powers from the phase space. We expect, then, that $\gamma \ll \Delta$. On the other hand, the overall decay rate Γ may have an arbitrary relation to these two parameters.

To visualize the roles of the three rates Δ, Γ, and γ, it is useful to think about the three possible extreme cases.

(1) $\Gamma \gg \Delta \gg \gamma$. In this case, the heavy hadrons decay so rapidly that the color magnetic-moment interactions of the heavy quark with the light degrees of freedom do not have time to work. If Γ is a rate of a strong-interaction decay process, then in this case the multiplet (H, H^*) would belong to the early stages of fragmentation, in the sense described above, and transitions through this multiplet would have no effect on the heavy-quark spin dynamics. Another possibility, if the quark mass is extremely large, is that the dominant contribution to Γ could come from the heavy-quark weak decay. In this circumstance, as long as $\Gamma \gg \Delta$, the weak-interaction decay will measure a spin orientation for the heavy quark, which is the same as that produced in the hard process, with no depolarization by fragmentation. This is the case which typically arises in studies of the top quark [6]. Notice that the approximation $\Gamma \gg \Delta$ can be valid even if $\Gamma \sim \Lambda_{QCD}$, so that the heavy quark partially hadronizes before it decays.

(2) $\Delta \gg \Gamma \gg \gamma$. In this case, the heavy hadron states H and H^* form distinct resonances. These resonances have width Γ and are well separated from one another. The decay products reflect the heavy-quark spin orientation in the separate states H and H^*. These two contributions must be added incoherently; thus the heavy quark is depolarized from its initial orientation. In Secs. IV and V we will give examples in which this limit applies, even though Γ is the rate of a strong-interaction decay process.

(3) $\Delta \gg \gamma \gg \Gamma$. In this case, the heavy hadrons H^* have time to make the transition to H before undergoing a decay out of the multiplet. In this case, the decay products of the multiplet reflect only the heavy-quark spin orientation in the state H. This leads to a substantial (and sometimes complete) depolarization. The simplest example of this situation arises in the production of B and B^* mesons in fragmentation; we will discuss this example in Sec. III.

In our arguments in the next few sections, we will be-

gin by assuming that the initial heavy quarks produced by the hard process are completely polarized. At some stage, though, we must go over to the realistic situation in which they are produced with partial polarization. We will denote the initial heavy-quark polarization by P. Since the Z^0 resonance provides the most accessible source of polarized heavy quarks, and since Z^0 decays produce mainly left-handed quarks, we will define the polarization to be positive in this case. At the Z^0,

$$P_q = A_{LR}^q = \frac{g_{Lq}^2 - g_{Rq}^2}{g_{Lq}^2 + g_{Rq}^2} , \tag{2.1}$$

so that

$$P_b = 0.94 , \quad P_c = 0.67 , \tag{2.2}$$

for $\sin^2\theta_{W*} = 0.232$. In the course of this paper, we will investigate what fractions of these very large values are actually visible to experimenters.

III. HEAVY PSEUDOSCALAR AND VECTOR MESONS

The simplest example with which to start is that in which the light degrees of freedom have spin-parity $j^P = \frac{1}{2}^-$. The constituent quark model would suggest that such a state, consisting of a light antiquark in an S wave, is the one of lowest energy, and in the charm and bottom systems this has indeed been observed to be the case. This light-quark system combines with the heavy-quark Q to form the multiplet (H, H^*) consisting of a heavy pseudoscalar meson and a heavy vector meson. The states are split by an amount of order Λ_{QCD}^2/m_Q. In the charm system, this is the (D, D^*) multiplet; for the bottom, it is the $(\bar{B}, \bar{B}^*)$ system. In the following discussion, we will refer to the spin of the light degrees of the freedom loosely as the "spin of the antiquark."

In the charm case, most of the parameters of this system are well determined. The D-D^* splitting Δ is approximately 140 MeV. Although $\Delta > m_\pi$ and the strong decay $D^* \to D\pi$ occurs, it is so suppressed by phase space that as yet there is only an upper limit on the intradouble transition width, $\gamma < 1.1$ MeV for D^{*0}, <2 MeV for D^{*+}. However, quark model estimates lead one to believe that γ should be no more than an order of magnitude smaller than this upper bound. Finally, since the D meson can only decay weakly, its width Γ is extremely small, of the order of 10^{-10} MeV. Hence we are safely within the region $\Delta \gg \gamma \gg \Gamma$ discussed above. A similar picture applies for the bottom mesons. Here $\Delta = 46$ MeV. Because the strong decay $\bar{B}^* \to \bar{B}\pi$ is prohibited, the transition must occur electromagnetically. The width γ for $\bar{B}^* \to B\gamma$ may be estimated from the upper limit on $D^* \to D\pi$ and the branching ratio for $D^* \to D\gamma$; we find an approximate value $\gamma \sim 0.01$ MeV. The multiplet width Γ is again due to a weak decay and so is many orders of magnitude smaller. In both cases, we are in the situation of case 3 described in Sec. II. For concreteness, we will refer to the bottom system in the following discussion.

We begin with the case in which the initial b quark is completely polarized in the left-handed direction. We would like to investigate whether any information on the

initial b polarization can be recovered experimentally. The fragmentation process leads to a heavy meson in which the b is combined with an antiquark (more carefully, with light degrees of freedom with $j = \frac{1}{2}$). We may assume that the fragmentation process occurs so rapidly that the color magnetic forces do not have time to act; thus the spin of the antiquark is uncorrelated from the spin of the b. In this case, there are only two choices for the spin orientation of the antiquark: $j^3 = \pm\frac{1}{2}$; we must sum over these possibilities incoherently. Since the fragmentation process conserves parity, the antiquark spin cannot be preferentially aligned in one direction along the axis of fragmentation; thus the two choices occur with equal probability. Hence the result of fragmentation is to produce meson states with the quark and antiquark spins

$$|\downarrow\rangle_b |\downarrow\rangle_{\bar{q}}, \quad |\downarrow\rangle_b |\uparrow\rangle_{\bar{q}} \tag{3.1}$$

with equal probability. Notice that the second state in (3.1) is a linear combination of a $\bar{B}$ and a $\bar{B}^*$ meson. The two components of this state propagate coherently up to a time Δ^{-1} and then go out of phase with one another. Since, in this example, $\Delta \gg \gamma \gg \Gamma$, the $\bar{B}$ and $\bar{B}^*$ components become completely incoherent before any decay occurs. This gives rise to the following table of probabilities for the occupation of the various possible helicity states:

$$\begin{bmatrix} p(\bar{B}^*, h) \\ p(\bar{B}, h) \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & 0 \\ & \frac{1}{4} & \end{bmatrix} . \tag{3.2}$$

The helicity of the $\bar{B}$ runs across the table from negative to positive values; for example, the table assigns the state $\bar{B}^*(h = -1)$ the probability $\frac{1}{2}$.

At a time γ^{-1}, the $\bar{B}^*$ mesons decay electromagnetically to $\bar{B}$'s. After this point, the $\bar{B}$'s will contain no polarization information, since the $\bar{B}$ meson has spin zero. Thus the polarization information can only be encoded in the photons emitted in the decay.

The decay $\bar{B}^* \to \bar{B}\gamma$ proceeds primarily through the light-quark magnetic-moment operator

$$\frac{e_q \sigma_q \cdot \mathbf{B}}{2m_q} , \tag{3.3}$$

since the b magnetic moment is suppressed by $1/m_b$. Let θ be the angle between the photon momentum and the fragmentation axis in the $\bar{B}^*$ rest frame. Then the differential partial widths $d\gamma/d\cos\theta$ for the various $\bar{B}^*$ helicity states are proportional to

$$\bar{B}^*(\pm 1): \quad \frac{1}{2}(1 + \cos^2\theta) ,$$
$$\bar{B}^*(0): \quad \sin^2\theta . \tag{3.4}$$

Multiplying these rates by the probabilities for producing the helicity states $\bar{B}^*(\pm 1)$ and $\bar{B}^*(0)$, we find that the total distribution is proportional to

$$\frac{1}{4}(1 + \cos^2\theta) + \frac{1}{4}\sin^2\theta = \frac{1}{2} . \tag{3.5}$$

Hence the photons are emitted isotropically, and their angular distribution gives no polarization information. The emitted photons are preferentially polarized left-

handed, but this polarization cannot be observed by a standard high-energy particle detector. We conclude that the polarization of the b quark is unobservable in fragmentation to $\bar{B}$ and $\bar{B}^*$ mesons.

This is our first example of a "no-win" theorem, to which we shall return. Under most conditions, the angular distribution of decay products gives no information on the polarization of the heavy quark. In reaching this conclusion, we do not assume that the heavy-quark spin is decoupled from the decay process. In this example, the heavy-quark spin couples to the light antiquark, giving it a net polarization $\frac{1}{2}$ on a time scale of order Δ^{-1}. However, the strong and electromagnetic interactions responsible for the decay conserve parity and thus cannot be sensitive to the direction of the heavy-quark spin. Thus the angular distribution of the decay products is the same as it would be if we averaged over the two possible directions of the heavy-quark spin. There is one amusing exception to this rule, which we will discuss in Sec. IV.

As a footnote to this section, we comment on the validity of the helicity distributions (3.2) for the charmed mesons. The heavy-quark limit predicts that, when we average over the direction of the heavy-quark spin, we recover the naive spin-counting predictions that the D and D^* mesons are produced in a 1:3 ratio, and that the D^* mesons are unpolarized. The latter result is confirmed by a CLEO measurement [7] which finds only a few percent longitudinal polarization in D^*'s produced directly from e^+e^- annihilation. However, many groups have measured the ratio $P_V=(D^*)/(D+D^*)$, which spin counting predicts to be 0.75, and find a substantially smaller number [10]:

$$P_V = 0.65 \pm 0.06 \ . \tag{3.6}$$

Such a value would not be unexpected in a thermodynamic model of particle production in which the higher-mass states are suppressed by a factor

$$\exp[-\Delta m / T_H] \ , \tag{3.7}$$

where Δm is the D^*-D mass difference and T_H is a hadronic "temperature," which should be expected to be about 300 MeV. Indeed, the central value of (3.6) is reproduced by setting $T_H = 280$ MeV. Notice that the suppression factor (3.7) does formally tend to 1 in the heavy-quark limit in which members of the same heavy-quark multiplet become degenerate. However, for the charmed mesons, it gives almost a factor 2 suppression. The correction results from the fact that the excited charm states which decay to D and D^* have widths which are comparable to the D^*-D mass difference, and so can resolve these two states and prefer the lighter D. This is a first example of the competition between decay rates and mass splittings which we will discuss quantitatively in the later sections of this paper.

In the examples discussed later in this paper, we will continue to ignore the thermodynamic factor (3.7) in the initial probability distributions of heavy mesons. In those later examples, this assumption will be justified by the fact that the states which decay to the (H,H^*) multiplet

in those cases typically have widths much larger than the H-H^* mass splitting.

IV. EXCITED HEAVY MESONS

We now turn to the more complicated case of heavy mesons in which the light degrees of freedom are in an excited state. We will focus on the charm system, and, in particular, on the observed excited charmed mesons $D_1(2420)$ and $D_2^*(2460)$. We will discuss the decay distributions of these states from the viewpoint of heavy-quark symmetry.

In the quark model, the lowest-energy excited states of the D and D^* mesons should be states in which the light antiquark has one unit of orbital angular momentum. By coupling this angular momentum to the antiquark spin, we find states in which the light degrees of freedom have $j^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$. In the $m_c \to \infty$ limit, the angular momentum j is a good quantum number irrespective of its quark-model interpretation.

It is reasonable to identify the spin-1 $D_1(2420)$ and the spin-2 $D_2^*(2460)$ as the heavy-meson multiplet (H,H^*) with $j^P = \frac{3}{2}^+$ [11,12]. The $j^P = \frac{1}{2}^+$ doublet, consisting of a spin-0 (D_0^*) and a spin-1 (D_1') meson, has not yet been identified. At order $1/m_c$, there may be mixing between the D_1 and the D_1' states, since they have identical quantum numbers.

It is likely that the (D_0^*, D_1') doublet has not been found because these states have a very large decay width to D and D^*. They should decay by emitting a pion in the S wave, a completely open channel. Kaidalov and Nogteva [13] have estimated the width Γ for this multiplet to be several hundred MeV. On the other hand, the mass splitting Δ should be smaller than 40 MeV, the mass splitting of the $j = \frac{3}{2}$ multiplet. Thus, this doublet corresponds to the uninteresting case 1 of Sec. II, $\Gamma \gg \Delta$.

The situation for the observed D_1 and D_2^* is more interesting. Since the j^P of the light degrees of freedom changes from $\frac{3}{2}^+$ to $\frac{1}{2}^-$, the decay pion must be emitted into an orbital D wave, and so the decay width is suppressed by angular momentum factors. The observed decay width Γ of the two members of the doublet is about 20 MeV, while the observed splitting Δ is approximately 35 MeV [14,15]. The intradoublet transition is an electromagnetic decay, so γ is much smaller than either of these rates. In the following discussion, we will treat the decays of D_1 and D_2^* in the limit $\Delta \gg \Gamma \gg \gamma$, case 2 of Sec. II. This is justified as a first approximation. Since the D_1 and D_2^* peaks are well separated compared to their width, their decays can be treated incoherently.

Because the experiments of Refs. [14] and [15] were carried out well below the Z^0, the charmed quarks were produced from e^+e^- annihilation with no polarization. Nevertheless, for full generality, we will begin our analysis by assuming that the charmed quarks have complete left-handed polarization. To this polarized charmed quark, we must add the light $j = \frac{3}{2}$ system. This system can be formed in one of four possible helicity states. Parity invariance requires that the probability of forming a given helicity state cannot depend on the sign of this heli-

city j^3. However, states with different magnitudes $|j^3|$ can have different probabilities. For the examples discussed in this paper, we can characterize these probabilities in the following way: *For a system of light degrees of freedom of spin j, let w_j be the probability that fragmentation leads to a state with the maximum value of $|j^3|$.* The parameter w_j takes values between 0 and 1.

In the case at hand, the various helicity states of the light degrees of freedom appear with the probabilities

$$p(\tfrac{3}{2},j^3)=(\tfrac{1}{2}w_{3/2},\tfrac{1}{2}(1-w_{3/2}) ,$$

$$\tfrac{1}{2}(1-w_{3/2}) ,\tfrac{1}{2}w_{3/2}) , \qquad (4.1)$$

where the helicity j^3 of the light degrees of freedom runs across the table from $-\tfrac{3}{2}$ to $\tfrac{3}{2}$. The state of definite left-handed c spin, combined with the state of the light degrees of freedom of definite j^3, produces a coherent linear superposition of the D_1 and D_2^* states of helicity $h=j^3-\tfrac{1}{2}$. In a time Δ^{-1} into the fragmentation process, the D_1 and D_2^* components of this state become incoherent, and it becomes appropriate to describe the original state as a mixed state containing D_1 or D_2^* with fixed probabilities. Following this logic, we find that the possible helicity states of D_1 and D_2^* should be populated with the probabilities shown by

$$\begin{bmatrix} p(D_2^*,h) \\ p(D_2,h) \end{bmatrix}$$

$$= \begin{bmatrix} \tfrac{1}{2}w_{3/2} & \tfrac{3}{8}(1-w_{3/2}) & \tfrac{1}{4}(1-w_{3/2}) & \tfrac{1}{8}w_{3/2} & 0 \\ & \tfrac{1}{8}(1-w_{3/2}) & \tfrac{1}{4}(1-w_{3/2}) & \tfrac{3}{8}w_{3/2} & \end{bmatrix} .$$

$$(4.2)$$

The notation is as in (3.2), with the values of the helicity running from negative to positive across the table. To find the probabilities for charmed quarks with initial polarization $P=0$, average the probabilities of states with equal and opposite helicities. Notice that for any value of $w_{3/2}$, and for any P, the total probabilities for producing the spin-2 and spin-1 states are $\tfrac{5}{8}$ and $\tfrac{3}{8}$, respectively.

Given these probabilities, we may now compute the angular distributions for the observed decays $(D_1,D_2^*)\to(D,D^*)+\pi(p)$. The general theory of pion transitions between heavy hadrons is due to Isgur and Wise [11], and is reviewed in Appendix A. According to this theory, the rate for the pion transition from a heavy hadron with light degrees of freedom with spin j to a heavy hadron with light degrees of freedom of spin j' depends on the total spins J,J' of the initial and final hadrons according to the factor

$$(2j+1)(2J'+1)\left|\begin{Bmatrix} j' & j & L \\ J & J' & \tfrac{1}{2} \end{Bmatrix}\right|^2 p_\pi^{2L+1} . \qquad (4.3)$$

In this equation, L is the pion orbital angular momentum, the brackets denote a 6-j symbol, and p_π is the pion three-momentum. For the transitions from (D_1,D_2^*) to (D,D^*), $L=2$. The last factor in (4.3) is the kinematic suppression factor for emitting pions of large L, which may vary significantly over the heavy multiplets even if their splitting is small. The purely group-theoretic factors give [11]

$$\Gamma(D_1\to D\pi) : \Gamma(D_1\to D^*\pi) : \Gamma(D_2^*\to D\pi) : \Gamma(D_2^*\to D^*\pi)=0 : 1 : \tfrac{2}{5} : \tfrac{3}{5} . \qquad (4.4)$$

The kinematic factor p_π^5 for these decays are

$$4.5 : 0.90 : 6.2 : 1.4 , \qquad (4.5)$$

in units of 10^{-2} GeV^5.

We can use these numbers to assess the experimental validity of the heavy-quark approach to (D_1,D_2^*) decays. Our discussion here follows the work of Lu, Wise, and Isgur (LWI) [16]. Assembling the factors above, one finds

$$\Gamma(D_2^*\to D\pi)/\Gamma(D_2^*\to D^*\pi)=3.0 , \qquad (4.6)$$

independent of charge assignments; this is in good agreement with the Particle Data Group average of 2.4 ± 0.7 for the relative rates of $D_2^{*0}\to D^+\pi^-,D^{*+}\pi^-$ [8]. From these values and the observed D_2^{*0} width of 19 ± 7 MeV, one predicts the total width of the D_1 meson to be $\Gamma(D_1)=5\pm2$ MeV, which is substantially smaller than the observed value of 20 ± 7 MeV for the D_1^0. LWI ascribed this discrepancy to a small mixing of the D_1 with

the D_1'. An increment of the D_0 width by 10 MeV, which would be accomplished by a mixing angle of order 0.2, would be quite sufficient. Such a mixing angle is not unreasonable, since the mixing is expected to be of order $(300 \text{ MeV}/m_c)$. LWI proposed an experimental test of this idea, which we will return to presently.

We now add to these results our understanding of fragmentation to heavy mesons. This will allow us to compute the angular distributions of the D_1 and D_2^* decay products in terms of the parameter $w_{3/2}$. We begin with the decay $D_2^*\to D\pi$. Let θ,ϕ denote the orientation of the pion with respect to the fragmentation axis, as measured in the D_2^* rest frame. The amplitude for the production of a pion at θ,ϕ from a D_2^* meson of helicity h is proportional to $Y_{2h}(\theta,\phi)$. Thus, the complete pion angular distribution should be proportional to

$$\sum_h p(D_2^*,h)|Y_{2h}(\theta,\phi)|^2 , \qquad (4.7)$$

where $p(D_2^*, h)$ are the probabilities from (4.2). Expanding and normalizing, we find

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta}(D_2^* \to D\pi)$$

$$=\tfrac{1}{4}[1+3\cos^2\theta-6w_{3/2}(\cos^2\theta-\tfrac{1}{3})] \ . \quad (4.8)$$

Note that this distribution is invariant under $\cos\theta \to -\cos\theta$, as required by parity, and thus gives no information on the c quark polarization. This accords with the "no-win" theorem discussed at the end of Sec. III. The pion angular distribution is generally anisotropic but becomes isotropic for isotropic fragmentation, $w_{3/2}=\tfrac{1}{2}$. In fact, the dependence of (4.8) on $w_{3/2}$ is fixed by this requirement and the requirement that the total rate be independent of $w_{3/2}$.

This angular distribution has been measured by ARGUS [14], and so the parameter $w_{3/2}$ can be extracted from experiment. The ARGUS data are shown in Fig. 1, along with the theoretical predictions for $w_{3/2}=0$ and 0.2. The ARGUS analysis found no significant population of the extreme helicity states $h=\pm2$. This implies that $w_{3/2}$ is small. Our best fit would come at $w_{3/2}=-0.3$, if this were meaningful. Assuming that $w_{3/2}>0$, we find

$$w_{3/2} < 0.24 \ , \quad 90\% \ \text{C.L.} \quad (4.9)$$

We will now discuss the physical interpretation of this result.

Once $w_{3/2}$ is known, we have definite predictions for the angular distributions of the remaining excited D meson decays. Consider next the decay $D_2^* \to D^*\pi$. The amplitude for a decay from the helicity state h to the D^* state of helicity k and a pion with orientation (θ,ϕ) is proportional to

$$Y_{2m}(\theta,\phi)\langle 2m\,1k|2h \rangle \ , \quad (4.10)$$

with $m=h-k$. Summing over D^* helicities, and summing over D_2^* helicities with the probabilities from (4.2), we find the following result for the pion angular distribution:

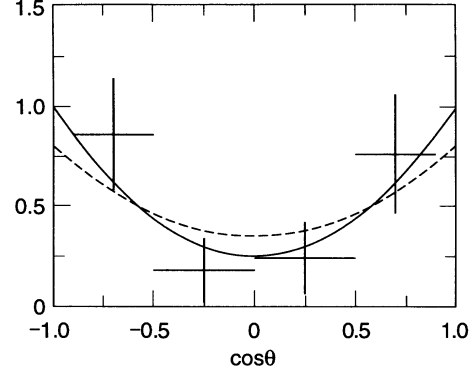FIG. 1. Angular distribution of pions in the decay $D_2^* \to D\pi$. The data are shown, along with theoretical predictions corresponding to $w_{3/2}=0$ (solid curve) and $w_{3/2}=0.2$ (dashed curve).

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta}(D_2^* \to D^*\pi)$$

$$=\tfrac{3}{8}[1+\cos^2\theta-2w_{3/2}(\cos^2\theta-\tfrac{1}{3})] \ . \quad (4.11)$$

This is a flatter distribution then we found for the direct decay to D. The two distributions are compared in Fig. 2 for the preferred value $w_{3/2}=0$.

Additional information can be obtained if the D^* is observed through its pion decay to D. The amplitude for this secondary decay is proportional to $Y_{1k}(\theta_2,\phi_2)$, where the angles give the orientation of the secondary pion. The joint angular distribution of the two pions is proportional to

$$\sum_h p(D_2^*,h)\left|\sum_{k=-1,0,1} Y_{2m}(\theta,\phi)Y_{1k}(\theta_2,\phi_2)\langle 2m\,1k|2h \rangle\right|^2 \ ,$$

$$(4.12)$$

where, again, $m=h-k$. In writing (4.12), we ignore the D^* recoil, as is appropriate in the heavy-quark limit. Simplifying this expression, we find

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta\,d\cos\theta_2\,d\phi_2}(D_2^* \to \pi\pi D)=\frac{9}{32\pi}[1+2\cos\theta\cos\theta_2\cos\alpha-\cos^2\alpha-\cos^2\theta_2-\cos^2\theta\cos^2\alpha$$

$$-2w_{3/2}(\tfrac{1}{3}+2\cos\theta\cos\theta_2\cos\alpha-\tfrac{1}{3}\cos^2\alpha-\cos^2\theta_2-\cos^2\theta\cos^2\alpha)] \ , \quad (4.13)$$

where

$$\cos\alpha=\cos\theta\cos\theta_2+\sin\theta\sin\theta_2\cos(\phi_2-\phi) \quad (4.14)$$

is the angle between the two pions in the D^* rest frame. The integral of this expression over θ_2,ϕ_2 reproduces (4.11), and the integral over orientations with α fixed gives the $\sin^2\alpha$ distribution characteristic of the spin-2 parent [14,15]. Notice that the complete distribution (4.13) is symmetric under $\cos\theta \to -\cos\theta$, so, again, all in-

formation about the heavy-quark polarization is lost.

The decay $D_1 \to D^*\pi$ can be analyzed in a similar fashion. In the ideal situation, we would ignore mixing of the D_1 with the D_1'. Then the decay amplitude from D_1 helicity h to D^* helicity k would be proportional to

$$Y_{2m}(\theta,\phi)\langle 2m\,1k|1h \rangle \ . \quad (4.15)$$

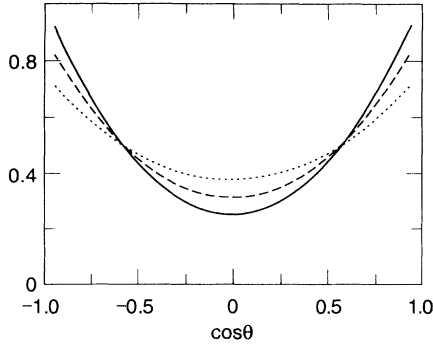This would lead to a pion angular distribution

FIG. 2. Angular distribution of pions from the decays $D_2^* \to D\pi$ (solid), $D_2^* \to D^*\pi$ (dashed), and $D_1 \to D^*\pi$. For D_1 decays, the dashed curve denotes the ideal case of zero mixing (and is the same as for $D_2^* \to D^*\pi$), while the dotted curve is computed for the more realistic situation $(S/D)^2 = 2$, $\eta = 0.45$. The curves assume the preferred value $w_{3/2} = 0$ and average over the polarization of the final D^*'s.

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos\theta}(D_1 \to D^*\pi)$$

$$= \tfrac{3}{8}[1 + \cos^2\theta - 2w_{3/2}(\cos^2\theta - \tfrac{1}{3})] . \qquad (4.16)$$

Curiously, this distribution is identical to (4.11).

However, we have argued above that the D_1 must also have some S wave component to its decay due to mixing. Following LWI, we modify (4.15) to

$$Y_{2m}(\theta,\phi)\langle 2m\,1k|1h\rangle - \frac{S}{D}e^{i\eta}Y_{00}(\theta,\phi)\delta(k,h) . \qquad (4.17)$$

The parameter S/D contains the D_1-D_1' mixing angle and the relative magnitudes of the D_1 and D_1' decay amplitudes. Note that S/D can be negative. The phase η of the interference term is approximately equal to the $D^*\pi$ $I = \tfrac{1}{2}$ S-wave phase shift; we do not call it $\delta_{1/2}$ to avoid confusion with the Kronecker delta symbol $\delta(k,h)$. Extrapolating linearly from the Weinberg value [17] of the phase shift at threshold, we would estimate

$$\eta = \frac{1}{4\pi}\frac{m_\pi^2}{f_\pi^2}\frac{p_\pi}{m_\pi} = 0.45 \qquad (4.18)$$

at the excitation energy of the D_1. However, there is a broad resonance in this partial wave at the D_1', and its influence will change this prediction in a manner difficult to predict if the D_1' overlaps the D_1 [18].

The inclusion of the S-wave amplitude increases the width of the D_1 by a factor $[1 + (S/D)^2]$. In our numerical examples, we will take $(S/D)^2 = 2$. The S-wave effect dilutes the angular dependence of (4.16) as follows:

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta}(D_1 \to D^*\pi) = \frac{1}{1+(S/D)^2}\frac{3}{8}\left\{1+\cos^2\theta+\frac{4}{3}\left[\frac{S}{D}\right]^2 - \frac{2}{3}\sqrt{2}\frac{S}{D}\cos\eta(1-3\cos^2\theta)\right.$$

$$\left. -2w_{3/2}\left[\cos^2\theta-\frac{1}{3}-\frac{2}{3}\sqrt{2}\frac{S}{D}\cos\eta(1-3\cos^2\theta)\right]\right\} . \qquad (4.19)$$

The corrected pion angular distribution is compared to the idealized form, and to our earlier results, in Fig. 2.

LWI suggested that the mixing parameter S/D can be measured from the properties of the joint pion angular distribution in $D_1 \to \pi D^* \to \pi\pi D$. They presented a number of useful partial distributions. But actually it is not difficult to construct the complete joint distribution of the two pion momenta, since it is simply proportional to

$$\frac{1}{1+(S/D)^2}\sum_h p(D_1,h)\left|\sum_{k=-1.0.1} Y_{1k}(\theta_2,\phi_2)\left[Y_{2m}(\theta,\phi)\langle 2m\,1k|2h\rangle - \frac{1}{\sqrt{4\pi}}\frac{S}{D}e^{i\eta}\delta(k,h)\right]\right|^2 \qquad (4.20)$$

The explicit formula for this angular distribution is

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta d\cos\theta_2 d\phi_2}(D_1 \to \pi\pi D)$$

$$= \frac{1}{32\pi}\frac{1}{1+(S/D)^2}\left[1 - 18\cos\theta\cos\theta_2\cos\alpha + 3\cos^2\alpha + 3\cos^2\theta_2 + 27\cos^2\theta\cos^2\alpha\right.$$

$$-2w_{3/2}(-1-18\cos\theta\cos\theta_2\cos\alpha-3\cos^2\alpha+3\cos^2\theta_2+27\cos^2\theta\cos^2\alpha)$$

$$+2\left[\frac{S}{D}\right]^2[1+3\cos^2\theta_2-2w_{3/2}(3\cos^2\theta_2-1)]$$

$$-2\sqrt{2}\frac{S}{D}\cos\eta[1-9\cos\theta\cos\theta_2\cos\alpha-3\cos^2\alpha+3\cos^2\theta_2$$

$$-2w_{3/2}(-1-9\cos\theta\cos\theta_2\cos\alpha+3\cos^2\alpha+3\cos^2\theta_2)]$$

$$\left. +6\sqrt{2}\frac{S}{D}\sin\eta\cos\alpha(1-4w_{3/2})(\hat{3}\times\hat{p}_\pi\cdot\hat{p}_{\pi 2})P\right] . \qquad (4.21)$$

The invariant in the last line is the triple product of the fragmentation axis with the directions of the two pion momenta. We have multiplied this term by the original charmed quark polarization P, since it is odd under reversal of the charmed quark spin direction. The remaining terms in (4.21) are independent of P. When the distribution (4.21) is integrated over angles with α fixed, it gives a distribution intermediate between the pure D-wave distribution $(1+3\cos^2\alpha)$ and the flat distribution expected from an S-wave decay. Unfortunately, the results on the α distribution reported in Refs. [14] and [15] are not yet sufficiently precise to give a useful constraint on (S/D).

The last term in (4.21) is a counterexample to the no-win theorem, the only one that we have found in the study of heavy meson fragmentation. It arises because the invariant

$$\vec{s}\times\hat{\mathbf{p}}_\pi\cdot\hat{\mathbf{p}}_{\pi_2} , \qquad (4.22)$$

where $\vec{s}$ is the heavy-quark spin, is parity-even and so can appear in the angular distribution formula [19]. This invariant is apparently T odd, but this simply means that the contribution of the invariant must be proportional to an absorptive phase. In this case, the phase is η, given approximately by (4.18). The phase is sufficiently large that this effect might someday be used to confirm that the c quarks emerging from the Z^0 are predominantly left handed.

Since the D_1 and D_2^* are prominent resonances of the charmed mesons, it is natural that bottom mesons should possess similar excited states. We now briefly discuss the properties of those resonances. The splitting of the heavy-quark multiplet should decrease by a factor $(m_b/m_c)\sim 3$ as we go from the charm to the bottom system, while the decay rates remain roughly constant, up to angular momentum factors. Thus, we expect that the bottom mesons should have a set of resonances located about 530 MeV above the centroid of the (B,B^*) system. These resonances should have widths of 20 MeV and a splitting of 10 MeV. The added width due to B_1-B_1' mixing should be down by a factor $(m_c/m_b)^2$ from the charm case; thus we can ignore this effect here. Note that the change to b quarks interchanges the relation of Γ and Δ that we had for charm.

Since the bottom system has $\Gamma > \Delta$, the two peaks associated with the initial B_1 and B_2^* should be merged. However, since the B-B^* splitting is 46 MeV, the separate decays to B and B^* should be resolved. Thus, we would expect that, when B mesons are produced in fragmentation, one should see two peaks in the pion energy distribution in the B meson frame, corresponding to pion energies of about 520 and 565 MeV, each peak having a width of about 20 MeV. The relative populations of the two peaks should be 2:1 in favor of the lower-energy transition $(B_1,B_2^*)\rightarrow B^*$; the 3:1 ratio from spin counting is partially balanced by a 1.5:1 ratio of the kinematic factors p_π^5. This experiment would allow both the discovery of the (B_1,B_2^*) multiplet and a nontrivial confirmation of the B-B^* mass splitting.

The fact that the B_1 and B_2 decay coherently has a cu-

rious effect on the angular distribution of the decay pion. In the limit $\Gamma \gg \Delta$, we should compute this distribution as a decay of the $j^P=\frac{3}{2}^+$ light antiquark configuration. The angular distribution for this decay is proportional to

$$\sum_h p(\tfrac{3}{2},j^3)|Y_{2m}(\theta,\phi)\langle 2m\tfrac{1}{2}j'^3|\tfrac{3}{2}j^3\rangle|^2 , \qquad (4.23)$$

where $p(\tfrac{3}{2},j^3)$ are the light antiquark probabilities from (4.1) and j'^3 is the helicity of the light antiquark after the decay. Our formalism predicts that the two helicity states $j'^3=\pm\frac{1}{2}$ are equally populated; these populations can then be combined with the heavy b quark spin to form B and B^* mesons. For any b polarization, the pion angular distribution follows from (4.23). Working this out explicitly, we find

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta}(B_1,B_2^*\rightarrow B,B^*\pi)$$
$$=\tfrac{1}{4}[1+3\cos^2\theta-6w_{3/2}(\cos^2\theta-\tfrac{1}{3})] , \qquad (4.24)$$

with the same distribution for the decay to B and B^*. This distribution is identical to (4.8), and that is easy to understand: We can think of the decay amplitude to B as a coherent sum of the decay amplitudes from B_1 and B_2^* to B; however, the amplitude for $B_1\rightarrow B\pi$ is zero, and so we revert to the earlier case. However, the relation of (4.24) to (4.11) and (4.16) is quite surprising. Naively, we might have expected the distribution in this case to be an average of (4.11) and (4.16) (which are actually identical). However, we find instead a sharper angular distribution, as the result of the coherent superposition of the two decay amplitudes. The difference between (4.24) and (4.11), (4.16) reflects the loss of information on the spin of the light degrees of freedom which occurs when the heavy-quark spin becomes involved in the dynamics. By observing this transition from the charm to the bottom system, we would effectively be timing the heavy-quark spin flip.

It should be noted that the calculation we have done applies to the asymptotic case $\Gamma \gg \Delta$. For Γ and Δ of the same order of magnitude, a more complicated formula is required. We give this formula in Appendix B.

We close this section with some speculations on the meaning of the result $w_{3/2}=0$. We have learned, in effect, that when a light spin-$\frac{1}{2}$ object forms in heavy-quark fragmentation, its angular momentum prefers to align transverse to, rather than along, the fragmentation axis. This is a striking result, and we have not been able to find an explanation for it. In models of string fragmentation, the physical degrees of freedom of the string are transverse oscillations, and so the orbital angular momentum would tend to point along the string direction, that is, along the fragmentation axis. Perturbative quark evolution by the Altarelli-Parisi equations can produce correlations between quark helicity and orbital angular momentum. For example, a polarized quark preferentially emits a gluon with the same helicity and opposite orbital angular momentum. Some, but not all, of this angular momentum can accompany an antiquark produced from the gluon. Neither viewpoint seems to lead to a crisp explanation of the phenomenon. In any event, this result on $w_{3/2}$, and related results for other values of w_j

that will be found in the near future, provide information on the process of fragmentation from a new perspective. Thus, they should provide incisive tests for proposed schemes of hadronization.

V. POLARIZATION OF HEAVY BARYONS

We will now carry over the insight we have gained from the study of heavy mesons to the phenomenology of heavy baryons. For heavy mesons, we saw that the "no-win" theorem prohibits any visible effects of an initial heavy-quark polarization, except under the special conditions described below (4.22). However, for heavy baryons, the situation is very different. The ground-state heavy baryon is built from a heavy quark combined with a $j=0$ combination of two light quarks. Since this system has no angular momentum to transfer to the heavy quark, the initial polarization cannot be diluted. Mannel and Schuler [4] and Close, Körner, Phillips, and Summers [5] have used this argument to conclude that the ground state b baryons produced in Z^0 decays will retain the initial high polarization P of the b quark. In this section, we will compute the first correction to this argument and find the depolarization of the b quark in this scheme of fragmentation. In the process, we will explore the polarization dependence of excited heavy baryon decays and find some further reactions which are sensitive to the competition between the decay and the spin splitting of a heavy-quark multiplet.

To begin, we review some basic properties of b baryons. Baryons are expected about 5% of the time in b fragmentation [20], so that about 10% of $b\bar{b}$ events or 2% of Z^0 hadronic events will contain baryons. In the nonrelativistic quark model, the lightest heavy baryons consist of a heavy quark together with a light-quark pair with zero orbital angular momentum. This pair can be either a ud system with isospin and spin $I=S=0$ or a uu, ud, or dd system with $I=S=1$. (We ignore strange heavy baryons.) In the heavy-quark effective theory, the lightest baryons should be formed from states of the light degrees of freedom with these quantum numbers. We will refer to such states as "diquarks" even when we do not assume that the quark model describes them accurately. By combining the $s=\frac{1}{2}$ b quark with the diquarks of $j^P=0^+$ and 1^+, we form the Λ_b baryon and the (Σ_b,Σ_b^*) baryon multiplet. We will treat these three sets of states as the final states of the rapid phase of b fragmentation to baryons.

Even if we ignore the coupling of the b-quark spin, as is appropriate to the heavy-quark approximation, the relative probabilities of finding these states in b fragmentation is still governed by two unknown parameters. The first of these, which we will call A, is the relative probability of producing an $I=S=1$ diquark as opposed to an $I=S=0$ diquark. This is the ratio of the total (Σ_b,Σ_b^*) production to primary Λ_b production, summed over the 9 possible spin and isospin states of the $I=S=1$ multiplet. The second of these is the parameter w_1, which gives the probability that the spin-1 diquark has maximum angular momentum $j^3=\pm1$ along the fragmentation axis. The parameter A is related, but not identical,

to a parameter of the Lund fragmentation model, which gives the relative probability of a spin-1 or a spin-0 diquark appearing when the color string breaks: $A\approx9\times\mathrm{PAR}(4)$ [21]. An important difference is that our parameter A is an output, rather than an input, of the fragmentation scheme, so that it is defined independently of any model. The parameter PAR(4) is not well determined experimentally. For example, in a recent study by the OPAL collaboration [22], this parameter could be varied by a factor 3 from the Lund default value of 0.05 by adjusting the other parameters of the baryon decay scheme. We know of no experimental determination of w_1. Nevertheless, it will be useful to have some definite values of these parameters for our numerical estimates. Motivated by the Lund default value and the results of the previous section, we will choose

$$A=0.45\ ,\quad w_1=0 \tag{5.1}$$

as our reference values. With these values, about 30% of b baryons are born initially as Σ_b or Σ_b^*.

We now consider the fragmentation of a b quark with complete left-handed polarization. Given values of A and w_1, the various helicity states of the b baryons are populated by fragmentation according to

$$
\begin{bmatrix} P(\Sigma_b^*,h) \\ p(\Sigma_b,h) \\ p(\Lambda_b,h) \end{bmatrix} = \frac{1}{1+A} \begin{bmatrix} \frac{1}{2}w_1A & \frac{2}{3}(1-w_1)A & \frac{1}{6}w_1A & 0 \\ & \frac{1}{3}(1-w_1)A & \frac{1}{3}w_1A & \\ & 1 & 0 & \end{bmatrix}.
$$
$$\tag{5.2}$$

The probabilities for the Σ_b and Σ_b^* helicity states represent the sum over the three isospin states. The relative production rate of $\Sigma_b:\Sigma_b^*$ is 1:2 independently of w_1.

We next consider the mass splittings of the b baryons. Unfortunately, in the b baryon system, only the Λ_b is known [23], and the only certain piece of information on any heavy baryon splitting is $m(\Sigma_c)-m(\Lambda_c)=168$ MeV [8]. The Σ_c^* has not yet been discovered. One can estimate its position from the splittings of the strange baryons; using quadratic mass relations, we find $m(\Sigma_c^*)-m(\Sigma_c)=100$ MeV; for comparison, Kwong, Rosner, and Quigg [24] find 64 MeV for this mass difference using linear relations. The experiments that give the Σ_c mass [25] would seem to exclude values of this mass difference below 80 MeV. Using our estimates, the centroid of the (Σ_c,Σ_c^*) multiplet is located 230 MeV above the Λ_c. The value of this mass splitting is expected to have only a weak dependence on the heavy-quark mass. Thus, we expect that the Σ_b and Σ_b^* should lie roughly 210 MeV and 240 MeV, respectively, above the Λ_b. Both splittings are well above the threshold for single-pion transitions to the Λ_b. Thus we expect that all b baryon states will eventually decay hadronically to Λ_b.

The decay rate for the transitions $(\Sigma_b,\Sigma_b^*)\rightarrow\pi+\Lambda_b$ can be estimated in the nonrelativistic quark model by using a pion-quark coupling estimated from the Goldberger-Trieman relation. This computation has

been done by Yan *et al.* [26]. They find

$$\Gamma = \frac{g_{Aq}^2}{6\pi f_\pi^2} p_\pi^3 = 28 \text{ MeV} \left[\frac{p_\pi}{200 \text{ MeV}} \right]^3 , \tag{5.3}$$

where p_π is the pion three-momentum, $f_\pi = 93$ MeV, and g_{Aq} is the axial vector coupling of the constituent quark. In the numerical estimate, we take $g_{Aq} = 0.75$ to give the correct g_A for the nucleon. The Σ_b and Σ_b^* have the same decay rate up to kinematic factors, since the decay mechanism does not directly involve the heavy quark.

It is curious that the predicted decay rate Γ and mass splitting Δ for the (Σ_b, Σ_b^*) multiplet are approximately equal. This is an accident, since Γ is independent of the heavy-quark mass while Δ is proportional to $1/m_b$. We have stressed that our estimates of Δ and Γ are quite uncertain. However, if they are correct, the Σ_b and Σ_b^* form two distinct resonances which thus decay incoherently. The two excited baryons can be observed together starting from a sample of (partially) reconstructed Λ_b's by plotting the distribution of pion energies in the Λ_b frame. The Σ_b and Σ_b^* should appear as two closely spaced peaks on this distribution. The proper values of Γ and Δ for the analysis to follow must eventually be determined experimentally by the measurement of this double-peak structure.

If it had turned out that $\Gamma \gg \Delta$, the Σ_b and Σ_b^* baryons could decay to Λ_b's without involving the heavy-quark spin. In this limit, there would be no depolarization of the b quark from its initial polarization P. However, our estimates make it reasonable to consider the opposite limit in which the two baryon resonances decay incoherently. After we analyze this limit in some detail, we will also present results for intermediate values of Γ/Δ.

Now we have all the ingredients we need to compute the properties of the excited baryon decays and the effect of these decays on the Λ_b polarization. We first consider the pion angular distributions. The amplitude for the decay of a Σ_b of helicity h to a Λ_b of helicity k is proportional to

$$Y_{1m}(\theta, \phi) \langle 1m\tfrac{1}{2}k | \tfrac{1}{2}h \rangle , \tag{5.4}$$

where θ, ϕ give the pion orientation with respect to the fragmentation axis and $m = h - k$. The amplitude for Σ_b^* decay is given by the analogous formula with $j = \tfrac{3}{2}$. Squaring and summing with the probabilities from (5.2), we find the pion angular distributions

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos\theta}(\Sigma_b \to \Lambda_b \pi) = \tfrac{1}{2} ,$$

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos\theta}(\Sigma_b^* \to \Lambda_b \pi)$$

$$= \tfrac{1}{4}[1 + 3\cos^2\theta - \tfrac{9}{2}w_1(\cos^2\theta - \tfrac{1}{3})] . \tag{5.5}$$

The first of these distributions is isotropic; the second becomes isotropic at $w_1 = \tfrac{2}{3}$. This second distribution can be used to determine w_1 experimentally. For comparison, the pion angular distribution in the case $\Gamma \gg \Delta$ is

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos\theta}(\Sigma_b, \Sigma_b^* \to \Lambda_b \pi)$$

$$= \tfrac{3}{2}[\cos^2\theta - \tfrac{1}{2}w_1(\cos^2\theta - \tfrac{1}{3})] . \tag{5.6}$$

The intermediate situation can be analyzed using the formulas provided in Appendix B.

On the other hand, we may integrate over the pion angles and look instead at the distribution of final Λ_b helicities which result from a sample of completely left-handed polarized b quarks. Again, we consider the extreme limit $\Delta \gg \Gamma$. From Σ_b decay, we find

$$\frac{\Lambda_b(+\tfrac{1}{2})}{\Lambda_b(-\tfrac{1}{2})} = \frac{2 - w_1}{1 + w_1} . \tag{5.7}$$

From Σ_b^* decay, we find

$$\frac{\Lambda_b(+\tfrac{1}{2})}{\Lambda_b(-\tfrac{1}{2})} = \frac{2 - w_1}{4 + w_1} . \tag{5.8}$$

Summing over all primary and secondary Λ_b's, we find

$$\frac{\Lambda_b(+\tfrac{1}{2})}{\Lambda_b(-\tfrac{1}{2})} = \frac{2(2 - w_1)A}{9 + A(5 + 2w_1)} . \tag{5.9}$$

To return to the situation of Z^0 decays, multiply the corresponding polarizations by the initial b polarization P given by (2.2). Thus, we find for the final Λ_b polarization P_Λ the values

$$P_\Lambda = \left\{ \frac{1 + (1 + 4w_1)A/9}{1 + A}P, \ \frac{1 + w_1}{3}P , \ -\frac{1 - 2w_1}{3}P \right\} , \tag{5.10}$$

for Λ_b's from the full sample, from Σ_b^* decays, and from Σ_b decays, respectively. Inserting the value from (5.1), we find

$$P_\Lambda = 0.72P, \ 0.33P, \ -0.33P ; \tag{5.11}$$

with (2.2), this implies a 68% polarization in the full sample of Λ_b's observed in Z^0 decay. The minus sign in the last entry of (5.1) is not a misprint but rather a curious prediction which would be very interesting to confirm. We emphasize again that these predictions are valid only if the Σ_b and Σ_b^* are distinct resonances and revert to the naive prediction $P_\Lambda = 1 \cdot P$ in the limit where these resonances completely overlap.

The intermediate case $\Gamma \sim \Delta$ can be treated by regarding the Σ_b and Σ_b^* as partially overlapping resonances. We present the formulas for this case in Appendix B. In Fig. 3(a), we show the pion energy spectrum for decays $(\Sigma_b, \Sigma_b^*) \to \Lambda_b + \pi$, and the contributions to the spectrum from each Λ_b helicity state, for the case $\Gamma = \Delta = 30$ MeV. In Fig. 3(b), we show how the three polarizations computed in (5.11) change as a function of the ratio Γ/Δ.

Since the extreme limit $\Delta \gg \Gamma$ is well satisfied in the case of charmed baryons, all of the results we have obtained in the preceding paragraphs should also apply to the $\Lambda_c, \Sigma_c, \Sigma_c^*$ system. We predict a polarization of 48% for Λ_c's produced in Z^0 decays. The parameter w_1 could
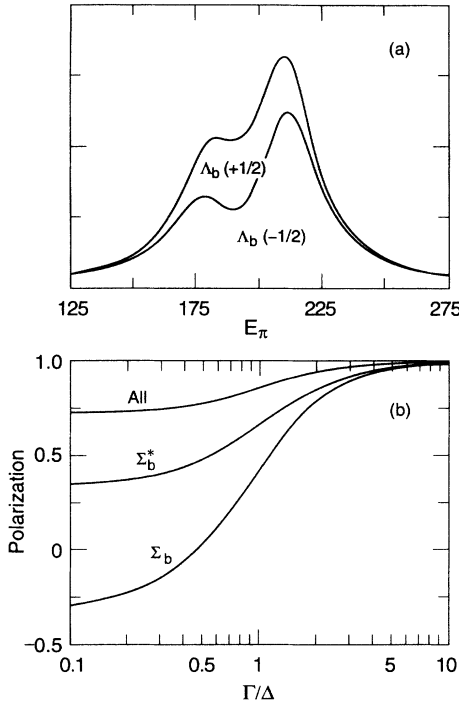
FIG. 3. (a) Pion energy spectrum for decays $(\Sigma_b, \Sigma_b^*) \to \Lambda_b + \pi$, for the case $\Gamma = \Delta = 30$ MeV. The upper curve is the total spectrum, while the lower curve is the contribution from the $\Lambda_b(-\frac{1}{2})$ helicity state. The spectrum is computed using the formula for $d\Gamma/dE_\pi$ given in Appendix B. (b) The polarization of the final Λ_b's as a function of Γ/Δ. We show the polarization of the full sample of Λ_b's as well as the separate contributions arising from Σ_b and Σ_b^* decays. These subsamples are defined carefully in Appendix B.

well be measured at the Cornell Electron Storage Ring (CESR) or in fixed target experiments, since the distributions (5.5) are independent of the heavy-quark polarization.

We should, finally, comment on the measurement of the polarization of Λ_b baryons. Close, Körner, Phillips, and Summers [5] and Amundson, Rosner, Worah, and Wise [27] have proposed that the absolute magnitude of the Λ_b polarization can be obtained by comparing the lepton distribution in semileptonic b decays to the spectator model, and the first set of authors have proposed additional methods using the $\Lambda_b \to \psi \Lambda$ decay mode. However, it is important to note as well that the *relative* polarization of two different samples of Λ_b's can be obtained more easily by observing any parity-violating forward-backward asymmetry with respect to the fragmentation axis in Λ_b decay. For example, the forward-backward asymmetry of Λ production in Λ_b decays should be proportional to P_Λ and can thus be used to check the relative magnitudes of P_Λ in the three samples described in (5.10).

VI. CONCLUSIONS

In this paper, we have discussed a number of phenomena connected to heavy hadron spectroscopy which are

sensitive to the competition between the rate of a hadronic decay and the rate of a heavy-quark spin flip. We have seen that this competition can affect the angular distributions observed for the decay of heavy hadrons and the degree of polarization of heavy baryons. Conversely, the properties of heavy hadron decays can be used to measure a new set of fragmentation parameters which we have called w_j, which provide nontrivial tests of schemes of hadronization.

We have added two contributions to the study of the observability of heavy-quark polarization as viewed from the final state of the hadronization process. For heavy baryons, one expects a large polarization; we have computed the leading effect of fragmentation which degrades this polarization. For heavy mesons, one generally expects no observable polarization effects, though we have identified one particular circumstance in which a polarization effect may be visible.

We look forward to further insight that will come from experiments on the excited states of hadrons containing heavy quarks.

ACKNOWLEDGMENTS

APPENDIX A: ISGUR-WISE THEORY OF HADRONIC TRANSITIONS BETWEEN HEAVY-QUARK STATES

In Ref. [11], Isgur and Wise presented the general theory of hadronic transitions between states containing a single heavy quark. This theory was presented in a telegraphic (Physical Review Letters) style, which somewhat concealed the elegant structure of their formalism. In this appendix, we review their theory and supply a few additional formulas which make this basic structure more clear. We apply these formulas in Secs. IV and V of this paper.

An excited state of a heavy hadron may decay to a lower-mass state containing the same heavy quark by a strong interaction process in which light hadrons are emitted. In the examples of this paper, the decay involves the emission of a single pion; however, the general formalism depends only on the angular momentum of the emitted system. To leading order as the heavy-quark mass goes to infinity, the heavy hadron does not recoil and the heavy quark does not flip its spin. Thus, we have the following general structure: The initial and final states are composed of a heavy quark with spin $s = \frac{1}{2}$, combined with light degrees of freedom of angular momentum j for the initial state and j' for the final state to form heavy hadrons of total spin J and J'. The transition from j to j' involves the emission of a light hadronic

system of angular momentum L and does not change the heavy-quark spin. These six angular momenta form a tetrahedron, and so the rate of the process is governed by a Wigner 6-j symbol.

More explicitly, we assign an invariant matrix element $\mathbf{M}$ as the strength of the $j \to L + j'$ transition. Then the

decay rate from any J state in the $j + s$ heavy hadron multiplet is given by decomposing the J state in the $j + s$ basis, setting the decay rate of the j state to be $\mathbf{M}$ times the appropriate Clebsch-Gordan coefficient, and then recombining the $j' + s$ states into the J' appropriate to the final state. Thus, the decay amplitude is given by

$$\mathcal{A}(J'J'^3 \to JJ^3 + Lm) = \mathbf{M} \cdot \langle J'J'^3 | j'j'^3 ss^3 \rangle \langle Lmj'j'^3 | jj^3 \rangle \langle jj^3 ss^3 | JJ^3 \rangle , \tag{A1}$$

summed over the intermediate values s^3, j^3, j'^3. This is Eq. (1) of Ref. [11]. This expression can be rewritten in the form [28]

$$\mathcal{A}(J'J'^3 \to JJ^3 + Lm) = M' \cdot (-1)^{L+j'+s+J}(2j+1)^{1/2}(2J'+1)^{1/2} \begin{Bmatrix} j' & j & L \\ J & J' & s \end{Bmatrix} \langle LmJ'J'^3 | JJ^3 \rangle , \tag{A2}$$

involving the Wigner 6-j symbol. The dependence on J'^3, m, J^3 is given by the angular momentum Clebsch-Gordan coefficient for the overall process, as must be so.

Formula (A2) decouples the angular dependence of the hadronic decay products from the dependence of the decay amplitudes on the position J in the $j + s$ heavy-quark multiplet. Both aspects of this equation are thus clarified. The angular distribution of the decay products is determined by the simple relation

$$\mathcal{A} \sim \sum_m Y_{Lm}(\Omega) \langle LmJ'J'^3 | JJ^3 \rangle , \tag{A3}$$

for fixed J^3, J'^3. The total rate of hadronic decays from a state J in the $j + s$ multiplet depends on J through the factor

$$\sum_{J'} (2j+1)(2J'+1) \left| \begin{Bmatrix} j' & j & L \\ J & J' & s \end{Bmatrix} \right|^2 = 1 \tag{A4}$$

by the standard orthogonality relation. Thus, the total decay rate is independent of J, as predicted by the physical picture of Isgur and Wise.

It is important to note, as Isgur and Wise do, that these relations apply formally to the limit in which the heavy hadrons in each of the $j + s$ multiplets are essentially degenerate. In realistic situations, there may be important corrections to these relations coming from kinematic factors in the amplitude. For example, a decay which emits a pion with angular momentum L has a rate proportional to p_π^{2L+1}. This factor may vary significantly over the heavy quark multiplet in cases of practical interest, for example, in the $(D_1, D_2^*) \to (D, D^*) + \pi$ transitions considered in Sec. IV. In addition, the emission of high-energy pions may be suppressed by form factors. Isgur and Wise assume a suppression factor $\exp[-p_\pi^2/(1 \text{ GeV})^2]$, but we omit this factor for simplicity. It gives at most a 15% correction to relative decay rates. We encourage the reader to keep this factor in mind, however, as contributing to the theoretical uncertainty of our heavy-quark predictions.

On the other hand, it is a major point of this paper that

these relations also do not apply when the splitting within a $j + s$ multiplet is much smaller than the hadronic widths of the heavy hadrons. The transition to this regime is discussed in Sec. IV.

APPENDIX B: PARTIAL COHERENCE OF HEAVY HADRON DECAYS

In this paper, we have mainly discussed heavy hadron decays in the extreme limits $\Gamma \gg \Delta$ or $\Delta \gg \Gamma$. However, it often happens that Γ and Δ are of the same order of magnitude, and so it is useful to have a formula which interpolates between these two limits. To obtain such a formula, we sum coherently over the heavy hadron states H and H^* as distinct resonances. In the following discussion, we will use a language in which the decay from (H, H^*) proceeds by emission of a single pion of angular momentum L. However, similar formulas apply to any strong interaction decay.

We consider transitions from H and H^*, of spin $J = j \pm \frac{1}{2}$, to a ground state hadron $\mathcal{H}$ of spin J'. Let E_π be the pion energy and let E_J be the excitation energy of the resonance: $E_J = m_H - m_{\mathcal{H}}$ for H, and similarly for H^*. In the heavy quark limit, H and H^* have the same width Γ. Assume first that the light system which leads to H and H' has angular momentum (j, j^3) with respect to the fragmentation axis, and that the heavy-quark spin is initially polarized left-handed. Then the amplitude for production of the state $\mathcal{H}$ in association with a pion of energy E_π in the angular momentum state (L, m) is

$$\mathcal{A}(j^3) = \sum_J \langle LmJ'J'^3 | JJ^3 \rangle$$

$$\times \frac{A_J}{E_\pi - E_J + i\Gamma/2} \langle jj^3 s - \tfrac{1}{2} | JJ^3 \rangle , \tag{B1}$$

with $J^3 = j^3 - \frac{1}{2}$, $m = J^3 - J'^3$. The factor A_J is the prefactor of the Clebsch-Gordan coefficient in (A2). Only the ratio of the two factors A_J is important. In the two examples analyzed here,

$$A_1 : A_2 = 1 : +\sqrt{\tfrac{3}{5}} \tag{B2}$$

for the $(D_1, D_2^*) \rightarrow D^* \pi$ transition, and

$$A_{1/2} : A_{3/2} = 1 : +1 \tag{B3}$$

for the $(\Sigma_b, \Sigma_b^*) \rightarrow \Lambda_b \pi$ transition.

To find the dependence of the pion emission rate on E_π, we square the amplitudes (B1) and sum them incoherently with the probability distributions of the light degrees of freedom:

$$\frac{d\Gamma}{dE_\pi} \sim \sum_{j^3} p(j, j^3) |\mathcal{A}(j^3)|^2 . \tag{B4}$$

For (D_1, D_2^*), we use (4.1); for (Σ_b, Σ_b^*), we use

$$p(1, j^3) = (\tfrac{1}{2} w_1, (1 - w_1), \tfrac{1}{2} w_1) . \tag{B5}$$

The resulting distribution of pion energies contains two overlapping resonances; thus, there is some ambiguity in the assignment of observed decays to one resonance or the other. In constructing Fig. 3, we have arbitrarily divided the distribution at the centroid of the (Σ_b, Σ_b^*) multiplet, $m_C = [m(\Sigma_b) + 2m(\Sigma_b^*)]/3$. Pions with energy less than $m_C - m(\Lambda_b)$ were assigned to the Σ_b sample; those with greater energy were assigned to the Σ_b^*.

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