Calculation of the $\Delta^+ p\gamma$ photon-decay amplitude $A_{1/2}$ and the E_{1^+}/M_{1^+} ratio in single-pion electroproduction: An algebraic approach

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A *nonperturbative* calculation of the $\Delta^+ \to P+\gamma$ transverse one-half helicity transition form factor $h_3(q^2)$ and the $A_{1/2}$ $\Delta^+ P\gamma$ photon-decay amplitude is made with results in good agreement with experiment. The ratio of the electric quadrupole amplitude to the magnetic dipole amplitude at resonance is calculated as a function of $G_M^*(0)$ and the Δ^+ mass. We confirm that $G_M^*(q^2)$ decreases more rapidly than the nucleon dipole form factor in the region where $G_E^*(q^2)$ is known to be small. Our treatment is completely relativistic and current conservation is guaranteed.

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The $\Delta N\gamma$ form factors [1–3] $G_M^*(q^2)$, $G_E^*(q^2)$, and $G_c^*(q^2)$ in elementary particle physics are very important in that they provide a basis for testing theories of effective quark forces or production models [4]. Specifically, they are important when one considers (1) perturbative QCD models involving gluon exchange mechanisms, tensor interactions, or possible hybrid baryonic states [5], (2) Skyrme models [6], (3) enhanced quark models [7] in which the $\Delta N\gamma$ transition form factors may be calculated as a function of q^2, (4) electroproduction and photoproduction processes [8], (5) symmetry schemes such as SU(6) and U(6,6) [9], and Melosh transformations [10], (6) bag model [11], (7) dispersion relations and Bethe-Salpeter approaches [12], (8) current algebra baryon sum rules [13], and (9) nonperturbative methods such as lattice QCD [14], QCD sum rules, and algebraic formulations [15].

The most important "physical observables" which are functions of the $\Delta N\gamma$ transition form factors, are the photon decay helicity amplitudes $A_{1/2}$ and $A_{3/2}$, the ratio $(E_{1^+}/M_{1^+})_{q^2=0} = -(G_E^*/G_M^*)_{q^2=0} \equiv$ electromagnetic ratio (EMR) of the electric quadrupole amplitude E_{1^+} to the magnetic dipole amplitude M_{1^+}, and the scalar quadrupole amplitude S_{1^+} [8]. The $A_{1/2}$ and $A_{3/2}$ amplitudes (which are linear combinations of M_{1^+} and E_{1^+}) can be obtained experimentally from the process $\gamma+N\to\pi+N$ and determine the radiative width $\Gamma(\Delta\to N+\gamma)$. The EMR, which is less model dependent than E_{1^+} or M_{1^+} [16], serves as a powerful and sensitive discriminant in filtering out viable theoretical models of hadron spectroscopy. The reasons for this are, of course, the theoretical fact that in the naive quark model (which is nonrelativistic), the EMR vanishes (i.e., no *D*-wave nucleon or Δ wave function components) for real photon production in the process $\Delta^+\to P+\gamma$ and the experimental fact that the EMR is small in magnitude (a few percent or less) and most likely negative.

The intent of this paper is to provide nonperturbative and completely relativistic theoretical results which shed new light on the behavior of the EMR as a function of $G_M^*(0)$ and the Δ mass and to determine theoretically the $A_{1/2}$ $\Delta^+ P\gamma$ photon decay amplitude for direct comparasion with the experimental data. We do not consider configuration mixing because it is quite interesting to study an uncomplicated scenario where the predictive power of a fully relativistic theory, which incorporates current conservation and whose basis is quantum chromodynamics, can be brought to bear on a model which incorporates nucleon and Δ ground states only.

In general [2,3] one may write for the $\Delta\to N+\gamma$ transition amplitude the expression

$$\langle N(\mathbf{p},\lambda_p)|j_\mu(0)|\Delta(\mathbf{p}^*,\lambda_\Delta)\rangle = \frac{e}{(2\pi)^3}\left[\frac{mm^*}{E_N E_\Delta}\right]^{1/2} \bar{u}_N(\mathbf{p},\lambda_p)[\Gamma_{\mu\beta}]u_\Delta^\beta(\mathbf{p}^*,\lambda_\Delta) \quad (1)$$

where

$$\Gamma_{\mu\beta} = -ih_3\Delta^{-1}\gamma\cdot p^* q_\beta \varepsilon_\mu(qp\gamma) + h_2\Delta^{-1}[2\varepsilon_{\beta\sigma}(p^*p)\varepsilon_\mu^\sigma(p^*p)\gamma_5 - i\gamma\cdot p^* q_\beta\varepsilon_\mu(qp\gamma)] + h_1\Delta^{-1}q_\beta[p\cdot q q_\mu - q^2 p_\mu]\gamma_5 \ . \quad (2)$$

In Eq. (2), the electromagnetic current is denoted by j_μ, $\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$, $q\equiv p^*-p$, p^* and p are the four-momenta of the Δ and nucleon, respectively, $\Delta^{-1}\equiv\{[(m^*+m)^2-q^2][(m^*-m)^2-q^2]\}^{-1}$ is a kinematic factor which depends on q^2, m^* (the Δ mass), and m (the nucleon mass); λ_p and λ_Δ are the helicities of the nucleon and Δ respectively; and the helicity form factors h_1, h_2, and h_3 include scalar, transverse $\frac{3}{2}$, and transverse $\frac{1}{2}$ transitions, respectively, in the rest frame of the Δ isobar. Also, h_1, h_2, and h_3 are related to the more familiar

*Deceased.

form factors $G_M^*(q^2)$, $G_E^*(q^2)$, and $G_E^*(q^2)$ by the relations [3]

$$h_3 = -\frac{3(m^*+m)}{2m}(G_M^* - 3G_E^*) ,$$
$$h_2 = -\frac{3(m^*+m)}{2m}(G_M^* + 3G_E^*) , \quad (3)$$
$$h_1 = \frac{3(m^*+m)}{m} G_C^* .$$

$G_M^*(q^2)$, $G_E^*(q^2)$, and $G_C^*(q^2)$ induce magnetic, electric, and Coulombic multipole transitions.

For the virtual process $p \to p + \gamma$, we have, similarly,

$$\langle P(\tilde{\mathbf{p}},\lambda)|j_\mu(0)|P(\tilde{\mathbf{p}}^*,\lambda^*)\rangle = \frac{e}{(2\pi)^3}\left[\frac{mm^*}{E_{\tilde{\mathbf{p}}}E_{\tilde{\mathbf{p}}^*}}\right]^{1/2} \bar{u}_p(\tilde{\mathbf{p}},\lambda) \times[\Gamma_\mu]u_p(\tilde{\mathbf{p}}^*,\lambda^*) , \quad (4)$$

where

$$\Gamma_\mu = [1-\tilde{q}^2/4m^2]^{-1} \times[(i/4m^2)G_M(\tilde{q}^2)\varepsilon_\mu(\tilde{P}\tilde{q}\gamma)\gamma_5 + \left[\frac{1}{2m}\right]G_E(\tilde{q}^2)\tilde{P}_\mu] ,$$
$$\tilde{P} \equiv \tilde{p} + \tilde{p}^* ,$$

and $\tilde{q} = \tilde{p}^* - \tilde{p}$. $G_M(\tilde{q}^2)$ and $G_E(\tilde{q}^2)$ are the familiar Sachs form factors.

The total Δ^+ radiative width $\equiv \Gamma_\gamma^T$, for decay into $p+\gamma$ is given by [17]

$$\Gamma_\gamma^T = \frac{mq_c^2}{2m^*\pi}\sum_{\lambda=1/2,3/2} A_\lambda^2 ; \quad (5)$$

where [18]

$$A_{1/2} = -e\left[\frac{\sqrt{3}}{12}\right]\left[\frac{3}{2}\frac{m^{*2}-m^2}{m^3}\right]^{1/2}[G_M^*(0) - 3G_E^*(0)] , \quad (6)$$

q_c = magnitude of the c.m. three-momentum, and

$$A_{3/2} = -e\left[\frac{1}{4}\right]\left[\frac{3}{2}\frac{(m^{*2}-m^2}{m^3}\right]^{1/2}[G_M^*(0) + G_E^*(0)] . \quad (7)$$

Experimentally [17], $A_{1/2} = (-141\pm5)\times10^{-3}$ GeV$^{-1/2}$ and $A_{3/2} = (-258\pm11)\times10^{-3}$ GeV$^{-1/2}$.

In the naive quark model, it can be shown that the EMR = 0, whereas in the naive Skyrme model [6], the EMR is large and of the order of -5%. Experimentally, [16,17], however, the EMR $\approx(-1.07\pm0.37)\%$.

We now relate theoretically the transition form factor $h_3(q^2)$ to the *isovector* part of $G_M(\tilde{q}^2)[\equiv G_M^V(\tilde{q}^2)]$ by considering asymptotic level realization [19,20] of the charge-current algebra $[j_V^\mu(0), A_{\pi^+}], A_{\pi^-} = 2j_V^\mu(0)$. We take as external states the ground-state baryons, i.e., the $1/2^+$ octet and $3/2^+$ decuplet, represented by the ket $|B(\alpha,s,\lambda)>$ or bra $<B(\alpha,s,\lambda)|$], with physical SU(3) index α, three-momentum $\mathbf{s}\to\infty$, and helicity λ. We define relevant axial-vector charge matrix elements ($s\to\infty$ understood) as

$$\langle p,\lambda=1/2|A_{\pi^+}|n,\lambda=1/2\rangle \equiv f = -\langle p,\lambda=-1/2|A_{\pi^+}|n,\lambda=-1/2\rangle ,$$
$$\langle \Delta^{++},\lambda=1/2|A_{\pi^+}|\Delta^+,\lambda=1/2\rangle \equiv -\sqrt{3/2}g = -\langle \Delta^{++},\lambda=-1/2|A_{\pi^+}|\Delta^+|\Delta^+,\lambda=-1/2\rangle ,$$
$$\langle \Delta^{++},\lambda=1/2|A_{\pi^+}|p,\lambda=1/2\rangle \equiv \sqrt{6}h = +\langle \Delta^{++},\lambda=-1/2|A_{\pi^+}|p,\lambda=-1/2\rangle .$$

For the spin-flip matrix elements of the isovector part of the electromagnetic current j_V^μ, we parametrize as follows (suppressing the Lorentz index μ):

$$\langle \Delta^+,\mathbf{s}\to\infty,\lambda=-1/2|j_V|\Delta^+,\mathbf{t}\to\infty,\lambda=1/2\rangle \equiv a ,$$
$$\langle p,\mathbf{s}\to\infty,\lambda=-1/2|j_V|p,\mathbf{t}\to\infty,\lambda=1/2\rangle \equiv b ,$$
$$\langle p,\mathbf{s}\to\infty,\lambda=-1/2|j_V|\Delta^+,\mathbf{t}\to\infty,\lambda=1/2\rangle \equiv c ,$$
$$\langle \Delta^+,\mathbf{s}\to\infty,\lambda=-1/2|j_V|p,\mathbf{t}\to\infty,\lambda=1/2\rangle \equiv d .$$

All other necessary SU(2) related spin-flip matrix elements of j_V^μ can then be obtained by considering the commutator $[[j_V^\mu(0), V_{\pi^+}], V_{\pi^-}] = 2j_V^\mu(0)$.

We now consider the ground-state contribution to the commutator $[[j_V^\mu(0), A_{\pi^+}], A_{\pi^-}] = 2j_V^\mu(0)$, by sandwiching it between all possible ground-state pairs, $\langle B(\alpha,\mathbf{s},\lambda=-1/2)|$, $|B'(\alpha,\mathbf{t},\lambda=+1/2)>$ with $\mathbf{s},\mathbf{t}\to\infty$. We obtain ten equations (not all independent):

$$\langle p,p\rangle: -2cgh + 8bh^2 - 16ah^2 - \sqrt{2}dfh - 3dgh + \cdots = 2b ,$$
$$\langle n,n\rangle: -3cgh + 8bh^2 + 16ah^2 - \sqrt{2}cfh - 2dgh + \cdots = -2b ,$$
$$\langle p,\Delta^+\rangle: 2cg^2 + 8ch^2 - 2bgh - 14agh + 2dh^2 - \sqrt{2}cfg + \cdots = 2c ,$$
$$\langle \Delta^+ p\rangle: 9agh - \sqrt{2}afh + 2dh^2 + df^2 - \sqrt{2dfg} + 2ch^2 - \sqrt{2}bfh + \tfrac{3}{2}dg^2 - 3bgh + \cdots = 2d ,$$

⋮

⋮

$$\langle \Delta^-,\Delta^-\rangle: -6ag^2 - 3dgh + 3cgh + 6bh^2 - 18ah^2 + \cdots = -16a .$$

These ten equations can be solved and yield the physical solution $a:b:c:d=1:-2:5\sqrt{2}/4:-5\sqrt{2}/4$, $g=-(\sqrt{2}/5)f$, and $h=\frac{2}{5}f$.

We thus obtain the important relation

$$\langle p,\mathbf{s},\lambda=-\tfrac{1}{2}|j^{\mu}_{\rm em}(0)|\Delta^+\mathbf{t},\lambda=\tfrac{1}{2}\rangle$$

$$=\varepsilon\langle p,\mathbf{s},\lambda=-\tfrac{1}{2}|j^{\mu}_{V}(0)|p,\mathbf{t},\lambda=\tfrac{1}{2}\rangle\ , \qquad (8)$$

where $|\mathbf{s}|,|\mathbf{t}|\to\infty$, and $\varepsilon=5\sqrt{2}/8$ [21].

From Eqs. (1), (4), and (8), we can now extract $h_3'(q^2)\equiv-\{2m/[3(m+m^*)]\}h_3(q^2)$ as a function of the *isovector* part of $G_M(\tilde{q}^2)$ by considering matrix elements (transverse 1/2) of $(j_1-ij_2)/\sqrt{2}$ (corresponding to a helicity $+1$ photon moving in the $+z$ direction) and Lorentz transformations ("Z boosts") [22] G and $\tilde{G}$ such that

$$p^{*\mu}=(p^{*0},0,0,t)\xrightarrow{G}p^{*\prime\mu}=(m^*,0,0,0)\ ,\ p^{\mu}(p^0,0,0,rt)\xrightarrow{G}p^{\prime\mu}=\left[\frac{r^2m^{*2}+m^2}{2rm^*},0,0,\frac{r^2m^{*2}-m^2}{2rm^*}\right]$$

and

$$\tilde{p}^{*\mu}=(\tilde{p}^{*0},0,0,t)\xrightarrow{\tilde{G}}\tilde{p}^{*\prime\mu}=(m,0,0,0),\tilde{p}^{\mu}=(\tilde{p}^0,0,0,rt)\xrightarrow{\tilde{G}}\tilde{p}^{\prime\mu}=\left[\frac{(r^2+1)m}{2r},0,0,-\frac{(1-r^2)m^2}{2r}\right]$$

where $\mathbf{t}=t\hat{\mathbf{z}},t\to\infty$. We find that $q^2=(1-r)(m^{*2}r-m^2)/r$, $\tilde{q}^2=-(1-r)^2m^2/r$, and $0<r\le m/m^*$. Note that if $r=r_0=m^2/m^{*2}$, then at the photon point, $q^2=0$ and $\tilde{q}^2=-[(m^{*2}-m^2)/m^*]^2$. After some algebra, we obtain

$$h_3'(q^2)\equiv G^*_M(q^2)-3G^*_E(q^2)$$

$$=\left[\frac{m5\sqrt{3}\sqrt{-\tilde{q}^2}}{3(m^*+m)\{(m^*-m)^2-q^2\}^{1/2}}\right]G^V_M(\tilde{q}^2)\ . \qquad (9)$$

In Eq. (9), one may use the excellent approximation that $G^V_M(\tilde{q}^2)\approx2.350[1-\tilde{q}^2/0.71]^{-2}$, so that $h_3'(0)$ and therefore $A_{1/2}$, are functions of the Δ^+ mass and $G^*_M(0)$.

If one uses, for example, the *less model-dependent* [23] *pole mass value* $m^*=1.211$ GeV/c^2, then one calculates numerically that

$$h_3'(0)=[G^*_M(0)-3G^*_E(0)]\approx2.972\ . \qquad (10)$$

This then implies that (see Fig. 1 for $m^*=1.211$ GeV/c^2)

$$A_{1/2}(m^*=1.211\ \text{GeV}/c^2)\approx-0.143\ \text{GeV}^{-1/2}\ . \qquad (11)$$

From Eq. (10), we can now calculate the EMR as a function of $G^*_M(0)$ and $h_3'(0)$, and we find that

$$\text{EMR}=-\frac{1}{3}\left[1-\frac{h_3'(0)}{G^*_M(0)}\right]\ ,$$

$$\text{EMR}\Big|_{m^*=1.211\ \text{GeV}}\approx-\frac{1}{3}\left[1-\frac{2.972}{G^*_M(0)}\right]\ . \qquad (12)$$

If for example, one takes the canonical value $G^*_M(0)\cong3.0$ and $m^*=1.211$ GeV/c, one obtains EMR $\approx-0.3\%$ (see Fig. 2 for other possibilities). Equation (12) reexpresses the EMR as a function of $G^*_M(0)$ and the quantity $h_3'(0)$ (≈2.972 for $m^*\approx1.211$ (GeV/c^2), the value of which is determined by Eqs. (9) and (10) (see Fig. 1 for other possibilities), based on an input value of the mass of the Δ^+.

Thus, the EMR is *reexpressed as a function of the parameters* $G^*_M(0)$ and the Δ^+ mass—as opposed to the parameters $G^*_M(0)$ and $G^*_E(0)$ and is given in Fig. 2. Note, however, as indicated in Figs. 1 and 2, that it is abundantly clear that current experimental data suffice only to fix the EMR in the range $\pm5\%$ corresponding to

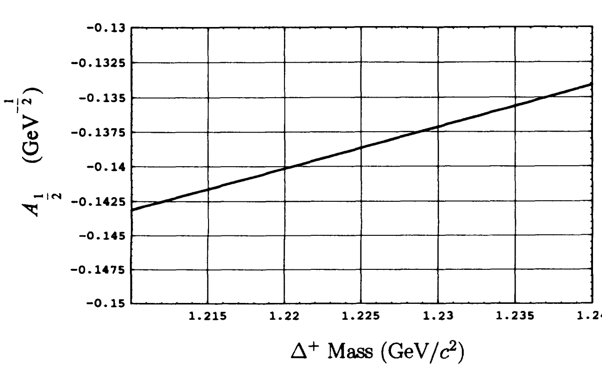

FIG. 1. The photon decay amplitude $A_{1/2}$ in units of GeV$^{-1/2}$ vs the Δ^+ mass m^*.

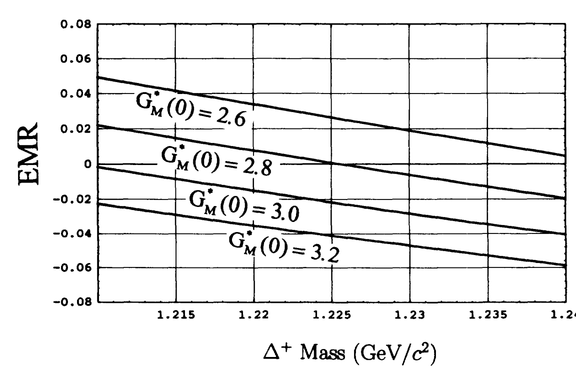

FIG. 2. The E_{1^+}/M_{1^+} ratio at $q^2=0$ (EMR) vs the Δ^+ mass m^*.

$[G_M^*(0)\approx 2.6,\ \Delta^+\approx 1.210\ \text{GeV}/c^2]$ and $[G_M^*(0)\approx 3.2,\ \Delta^+\approx 1.240\ \text{GeV}/c^2]$. It is equally clear from Eq. (2), however, that a much more precise measurement (or theoretical estimate) of $G_M^*(0)$ *and* some method of determining *unambiguously* the appropriate Δ^+ mass to use will be required before one can obtain a very reliable value for the EMR due to its sensitivity to the Δ^+ mass and $G_M^*(0)$. Note that while the EMR is a very sensitive function of $G_M^*(0)$ and Δ^+ mass, the $A_{1/2}$ helicity amplitude is not, thus making it a less useful tool for determining the EMR. In addition, we point out that in agreement with experiment, Eq. (9) predicts a faster than dipole fall-off behavior for $G_M^*(q^2)$ consistent with experimental data in the region where $G_E^*(q^2)$ is known to be small [24]. Unfortunately, because we have only one sum rule result at our disposal (Eq. 8), we are not able to separate $G_M^*(q^2)$ from $G_E^*(q^2)$ and thus we cannot comment definitively at this time on the high q^* perturbative QCD (PQCD) predicted behavior of E_{1^+} versus that of M_{1^+} [25].

We have demonstrated that the transverse 1/2 helicity form factor $h_3(q^2)$ and the $A_{1/2}$ photon decay amplitude can be calculated *nonperturbatively* and is in good agreement with experiment. We have demonstrated that an EMR value ranging from -5% to +5% is obtainable depending on the precise values of $G_M^*(0)$ and the Δ^+ mass that one ultimately uses in one's calculations. We have theoretically confirmed that $G_M^*(q^2)$ *decreases faster* than the nucleon dipole form factor in the q^2 region where the condition $|G_E^*(q^2)| \ll G_M^*(q^2)$ is valid, in agreement with available experimental data and PQCD. Our treatment is *completely relativistic.* Current conservation is guaranteed. Additionally, the correct transition operator is used in all calculations. Our treatment is *nonperturbative* and performed in a broken symmetry hadronic world without the use of "mean" mass approximations since physical masses are used at all times. Thus, $G_E^*(q^2)$ is not constrained to equal zero (i.e., no *D*-wave state) as in the naive Skyrme model and in the naive quark model; the EMR is reexpressed as a function of $G_M^*(0)$ and the Δ^+ mass and is found to be *very sensitive* to their values; $A_{1/2}$ is computed *nonperturbatively* and is in good agreement with experiment.

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[18] When $q^2 \neq 0$, note that $A_{1/2}(q^2) \propto [G_M^*(q^2) - 3G_E^*(q^2)]\sinh(\zeta^*/2)$, where $\sinh(\zeta^*/2) = \sqrt{[(m^* - m)^2 - q^2]/[4m^*m]}$. Similarly, $A_{3/2}(q^2) \propto [G_M^*(q^2) + G_E^*(q^2)]\sinh(\zeta^*/2)$. Also note that, we use the same $\Delta^+ P\gamma$ isospin normalization convention as in Ref. [2]. Thus, the right-hand side (RHS) of Eq. (1) must be multiplied by a factor of $\sqrt{2/3}$ when considering the specific process $\Delta^+ \to P + \gamma$. See some relevant comments on normalization by C. H. Llewellyn-Smith, Phys. Rep. C **3**, 261 (1972) (p. 328 in particular) and by W. Pfeil and D. Schwela, in *Low Energy Hadron Interactions,* Proceedings of the Meeting Ruhestein, 1970, Springer Tracts in Modern Physics Vol. 55 (Springer, Berlin, 1970), p. 213; see also F. D. Gault and M. D. Scadron, Nucl. Phys. **B157**, 517 (1979).

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[21] Using the notation of Ref. [20], a particular solution reported there [Eq. (14) of Ref. [20]] was $a:b:c:d=1:5/2:-\sqrt{2}:\sqrt{2}$ (which implies $\varepsilon=2\sqrt{2}/5$ and $f^{L=0}=0$). Recently, one of us (M.D.S.) obtained the complete solution—there are only two corresponding to $h/f=+2/5$—resulting in the additional solution $a:b:c:d=1:-2:5\sqrt{2}/4:-5\sqrt{2}/4$ (which implies $\varepsilon=5\sqrt{2}/8$ and $f^{L=0}=27f^2/25\neq 0$). This second solution is physical since it implies that $f^{L=0}\neq 0$ as is required in our model. We note that all results of Ref. [20] remain intact since the relation $c:d=-1$ is true for both solutions. We also note that the relation $a:b=1:-2$ (physical solution) results in a value for the Δ^{++} total magnetic moment $\mu^*_{\Delta^{++}}$ (to be published) consistent with experiment and the lattice gauge results of Ref. [14].

[22] L. Durand III, P. C. DeCelles, and R. B. Marr, Phys. Rev. **126**, 1882 (1962). The Lorentz transformations G and $\tilde{G}$ that we utilize are the so-called Z-boost transformations. In particular, the boost G is constructed so that it (active sense) transforms a Δ^+ with four-momentum $p^{*\mu}=(p^{*0},0,0,t)$, where $t\equiv|\mathbf{t}|$ and $p^{*0}=\sqrt{m^{*2}+t^2}$ into a Δ^+ at rest with mass m^*. The same boost G transforms a proton p with four-momentum $p^\mu=(p^0,0,0,rt)$, where $rt=|\mathbf{s}|$ and $p^0=\sqrt{m^2+r^2t^2}$ into a proton with energy $p'^0=(r^2m^{*2}+m^2)/2rm^*$ and z component of three-momentum $P'_z=(r^2m^{*2}-m^2)/2rm^*$, where $0<r\leq m/m^*$. The inverse of G or G^{-1} then has the property that it will take the kinematical configuration where a Δ^+ at rest decaying into a photon (real or virtual) moving in the positive z direction with a proton moving in the negative z direction into the kinematical configuration described by the LHS of Eq. (8), where all particles are moving in the directions specified by $\mathbf{s}(=s\hat{\mathbf{z}}=rt)$ and $\mathbf{t}=t\hat{\mathbf{z}}$. Similarly, the boost $\tilde{G}$ is constructed so that it transforms a proton with four-momentum $\tilde{p}^{*\mu}=(\tilde{p}^{*0},0,0,t)$, where $\tilde{p}^{*0}=\sqrt{m^2+t^2}$ into a proton at rest with mass m.

[23] See the comments on page VIII.13, Particle Data Group, K. Hikasa *et al.*, Phys. Rev. D **45**, S1 (1992).

[24] For instance, if one defines $R\equiv h'_3(q^2)/[h'_3(0)G_{\text{dipole}}]$, $G_{\text{dipole}}\equiv(1-q^2/0.71)^{-2}$, and uses $G_M^V(\tilde{q}^2)\approx 2.350[1-\tilde{q}^2/0.71]^{-2}$ and $m^*=1.232$ GeV/c^2 in Eq. (9), then $R\cong 0.76$ for $-q^2\cong 0.5$ GeV$^2/c^2$. Equation (9) also predicts that $R\to 1$ as $q^2\to\infty$.

[25] G. A. Warren and C. E. Carlson, Phys. Rev. D **42**, 3020 (1990).