Dynamical confinement in bosonized two-dimensional QCD

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In the bosonized version of two-dimensional theories nontrivial boundary conditions (topology) play a crucial role. They are inevitable if one wants to describe nonsinglet states. In Abelian bosonization, color is the charge of a topological current in terms of a nonlinear meson field. We show that confinement appears as the dynamical collapse of the topology associated with its nontrivial boundary conditions.

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I. INTRODUCTION

The study of confinement in two-dimensional quantum chromodynamics has been a much discussed problem in the literature since the pioneering work of t' Hooft [1]. Many controversies about the realization of confinement in two-dimensional QCD (QCD_2) have arisen over the years. Contrary to 't Hooft's results, other approaches show several phases in parameter space [M (quark mass), e (coupling constant), and N (number of colors)] [2–4]. The large-N approximation appears responsible for the exceptionality of the confinement mechanism in the model of 't Hooft.

In the large-N limit strong suppressions occur in the set of diagrams of the theory. Only planar diagrams contribute; there are no vertex corrections and quark loops disappear. Under these severe restrictions the quark-antiquark interaction is controlled by the one-gluon exchange potential (OGEP) exclusively. Unlike in the four-dimensional (4D) case the 2D OGEP is confining. The 2D Poisson equation in the presence of a static charge tells us that the zero component of the gauge field must rise linearly with distance. The gluon propagator is then

$$\partial_1^{-2}(x,y)=\tfrac{1}{2}|x_1-y_1|\delta(x_0-y_0) , \tag{1}$$

which to first order in $1/N$ is proportional to the $q\bar{q}$ interaction. Thus, confinement here is a peculiarity of the dimensionality of space-time [5]. The resolution of 't Hooft's equation confirms this mechanism leading to a discrete, stable, and infinite spectrum [1]. However, the formalism is much more powerful, eliminating those amplitudes which would violate confinement explicitly [6,7]. Let us study, for example, the process meson $\rightarrow q\bar{q}$. It can be shown that the amplitude for it is given by

$$F_n^{a\bar{b}}(t,r)=\frac{e^2}{r_-\sqrt{\pi N}}P\int_0^1 dt'\frac{\varphi_n^{a\bar{b}}(t')}{(t-t')^2} \tag{2}$$

if $t\in[0,1]$. The notation follows that of Ref. [1]. In order to obtain the physical amplitude one has to impose the on-mass-shell restrictions, i.e.,

$$p^2=M_a^2 , \quad (p-r)^2=M_{\bar{b}}^2 , \quad r^2=r_n^2 , \tag{3}$$

where M_i are the renormalized quark masses and r_n the corresponding meson mass. The on-mass-shell condition leads to the following relation for the dimensionless momentum t:

$$\mu^2=\frac{\alpha_a}{t}+\frac{\alpha_{\bar{b}}}{1-t} , \tag{4}$$

and therefore 't Hooft's equation becomes

$$P\int_0^1\frac{\varphi_n^{a\bar{b}}}{(t-t')^2}=0 , \tag{5}$$

which implies the vanishing of the amplitude for the process. Therefore, an expected consequence of the confinement mechanism is that no quarks can be liberated from a bound state.

Another limit in which the confinement mechanism can be understood is that of very heavy quarks. In leading order, that is, when quarks are infinitely heavy $M\rightarrow\infty$, QCD_2 becomes a pure gauge theory with static external color sources. Confinement for such a theory is trivial. As in the large-N limit, the quark propagator, as well as the $q\bar{q}$ potential, linearly rise with distance. Analogous to the $N\rightarrow\infty$ limit, in a pure gauge theory with static external sources there are no sea quark contributions, no nonplanarity effects, and no vertex corrections. Therefore, no special assumptions are needed to prove confinement when $e \ll M\rightarrow\infty$. In the next-to-leading order in $1/M$, the eigenvalue equation for the mesonic amplitudes can be reduced to a one-dimensional Schrödinger equation [8]. The $q\bar{q}$ potential appearing in this equation is again linear. In this case only the zero component of the gauge field survives the nonrelativistic limit and it is proportional to the gluon propagator. The spectrum exhibits the same features as t' Hooft's model. It is discrete, infinite, and stable. No quarks are allowed.

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Nevertheless, once we move away from these two limits in the space of parameters, QCD_2 becomes extremely complex. The aforementioned sea quark excitations, vertex corrections, and nonplanar contributions are now of great relevance. Consequently, the confinement mechanism becomes much more complicated than in the above cases.

In this paper we give an alternative description of the confinement mechanism for two-dimensional quantum chromodynamics, which has the advantage of being universal in the space of parameters. We develop it in an SU(2) gauge theory, without any restriction associated with large N or large mass. Therefore, our results hold in any regime of the theory including those where *naive* confinement cannot take place.

According to our investigation the crucial ingredients to understand confinement in QCD_2 are the following.

(i) Boundary conditions. Nontrivial boundary conditions (NTBC's) allow the existence of a nonlinear nonlocal realization of the SU(2) color symmetry in 2D. This realization is built upon one single-colored field transforming under a *nonlinear nonlocal representation* of the group. The color content of the theory *dramatically* depends on the structure of the BC's of this field.

(ii) Vacuum invariance. The variance of the QCD_2 vacuum under SU(2) *global* transformations is an *exact* statement (Coleman's theorem [9]). Its validity is universal for any value of the parameters (M and e and easily generalizable to any N).

We will show in the following sections that these two statements lead to a unique conclusion: There is only one phase of permanent confinement in QCD_2 for every value of M, e, and N. No colored states are allowed for any value of the parameters.

II. REALIZATION OF COLOR SYMMETRY

Nonlinear nonlocal realizations of a non-Abelian symmetry were used initially by Halpern in the study of two-dimensional gauge theories [10]. This peculiar realization of the SU(N) symmetry in 2D has been also extensively used in conformal field theory under the name of Frenkel-Kac-Segal or "vertex operator" construction [11]. In order to understand this special representation of color symmetry we start with the simplest example: the theory of a single massless Dirac fermion with two internal degrees of freedom (one flavor, two colors), defined by

$$\mathcal{L}_F=\bar{q}_\alpha i\gamma_\mu\partial^\mu q_\alpha\ ,\quad \alpha=1,2\ . \tag{6}$$

Using Abelian bosonization we can write a *completely* equivalent theory in terms of *two* bosons, one for every internal fermionic degree of freedom, verifying

$$\mathcal{L}_B=\tfrac{1}{2}\partial_\mu\varphi_\alpha\partial^\mu\varphi_\alpha\ ,\quad \alpha=1,2 \tag{7}$$

and

$$:\bar{q}_\alpha\gamma^\mu q_\alpha:=\frac{1}{\sqrt{\pi}}\epsilon^{\mu\nu}\partial_\nu\varphi_\alpha\ . \tag{8}$$

Introducing the combinations $\varphi=(1/\sqrt{2})(\varphi_1+\varphi_2)$ and $\eta=(1/\sqrt{2})(\varphi_1-\varphi_2)$, we can rewrite the U(1) flavor current and the third component of color isospin in a more suggestive form,

$$J^\mu=\frac{1}{2}\sum_\alpha :\bar{q}_\alpha\gamma^\mu q_\alpha:=\frac{1}{\sqrt{2\pi}}\epsilon^{\mu\nu}\partial_\nu\varphi\ , \tag{9}$$

$$J_3^\mu=\mathrm{Tr}:\bar{q}_\alpha\gamma^\mu\frac{\tau_3}{2}q_\alpha:=\frac{1}{\sqrt{2\pi}}\epsilon^{\mu\nu}\partial_\nu\eta\ , \tag{10}$$

and the (topologically) conserved charges as

$$B=\frac{1}{\sqrt{2\pi}}[\varphi(+\infty)-\varphi(-\infty)]\ , \tag{11}$$

$$T_3=\frac{1}{\sqrt{2\pi}}[\eta(+\infty)-\eta(-\infty)]\ . \tag{12}$$

The bosonized Lagrangian shows an explicit splitting of flavor and color degrees of freedom when expressed in terms of these new fields:

$$\mathcal{L}_B=\tfrac{1}{2}\partial_\mu\varphi\partial^\mu\varphi+\tfrac{1}{2}\partial_\mu\eta\partial^\mu\eta\ . \tag{13}$$

There is nothing special about the realization of the U(1) flavor symmetry of the original fermion model (6). The φ field takes care of the $U(1)_V\otimes U(1)_A$ flavor symmetry in a complete manner. The vector symmetry is conserved in a topological way (that is, without resorting to the φ equation of motion) thanks to the presence of the fully antisymmetric tensor in (9). On the contrary, the axial flavor current is a real Noether current. Its conservation is guaranteed by the massless character of the φ field.

The φ field realizes the U(1) flavor symmetry in a *linear and local way.*[1] Nevertheless it holds NTBC solutions, as is required by the vector current bosonization rule (9). The full equivalence of the fermion and boson theories forces its bosonized version to have operators generating states carrying fermion quantum numbers [12]. Because baryon number (as well as the third component of color isospin) is a topological charge [Eq. (11)], the only way to achieve baryonic charged operators in the $U(1)_F$ sector of the bosonized model is through NTBC ($\Delta_\infty\varphi=\sqrt{2\pi}B\neq 0$).

On the contrary, the realization of color symmetry is much more subtle. It is clear from the bosonization rules and the bosonized action that the whole SU(2) color structure must be built upon the η field. However, we know from group theory that SU(N) cannot be represented *linearly* on N real fields. Just on these general grounds we can already predict the nonlinear behavior of the η field under a general SU(2) transformation.[2]

In fact, η transforms under a nonlinear nonlocal representation of SU(2). However, it can be related to linearly transforming objects through nonlinear nonlocal expressions. For example, it can be related to the following.

(i) Objects in the fundamental representation:

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[1]This can be easily seen if we rewrite the U(1) part of the bosonized Lagrangian in its "group form:" $\partial_\mu\varphi\partial^\mu\varphi=(1/2\pi)\partial_\mu(e^{i\sqrt{2\pi}\varphi})\partial^\mu e^{-i\sqrt{2\pi}\varphi}$.

[2]Non-Abelian bosonization would lead to an action in terms of the SU(2) fields $g=\exp[i(\tau^a\pi^a/2)]$, $a=1,2,3$, with three color degrees of freedom, where the π^a fields transform according to the adjoint representation.

$$S^{\alpha}(x,t)=:\exp\left[i(-)^{\alpha+1/2}\left[\frac{\pi}{2}\right]^{1/2}\left[\int_{-\infty}^{x}d\xi\dot{\eta}(\xi,t)+\eta(x,t)\right]\right]: , \tag{14}$$

where $\alpha=-\frac{1}{2},+\frac{1}{2}$.

(ii) Objects in the adjoint representation:[3]

$$J_{\pm}(x,t)=:\exp\left[\pm i\sqrt{2\pi}\left[\int_{-\infty}^{x}d\xi\dot{\eta}(\xi,t)+\eta(x,t)\right]\right]: ,$$
$$\bar{J}_{\pm}(x,t)=:\exp\left[\pm i\sqrt{2\pi}\left[\int_{-\infty}^{x}d\xi\dot{\eta}(\xi,t)-\eta(x,t)\right]\right]: , \tag{15}$$
$$J_3^0(x,t)=\frac{1}{\sqrt{2\pi}}\partial_x\eta(x,t) ,$$

as can be checked explicitly.

By means of the canonical commutation relations of the η field one can prove that the operators in (15) close an SU(2)⊗SU(2) current algebra:

$$[J_+(x),J_+(y)]=4J^3(x)\delta(x-y)+i\frac{2}{\pi}\partial_x\delta(x-y) \tag{16}$$

(identically for $\bar{J}$) [10]. Their conservation is guaranteed by topology in the case of J_3^{μ} [Eq. (9)] and by the equations of motion in the case of $J_{\pm}$ and $\bar{J}_{\pm}$. The spatial integrals of the zero components of the $(J_{\pm}^{\mu},J_3^{\mu})$ currents define the conserved SU(2) color charges $(T_{\pm},T_3)$. Analogously, by using the same techniques leading to the current algebra (16), one finds that the soliton operator (14) is *really* in the fundamental representation of SU(2) because

$$[T^a,S^{\alpha}(x,t)]=\left[\frac{\tau^a}{2}\right]_{\alpha\beta}S^{\beta}(x,t) . \tag{17}$$

In this way η transforms under the action of the lowering and raising operators of SU(2) as [10]

$$[T_{\pm},\eta(x,t)]=\pm\sqrt{2\pi}\int_{-\infty}^{x}d\xi J_{\pm}^0(\xi,t) . \tag{18}$$

The fact that the S^{α} and $J_{\pm}$ operators lie on irreducible representations of the color group prevents the η field from transforming *linearly and locally* under the group. The complexity of the relation between $T_3\neq 0$ tensorial objects and the field supporting the symmetry [Eqs. (14) and (15)] causes this particular realization of the non-Abelian structure. Observe that is *only* the value of η at $+\infty$, not η itself, which transforms properly under the adjoint representation of SU(2). This is because $\eta(+\infty)=\sqrt{2\pi}T_3$, Eq. (12).[4] Thus, the BC's contain crucial information associated with the realization of color symmetry.

Now it is easy to understand how the bosonization procedure operates. We have just learned that it is possible to build nontrivial color operators based upon the η field. For this reason the original Fermi field can be expressed in terms of soliton operators carrying $(B=T=\frac{1}{2}, T_3=\pm\frac{1}{2})$ quantum numbers. That is to say,[5]

$$q_{\mp}^{\alpha}(x,t)=k\mu^{1/2}S(x,t)S^{\alpha}(x,t) , \tag{19}$$

where k is a numerical constant and μ a renormal ordering mass [13]. The S operator creates the $B=\frac{1}{2}$ "flavor" soliton in terms of the φ field, whereas S^{α} does the same for the color soliton. Thus, any fermion operator is capable of being expressed in terms of the φ and η boson fields by means of the previous relation. Diagonal operators in color $(T_3=0)$ will be local in η. In the same way $B=0$ operators will be local in φ. Any other operator will contain nonlocal pieces, as those occurring in the charge operators of Eqs. (14) and (15).

In the more modern language of conformal field theories (CFT's), the above construction expresses the possibility of writing the same CFT, defined by its central charge c and its level k, in different free field representations. For a theory realizing an affine U(1)⊗SU(N) symmetry at level $k=1$, the central charge is $c=N$. This is satisfied both for a theory of N free complex fermions and for a theory of N free bosons. Because c and k define completely the current algebra of the theory (Kac-Moody algebra), both representations have the same current algebra [Eq. (16)] and therefore preserve the same symmetries. Our diagonal current J_3^{μ} [Eq. (9)] represents the Cartan subalgebra of color SU(2), whereas the charged currents $J_{\pm}^{\mu}$, are "vertex operators" with conformal weights (1,0) and (0,1) representing the remaining SU(2) currents. The η field itself is not a conformal field because of its 2D infrared behavior. It is not a primary field and it has no well-defined conformal dimensions. Only its derivative (J_3^{μ}) or its exponential $(J_{\pm}^{\mu})$ have good conformal properties. Because of the deep link between conformal symmetry (Virasoro algebra) and internal symmetry (Kac-Moody algebra) in 2D, the strong IR behavior of the η field also spoils its SU(2) properties. Because η is not a primary field it cannot provide a linear representation of the color group either [14].[6] This peculiar regeneration of the whole SU(N) structure out of its Cartan subalgebra in 2D is called the Frenkel-Kac-Segal or "vertex operator" construction [11].[7]

At this stage the importance of BC's and topology should be clear. When considering the action (13) we assume φ and η are bosonic coordinates compactified on a circle. In other words, its action can be written in terms of the $U(1)_F\otimes U(1)_C$ fields $(e^{i\sqrt{2\pi}\varphi},e^{i\sqrt{2\pi}\eta})$ using

$$\partial_{\mu}\phi\partial^{\mu}\phi=\frac{1}{2\pi}\partial_{\mu}(e^{i\sqrt{2\pi}\phi})\partial^{\mu}e^{-i\sqrt{2\pi}\phi} , \tag{20}$$

where $\phi=\varphi$, η. This means that, when calculating any

[3]In our notation $J=J_0+J_1$ and $\bar{J}=J_0-J_1$. The subindex $\pm$ stands for the lowering and raising color-isospin currents.

[4]We have chosen $\eta(-\infty)=0$.

[5]In 2D q is a bispinor, $q=(1/2^{1/4})(q_+,q_-)$.

[6]Note the difference between η and the non-Abelian bosonization field g_{β}^{α}. The latter is *really* a primary field of the WZWN model and thus it belongs to the $(1/N,1/N)$ representation of SU(N)⊗SU(N).

[7]We are grateful to E. Alvarez for this reference.

functional integral of this theory in a finite volume (finite length L), we have to consider all different sectors induced by the topological nontrivial mapping from the S^1 sphere (the compactified one-dimensional space) into the $U(1)_F \otimes U(1)_C$ group space ($S^1 \otimes S^1$). Each of the different homotopic solutions can be characterized by two integers (ν_F, ν_C). These topological charges are obtained by means of the integral formulas in terms of group elements [5]:

$$\nu_i = \frac{i}{2\pi}\int_0^{2\pi} d\theta\, g_i \frac{d}{d\theta} g_i^{-1} , \tag{21}$$

where $g_i \in U_i(1)$, $i = F, C$, and therefore

$$\begin{aligned} \nu_F &= \sqrt{2/\pi}[\varphi(2\pi) - \varphi(0)] = 2B , \\ \nu_C &= \sqrt{2/\pi}[\eta(2\pi) - \eta(0)] = 2T^3 . \end{aligned} \tag{22}$$

The nontrivial boundary conditions produce the *winding numbers* associated with the homotopy classes of these mappings.

As we have seen the realization of the U(1) "flavor" symmetry by the φ field is linear. In the case of the η field *the existence of NTBC's* allows the enlargement of the "explicit" $U(1)_C$ to the whole SU(2) color symmetry through the vertex operator construction (15). NTBC's *are crucial* for the realization of the color symmetry in this special way. If they did not exist the nonlocal pieces of the nondiagonal operators would disappear leading us to a *trivial Abelian current algebra* (16). The vertex operator construction would not be possible anymore. This is precisely the crux of the matter which we will exploit systematically in the next section.

The introduction of a mass term in the fermion Lagrangian (6) does not alter the previous construction [10,12]. The bosonized form of the mass operator is easily obtainable from the soliton-fermion operator correspondence (19) leading to

$$S_M = \int d^2x \{ k^2 \mu M (\cos\sqrt{2\pi}\varphi)_\mu (\cos\sqrt{2\pi}\eta)_\mu \} , \tag{23}$$

M being the fermion mass. The classical potential has a minimum at

$$V_M(\tilde{\varphi}, \tilde{\eta}) = -k^2 \mu M , \tag{24}$$

which implies a degenerate minima structure formed by the infinite set of points [10]

$$\varnothing = \varnothing_{\rm I} \cup \varnothing_{\rm II} , \tag{25}$$

where

$$\varnothing \equiv \begin{cases} (\sqrt{2\pi} n \ , \sqrt{2\pi} m) \in \phi_{\rm I} , \\ (\sqrt{2\pi}(n+\frac{1}{2}) \ , \sqrt{2\pi}(m+\frac{1}{2})) \in \phi_{\rm II} , \\ (n,m) \in Z . \end{cases} \tag{26}$$

Although the mass term has no effect on the vertex operator construction, it possesses a very appealing property: namely, it allows us to relate the minima structure to the SU(2) content of the theory. The minima give us precise information about the solutions of the bosonized theory, since they are related to the possible boundary conditions [5,15], i.e.,

$$\lim_{x \to \pm\infty} (\varphi(x), \eta(x)) = (\varphi_{n\pm}, \eta_{m\pm}) \in \varnothing . \tag{27}$$

There also exists a close link between the minima structure and the nonlinear nonlocal realization of color symmetry. Charge operators ($B \neq 0$, $T_3 \neq 0$) connect different minima (different BC's) in the infinite lattice defined by $\varnothing$. Thus, any charge operator of the fermionic theory can be represented by means of solitonic operators linking different lattice points (see Fig. 1). In this way the realization of the whole SU(2) color symmetry is guaranteed. Every state generated by any of these operators will transform linearly under the color group.

We can use the (φ, η) minima structure plane $\varnothing$ as a diagram for physical states, just recalling the relation between (B, T_3) charges and the asymptotic conditions of the fields [Eqs. (11) and (12)]. Figure (1) shows that all states described by arrows of the same length and direction are equivalent; i.e., they have the same (B, T_3) charges. Thus, we can define an equivalence relation and choose just one representative per class. For example, we proceed by attaching the arrows to the same point $[\varphi(-\infty) = \eta(-\infty) = 0]$, since we have the freedom to select one of the boundary conditions. With this restriction all the physical states will be represented by the points of the vacuum structure lattice, which becomes in this way a (B, T_3) plane (see Fig. 2).

Global SU(2) transformations leave the potential invariant. This is easily seen in the non-Abelian bosonization scheme where the potential is proportional to $\text{tr}(g) = \cos\sqrt{2\pi}\eta$, a SU(2) invariant, and $\cos\sqrt{2\pi}\varphi$, a function of φ, invariant by construction [16]. Therefore, the set of physical states, the lattice, is invariant under these transformations. Moreover, it is also invariant under the discrete "orthogonal" translations (horizontal and vertical shifts) [10]

$$\begin{aligned} &\varphi \to \varphi + \sqrt{2\pi} n , \\ &\eta \to \eta + \sqrt{2\pi} m , \\ &(n,m) \in Z , \end{aligned}$$

and the "diagonal" ones

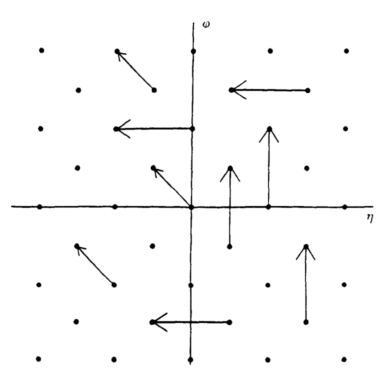

FIG. 1. Examples of allowed states: (a) diagonal arrows: down-color quark state ($B = \frac{1}{2}$, $T^3 = -\frac{1}{2}$); (b) horizontal arrows: down-color vector state ($B = 0$, $T^3 = -1$); (c) vertical arrows: baryon state ($B = 1$, $T^3 = 0$).

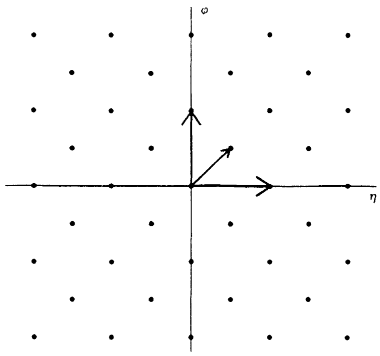

FIG. 2. Representation of the physical states in the (B,T^3) plane.

$$\varphi \rightarrow \varphi + \sqrt{\pi/2}n \ ,$$
$$\eta \rightarrow \eta + \sqrt{\pi/2}m \ , \quad (28)$$
$$n+m = \text{even} \ .$$

Before closing this section it is important to emphasize that the topological structure just studied coincides with that of the chiral limit of the theory. The topological charges of the physical states are mass independent. They survive in the chiral limit together with the asymptotic conditions that generate them. The minima of the potential become the NTBC's, which are allowed by energy considerations. NTBC's are a consequence of the nontrivial topology of the $U(1)_F \otimes U(1)_C$ bosonized theory (13) on the circle. This is *independent* of the existence of the mass term. When the mass potential is present there exists, in addition, a one-to-one correspondence between its minima and the BC's of the U(1) fields. In this way the lattice of physical state $(=\varnothing)$ is just given by the first homotopy class of the group:

$$\varnothing \approx \Pi_1[U(1)_F \otimes U(1)_C]$$
$$\approx \Pi_1[U(1)_F] \otimes \Pi_1[U(1)_C] \approx Z_F \otimes Z_C \ . \quad (29)$$

This property is a *necessary* condition for the nonlinear nonlocal realization of the *whole* color symmetry.

III. THE VACUUM STRUCTURE OF QCD_2

The introduction of the color gauge interaction is carried out in the usual way, that is, by minimally coupling gluons to matter in the free fermionic Lagrangian (6) and by adding a gauge-invariant gluon kinetic term. The new Lagrangian is locally gauge invariant and thus is, in particular, globally gauge invariant. Therefore, there must exist conserved charges associated with this global symmetry. Certainly these charges are nothing but the color charges of the SU(N) symmetry and they are conserved.[8] But now we have two different sources generating this internal degree of freedom: the fermions (quarks) and the colored gauge particles (gluons). The conserved color current is the sum of both contributions:

$$J_a^\nu = j_a^\nu + G_a^\nu \ , \quad a=1,\ldots,N^2-1 \ , \quad (30)$$

where j_a^ν is the quark current and

$$G_a^\nu = i[A_\mu, F^{\mu\nu}]_a \ . \quad (31)$$

The J_a^ν current is conserved through the equations of motion of the gluon field $D_\mu F^{\mu\nu} = ej^\nu$,

$$\partial_\mu F_a^{\mu\nu} = ej_a^\nu + eG_a^\nu = eJ_a^\nu \quad (32)$$

and thus $\partial_\mu J_a^\mu \equiv 0$. The quark current is a covariant object under local gauge transformations. This is not the case of the gluon current which does not transform as a tensor, as one can see from its definition (31) $(\delta_U G^\nu = e^{-1}[(\partial_\mu U)U^{-1}, F_{\mu\nu}])$. However, global color symmetry is preserved in any gauge (32). Therefore, also in any gauge, we have at our disposal a set of N^2-1 different color charges commuting with the Hamiltonian. These charges will be related in different gauges by local gauge transformations.

In order to perform our calculation we proceed to fix the gauge. The techniques developed in the previous section require gauges in which the topological character of the color current shows up explicitly. The so-called "hybrid gauges," involving restrictions on both the gauge field and the field strength, can be used for our purpose. In particular, to generate, for $N=2$, a topological J_3^ν current it is enough to demand $G_3^\nu = 0$ in the desired gauge. In this way, $J_3^\nu = j_3^\nu$.

Once we integrate out the gluons, we are left with a theory with only fermionic degrees of freedom. By means of the bosonization techniques we will obtain a theory for the η field possessing the topological color conservation law, $\partial_\mu J_3^\mu \equiv 0$, since

$$J_3^\nu = j_3^\nu = \frac{1}{\sqrt{2\pi}} \epsilon^{\mu\nu} \partial_\nu \eta \ . \quad (33)$$

A vertex operator construction provides the conditions to understand the effects of the η dynamics on the $U(1)_C$ topology, i.e., the color structure of the theory.

If we write the $F^{\mu\nu} = \epsilon^{\mu\nu} F$ and A^μ adjoint fields in the spherical basis,[9] then the condition $G_3^\nu = 0$ is equivalent to requiring

$$F_+ A_-^\mu = F_- A_+^\mu \ , \quad \mu = 0,1 \ . \quad (34)$$

There are many gauges which satisfy (34) and thus the topological condition (33). We take the gauge of Baluni, in which a complete bosonization of the theory has been already worked out [3]. This gauge generates a very convenient framework to study the topological realization of color symmetry in QCD_2. Baluni's gauge clearly satisfies the two above conditions, since it requires that the nondiagonal terms of the field strength (F_+, F_-) be zero. To fix the gauge completely another independent condition is needed, which in Baluni's gauge is $A_3^1 = 0$.

The bosonized form of the third component of the field strength is the same in any "topological" gauge (34). From the equation of motion (32) and the bosonization rule (33) it is clear that

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[8]Because the SU(N) *vector* symmetry is anomaly-free this holds also at the quantum level.

[9]$F = F_- T_- + F_+ T_+ + F_3 T_3$ (the same for A^μ).

$$\partial_\mu F_3 = \frac{e}{\sqrt{2\pi}} \partial_\mu \eta \tag{35}$$

and then

$$F_3(x) = \frac{e}{\sqrt{2\pi}} \eta(x) + E \ , \tag{36}$$

where E is a constant color background field which can be attributed to the existence of classical charges at spatial infinity. In the Abelian case such a background field has physical relevance. It generates the θ angle of the massive Schwinger model. θ is a mass and coupling constant independent parameter possessing important physical properties [17]. However, in the non-Abelian case, the corresponding angle associated with E has no physical consequences. Thus, we consider that there is no color background field by setting $E=0$ [5]. This bosonizes unambiguously the third component of the gluon kinetic term producing a mass term for the η field.

The bosonization of the other terms is more involved but straightforward. Nevertheless, we would like to draw attention to an important detail of the bosonization procedure. After the integration of the gluons in any "topological" gauge, one is able to bosonize completely the effective fermionic action by means of the fermion-boson correspondence (19). The gauge-dependent effective fermion interaction can be extremely complicated, but it has to verify a very nice property when bosonized, namely, that it must be *local* in the color field η. This has to be so because the Hamiltonian is a *diagonal* operator in color ($T_3=0$). An analogous argument holds for the "flavor" field φ. Therefore, in any "topological" gauge the final outcome of the bosonization procedure is a local action in the bosonic fields φ and η. Moreover, this action must include a mass term for the color field η, corresponding to the bosonized form of the pure gauge action.

Baluni's bosonized action is an explicit example of a "topological" gauge and therefore satisfies the features just described. In this case the potential arising from the quark-gluon interaction is

$$V_i(\eta) = \frac{e^2}{32\pi}\eta^2 + \sqrt{\pi} k^4 \mu^2 \left[1 - \frac{\sin\sqrt{2\pi}\eta}{\sqrt{2\pi}\eta} \right] , \tag{37}$$

where k and μ are the parameters appearing in the bosonization formula (19). What are the implications of the above potential in the $U(1)_C$ color topology of the gauge theory? We proceed to provide an answer.

Because of the relation between the minima structure and NTBC's [Eq. (27)], our first task is to find the constraints induced by $V_i(\eta)$ into the full action:

$$S = \int d^2x \{ \tfrac{1}{2}\partial_\mu \varphi \partial^\mu \varphi + \tfrac{1}{2}\partial_\mu \eta \partial^\mu \eta + k^2 \mu M (\cos\sqrt{2\pi}\varphi)_\mu (\cos\sqrt{2\pi}\eta)_\mu - V_i(\eta) \} \ . \tag{38}$$

The potential V_i is positive definite and it has only one absolute minimum $V_i(\eta)=0$ which occurs for $\eta=0$:

$$V_i(\eta) = 0 \leftrightarrow \eta = 0 \ . \tag{39}$$

Therefore, the full potential has a lower bound

$$V(\varphi,\eta) = V_M(\varphi,\eta) + V_i(\eta) \geq -k^2 \mu M \ , \quad \forall (\varphi,\eta) \ . \tag{40}$$

The equality is only saturated if the following conditions are met:

$$\cos\sqrt{2\pi}\varphi \cos\sqrt{2\pi}\eta = 1 \ \text{ and } \ V_i(\eta) = 0 \ . \tag{41}$$

Thus, the minima form the set

$$\varnothing_{QCD_2} = \{ (\varphi_n, \eta_0 = 0) n \in Z \} \ , \tag{42}$$

which correspond to the shaded lattice points shown in Fig. 3.

So far only mathematics have been invoked. Now we have to put the previous properties into physical words. This will require a careful analysis of both the global SU(2) properties of the vacuum and the nonlinear nonlocal realization of color symmetry.

We start by paying attention to the first of these two important issues. The full potential V is SU(2) invariant just by construction. In order to obtain it, we have rewritten the invariant interaction Hamiltonian of the fermion model in terms of the η field by means of the fermion-boson equivalence (19). Because bosonization is exact, when writing V in terms of η-dependent tensorial objects we must obtain the original SU(2)-invariant fermion interaction, no matter how complicated the transformation law of the η field is.

Consequently, in any "topological" gauge the shape of the full potential must be preserved by global color transformations (this is exactly what happened in the free case too). Because the V_i potential is exactly the same before and after a SU(2) transformation its minimum in $\eta_0=0$ must stay. In other words, a global SU(2) transformation cannot move the V_i minimum from $\eta_0=0$. Certainly, a color transformation will not shift a "flavor" minimum φ_n into another for the simple reason it does not act on the φ field. Thus, any of the $\varnothing_{QCD_2}$ minima will remain unaltered by a global color rotation.

The special realization of color symmetry has provided us with a potential depending on *one single real scalar* field. This means that the set of minima $\varnothing_{QCD_2}$ can only be a discrete set (no typical 2D "Mexican hat" potential is allowed because we have a function depending only on *one* real variable). Therefore, only *discrete* transforma-

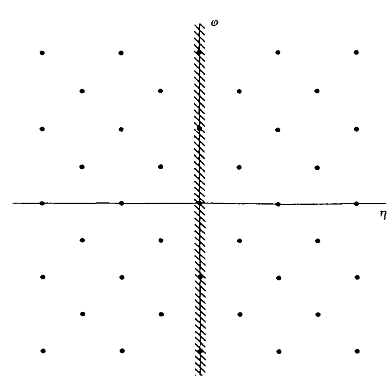

FIG. 3. The color singlet sector of QCD_2.

tions can shift one of this minima to another. A global continuous transformation will leave these minima untouched. If we choose one of them as the real vacuum, it will be necessarily invariant under *continuous* global color transformations. This is nothing more than an explicit example of the more general result that a continuous symmetry cannot be spontaneously broken in 2D [9].[10] This is a general statement valid for any value of the quark mass, the coupling constant, and the number of colors.

The second issue is to unveil the consequences of the new interaction and the new vacuum structure on the realization of color symmetry. This cannot be done in a standard group theory fashion because η does not realize the color symmetry in a standard way.

To see this, in a more transparent way, let us proceed *ad absurdum*. Assume that η can be *naively* interpreted as a conventional color field, which does not carry a third component of color charge [recall $[T_3,\eta]=0$ and (18)]. Furthermore, let it belong to an irreducible representation of SU(2), e.g., η behaves like π_0 in the pion triplet. Then the Hamiltonian cannot be a color singlet. To see this, one realizes that the Hamiltonian may be expanded in powers of $\eta^2(\sim\pi_0^2)$, but $\eta^2(\sim\pi_0^2)$ is not a SU(2) invariant object (only $\pi_a\pi_a$ is). Thus, we arrive to the absurd result that the bosonization procedure is inconsistent.[11]

The η field is not adequate to study the color symmetry, but its BC's are. The η field NTBC's determine not only the topology of the $U(1)_C$ subgroup of color SU(2) but its whole structure. We therefore have to establish the new $U(1)_C$ topology induced by the interaction. This is easy using the relation (27) between BC's and the minima structure of the potential. We can understand what happens just by comparing the set of minima of the free theory $\varnothing$ to that of QCD_2. Because of the interaction the $U(1)_C$ topology experiments a complete collapse. Only *trivial* BC solutions, i.e., $\eta(+\infty)=\eta(-\infty)=\eta_0=0$, are allowed by the minima structure of QCD_2.

This result can be expressed in a more dynamical way. If we wish to calculate the mass of a $T_3=n/2$ $(n\in Z)$ charged particle, we need to evaluate the expectation value $\langle\psi_n|\hat{V}|\psi_n\rangle$, $\hat{V}$ being the interaction potential operator and $|\psi_n\rangle$ a generic charged state. In bosonized language such a state will be built upon the vacuum by means of "vertex operators" of the kind S^α or $J_\pm$. Because the vacuum is not charged, it is color invariant, the T_3 charge of this state will coincide with that of the "vertex operators." These states are certainly responsible for NTBC's solutions because they produce the soliton "jump" of the η operator [12]:

$$\langle\psi_n|\hat{\eta}(x)|\psi_n\rangle\to\begin{cases}0, & x\to-\infty\ ,\\ \sqrt{\pi/2}n\ , & n\in Z\ ,\ x\to+\infty\ .\end{cases} \tag{43}$$

If we expand this expectation value in powers of Planck's constant, the leading term will be just the classical soliton solution

$$\langle\psi_n|\hat{\eta}(x)|\psi_n\rangle=\eta_n^{\rm cl}(x)+O(h)\ . \tag{44}$$

Analogously we can expand the total energy of the charged particle (at rest) in powers of h:

$$\langle\psi_n|\hat{V}|\psi_n\rangle=\mathcal{E}(\eta_n^{\rm cl})+O(h)\ , \tag{45}$$

where the static classical energy $\mathcal{E}$ can be calculated just by means of the classical potential [15]

$$\mathcal{E}(\eta_n^{\rm cl})\sim\int_{-\infty}^{\infty}dx\ V(\eta_n^{\rm cl})\ . \tag{46}$$

But this integral is necessarily divergent unless $\eta_n^{\rm cl}(+\infty)$ is an absolute minimum η_n of the classical potential V. Then it becomes clear from the $\varnothing_{QCD_2}$ structure that only $n=0$ solutions can remain with finite energy. The interaction has eliminated all the possible $(\eta_n, n\neq0)$ minima, thus giving an infinite mass to any $(T_3=n/2\neq0)$ state.[12]

Apparently the previous mechanism only allows us to eliminate the T_3 charged particles from the physical spectrum. In principle, finite energy color states are still possible. Those states belonging to nonsinglet multiplets but with a $T_3=0$ charge still have a chance of remaining in the spectrum. We will prove next that these states also have an infinite mass. We only need to remember two basic properties of the QCD_2 interaction which we have extensively discussed.

(i) The interaction is SU(2) global color invariant:

$$[\hat{V},T^a]=0\ .$$

(ii) It gives rise to a neutral color vacuum:

$$T_a|0\rangle=0\ . \tag{47}$$

These properties force the physical states to accommodate in representations of the SU(2) color group. Let us now consider a representation having total color charge $T=m$ $(m=1,2,3,\ldots)$. It certainly contains a state with $T_3=0$. According to our previous arguments, states described by a nontrivial boundary condition get through the dynamics infinite energy. Consequently, all the states of this representation with $T_3\neq0$ acquire an infinite mass. But so does the $T_3=0$ state because it is *degenerate* with its multiplet partners since the color symmetry is not broken. However, there is one representation which evades this mechanism: namely, the singlet one $(T=0)$. It only contains a *single* state arising from trivial boundary conditions $(T_3=0)$ and therefore has finite energy.

The previous result is not a surprise. As we showed in the last section, NTBC's are *necessary* to recover the

[10] In Coleman's paper Noether currents are used. Nevertheless, it is easy to see that a topological current like (32) verifies also the theorem.

[11] Note that we could use the same argument for the free massive action [see Eq. (23)].

[12] In a recent paper by Ellis *et al.* [4] they find color solitons in bosonized QCD_2 with infinite energy. We claim that their *constituent quarks* correspond to solitons which do not connect absolute minima.

whole SU(2) structure in the vertex operator framework. Now the dynamics does not allow us to keep NTBC's and therefore the non-Abelian character of the charge operators disappears in the realization of the spectrum.

Let us show how this mechanism works in an explicit way. In QCD_2, dynamics forces any state to verify ordinary BC's ($n=0$). According to (43) this implies that

$$\langle \psi|\hat{\eta}(x)|\psi\rangle \xrightarrow{x\to\pm\infty} 0 \tag{48}$$

for every physical state $|\psi\rangle$. The question is what kind of η-dependent operators are allowed now under the restriction (48)?

The $|\psi\rangle$ state will be generated out of the vacuum by some generic operator $\hat{S}$ depending on η. The color quantum numbers of the state will be the same as those of the operator since the vacuum is a singlet. The topological current (33) is conserved independently of the dynamics and does not generate charged states when applied to the vacuum [it preserves (48)]. Thus, the T_3 charge is a good quantum number. What happens to T_3 charged operators when the new BC's (48) are dynamically imposed?

A $T_3=n/2$ charged operator has the following commutation relation with the T_3 current:

$$[\hat{S}^n(x),J_3^0(y)]=\frac{n}{2}\delta(x-y)\hat{S}^n(x) . \tag{49}$$

This implies that the η and S^n operators will verify the nontrivial commutation relation

$$[\hat{S}^n(x),\hat{\eta}(y)]=\sqrt{\pi/2}\,n\,\theta(x-y)\hat{S}^n(x) . \tag{50}$$

But if this relation holds, then

$$\langle \psi_n|\hat{\eta}(x)|\psi_n\rangle \xrightarrow{x\to+\infty} \sqrt{\pi/2}\,n\neq 0 , \quad |\psi_n\rangle=\hat{S}^n|0\rangle \tag{51}$$

in contradiction with the dynamical constraint (48).

From the preceding argument we conclude that charge operators are no longer allowed. In particular, we cannot construct the nondiagonal currents $J^{\mu}_{\pm}$ to enlarge the symmetry from $U(1)_C$ to $SU(2)_C$, as we did in the free case. Therefore, the only symmetry that remains is the topological $U(1)_C$ *restricted* to zero charge particles.

The η field does not transform under color transformations, since, if it did, the existence of the nondiagonal currents $J^{\mu}_{\pm}$ would be required, Eq. (18), and those cannot be constructed due to the dynamical restrictions. Thus, the color symmetry has disappeared from the bosonized QCD_2 action.

It is clear why we do not run into any inconsistency with the color invariance of the Hamiltonian. The η is not a $T_3=0$ particle in the adjoint representation. The special realization of the color symmetry on η along QCD_2 boundary conditions tells us that the η is really a *color singlet field.* Consequently, the potential $V(\eta)$ is trivially invariant, as it must be according to the bosonization procedure.

Let us draw your attention to a very important point. Note that in the nonlinear nonlocal realization of color symmetry used it is not possible to say *a priori* what the color properties of the fundamental field η are. One *cannot* read the color quantum numbers of the η field *independently* of the dynamics as in the linear case [i.e., in the Wess-Zumino-Witten-Novikov (WZWN) model]. In our nonlinear nonlocal realization of the color symmetry it is the value of the η field at $+\infty$, the η field BC, which transforms linearly under the group. Therefore, *nothing* can be said about the color properties of the fundamental field until its BC's are not established. We have to learn first what kind of $U(1)_C$ topology, and, thus, what kind of BC's the dynamics allows. Then, and only then, are we able to know the real color structure of the fundamental field η.

Since η is a color singlet it must have simple relations with SU(2) invariant quantities. This is easily established when we compare simple operators in their Abelian and non-Abelian forms. For example, we can express the non-Abelian bosonization field g in its Abelian form as

$$\mathrm{tr}g=g_1^1+g_2^2=2\cos\sqrt{2\pi}\eta . \tag{52}$$

But because $g\in SU(2)_C$, it also has a standard representation in terms of the adjoint fields π_a,

$$g^{\alpha}_{\beta}=\exp\left[i\pi_a\left[\frac{\tau_a}{2}\right]_{\alpha\beta}\right] ,$$

and therefore $\eta(x)=[\mathrm{sgn}(x)/2\sqrt{2\pi}]\sqrt{\pi_a\pi_a}$.[13] We may write the full potential $V(\eta)$ in terms of the scalar singlet $|\pi|$ thus generating an *explicit* color invariant interaction $V(|\pi|)$. We could proceed to calculate straight off the spectrum of the theory.

In the chiral limit the φ and $|\pi|$ actions decouple. The first gives rise to a massless pseudoscalar particle; the second is responsible for the resonant meson masses [16]. Resonant states appear as *massive*[14] bound states of the color singlet field $|\pi|$. Obviously, every eigenstate of this Hamiltonian will be a singlet. The spectrum of $\hat{V}(|\pi|)$ contains *no trace* of colored particles.

The spectrum is an observable and therefore cannot depend on the chosen gauge. Thus, if the spectrum of the Hamiltonian is free of colored states in this gauge, so it is in any other gauge. The introduction of a mass term does not change the picture. The full Hamiltonian will depend now on the color singlet fields φ and $|\pi|$. No colored state can be built out of these singlet fields either, so the above arguments still hold.

As we have just seen, color disappears from the bosonized action completely. However, the $U(1)_F$ nontrivial topology remains since we did not gauge the "flavor" degree of freedom. Nevertheless, it is still affected by the way $U(1)_C$ collapses. If we compare to the free case, due to the minima structure of the free theory ∅ [Eq. (26)], the allowed "flavor" states have $B=m/2$ $(m\in Z)$ charges. Once we turn on the interaction ∅ becomes

[13]The sign function is necessary because η is a pseudoscalar Eq. (10), whereas $|\pi|\equiv\sqrt{\pi_a\pi_a}$ is a scalar. Notice that the cosine in (52) is invariant under $\eta\to-\eta$.

[14]Recall that $|\pi|$ is massive.

$\varnothing_{\mathrm{QCD}_2}$ [Eq. (42)] and only $B=m$ states can survive. Thus, the singlet spectrum is formed by particles of *integer* baryon number $B=m$, i.e., by mesons ($m=0$) and by baryons ($m=1,2,\ldots$) or antibaryons ($m=-1,-2,\ldots$). Particles with half-integer baryon number have disappeared from the spectrum, as they should. This result agrees with what is expected from the fermionic theory. Color singlet states are operators which in color space are of the form

$$q_\alpha^\dagger q_\alpha \;, \varepsilon^{\alpha\beta} q_\alpha q_\beta \;, q_\alpha^\dagger q_\alpha \varepsilon^{\beta\gamma} q_\beta q_\gamma \;, \ldots . \tag{53}$$

An important remark becomes necessary before concluding. We have learned that the cornerstone of the confinement mechanism in QCD_2 is the minima structure $\varnothing_{\mathrm{QCD}_2}$ induced by the interaction. However, this set of minima is obtained by looking at the shape of a *classical* potential V. This fact can be worrisome because we know examples in which radiative corrections can shift away the classical minimum from its original position [18]. We have to take into account that Coleman's theorem does not prevent the spontaneous breaking of the *discrete* symmetry $\eta \to -\eta$. Thus, we cannot exclude *a priori* the possibility of a radiatively induced spontaneous symmetry breakdown.

Let us assume that this possibility really occurs. For the sake of simplicity we consider the case that radiative corrections induce only two new symmetric minima. That is, we deal with the case of a 1D "Mexican hat" effective potential. We call this couple of minima $(v,-v)$, v being some dimensionless function of the action parameters M and e. Because of this new topology we are now energetically allowed to construct two soliton operators connecting these two minima. A state generated by this operator will have finite energy and $T_3=\sqrt{2/\pi}v$. For $v=\frac{1}{2}\sqrt{\pi/2}$ we would get a couple of finite energy "quarklike" solutions, $T_3=\pm\frac{1}{2}$. Once we choose one of the two minima as the real vacuum we break the $\eta\to-\eta(Z_2)$ symmetry of the action. We define a new η' field having zero vacuum expectation value (VEV) just by shifting η (we chose $-v$ as the real vacuum):

$$\eta'=\eta+v \; . \tag{54}$$

If we look now at the bosonization rule of the field strength [Eq. (36)], we realize that the previous operation is equivalent to a shift in F_3:

$$F_3'=F_3+\frac{e}{\sqrt{2\pi}}v \; . \tag{55}$$

The VEV of the η field gives rise to a constant color background field $E=(e/\sqrt{2\pi})v$. We already argued that such a constant background field had no physical sense in a non-Abelian theory, which motivated our choice, $E=0$.

The reason to set this background field equal to zero in the non-Abelian case arises when we compute the above shift for the pure gauge action. Since the pure gluon term is quadratic in the field strength, we pick up after the shift one term of the form

$$\int d^2x\left[-\tfrac{1}{2}E_a\tilde{F}_a(x)\right] \; , \tag{56}$$

where $\tilde{F}_a$ is the dual field strength and $E_a=(0,0,E)$. E_a is an *external* constant background field which does *not* change under local gauge transformations. A term such as (56) *violates* local gauge invariance.[15] Local gauge invariance prevents the interaction to produce the violating term (56) *to all orders* in h. The classical potential cannot have such a term and quantum corrections are not allowed to produce it. Thus, we are forced to take $E=0$ and consequently $v=0$ as well.

Local gauge invariance teaches us that the original Z_2 symmetry of the bosonized Lagrangian cannot be spontaneously broken. The original $\varnothing_{\mathrm{QCD}_2}$ structure is thus preserved and the mechanism of topological confinement is valid in its full extent.

It must be clear that no restriction on the range of the QCD_2 parameters (M and e) has been made. The minima structure $\varnothing_{\mathrm{QCD}_2}$ of the theory is the same [Eq. (42)] *no matter* what the values of M and e are. Even in the massless limit the topology stays unaltered. Moreover, it is easy to generalize our results to any N. The main features of the confinement mechanism remain: the nonlocal nonlinear realization of color symmetry in terms of N boson fields, the vacuum invariance under global $\mathrm{SU(N)}_C$, and the presence of quadratic terms in the bosonized action.

Finally, let us point out that the formalism used to explore confinement in QCD_2 can also be extended to any 2D SU(N) theory. If we add to the *free* action a nonconfining interaction, then the quarks must be the only asymptotic in/out states. Thus, asymptotically the action is that of a free (φ,η) fields with nontrivial boundary conditions and therefore we obtain a free vacuum structure. The topological conservation law of the $\mathrm{U(1)}_F\otimes\mathrm{U(1)}_C$ charges ensures that the minima structure of the asymptotic theory is *identical* to that of the interaction theory. Thus, the addition of a nonconfining interaction leaves the vacuum structure unaltered and therefore one can characterize these interactions topologically by Eq. (29). For $N=2$ the bosonized potential of a nonconfining SU(2) theory has to be local in η and support the free minima structure $\varnothing$. That is to say, it must be periodic in η and invariant under "diagonal" translations $\eta\to\eta+\sqrt{\pi/2m}$ [see Eq. (28)].

From this exhaustive survey on the topology and color structure of QCD_2 only the following conclusion can be drawn: The confinement mechanism in QCD_2 is purely topological. Confinement occurs for any value of the quark mass, the gauge coupling constant, and for any number of colors. There is only one phase of permanent confinement.

IV. CONCLUSIONS

The problem of confinement in the fermionic formulation of QCD_2 becomes extremely complex beyond the

[15]Notice the peculiar fact that in the Abelian case this term is perfectly gauge invariant and therefore allowed.

large-N and weak-coupling limits. In these two limits the spectrum, as well as the vanishing of quark creation amplitudes, corroborate the nonexistence of *free* color nonsinglet states. The quark self-energy shows a confining behavior.

However, sea quark excitations, vertex corrections, and nonplanar contributions turn out to be of great relevance in the most general case, complicating our understanding of the confinement mechanism. Nevertheless, the fermionic description seems to indicate in a qualitative manner that QCD_2 is a confining theory beyond leading order in the $1/N$ expansion [6]. In this paper we have given an alternative general method to solve the confinement problem beyond the large-N and large mass restrictions.

In the bosonized version of fermionic two-dimensional theories topology plays a crucial role. States with baryon number and color charge are described by solitons. The properties of the vacuum, which give rise to nontrivial boundary conditions, determine the quantum number structure of the Fock space. We have analyzed initially the rich spectrum of nonconfining theories by discussing the role of boundary conditions. The existence of nontrivial boundary conditions is a consequence of the nontrivial topology of the $SU(N)$ maximal Abelian subgroup $U(1)^N$ on the S^1 compactified 1D space. This nontrivial topology allows the enlargement of the explicit $U(1)^N$ symmetry into a complete color $SU(N)$ symmetry by means of the so-called vertex operator construction. The discussion of color symmetry can be reduced to the study of the minima structure of the bosonized potential when written in terms of the $U(1)^N$ color Abelian fields. It is appealing that in the bosonized version of the theory this discussion can be carried out purely at the classical level.

When the interaction is switched on the $U(1)^N$ color topology breaks down. The $U(1)^N$ color Abelian charges are dynamically screened and the $SU(N)_C$ symmetry cannot be reconstructed anymore. The invariance of the vacuum under $SU(N)_C$ transformations ensures that all $SU(N)_C$ charges are likewise screened. Only singlet states can remain in the spectrum. This topological mechanism is independent of the relation between the coupling constant and the quark mass and it is valid to all orders in h. QCD_2 shows only one single phase of permanent confinement. The mechanism can be cast in a more mathematical language by invoking homotopy groups, but one should not avoid the very naive dynamical statement, namely, that color solitons are given infinite energy [4].

The simplicity of the topological description and its generality can be used in a wider spectrum of 2D $SU(N)$ theories. They can be classified according to its $U(1)^N$ minima structure. Then very strong statement can be made about their confining properties.

Unluckily at this moment, four-dimensional calculations seem to be pretty unrelated to two-dimensional theories. Recent investigations (string theory [19], dimensional reduction techniques [20], observable effects of stringlike confining mechanisms [21], etc.) suggest that it might be possible to make a connection with realistic problems in the near future. The deep knowledge of QCD_2 and related theories could be a big help.

Moreover, the abstraction associated with the mathematical language might guide one into the four-dimensional case. It is not possible to write an approximate bosonized theory in four dimensions? Skyrme-type models have been extremely successful in describing the low-energy flavor properties of the theory, but they avoid confinement simply by assuming its existence [22]. There already have been attempts at writing an approximate bosonized Lagrangian in terms of the color degrees of freedom. Quarklike solutions appear as color Skyrmions of this effective action [23]. Could we find here an analogous mechanism that produced the collapse of the spectrum? The work of 't Hooft [24] has been pioneering in this respect, but again the dimensionality of space-time makes difficult the connection. No relation between our topological scheme and his has been as of yet found, but the endeavor seems sufficiently appealing to continue embarking upon it.

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