Radiative decay of light and heavy mesons

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(Received 25 January 1993)

The $M1$ transition among the vector (V) and pseudoscalar (P) mesons in the light and heavy flavor sectors has been investigated in a potential model of independent quarks. Going beyond the static approximation, to add some momentum dependence due to the recoil effect in a more realistic calculation, we find an improvement in the results for the radiative decay of light flavored mesons. However, our prediction on the decay rates for the mesons (D^* and B^*) in the heavy flavor sector remains unaffected and compares well with those of other model calculations.

PACS number(s): 13.40.Hq, 12.39.Pn

I. INTRODUCTION

Study of radiative transitions among the low-lying vector (V) and pseudoscalar (P) mesons has been considered as a very useful testing ground for various phenomenological quark models, which provide effective methods of investigation for such low-lying hadronic phenomena for which a rigorous field-theoretic formulation with a first principles application of QCD has not so far been possible. Therefore, to gain a clear understanding of these radiative transitions there have been several attempts by various authors starting with the pioneering work of Gell-Mann and others [1]. As it appears from the existing literature, there is no single description of one-photon radiative transitions among the vector and pseudoscalar mesons that can successfully account for all the experimentally observed decay widths. In a recent work [2] based on the scalar-vector confining potential of harmonic nature [3] for the independently confined quarks inside hadrons, we have performed a static calculation of the partial decay widths of several possible $M1$ transitions such as $V \rightarrow P\gamma$ and $P \rightarrow V\gamma$ in the light flavor sector only. In this work we have followed the traditional picture of photon emission induced by the electromagnetic current of the confined quark and antiquark inside the meson. The results so obtained were found to be reasonably satisfactory except for the decays such as $(\omega,\rho,\phi) \rightarrow \pi\gamma$. We believe that this discrepancy might be due to our static approximation adopted in the calculation. The dichotomic nature of the pion in being a Goldstone boson with a quark-antiquark structure may also have some bearing which cannot be treated in this simplistic traditional picture. Taking advantage of this aspect in the intriguing nature of pion, Singer and Miller [4] have considered three different dynamical mechanisms such as (i) photon emission by quarks, (ii) photon emission by the pion cloud, and (iii) transition of a vector bag to a photon accompanied by pion emission, which contribute either singly or in combinations. However, following this approach the decay rates for $V \rightarrow \pi\gamma$ in the cloudy bag model (C.B.M.) were found to be rather overestimated. Therefore, our purpose here is only to investigate the limitations of our static calculation, which is based on the assumption that the momentum transfer involved in the radiative transitions is low or moderate. We have also seen that our potential model can most suitably be applicable to the non-self-conjugate heavy mesons (D,B) consisting of one heavy and the other light flavored quark or antiquark, where the confining interaction can still play the dominant role so as to successfully generate the ground-state mass spectrum in a perturbative manner [5]. In the radiative transitions of these mesons, the momentum transfer involved turns out to be much less compared to those in the case of light mesons. Therefore, a static calculation can be all the more reliable in this non-self-conjugate heavy meson sector. With this contention in mind we would first extend our static calculation more appropriately to the radiative transitions involving D and B mesons. Since the experimental data available in this sector are very meager, predictions of a model, if found reliable, can be utilized quite fruitfully. We may as well attempt a general formulation incorporating to some extent the recoil effect in order to have a more realistic calculation beyond the static approximation. Since a decay process occurs physically in the definite momentum eigenstates of the participating mesons, such a calculation of the radiative decay widths requires suitable expressions for the initial- and final-state mesons reflecting appropriate momentum distribution of the constituent quark and antiquark in their corresponding spin-flavor configuration. Such a procedure has already been tested successfully in describing the leptonic decay of light vector mesons [6] as well as the weak leptonic decay of light and heavy pseudoscalar mesons [5] in the same independent quark potential model. Therefore, we would like to use this generalized approach to calculate the radiative decay widths in the light as well as heavy flavor sector which would take into account more effectively some momentum-dependent effects due to recoil.

In Sec. II, we provide a brief account of the potential

0556-2821/94/49(1)/299(10)/$06.00

model adopted here with its notation and conventions relevant for the present investigation. Since our purpose here is not to make a parametric best fit for the results of our investigation, we would only quote at appropriate places the model quantities and parameters from earlier works which are to be used subsequently in the present investigation. Such an approach can establish the extent of applicability of the present model to wider ranging hadronic phenomena. In Sec. III, we present our results for the radiative decay widths involving heavy flavor mesons such as D and B, obtainable in a static calculation. These results are compared with the predictions of several other models. Section IV describes the momentum wave-packet representation of the initial and final meson states leading to a more realistic framework incorporating the recoil effects for the calculation of radiative decay widths in the light as well as heavy meson sector. Finally, Sec. V embodies the results of this calculation with relevant discussion.

II. THE POTENTIAL MODEL

The model adopted here pictures a meson as a color-singlet assembly of a quark and an antiquark independently confined by an effective flavor-independent potential taken phenomenologically in the scalar-vector-harmonic form [3]. The quark-gluon interaction at a short distance originating from one-gluon-exchange and quark-pion interaction required in the nonstrange sector to preserve chiral symmetry in such a model, are presumed to be residual interactions compared to the dominant confining interaction. Although these residual interactions treated perturbatively in the model are crucial in generating mass splittings [7,8], their role in the hadronic decay processes are considered less significant.

Therefore, to a first approximation, the confining part of the interaction taken in this model as

$$U(r)=\tfrac{1}{2}(1+\gamma^0)(ar^2+V_0) \tag{1}$$

is believed to provide the zeroth-order quark dynamics inside the hadron core through the Lagrangian density

$$\mathcal{L}_q^0(x)=\bar{\psi}_q(x)\left[\frac{i}{2}\gamma^\mu\overleftrightarrow{\partial}_\mu-m_q-U(r)\right]\psi_q(x) . \tag{2}$$

The ensuing Dirac equation with $E_q'=E_q-V_0/2$, $m_q'=m_q+V_0/2$, $\lambda_q=(E_q'+m_q')$, and $r_{0q}=(a\lambda_q)^{-1/4}$ admits static solutions of positive and negative energy in zeroth order with the independent-quark bound-state condition

$$\sqrt{\lambda_q/a}\,(E_q'-m_q')=(4n+2l-1) . \tag{3}$$

The solution of this cubic equation provides the zeroth-order binding energies of the confined quark for the various possible eigenmodes. The explicit form of the confined quark orbitals in the lowest eigenmode corresponding to the positive and negative energy state, which are relevant for the present calculation, can be expressed as

$$\Phi_{q\lambda}^{(+)}(\mathbf{r})=\frac{1}{\sqrt{4\pi}}\begin{pmatrix} ig_q(r)/r \\ \sigma\cdot\hat{\mathbf{r}}f_q(r)/r\end{pmatrix}\chi_\lambda ,$$
$$\Phi_{q\lambda}^{(-)}(\mathbf{r})=\frac{1}{\sqrt{4\pi}}\begin{pmatrix} \sigma\cdot\hat{\mathbf{r}}f_q(r)/r \\ -ig_q(r)/r\end{pmatrix}\tilde{\chi}_\lambda . \tag{4}$$

The two component spinors χ_λ and $\tilde{\chi}_\lambda$ stand for

$$\chi_\uparrow=\begin{pmatrix}1\\0\end{pmatrix},\quad \chi_\downarrow=\begin{pmatrix}0\\1\end{pmatrix}$$

and

$$\tilde{\chi}_\uparrow=\begin{pmatrix}0\\-i\end{pmatrix},\quad \tilde{\chi}_\downarrow=\begin{pmatrix}i\\0\end{pmatrix} ,$$

respectively. The reduced radial parts in the upper and lower component solutions corresponding to the quark flavor q are

$$g_q(r)=\mathcal{N}_q(r/r_{0q})\exp(-r^2/2r_{0q}^2) ,$$
$$f_q(r)=-\left(\frac{\mathcal{N}_q}{\lambda_q r_{0q}}\right)\left(\frac{r}{r_{0q}}\right)^2\exp\left(-\frac{r^2}{2r_{0q}^2}\right) , \tag{5}$$

where the normalization factor $\mathcal{N}_q$ is given by

$$\mathcal{N}_q^2=8\lambda_q/[\sqrt{\pi}r_{0q}(3E_q'+m_q')] . \tag{6}$$

The quark binding energy of zeroth order in the meson ground state is obtained from the bound-state condition in Eq. (3) with $n=1$ and $l=0$.

This provides a brief outline of the potential model and its conventions which is to be adopted in the present investigation of the radiative transitions involving light as well as heavy mesons. This model has been very successfully employed to study several aspects in baryon as well as meson sectors [3,7–9]. We would therefore use the model parameters and other relevant model quantities obtained in these earlier studies to realize, in a way, a parameter-free calculation of the decay rates.

III. RADIATIVE DECAY WIDTHS OF HEAVY MESONS IN THE STATIC APPROXIMATION

In this section we extend our previous calculation [2] of the $M1$ transition rates to the heavy meson sector of D^* and B^* with the respective quark-flavor configurations such as $(c\bar{u},c\bar{d},c\bar{s})$ and $(u\bar{b},d\bar{b},s\bar{b},c\bar{b})$. Although radiative transitions $D^*\to D\gamma$ and $B^*\to B\gamma$ have already been observed experimentally [10,11], the data available only in the case of D^* have very large uncertainties. Nevertheless, possible decays of this type have the potential of revealing the static magnetic properties of c and b-flavor quarks. The strength of these radiative transitions are also required in the calculation of the electroweak radiative decays of these heavy quarks which provide a fruitful ground for testing various aspects of the standard model including the gluonic corrections to electroweak processes. Therefore, it is important to have a reliable estimate of the partial decay widths for all possible radiative decays involving D and B mesons.

We have shown in our recent work on weak leptonic decays of pseudoscalar mesons [5] that the independent quark model with the scalar-vector confining potential in harmonic form of Eq. (1) can successfully generate the ground-state hyperfine splitting of (D^*,D) and (B^*,B) mesons by appropriately taking into account the center-of-mass corrections followed by the one-gluon-exchange corrections as per Ref. [7]. The ground-state mass values obtained in this manner for (D^*,D), (D_s^*,D_s), (B^*,B), and (B_s^*,B_s) have very good agreement with available experimental data. The weak decay constants calculated in this model for these heavy pseudo-scalar mesons are also found to be quite consistent with those of lattice [12,13] as well as other model [13] calculations. Therefore, we can very well apply the same model as outlined in Sec. II to estimate the partial decay widths $\Gamma(V\rightarrow P\gamma)$ of the radiative transitions $D^*\rightarrow D\gamma$ and $B^*\rightarrow B\gamma$.

Since the momentum transfer in these transitions remain within an approximate range of 40–140 MeV, it must be quite reliable to use the results of our static calculation in Ref. [2] in the general form as

$$\Gamma(V\rightarrow P\gamma)=\tfrac{4}{3}\alpha\bar{k}^3[\mu_{VP}(\bar{k})]^2 , \tag{7}$$

where the kinematically allowed energy of the outgoing photon is given by

$$\bar{k}=(M_V^2-M_P^2)/2M_V . \tag{8}$$

The transition magnetic moments $\mu_{VP}(\bar{k})$ for various possible radiative transitions in these heavy meson sector are expressed as

$$\begin{aligned}
&\text{(i)}\ \ \mu_{D^{*+}D^+}(\bar{k})=\tfrac{2}{3}\mu_c^0(\bar{k})-\tfrac{1}{3}\mu_d^0(\bar{k}) ,\\
&\text{(ii)}\ \ \mu_{D^{*0}D^0}(\bar{k})=\tfrac{2}{3}\mu_c^0(\bar{k})+\tfrac{2}{3}\mu_u^0(\bar{k}) ,\\
&\text{(iii)}\ \ \mu_{D_s^{*+}D_s^+}(\bar{k})=\tfrac{2}{3}\mu_c^0(\bar{k})-\tfrac{1}{3}\mu_s^0(\bar{k}) ,\\
&\text{(iv)}\ \ \mu_{B^{*+}B^+}(\bar{k})=\tfrac{2}{3}\mu_u^0(\bar{k})-\tfrac{1}{3}\mu_b^0(\bar{k}) ,\\
&\text{(v)}\ \ \mu_{B^{*0}B^0}(\bar{k})=-\tfrac{1}{3}\mu_d^0(\bar{k})-\tfrac{1}{3}\mu_b^0(\bar{k}) ,\\
&\text{(vi)}\ \ \mu_{B_s^{*0}B_s^0}(\bar{k})=-\tfrac{1}{3}\mu_s^0(\bar{k})-\tfrac{1}{3}\mu_b^0(\bar{k}) ,\\
&\text{(vii)}\ \ \mu_{B_c^{*+}B_c^+}(\bar{k})=\tfrac{2}{3}\mu_c^0(\bar{k})-\tfrac{1}{3}\mu_b^0(\bar{k}) ,
\end{aligned} \tag{9}$$

where

$$\mu_q^0(\bar{k})=2\exp(-\bar{k}^2r_{0q}^2/4)/(3E_q'+m_q') . \tag{10}$$

The present model assumes SU(2)-flavor symmetry with $m_u=m_d\neq m_s$, $\mu_u^0(\bar{k})=\mu_d^0(\bar{k})\neq\mu_s^0(\bar{k})$. In fact, $\mu_q^0(\bar{k}=0)$ is the magnetic moment of the bound quark in this model [3].

Now to estimate the partial decay widths from Eqs. (7)–(10), we would be using here the model parameters (a,V_0), the quark masses $(m_u=m_d,m_s,m_c,m_b)$ and the other corresponding relevant quantities such as (E_q,r_{0q}) as obtained earlier in the application of this model to baryon as well as meson sectors [2,3,5–7,9]. Accordingly, we shall take the potential parameters as

$$(a,V_0)\equiv(0.017\,166\ \text{GeV}^3, -0.1375\ \text{GeV}) . \tag{11}$$

However, we would consider two sets of quark masses used in connection with different aspects of hadronic phenomena studied earlier.

Considering first the quark masses as according to [3,5–7,9] along with (a,V_0) given in Eq. (11), called hereafter parameter set (1), we have

$$\begin{aligned}
&(m_u=m_d,m_s)\equiv(78.75\ \text{MeV},\ 315.75\ \text{MeV}) ,\\
&(m_c,m_b)\equiv(1492.76\ \text{MeV},\ 4776.59\ \text{MeV}) .
\end{aligned} \tag{12}$$

Then the model dynamics as outlined in Sec. II would provide the ground-state confined quark energy E_q and the scale factor r_{0q} relevant for the present calculation in the following manner:

$$\begin{aligned}
&(E_u=E_d,E_s)\equiv(471\ \text{MeV},\ 591\ \text{MeV}) ,\\
&(E_c,E_b)\equiv(1579.51\ \text{MeV},\ 4766.33\ \text{MeV}) ,
\end{aligned} \tag{13}$$

$$\begin{aligned}
&(r_{0u}=r_{0d},r_{0s})\equiv(3.208\ \text{GeV}^{-1},2.831\ \text{GeV}^{-1}) ,\\
&(r_{0c},r_{0b})\equiv(2.087\ \text{GeV}^{-1},1.572\ \text{GeV}^{-1}) .
\end{aligned} \tag{14}$$

As shown in Ref. [5], this choice of model parameters yields the ground-state masses of the heavy mesons $(D^{*+},D^+,D^{*0},D^0,D_s^{*+},D_s^+)$ and $(B^{*+},B^+,B^{*0},B^0,B_s^{*0},B_s^0)$ in good agreement with the experimental values. However, since the theoretical uncertainty due to the perturbative calculation cannot be overlooked here, we would be prefering the experimental meson masses in the computation of the outgoing photon energy $\bar{k}$ in Eq. (8). But in case of (B_c^{*+},B_c^+), for which experimental data are not yet available, we would be using the model masses as $M_{B_c^{*+}}=6.3078$ GeV and $M_{B_c^+}=6.2642$ GeV as per Ref. [5]. Then the partial decay widths $\Gamma(V\rightarrow P\gamma)$ in the D- and B-meson sectors can be calculated to yield the results

$$\begin{aligned}
&\text{(i)}\ \ \Gamma(D^{*+}\rightarrow D^+\gamma)=1.15\ \text{keV} ,\\
&\text{(ii)}\ \ \Gamma(D^{*0}\rightarrow D^0\gamma)=27.94\ \text{keV} ,\\
&\text{(iii)}\ \ \Gamma(D_s^{*+}\rightarrow D_s^+\gamma)=0.32\ \text{keV} ,\\
&\text{(iv)}\ \ \Gamma(B^{*+}\rightarrow B^+\gamma)=0.67\ \text{keV} ,\\
&\text{(v)}\ \ \Gamma(B^{*0}\rightarrow B^0\gamma)=0.21\ \text{keV} ,\\
&\text{(vi)}\ \ \Gamma(B_s^{*0}\rightarrow B_s^0\gamma)=0.11\ \text{keV} ,\\
&\text{(vii)}\ \ \Gamma(B_c^{*+}\rightarrow B_c^+\gamma)=0.02\ \text{keV} .
\end{aligned} \tag{15}$$

If, however, we choose our light flavor quark masses differently as per Refs. [2,6] along with the same potential parameters (a,V_0), which we call our parameter set (2), we would have

$$\begin{aligned}
&(m_u=m_d,m_s)\equiv(10\ \text{MeV},240\ \text{MeV}) ,\\
&(E_u=E_d,E_s)\equiv(451\ \text{MeV},546\ \text{MeV}) ,\\
&(r_{0u}=r_{0d},r_{0s})\equiv(3.352\ \text{GeV}^{-1},\ 2.934\ \text{GeV}^{-1}) .
\end{aligned} \tag{16}$$

Then the quark masses (m_c,m_b) with their corresponding model solutions (E_c,E_b) and (r_{0c},r_{0b}) required to generate the ground-state mass spectrum of these heavy mesons (D,B) equally well as in case of parameter set (1) can be found as

$$(m_c, m_b) \equiv (1498.76 \text{ MeV}, 4777.59 \text{ MeV}) ,$$
$$(E_c, E_b) \equiv (1585.09 \text{ MeV}, 4767.32 \text{ MeV}) , \tag{17}$$
$$(r_{0c}, r_{0b}) \equiv (2.085 \text{ GeV}^{-1}, 1.572 \text{ GeV}^{-1}) .$$

This set of parameters, when used to calculate the partial decay widths $\Gamma(V \to P\gamma)$, again leads us to

$$\begin{aligned}
&\text{(i)} \quad \Gamma(D^{*+} \to D^{+}\gamma) = 0.83 \text{ keV} ,\\
&\text{(ii)} \quad \Gamma(D^{*0} \to D^{0}\gamma) = 24.71 \text{ keV} ,\\
&\text{(iii)} \quad \Gamma(D_s^{*+} \to D_s^{+}\gamma) = 0.17 \text{ keV} ,\\
&\text{(iv)} \quad \Gamma(B^{*+} \to B^{+}\gamma) = 0.57 \text{ keV} ,\\
&\text{(v)} \quad \Gamma(B^{*0} \to B^{0}\gamma) = 0.18 \text{ keV} ,\\
&\text{(vi)} \quad \Gamma(B_s^{*0} \to B_s^{0}\gamma) = 0.11 \text{ keV} ,\\
&\text{(vii)} \quad \Gamma(B_c^{*+} \to B_c^{+}\gamma) = 0.02 \text{ keV} .
\end{aligned} \tag{18}$$

We find that the results obtained with either set of the parameters in the present model are not much different from each other. With the experimental meson masses determining the outgoing photon energy, the transition moments obtained by the relativized quark model of Godfrey and Isgur [14] would yield the decay widths for the processes (i)–(vi) in Eq. (18) as 0.85, 22.98, 0.12, 0.50, 0.16, and 0.09 keV, respectively, which compares very well with our results. In a bag model calculation, Hackman *et al.* [15] also find similar results with $\Gamma(D^{*+} \to D^{+}\gamma) = 1.0$ keV, $\Gamma(D^{*0} \to D^{0}\gamma) = 27.73$ keV, and $\Gamma(D_s^{*+} \to D_s^{+}\gamma) = 0.12$ keV. Singer and Miller [16,17] have also studied $D^{*} \to D\gamma$ and $B^{*} \to B\gamma$ in CBM with a very different approach incorporating additional contributions due to pion exchange current. With the heavy quarks having no anomalous magnetic moments their results for the first six of the above processes would be 1.0, 21.2, 1.2×10^{-4} keV (Ref. [16] for $\lambda = 1$) and 0.62, 0.28, 0.10 keV (Ref. [17]), respectively. We find that except for $D_s^{*+} \to D_s^{+}\gamma$ having very unusually small decay width, the rest are not much different from our results. If we consider the ratio of radiative decay widths of $(D^{*+} \to D^{+}\gamma)$ and $(D^{*0} \to D^{0}\gamma)$ as well as the same for $(B^{*+} \to B^{+}\gamma)$ and $(B^{*0} \to B^{0}\gamma)$ our calculation for parameter set 1 (set 2) gives

$$\begin{aligned}
R_{D^*} &= \Gamma(D^{*+} \to D^{+}\gamma)/\Gamma(D^{*0} \to D^{0}\gamma) = 0.041 \ (0.034) ,\\
R_{B^*} &= \Gamma(B^{*+} \to B^{+}\gamma)/\Gamma(B^{*0} \to B^{0}\gamma) = 3.19 \ (3.17) .
\end{aligned} \tag{19}$$

This quantity does not differ from Godfrey and Isgur's [14] prediction of $R_{D^*} = 0.037$ and $R_{B^*} = 3.13$. A recent calculation of R_{D^*} based on an effective field theory with expansion in external momentum and heavy quark mass predicts it to be between 0.01 and 0.056 [18]. However, the prediction ($R_{D^*} = 0.047$, $R_{B^*} = 2.2$) obtained in a confined Coulombic model [16,17] with effects of pion exchange current shows a marked departure in R_{B^*}.

To minimize any possible uncertainty in our theoretical prediction within the limits of the model adopted, one may need to look beyond the static approximation in making a more realistic estimate with the inclusion of some momentum dependence due to recoil effects. Such a calculation is attempted in the following section.

IV. $\Gamma(A \to B\gamma)$-BEYOND STATIC APPROXIMATION

In this section we attempt a more realistic calculation of the radiative decay widths for transitions of the type $A \to B\gamma$ among the vector mesons $V \equiv (\rho, \omega, \phi, K^{*+}, K^{*0}, D^{*+}, D^{*0}, D_s^{*+}, B^{*+}, B^{*0}, B_s^{*0}, B_c^{*+})$ and their corresponding pseudoscalar mesons P in the light as well as heavy flavor sector. Since any such decay physically occurs between the momentum eigenstates of the participating mesons, a more exact field-theoretic calculation should have the initial and final mesons states represented as appropriate momentum wave packets reflecting their respective constituent quark-antiquark momentum distribution. Although the bound quark and the antiquark inside the meson are in definite energy states having no definite momenta, one can always obtain a momentum probability amplitude by suitable momentum-space projection of the corresponding bound quark or antiquark orbital derivable in the model as in Eq. (4). Then using a momentum profile function constructed suitably from the quark-antiquark momentum probability amplitude, one can represent a meson $M(q_1\bar{q}_2)$ in its momentum state $(\mathbf{P})$ as a momentum wave packet. Taking such wave packets for the initial and final meson states in the radiative decay processes, one can evaluate the S-matrix elements leading to the field-theoretic calculation of the decay rates.

We represent the state of a meson $M(q_1\bar{q}_2)$ with an arbitrary momentum $\mathbf{P}$ and spin projection S_V as

$$|M(\mathbf{P}), S_V\rangle = \frac{1}{\sqrt{N(\mathbf{P})}} \sum_{\lambda_1\lambda_2 \in S_V} \zeta^{M}_{q_1 q_2}(\lambda_1, \lambda_2) \int d\mathbf{p}_1 d\mathbf{p}_2 \delta^{(3)}(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{P}) G_M(\mathbf{p}_1, \mathbf{p}_2) b^{\dagger}_{q_1}(\mathbf{p}_1, \lambda_1) \tilde{b}^{\dagger}_{q_2}(\mathbf{p}_2, \lambda_2)|0\rangle , \tag{20}$$

where, $b^{\dagger}_{q_1}(\mathbf{p}_1, \lambda_1)$ and $\tilde{b}^{\dagger}_{q_2}(\mathbf{p}_2, \lambda_2)$ are, respectively, the quark and antiquark creation operators. $\zeta^{M}_{q_1 q_2}(\lambda_1, \lambda_2)$ stands for the appropriate SU(6)-spin-flavor coefficients for the pseudoscalar meson $M(q_1\bar{q}_2)$. $N(\mathbf{P})$ represents the overall normalization factor, which can be expressed in an integral form as

$$N(\mathbf{P}) = \int d\mathbf{p} |G_M(\mathbf{p}, \mathbf{P} - \mathbf{p})|^2 . \tag{21}$$

This is obtainable from the meson-state normalization considered here in the form as

$$\langle M(\mathbf{P})|M(\mathbf{P}')\rangle=\delta^{(3)}(\mathbf{P}-\mathbf{P}')\ . \tag{22}$$

Finally, $G_M(\mathbf{p}_1,\mathbf{p}_2)$ in Eq. (20) provides the effective momentum distribution amplitude for the quark and antiquark inside the meson. In an independent particle picture of the present model, $G_M(\mathbf{p}_1,\mathbf{p}_2)$ can be expressed in terms of individual momentum distribution amplitudes $G_{q_1}(\mathbf{p}_1)$ and $\tilde{G}_{q_2}(\mathbf{p}_2)$ of the quark q_1 and antiquark $\bar{q}_2$, respectively. We follow here the simple ansatz as in Ref. [5] in a straightforward extension of the idea of Margolis and Mendel [19] to express $G_M(\mathbf{p}_1,\mathbf{p}_2)$ as the geometric mean of the individual quark and antiquark momentum distribution amplitudes $G_{q_i}(\mathbf{p}_1)$ and $\tilde{G}_{q_2}(\mathbf{p}_2)$, so that

$$G_M(\mathbf{p}_1,\mathbf{p}_2)=\sqrt{G_{q_1}(\mathbf{p}_1)\tilde{G}_{q_2}(\mathbf{p}_2)}\ . \tag{23}$$

Here $G_{q_1}(\mathbf{p}_1)$ can be obtained by a suitable momentum-space projection of the bound-quark orbital $\Phi_{q\lambda}^{(+)}(\mathbf{r})$ in Eqs. (4)–(6) corresponding to the lowest eigenmode. If $G_q(\mathbf{p};\lambda,\lambda')$ is the amplitude of a bound quark in its eigenmode $\Phi_{q\lambda}^{(+)}(\mathbf{r})$ for being found in a state of definite momentum $\mathbf{p}$ and spin projection λ', then [6]

$$G_q(\mathbf{p};\lambda,\lambda')=\frac{U_q^\dagger(\mathbf{p},\lambda')}{\sqrt{2E_p}}\int d\mathbf{r}\,\Phi_{q\lambda}^{(+)}(\mathbf{r})\exp(-i\mathbf{p}\cdot\mathbf{r})\ , \tag{24}$$

where $E_p=\sqrt{(\mathbf{p}^2+m_q^2)}$ and $U_q(\mathbf{p},\lambda')$ is the usual free Dirac spinor which are normalized according to the relations

$$\begin{aligned}U_q^\dagger(\mathbf{p},\lambda_1)U_q(\mathbf{p},\lambda_2)&=2E_p\delta_{\lambda_1\lambda_2}=V_q^\dagger(\mathbf{p},\lambda_1)V_q(\mathbf{p},\lambda_2)\ ,\\ \bar{U}_q(\mathbf{p},\lambda_1)U_q(\mathbf{p},\lambda_2)&=2m_q\delta_{\lambda_1\lambda_2}=\bar{V}_q(\mathbf{p},\lambda_1)V_q(\mathbf{p},\lambda_2)\ ,\end{aligned} \tag{25}$$

and

$$\begin{aligned}\sum_\lambda U_q(\mathbf{p},\lambda)\bar{U}_q(\mathbf{p},\lambda)&=(\not{p}+m_q)\ ,\\ \sum_\lambda V_q(\mathbf{p},\lambda)\bar{V}_q(\mathbf{p},\lambda)&=(\not{p}-m_q)\ .\end{aligned} \tag{26}$$

Then Eq. (24) can be simplified to give

$$G_q(\mathbf{p};\lambda,\lambda')=G_q(\mathbf{p})\delta_{\lambda\lambda'} \tag{27}$$

when, with $\alpha_q=1/2r_{0q}^2$,

$$G_q(\mathbf{p})=\frac{i\pi\mathcal{N}_q}{2\alpha_q\lambda_q}\sqrt{(E_p+m_q)/E_p}\,(E_p+E_q)\exp(-p^2/4\alpha_q)\ . \tag{28}$$

Thus $G_q(\mathbf{p})$ essentially provides the momentum probability amplitude for a quark q in its eigenmode $\Phi_{q\lambda}^{(+)}(\mathbf{r})$ to have a definite momentum $\mathbf{p}$ inside the meson. In a similar manner one can obtain the momentum probability amplitude $\tilde{G}_q(\mathbf{p})$ for an antiquark in its eigenmode $\Phi_{q\lambda}^{(-)}(\mathbf{r})$ to realize that, for like flavors,

$$\tilde{G}_q(\mathbf{p})=G_q^*(\mathbf{p})\ . \tag{29}$$

Such an ansatz for the effective momentum distribution amplitude as in Eq. (23) has been very successfully adopted in our earlier study of leptonic decays of light neutral vector mesons [6] and the weak leptonic decays of light and heavy pseudoscalar mesons [5].

Now assuming that $A\to B\gamma$ transitions are predominantly single vertex processes governed mainly by the photon emission from independently confined quark or antiquark inside the meson [Figs. 1(a) and 1(b)], the S-matrix element in the configuration space can be written as

$$S_{BA}=\left\langle B\gamma\left|-ie\int d^4x\ T\left[\sum_q e_q\bar{\psi}_q(x)\gamma^\mu\psi_q(x)A_\mu(x)\right]\right|A\right\rangle\ . \tag{30}$$

Using the quark and photon field expansions as

$$\begin{aligned}\psi_q(x)&=\sum_\lambda\frac{1}{(2\pi)^{3/2}}\int\frac{d\mathbf{p}}{\sqrt{2E_p}}[b_q(\mathbf{p},\lambda)U_q(\mathbf{p},\lambda)\exp(-ipx)+\tilde{b}_q^\dagger(\mathbf{p},\lambda)V_q(\mathbf{p},\lambda)\exp(ipx)]\ ,\\ A_\mu(x)&=\sum_\delta\frac{1}{(2\pi)^{3/2}}\int\frac{d\mathbf{k}}{\sqrt{2k_0}}[a(\mathbf{k},\delta)\exp(-ikx)+a^\dagger(\mathbf{k},\delta)\exp(ikx)]\epsilon_\mu(\mathbf{k},\delta)\end{aligned} \tag{31}$$

Eq. (30) can be reduced to

$$S_{BA}=i\sqrt{\alpha/k_0}\left\langle B\left|\sum_{q\lambda\lambda'}\frac{e_q}{e}\int\frac{d\mathbf{p}\,d\mathbf{p}'}{\sqrt{4E_pE_{p'}}}\delta^{(4)}(p'+k-p)\Lambda(\mathbf{p}'\lambda',\mathbf{p}\lambda,\mathbf{k}\delta)\right|A\right\rangle\ , \tag{32}$$

where we have set

$$\begin{aligned}C(\mathbf{p}'\lambda',\mathbf{p}\lambda,\mathbf{k}\delta)&=\bar{U}(\mathbf{p}'\lambda')\boldsymbol{\gamma}\cdot\boldsymbol{\epsilon}(\mathbf{k}\delta)U(\mathbf{p}\lambda)\ ,\\ \tilde{C}(\mathbf{p}\lambda,\mathbf{p}'\lambda',\mathbf{k}\delta)&=\bar{V}(\mathbf{p}\lambda)\boldsymbol{\gamma}\cdot\boldsymbol{\epsilon}(\mathbf{k}\delta)V(\mathbf{p}'\lambda')\end{aligned} \tag{33}$$

in order to write

$$\Lambda(\mathbf{p}'\lambda',\mathbf{p}\lambda,\mathbf{k}\delta)=[C(\mathbf{p}'\lambda',\mathbf{p}\lambda,\mathbf{k}\delta)b_q^\dagger(\mathbf{p}'\lambda')b_q(\mathbf{p}\lambda)-\tilde{C}(\mathbf{p}\lambda,\mathbf{p}'\lambda',\mathbf{k}\delta)\tilde{b}_q^\dagger(\mathbf{p}'\lambda')\tilde{b}_q(\mathbf{p}\lambda)]\ . \tag{34}$$

Now incorporating the initial and final meson states as per Eq. (20) the S-matrix element in the rest frame of the decaying meson A can be expressed as

$$S_{BA}=i\sqrt{\alpha/k_0}\delta^{(4)}(P+k-\hat{O}M_A)[Q(\mathbf{P},\mathbf{k})-\tilde{Q}(\mathbf{P},\mathbf{k})]\ , \tag{35}$$

where, $P\equiv(E_B,\mathbf{P})$, $\hat{O}\equiv(1,0,0,0)$ and

$$\begin{aligned}Q(\mathbf{P},\mathbf{k})&=\sum\frac{e_{q_1}}{e}\zeta^A_{q_1q_2}(\lambda_1\lambda_2)\zeta^B_{q_1q_2}(\lambda_1'\lambda_2)\int d\mathbf{p}\frac{G_A(\mathbf{p},-\mathbf{p})G_B(\mathbf{P}+\mathbf{p},-\mathbf{p})C(\mathbf{p}'\lambda_1',\mathbf{p}\lambda_1,\mathbf{k}\delta)}{\sqrt{4E_1E_{1P}N_A(0)N_{B_1}(\mathbf{P})}}\ ,\\ \tilde{Q}(\mathbf{P},\mathbf{k})&=\sum\frac{e_{q_2}}{e}\zeta^A_{q_1q_2}(\lambda_1\lambda_2)\zeta^B_{q_1q_2}(\lambda_1\lambda_2')\int d\mathbf{p}\frac{G_A(\mathbf{p},-\mathbf{p})G_B(\mathbf{P}+\mathbf{p},-\mathbf{p})\tilde{C}(-\mathbf{p}\lambda_2,\mathbf{P}-\mathbf{p}\lambda_2',\mathbf{k}\delta)}{\sqrt{4E_2E_{2P}N_A(0)N_{B_2}(\mathbf{P})}}\ .\end{aligned} \tag{36}$$

Here $E_i=\sqrt{(\mathbf{p}^2+m_{q_i}^2)}$ and $E_{iP}=\sqrt{(\mathbf{P}+\mathbf{p})^2+m_{q_i}^2}$, $i=1,2$. We must mention that in extracting the correct δ function relating to the energy conservation at the photon-hadron vertex, we have used the usual approximation [5,6] in setting $(E_1+E_2)\simeq M_A$ and $(E_{1k}+E_2)\simeq(E_{2k}+E_1)\simeq E_B$. Now making use of the explicit form of the Dirac spinors $U(\mathbf{p},\lambda)$ and $V(\mathbf{p},\lambda)$, Eq. (36) can be further simplified as

$$\begin{aligned}Q(\mathbf{P},\mathbf{k})&=\sum\frac{e_{q_1}}{e}\zeta^A_{q_1q_2}(\lambda_1\lambda_2)\zeta^B_{q_1q_2}(\lambda_1'\lambda_2)\chi^\dagger_{\lambda_1'}(\sigma\cdot\mathbf{K})\chi_{\lambda_1}J_{q_1}(\mathbf{P},\mathbf{k})\ ,\\ \tilde{Q}(\mathbf{P},\mathbf{k})&=\sum\frac{e_{q_2}}{e}\zeta^A_{q_1q_2}(\lambda_1\lambda_2)\zeta^B_{q_1q_2}(\lambda_1\lambda_2')\tilde{\chi}^\dagger_{\lambda_2}(\sigma\cdot\mathbf{K})\tilde{\chi}_{\lambda_2'}J_{q_2}(\mathbf{P},\mathbf{k})\ ,\end{aligned} \tag{37}$$

where, $\mathbf{K}=\mathbf{k}\times\epsilon(\mathbf{k},\delta)$ and

$$\begin{aligned}J_{q_1}(\mathbf{P},\mathbf{k})&=[N_A(0)N_{B_1}(\mathbf{P})]^{-1/2}\int d\mathbf{p}\,G_A(\mathbf{p},-\mathbf{p})G_B(\mathbf{p}-\mathbf{k},-\mathbf{p})\left[\frac{E_1+m_{q_1}}{4E_1E_{1k}(E_{1k}+m_{q_1})}\right]^{1/2}\ ,\\ J_{q_2}(\mathbf{P},\mathbf{k})&=[N_A(0)N_{B_2}(\mathbf{P})]^{-1/2}\int d\mathbf{p}\,G_A(-\mathbf{p},\mathbf{p})G_B(-\mathbf{p},\mathbf{p}-\mathbf{k})\left[\frac{E_2+m_{q_2}}{4E_2E_{2k}(E_{2k}+m_{q_2})}\right]^{1/2}\ .\end{aligned} \tag{38}$$

In view of the four-δ function appearing in Eq. (35) the recoil momentum $\mathbf{P}$ of the product meson B is obviously implied to be $-\mathbf{k}$, which has been incorporated in Eq. (38). Therefore, E_{ik} in the above expressions for $i=1,2$ stands for $\sqrt{(\mathbf{p}-\mathbf{k})^2+m_{q_i}^2}$. Considering the fact that all possible directions of quark and photon momenta are ultimately included in the calculation, it would be a good approximation to take $(\mathbf{p}-\mathbf{k})^2\simeq(\mathbf{p}^2+\mathbf{k}^2)$ in the integrands of the expressions in Eq. (38). The same approximation also becomes useful in evaluating the meson-state normalization factors using Eq. (21). Then the integrals $J_{q_1}(\mathbf{P},\mathbf{k})$ and $J_{q_2}(\mathbf{P},\mathbf{k})$ in Eq. (38) can be effectively expressed as

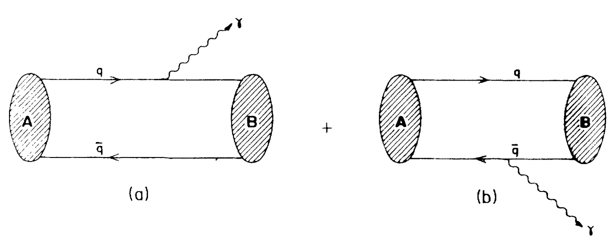

FIG. 1. Lowest-order graphs contributing to mesonic $M1$ transitions.

$$J_{q_i}(\mathbf{k})=[\bar{N}_A(0)\bar{N}_{B_i}(\mathbf{P})]^{-1/2}\times\int_0^\infty dp\,p^2X_i(\mathbf{p},\mathbf{k})\exp(-\beta p^2)\ , \tag{39}$$

where $\beta=\frac{1}{4}(1/\alpha_{q_1}+1/\alpha_{q_2})$ and

$$\begin{aligned}\bar{N}_A(0)&=\int_0^\infty dp\,p^2R_A(p)\exp(-\beta p^2)\ ,\\ \bar{N}_{B_i}(\mathbf{P})&=\int_0^\infty dp\,p^2R_{B_i}(p,k)\exp(-\beta p^2)\ ,\end{aligned} \tag{40}$$

when

$$\begin{aligned}R_A(p)&=\prod_{j=1}^{2}\left[(E_j+E_{q_j})\left[1+\frac{m_{q_j}}{E_j}\right]^{1/2}\right]\ ,\\ R_{B_i}(p,k)&=\frac{E_{ik}+E_{q_i}}{E_i+E_{q_i}}\left[\frac{E_i(E_{ik}+m_{q_i})}{E_{ik}(E_i+m_{q_i})}\right]^{1/2}R_A(p)\ ,\\ X_i(p,k)&=\frac{R_A(p)}{2}\left[\frac{(E_i+m_{q_i})(E_{ik}+E_{q_i})^2}{(E_{ik}+m_{q_i})(E_i+E_{q_i})^2}\frac{1}{E_iE_{ik}^3}\right]^{1/4}\ .\end{aligned} \tag{41}$$

Now following the mixing angle conventions as detailed in our earlier work [2] and specifying the appropriate spin-flavor coefficients $\zeta^M_{q_1q_2}(\lambda_1\lambda_2)$ for the pseudoscalar meson states and vector-meson states of different spin projections $S_V=(\pm 1,0)$, Eq. (37) can be further simplified so as to reduce Eq. (35) to a form

$$S_{BA}=\sqrt{\alpha/k_0}\,\delta^{(3)}(\mathbf{P}+\mathbf{k})\delta(E_B+k_0-M_A)\tilde{\mu}_{BA}(\mathbf{k})K_{S_V} , \tag{42}$$

where K_{S_V} for $S_V=(\pm 1,0)$ stands for the following combinations expressed separately for $V\to P\gamma$ and $P\to V\gamma$ type transitions:

$$\begin{aligned} K_{S_V}(V\to P\gamma)&=[\mp(K_1\pm iK_2)/\sqrt{2},K_3] ,\\ K_{S_V}(P\to V\gamma)&=[\pm(K_1\mp iK_2)/\sqrt{2},K_3] . \end{aligned} \tag{43}$$

Nevertheless, the summation over the photon polarization index δ and the meson spin S_V yields a general relation

$$\sum_{\delta,S_V}|K_{S_V}|^2=2k^2 . \tag{44}$$

Finally, $\tilde{\mu}_{BA}(\mathbf{k})$ in Eq. (42) is related to the transition magnetic moment which can be expressed appropriately in terms of $J_{q_1}(\mathbf{k})$ and $J_{q_2}(\mathbf{k})$. Now summing over the photon polarization index δ and the final meson spin appropriately while averaging over initial meson spin when necessary, the partial decay widths for the transition $A\to B\gamma$ can be obtained as

$$\Gamma(A\to B\gamma)=\sum_{\delta,S_V}\int d\mathbf{k}\,d\mathbf{P}|S_{BA}|^2/[VT/(2\pi)^3] . \tag{45}$$

Then for $V\to P\gamma$ and $P\to V\gamma$ processes, we can realize the partial decay widths in the standard form in terms of the outgoing photon energy $\bar{k}=(M_A^2-M_B^2)/2M_A$:

$$\begin{aligned} \Gamma(V\to P\gamma)&=\tfrac{4}{3}\alpha\bar{k}^3[\sqrt{E_P(\bar{k})/M_V}\tilde{\mu}_{PV}(\bar{k})]^2 ,\\ \Gamma(P\to V\gamma)&=4\alpha\bar{k}^3[\sqrt{E_V(\bar{k})/M_P}\tilde{\mu}_{VP}(\bar{k})]^2 . \end{aligned} \tag{46}$$

One must note that the phase-space factor $\sqrt{E_B(\bar{k})/M_A}$ in Eq. (46) is arising here out of the argument factorization of the energy δ function which was extracted from a constituent level integration in Eq. (32) under certain approximations required to realize the correct photon energy at the mesonic level. Therefore, before taking this factor so seriously, one must compare the expression (46) with the ones obtainable from a formal relativistic calculation at the mesonic level.

In fact, starting with a relativistic effective interaction of the form

$$\mu_{PV}\epsilon^{\mu\nu\lambda\sigma}\partial_\mu A_\nu(x)\partial_\lambda V_\sigma(x)P(x) , \tag{47}$$

where $A_\nu(x)$, $V_\sigma(x)$, and $P(x)$ are, respectively, the fields of photon, vector meson, and pseudoscalar meson, one can arrive at an expression for $\Gamma(V\to P\gamma)$ in terms of the transition moment μ_{PV} without the mesonic level phase-space factor (E_P/M_V) as found in Eq. (46). One can also do the same thing by considering the covariant expansion

$$\langle P(P')|J^\mu_{\rm em}|V(P)\rangle=\mu_{PV}\epsilon^{\mu\nu\lambda\sigma}\epsilon_\nu(P+P')_\lambda(P-P')_\sigma \tag{48}$$

in Eq. (30) together with the appropriate relativistic phase space in Eq. (45). This is therefore a pathological problem common to all such models attempting to explain hadronic level decay in terms of its constituent level dynamics considered in zeroth order. Hence it requires appropriate corrective measures for eliminating this spurious phase-space factor $\sqrt{E_B/M_A}$ at the mesonic level. For the transitions involving photon energy much less compared to the decaying meson mass, this factor is

$$E_B(\bar{k})/M_A=1-\bar{k}/M_A\simeq 1 .$$

Hence setting $[E_B(\bar{k})/M_A]\simeq 1$ and interpreting $\tilde{\mu}_{BA}(\bar{k})$ as the corresponding transition moment as has been done by many authors [14] in the past may be justified only for static calculations. But such a prescription is not, in general, correct when one wants to look beyond the static calculation. However, there is a clear possibility of explicit cancellation of this phase-space factor taken appropriately along with the contribution of the quark spinors. This has been shown explicitly in the works of Altomari [22] and also has been discussed in a recent work by O'Donnell and Tung [23] within the scope of their model based on the description of the meson states in the loose binding approximation in terms of relative momentum wave function of the constituent quarks in the nonrelativistic Gaussian forms. In view of this observation, we prefer here to push back this phase-space factor from the mesonic level to the quark level integrals $J_{q_i}(\mathbf{k})$ describing $\tilde{\mu}_{BA}(\bar{k})$ under the same approximation with which it was extracted out through the energy argument of the δ function. Hence

$$\sqrt{E_B(\bar{k})/M_A}\simeq\left[\frac{E_{ik}+E_j}{E_1+E_2}\right]^{1/2} ,$$

when $i\neq j=1,2$ taken inside the quark level integrals $J_{q_i}(\mathbf{k})$ in Eq. (39) effectively modifies it to $I_{q_i}(\bar{k})$ in the form

$$I_{q_i}(\bar{k})=[\bar{N}_A(0)\bar{N}_{B_i}(\mathbf{P})]^{-1/2}\times\int_0^\infty dp\,p^2\left[\frac{E_{ik}+E_j}{E_1+E_2}\right]^{1/2}X_i(\mathbf{p},\mathbf{k})\exp(-\beta p^2) . \tag{49}$$

Since we have considered flavor SU(2) symmetry with $m_u=m_d\neq m_s$, the integral expression in Eq. (49) corresponding to any particular transition can provide $I_d(\bar{k})=I_u(\bar{k})$. Then

$$\begin{aligned} \Gamma(V\to P\gamma)&=\tfrac{4}{3}\alpha\bar{k}^3[\mu_{PV}(\bar{k})]^2 ,\\ \Gamma(P\to V\gamma)&=4\alpha\bar{k}^3[\mu_{VP}(\bar{k})]^2 , \end{aligned} \tag{50}$$

where the transition moments $\mu_{BA}(k)$ for various processes can now find separate explicit expressions in terms of $I_{q_1}(\bar{k})$ and $I_{q_2}(\bar{k})$ in the following manner.

For transitions involving mesons in ordinary light flavor sector, we can have

$$
\begin{aligned}
&(\mathrm{i})\ \mu_{\pi\rho}(\bar{k})=\tfrac{1}{3}I_u(\bar{k})\ ,\\
&(\mathrm{ii})\ \mu_{\pi\omega}(\bar{k})=\cos\delta_v I_u(\bar{k})\ ,\\
&(\mathrm{iii})\ \mu_{\pi\phi}(\bar{k})=\sin\delta_v I_u(\bar{k})\ ,\\
&(\mathrm{iv})\ \mu_{\eta\rho}(\bar{k})=\sin\delta_p I_u(\bar{k})\ ,\\
&(\mathrm{v})\ \mu_{\eta\omega}(\bar{k})=\tfrac{1}{3}[\cos\delta_v\sin\delta_p I_u(\bar{k})\\
&\qquad\qquad +2\sin\delta_v\cos\delta_p I_s(\bar{k})]\ ,\\
&(\mathrm{vi})\ \mu_{\eta\phi}(\bar{k})=\tfrac{1}{3}[\sin\delta_v\sin\delta_p I_u(\bar{k})\\
&\qquad\qquad -2\cos\delta_v\cos\delta_p I_s(\bar{k})]\ ,\\
&(\mathrm{vii})\ \mu_{\eta'\phi}(\bar{k})=\tfrac{1}{3}[\sin\delta_v\cos\delta_p I_u(\bar{k})\\
&\qquad\qquad +2\cos\delta_v\sin\delta_p I_s(\bar{k})]\ ,\\
&(\mathrm{viii})\ \mu_{\rho\eta'}(\bar{k})=\cos\delta_p I_u(\bar{k})\ ,\\
&(\mathrm{ix})\ \mu_{\omega\eta'}(\bar{k})=\tfrac{1}{3}[\cos\delta_p\cos\delta_v I_u(\bar{k})\\
&\qquad\qquad -2\sin\delta_p\sin\delta_v I_s(\bar{k})]\ ,\\
&(\mathrm{x})\ \mu_{K^+K^{*+}}(\bar{k})=\tfrac{1}{3}[2I_u(\bar{k})-I_s(\bar{k})]\ ,\\
&(\mathrm{xi})\ \mu_{K^0K^{*0}}(\bar{k})=\tfrac{1}{3}[I_u(\bar{k})+I_s(\bar{k})]\ .
\end{aligned}
\tag{51}
$$

In the above expressions δ_v and δ_p are the mixing angle deviations from the ideal mixing such that $\delta_M=(\arcsin 1/\sqrt{3}-\theta_M)$. Similarly for the transitions involving the heavy mesons (D,B), we find appropriate expressions as

$$
\begin{aligned}
&(\mathrm{i})\ \mu_{D^+D^{*+}}(\bar{k})=\tfrac{1}{3}[2I_c(\bar{k})-I_u(\bar{k})]\ ,\\
&(\mathrm{ii})\ \mu_{D^0D^{*0}}(\bar{k})=\tfrac{2}{3}[I_c(\bar{k})+I_u(\bar{k})]\ ,\\
&(\mathrm{iii})\ \mu_{D_s^+D_s^{*+}}(\bar{k})=\tfrac{1}{3}[2I_c(\bar{k})-I_s(\bar{k})]\ ,\\
&(\mathrm{iv})\ \mu_{B^+B^{*+}}(\bar{k})=\tfrac{1}{3}[2I_u(\bar{k})-I_b(\bar{k})]\ ,\\
&(\mathrm{v})\ \mu_{B^0B^{*0}}(\bar{k})=\tfrac{1}{3}[I_u(\bar{k})+I_b(\bar{k})]\ ,\\
&(\mathrm{vi})\ \mu_{B_s^0B_s^{*0}}(\bar{k})=\tfrac{1}{3}[I_s(\bar{k})+I_b(\bar{k})]\ ,\\
&(\mathrm{vii})\ \mu_{B_c^+B_c^{*+}}(\bar{k})=\tfrac{1}{3}[2I_c(\bar{k})-I_b(\bar{k})]\ .
\end{aligned}
\tag{52}
$$

V. RESULTS AND DISCUSSION

In this section we would use the formalism in the previous section to evaluate the partial decay widths of several energetically possible $M1$ transitions among the vector and pseudoscalar mesons in the heavy as well as light flavor sector. The input parameters of the model would be the same in two sets as described in Sec. III. In our static calculation of the radiative decay widths of light mesons [2] we have used the parameter set 2 only, whereas for heavy mesons both the sets have been considered in Sec. III. The mixing angle parameters for (ω,ϕ) and (η,η') mixing are also taken in the same manner as in Ref. [2] in accordance with the quadratic mass formula so as to have $(\delta_v,\delta_p)\equiv(-3.7°,45°)$. Then the present calculation would demonstrate the extent to which our previous results of the static calculation can get modified after incorporating some momentum dependent effects.

First of all we calculate the transition moments $\mu_{BA}(\bar{k})$ for various processes provided through Eqs. (51) and (52). The integral expressions $I_{q_1}(\bar{k})$ and $I_{q_2}(\bar{k})$ for each of these processes required for the above calculation are obtained from Eqs. (40), (41), and (49) through numerical evaluation by Gaussian quadrature technique. The outgoing photon energy $\bar{k}$ used for each individual process is computed from the kinematic expression in Eq. (8) using the respective experimental meson masses. The results of our calculation for the transition moments expressed in nuclear magneton units are presented in Table I in comparison with the corresponding experimental quantities. Godfrey and Isgur [14] in their relativized quark model with the mock-meson approach have also estimated these transition moments. They have considered an effective exponent for the fraction (m_{q_i}/E_i) in their expression for $I_{q_i}(\bar{k})$ so as to fit $\mu_{\rho\pi}(\bar{k})$ with the experimental data. Table I provides a detailed comparative account of their estimation with the present ones.

For a more direct comparison with the available experimental data, we finally calculate the partial decay widths from Eq. (50). Table II embodies these results along with our earlier predictions based on the simple static calculation. We observe that in going beyond the static approximation to include the momentum dependence due to recoil effect, the results in the heavy flavor sector stand almost unaffected, justifying our earlier contention in Sec. III. On the other hand, in the light meson sector, the change appears to be very encouraging almost in all cases. Considering for example the $V\to\pi\gamma$ transitions, we find that the present predictions are significantly improved in comparison with static results to be in good agreement with experiment. Godfrey and Isgur [14], using $\rho\to\pi\gamma$ as the input to fit the adjustable parameter in their model, find the decay widths for other processes involving pion in the final state to be somewhat underestimated. It may be worthwhile to mention here the work of Geffen and Wilson [1] where they find a very good fit for most of the radiative decay processes in the light meson sector by allowing arbitrary effective quark moments. But these effective quark moments failed to describe the observed hyperon moments. In the CBM [4] treatment of $V\to\pi\gamma$, as processes involving the emission of the elementary pion in combination with $V\to\gamma$ transitions, the decay rates are found to be rather overestimated. In that sense the present model, based on the conventional picture of photon emission with appropriate recoil effects taken into consideration in a purely relativistic calculation, serves as an adequate description for radiative decays like $V\to\pi\gamma$ in the light meson sector. The only discrepancy to be noticed here is that the partial decay width $\Gamma(\rho^0\to\pi^0\gamma)$, being equal to $\Gamma(\rho^+\to\pi^+\gamma)$ as a consequence of isospin invariance, comes out to be much less compared to its experimental value 121 ± 31 keV. This may imply some significant isospin mixing between $\rho^0(770)$ and $\omega^0(783)$ due to the accidental near degenera-

TABLE I. Transition moment $\mu_{BA}(\bar{k})$ in the $M1$ transitions in units of μ_N in comparison with the results of the calculation [14] and the experiment [10].

Physical process $A\to B\gamma$	$\bar{k}$ (MeV)	Present calculation Set 1	Set 2	Calculation [14]	Experiment [10]
$\rho^+\to\pi^+\gamma$	370.75	0.70	0.70	0.69	0.695±0.223
$\rho^0\to\pi^0\gamma$	372.35	0.70	0.70		0.921±0.466
$\omega\to\pi\gamma$	379.33	2.08	2.07	2.07	2.18±0.576
$\phi\to\pi\gamma$	500.77	0.12	0.12	0.06	0.129±0.043
$\rho\to\eta\gamma$	189.18	1.76(2.06)	1.80(2.10)	1.53	1.82±0.953
$\omega\to\eta\gamma$	199.34	0.50(0.61)	0.51(0.62)	0.50	0.427±0.281
$\phi\to\eta\gamma$	362.71	0.82(0.66)	0.87(0.72)	0.71	0.656±0.161
$\phi\to\eta'\gamma$	59.80	0.85(1.01)	0.96(1.14)	0.66	<1.764
$\eta'\to\rho\gamma$	170.63	1.79(1.41)	1.83(1.45)	1.85	1.203±0.454
$\eta'\to\omega\gamma$	159.67	0.64(0.52)	0.66(0.55)	0.63	0.422±0.185
$K^{*+}\to K^+\gamma$	309.14	0.84	0.79	0.91	0.782±0.247
$K^{*0}\to K^0\gamma$	309.85	1.28	1.35	1.20	1.193±0.349
$D^{*+}\to D^+\gamma$	135.78	0.37	0.39	0.35	<5.91
$D^{*0}\to D^0\gamma$	137.91	1.94	1.98	1.78	<12.152
$D_s^{*+}\to D_s^+\gamma$	136.57	0.17	0.23	0.13	<25.56
$B^{*+}\to B^+\gamma$	45.80	1.50	1.59	1.37	
$B^{*0}\to B^0\gamma$	45.80	0.85	0.89	0.78	
$B_s^{*0}\to B_s^0\gamma$	46.80	0.62	0.68	0.55	
$B_c^{*+}\to B_c^+\gamma$	43.45	0.32	0.32		

cy in their mass. Consideration of such a mixing with a mixing angle θ_I can lead to the modified expressions [20]

$$\bar{\Gamma}(\rho^0\to\pi^0\gamma)=\Gamma(\rho^0\to\pi^0\gamma)(\cos\theta_I+3\sin\theta_I\cos\delta_v)^2\,,$$
$$\bar{\Gamma}(\omega^0\to\pi^0\gamma)=\Gamma(\omega^0\to\pi^0\gamma)\left[\cos\theta_I-\frac{\sin\theta_I}{3\cos\delta_v}\right]^2. \quad (53)$$

Then

$$\Gamma_{\rm expt}(\rho^0\to\pi^0\gamma)=(121\pm31)\text{ keV}$$

would imply a mixing angle $\theta_I\simeq 6.4°$. But, on the other hand, $\Gamma(\omega\to\pi\gamma)$ gets suppressed further from its calculated value of 651 to 596 keV which is to some extent below the experimental data. But, nevertheless, this result is rather closer to the experiment as compared to one obtained in our static calculation.

The decay widths of $V\to\eta\gamma$ and $V\to\eta'\gamma$ type transitions obtained here on the basis of the mixing angle sanctioned by the quadratic mass formula are quite satisfactory except for $\phi\to\eta\gamma$ and $\eta'\to V\gamma$ transitons. If, on the

TABLE II. Partial decay widths $\Gamma(A\to B\gamma)$ in comparison with the results of the static [2] and the CBM calculation [4,16,17] together with the experiment [10] with $\delta_v=-3.7°$ and $\delta_p=45°(56°)$.

Physical process $A\to B\gamma$	Present result (keV) Set 1	Set 2	Static result [2] (keV)	CBM result [4,16,17] (keV)	Expt. [10] (keV)
$\rho^+\to\pi^+\gamma$	68.84	68.34	45.80	124	68±7
$\rho^0\to\pi^0\gamma$	69.53	69.00	45.40	124	121±31
$\omega\to\pi\gamma$	651	645	419	1180	717±50
$\phi\to\pi\gamma$	5.10	4.94	2.20	4.70	5.81±0.65
$\rho\to\eta\gamma$	57.78(79.43)	60.34(82.94)	46.90(64.50)	23	62±17
$\omega\to\eta\gamma$	5.51(8.13)	5.70(8.36)	4.90(7.20)	2.30	4±1.73
$\phi\to\eta\gamma$	89.62(58.07)	99.87(69.12)	62(40.20)	43	56.74±3.43
$\phi\to\eta'\gamma$	0.42(0.60)	0.54(0.77)	0.40(0.60)	0.29	<1.84
$\eta'\to\rho\gamma$	131.65(82.33)	138.40(86.55)	110.20(68.90)	53	59.67±8.30
$\eta'\to\omega\gamma$	13.64(9.27)	14.82(10.13)	12.10(8.20)	6	6±1.16
$K^{*+}\to K^+\gamma$	57.07	51.13	48.30	47	50±5
$K^{*0}\to K^0\gamma$	135.39	149.80	107.10	98	117±10
$D^{*+}\to D^+\gamma$	0.95	1.08	1.15	1.00	<198
$D^{*0}\to D^0\gamma$	27.18	28.41	27.94	21.20	<945
$D_s^{*+}\to D_s^+\gamma$	0.20	0.38	0.32	1.2×10^{-4}	<4500
$B^{*+}\to B^+\gamma$	0.59	0.67	0.67	0.62	
$B^{*0}\to B^0\gamma$	0.19	0.21	0.21	0.28	
$B_s^{*0}\to B_s^0\gamma$	0.11	0.13	0.11	0.10	
$B_c^{*+}\to B_c^+\gamma$	0.02	0.02	0.02		

other hand, we take $\delta_p = 56°$ instead of 45°, which finds support from the reported value of $\theta_p = -17.6° \pm 3.6°$ obtained from $\Gamma_{\text{expt}}(\eta \to 2\gamma) = 0.56 \pm 0.16$ keV [21], the margin of discrepancy is considerably reduced. But such a value of θ_P obtained from the analysis of the two γ decay of η based on SU(3) flavor symmetry, may not be strictly applicable in the present context where the strange quark is taken to have a mass different from those of the up and down quarks. Nevertheless, this value of $\delta_p = 56°$ is close to the mixing angle sanctioned by the linear mass formula and it does not change other predictions beyond the experimental limits. Since our purpose here is not so much to obtain a detailed fit to the experimental data, but to demonstrate the wider applicability of our model with the relevant parameters taken from its earlier applications, we rest content in showing only the variation in our model predictions here within an acceptable range of values for δ_P. Results for $\delta_p = 56°$ are provided within parenthesis in Table II to give an idea about the range of variation in our results for δ_p taken between 45° and 56°.

Finally, we notice that the decay widths for $K^* \to K\gamma$ transitions are in good agreement with the experimental values proving the adequacy of the simple traditional picture of the photon emission induced by the quark electromagnetic current.

On the whole, the momentum dependent effect incorporated through the present formalism, is found to be quite adequate and relevant for the real dynamical involved in the process. Thus we find that within the working approximation adopted here, the present model provides a more realistic calculational framework to describe the mesonic $M1$ transitions based on the conventional picture of photon emission.

ACKNOWLEDGMENTS

One of us (P.C.D.) gratefully acknowledges the support of the Department of Education, Government of Orissa, India for providing study leave for the purpose.

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