Global symmetries of open strings in an electromagnetic background

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The global symmetries of open bosonic strings in an electromagnetic background are investigated. The Poincaré subalgebra and the mass of the open charged string are derived. These results are useful for computing the background electric field dependence of the one-loop free energy and Hagedorn temperature of a neutral string gas.

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I. INTRODUCTION

In spite of the innumerable difficulties found during the development of string theories, they still remain as the only promising candidate for a "theory of everything." Such a desirable trait and the internal beauty of their theoretical scheme serve as motivation for many physicists not give up without reaching a complete understanding and eventually a solution of the challenging problems of the theory.

Within the above-mentioned difficulties, the determination of a unique vacuum, a basic string theory problem has encouraged many investigations. An approach to this question has been implemented through the study of strings propagating in background fields [1–10]. In particular, the effective action for an Abelian gauge field coupled to open bosonic strings has been found in [2] and also in [7] by using two independent techniques: a Polyakov path integral and the β function, respectively. The general aim of these investigations is to gain deeper knowledge of the nonperturbative properties of the system. In principle, an effective action for an infinite number of fields, corresponding to excitation modes of a string, can serve as a generating functional for all possible amplitudes on an arbitrary background. By extremizing it with respect to the background fields, the true vacuum of the theory could be established.

If (super)string theories really permit us to unify all the fundamental interactions, they must describe the physics at the Planck energy region. This scale of energies corresponds to the early Universe scales. As temperatures and densities were both very high in the early Universe, it is crucial to investigate the behavior of strings under such extreme conditions.

The thermodynamics of strings has been studied by many authors [11–15]. Strings at finite densities have also been considered (see, for example, Ref. 16). More recently, the interest in understanding the quantum behavior of strings under the condition of the early Universe, as well as the role of the many fields that form the string spectrum, has motivated different investigations where the presence of a background field is considered simultaneously with the finite temperature [17,18].

In the investigation of strings at finite temperature, the string mass operator plays a key role in the calculation of the free energy of the string gas. If a background field is present, the string mass operator may be very different from that obtained in the free-string case. Such a modification of the mass may give rise to very nontrivial physical consequences. In fact, as has been shown in a previous paper [18], the background dependence that appears in the Hagedorn temperature of an open neutral string gas in an electromagnetic background is produced by the change in the string mass due to the background field. Therefore, the correct determination of the string mass in problems that involve strings propagating in background fields is important in itself, for a complete description of the fundamental characteristics of such systems, as well as for the investigation of their thermodynamical properties.

In the present paper we investigate the global symmetries of an open charged bosonic string in a constant and homogeneous electromagnetic background. We derive the Poincaré subalgebra that characterizes the theory and obtain the string mass as one of the invariants of this group of symmetries. An amazing result is that the above algebra coincides with the one associated with the global symmetries of a charged relativistic particle in a constant and homogeneous electromagnetic background. One might think that this is the natural extension of the known analogy between the space-time symmetries of a free noncharged string and a free relativistic particle. However, as we show in this paper, such an analogy is not complete: in the neutral string case ($q_1=-q_2\neq 0$), the global symmetries are described by a subalgebra of Poincaré because some space-time symmetries are still broken, while in the neutral particle case the whole Poincaré group is recovered.

The paper is organized as follows. In Sec. II, we determine the subset of Poincaré transformations that leave invariant the action of an open charged string in a constant and homogeneous electromagnetic background. Then, we derive the subalgebra of Poincaré corresponding to such global symmetries. We also discuss the arguments that permit us to obtain the correct charged string mass. In Sec. III, the square mass operator for the neutral case is obtained in terms of the transversal string modes. Finally, in Sec. IV we discuss the possible implications of our results and give the concluding remarks.

II. OPEN BOSONIC STRING IN ELECTROMAGNETIC BACKGROUND

A. General features

The interaction of an open bosonic string with an electromagnetic background A_μ can be implemented by attaching charges q_1 and q_2 at the string's ends. The reparametrization-invariant action of the string in this background can be written as

$$S=-\frac{T}{2}\int_{M^2} d\tau\, d\sigma \sqrt{|g|}\, g^{ab}\partial_a X_\mu \partial_b X^\mu + \sum_{i=1}^{2} q_i \int_{\partial M_i} dx_\mu A^\mu . \tag{2.1}$$

The world sheet M^2 traced out by the string propagation and its boundaries ∂M_i can be simplified without loss of generality to those defined by the strip

$$\tau_1 \le \tau \le \tau_2, \quad 0 \le \sigma \le \pi .$$

In Eq. (2.1) the boundary term describes the coupling with the background. The usual notation in which T is the string tension and g^{ab} is the bi-dimensional metric with $g=\det g^{ab}$ has been used. We assume that the dimension D of the embedding space-time is an even number and that the metric of this D-dimensional Minkowski space is flat with signature $(1,-1)$.

It is easy to see that the action (2.1) has the same local symmetries as the free-string action; thus, it gives rise to the same constraints. Considering the conformal gauge

$$g^{ab}=\eta^{ab}e^{\phi(\tau,\sigma)}, \quad \eta^{ab}=\mathrm{diag}(-1,+1) \tag{2.2}$$

and defining τ as the boundary parameter, the action takes the form (see [9])

$$S=\frac{T}{2}\int d\tau \int_0^\pi d\sigma (\dot X_\mu \dot X^\mu - X'_\mu X'^\mu) + 2\int d\tau \int_0^\pi d\sigma\, A^\mu \dot X_\mu [q_1 \delta(\sigma)+q_2\delta(\sigma-\pi)] , \tag{2.3}$$

where

$$\dot X_\mu = \partial_\tau X_\mu, \quad X'_\mu=\partial_\sigma X_\mu$$

and we have used

$$\int_a^b f(x)\delta(x-y)dx=\frac{f(y)}{2} \quad \text{if } y=a \text{ or } y=b . \tag{2.4}$$

Any arbitrary transformation $X_\mu \to X_\mu+\delta X_\mu$ leaves the action stationary if the equations of motion

$$X''_\mu - \ddot X_\mu = 0 \tag{2.5}$$

and boundary conditions

$$TX'^\mu - q_1 \partial_\tau A^\mu + q_1 \frac{\partial A^\nu}{\partial X_\mu}\dot X_\nu = 0, \quad \sigma=0 , \tag{2.6a}$$

$$TX'^\mu + q_2 \partial_\tau A^\mu - q_2 \frac{\partial A^\nu}{\partial X_\mu}\dot X_\nu = 0, \quad \sigma=\pi \tag{2.6b}$$

are satisfied.

In the conformal gauge the constraints of the action (2.3) take the usual form

$$(\dot X \pm X')^2 = 0 . \tag{2.7}$$

Henceforth we will consider an electromagnetic background of constant strength $F_{\mu\nu}$:

$$A_\mu = -\tfrac{1}{2}F_{\mu\nu}X^\nu . \tag{2.8}$$

For such a field, the boundary conditions (2.6) take the form

$$TX'_\mu + q_1 F_{\mu\nu}\dot X^\nu = 0, \quad \sigma=0 , \tag{2.9a}$$

$$TX'_\mu - q_2 F_{\mu\nu}\dot X^\nu = 0, \quad \sigma=\pi . \tag{2.9b}$$

It is always possible to transform the antisymmetric tensor $F_{\mu\nu}$ in order to put it in a block diagonal form:

$$F_\mu^\nu = \begin{bmatrix} 0 & -E & & \\ -E & 0 & & \\ & & 0 & H_i \\ & & -H_i & 0 \end{bmatrix} , \quad i=1,2,\ldots,(D-2)/2 . \tag{2.10}$$

This tensor is characterized by the Lorentz invariant, which for $D=4$ can be written in terms of $F_{\mu\nu}$ and its dual $F^*_{\mu\nu}$:

$$\tfrac{1}{2}F_{\mu\nu}F^{\mu\nu}, \quad \tfrac{1}{2}F_{\mu\nu}F^{*\mu\nu} .$$

For any arbitrary dimension D, the tensor $F_{\mu\nu}$ is characterized by its eigenvalues. The eigenvalues are the roots of the equation

$$\mathrm{Det}(F_{\mu\nu}-\lambda\eta_{\mu\nu})=0 . \tag{2.11}$$

B. Global symmetries

It is well known that the free-string action remains invariant under the set of global D-dimensional space-time transformations: the D-dimensional Poincaré group. When the background field (2.8) is present, the D-dimensional isotropy is broken. One may expect that in this case the Poincaré symmetry is somehow reduced. In this section we will see that this is indeed the case.

1. Conserved quantities

To determine the group of global symmetries that characterize the theory (2.3) we must find out the conserved quantities associated to the D-dimensional space-time transformations. With this aim, we start by writing the action (2.3) in a more convenient way:

$$S=\int d\tau\, d\sigma \left[\frac{T}{2}(\dot X_\mu \dot X^\mu - X'_\mu X'^\mu) - F_{\mu\nu}\dot X^\mu X^\nu [q_1\delta(\sigma)+q_2\delta(\sigma-\pi)] \right] . \tag{2.12}$$

a. Translations. Let us make the following transformation in the string coordinates:

$$X_\mu \rightarrow X_\mu + \epsilon_\mu, \quad \epsilon_\mu = \text{const} , \tag{2.13}$$

which corresponds to an infinitesimal translation. It is easy to see that under the transformation (2.13) the action (2.12) changes as

$$\delta S = \int d\tau\, d\sigma\, \epsilon^\mu \bar{q} F_{\mu\nu} \dot{X}^\nu , \tag{2.14}$$

where

$$\bar{q} = \bar{q}(\sigma) = q_1 \delta(\sigma) + q_2 \delta(\sigma - \pi) . \tag{2.15}$$

By using the boundary conditions (2.9), the integral (2.14) can be written as the integral of a divergence. This means, according to the Noether theorem, that a conserved quantity, associated to translations, must exist. In order to find this conserved magnitude, let us transform (2.12) with (2.13) again, but now we integrate by parts before using ϵ_μ as a constant,

$$\delta S = \int d\tau\, d\sigma \{ \epsilon^\mu [-\partial_\tau (T\dot{X}_\mu) + \partial_\sigma (TX'_\mu) + 2\bar{q} F_{\mu\nu} \dot{X}^\nu] + \partial_\tau [(T\dot{X}_\mu - \bar{q} F_{\mu\nu} X^\nu) \epsilon^\mu] + \partial_\sigma (-TX'_\mu \epsilon^\mu) \} . \tag{2.16}$$

Equating (2.14) to (2.16) we obtain

$$\int d\tau\, d\sigma \{ \epsilon^\mu [-\partial_\tau (T\dot{X}_\mu) + \partial_\sigma (TX'_\mu) + \bar{q} F_{\mu\nu} \dot{X}^\nu] + \partial_\tau [(T\dot{X}_\mu - \bar{q} F_{\mu\nu} X^\nu) \epsilon^\mu] + \partial_\sigma (-TX'_\mu \epsilon^\mu) \} = 0 . \tag{2.17}$$

Using now the equations of motion and taking into account that

$$\partial_\tau \bar{q}\, F_{\mu\nu} X^\nu \epsilon^\mu = \bar{q} F_{\mu\nu} \dot{X}^\nu \epsilon^\mu ,$$

we obtain

$$\int d\tau\, d\sigma [\partial_\tau (T\dot{X}_\mu \epsilon^\mu) + \partial_\sigma (-TX'_\mu \epsilon^\mu)] = 0 . \tag{2.18}$$

Therefore, we arrive at the conserved current equation

$$\partial_\tau (T\dot{X}_\mu) + \partial_\sigma (-TX'_\mu) = 0 . \tag{2.19}$$

Integrating (2.19) in σ,

$$\partial_\tau \int d\sigma\, T\dot{X}_\mu - TX'_\mu(\pi) + TX'_\mu(0) = 0 .$$

Using the boundary conditions (2.9), we find

$$\partial_\tau \int d\sigma (T\dot{X}_\mu - 2\bar{q} F_{\mu\nu} X^\nu) = 0 . \tag{2.20}$$

It implies that the momentum

$$\pi_\mu = \int d\sigma (T\dot{X}_\mu - 2\bar{q} F_{\mu\nu} X^\nu) \tag{2.21}$$

is conserved. The quantum magnitude corresponding to π_μ in the quantized theory of the string in an electromagnetic background will be the generator of translations. Note that the conserved momentum π_μ does not coincide with the canonical momentum:

$$P_\mu = \int d\sigma\, p_\mu , \tag{2.22}$$

$$p_\mu = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu} = T\dot{X}_\mu - \bar{q} F_{\mu\nu} X^\nu . \tag{2.23}$$

The canonical momentum P_μ is not conserved now and, of course, its quantum counterpart will not generate translations anymore.

b. Lorentz transformations. Similarly to the translation case, we have that, under an infinitesimal Lorentz transformation,

$$X_\mu \rightarrow X_\mu + \lambda_{\mu\nu} X^\nu, \quad \lambda_{\mu\nu} = -\lambda_{\nu\mu} , \tag{2.24}$$

the action (2.12) varies as

$$\delta S = \int d\tau\, d\sigma\, \bar{q} F_{\mu\nu} \lambda^\nu_\rho (\dot{X}^\rho X^\mu - \dot{X}^\mu X^\rho) . \tag{2.25}$$

This term cannot be written as a divergence. It must vanish in order that the theory remains invariant under such transformation.

Therefore, the invariance of the theory (2.12) under the Lorentz transformations takes place only for those $\lambda_{\mu\nu}$ such that the product $F_{\mu\nu}\lambda^\mu_\rho$ is diagonal. This condition implies that from the whole set of $D(D-1)/2$ Lorentz transformations only $D/2$ of them leave the action (2.12) invariant. In the system where $F_{\mu\nu}$ is block diagonal, these $D/2$ transformations must be also block diagonal. Thus,

$$\lambda_{\mu\nu} = \begin{bmatrix} 0 & \lambda_0 & & \\ -\lambda_0 & & & \\ & & 0 & \lambda_i \\ & & -\lambda_i & 0 \end{bmatrix} , \quad i = 1, 2, \ldots, (D-2)/2 . \tag{2.26}$$

It is not difficult to find, using the Noether theorem, the conserved magnitudes associated with the Lorentz transformations (2.26):

$$M_{2n\,2n+1} = \int d\sigma [(T\dot{X}_{2n} - \bar{q} F_{2n\,2n+1} X^{2n+1}) X_{2n+1} - (T\dot{X}_{2n+1} - \bar{q} F_{2n+1\,2n} X^{2n}) X_{2n}] . \tag{2.27}$$

They satisfy

$$\partial_\tau M_{2n\,2n+1} = 0, \quad n = 0, 1, \ldots, \frac{D}{2} - 1 . \tag{2.28}$$

In terms of the canonical momentum density p_μ, the conserved momentum and angular momentum take the form

$$\pi_\mu = \int d\sigma (p_\mu - \bar{q} F_{\mu\nu} X^\nu) , \tag{2.29a}$$

$$M_{2n\,2n+1} = \int d\sigma (p_{2n} X_{2n+1} - p_{2n+1} X_{2n}) . \tag{2.29b}$$

2. *Poincaré subalgebra and string mass*

We have already found the subset of Lorentz transformations that leave invariant the theory of the string in an electromagnetic background. This reduction of the Lorentz symmetries must also be reflected in the Poincaré algebra of the conserved quantities. In order to find this algebra we have to consider the Poisson brackets of the theory. As usual, they must be defined by using the conjugate magnitudes X_μ and p_μ. Thus, the equal time

Poisson brackets are

$$\{p_\mu(\tau,\sigma),X_\nu(\tau,\sigma')\}=\eta_{\mu\nu}\delta(\sigma-\sigma') , \qquad (2.30a)$$

$$\{p_\mu,(\tau,\sigma),p_\nu(\tau,\sigma')\}=0 , \quad \{X_\mu(\tau,\sigma),X_\nu(\tau,\sigma')\}=0 . \qquad (2.30b)$$

Using (2.30) one can show that the algebra of the conserved quantities is given by the relations

$$\{\pi_\mu,\pi_\nu\}=QF_{\mu\nu} , \qquad (2.31a)$$

$$\{\pi_\mu,M_{2n\,2n+1}\}=\eta_{\mu 2n+1}\pi_{2n}-\eta_{\mu 2n}\pi_{2n+1} , \qquad (2.31b)$$

$$\{M_{2n\,2n+1},M_{2m\,2m+1}\}=0 , \qquad (2.31c)$$

where $Q=q_1+q_2$ is the total charge of the string.

It is timely to comment on the algebra (2.31). First, this algebra is essentially the same one obtained in the case of a charged relativistic particle in a constant and homogeneous electromagnetic background [19]. Second, it is a central extension of the Lie algebra of the Poincaré subgroup that leaves invariant the electromagnetic field under consideration.

As we mentioned in Sec. II, this is an expected result. The background field $F_{\mu\nu}$ breaks the isotropy of the D-dimensional space; therefore, only a subgroup of the original symmetry group must remain. The fact that the algebra (2.31) coincides with the one obtained for the charged relativistic particle seems to reflect here also the analogy already found between a free string and a free relativistic particle in relation to their properties under D-dimensional space-time transformations.

Nevertheless, one should be very careful to avoid making naive conclusions. Although for the interacting charged string and the charged particle this analogy seems to work, their correspondent neutral cases have algebras (or symmetries) completely different. This can be realized if one evaluates $Q=0$ (neutral string) in Eq. (2.31). In this case one still has a subalgebra of the Poincaré algebra. In the neutral particle case, however, the global symmetries of the theory are described by the whole Poincaré group, i.e., the Poincaré algebra.

From a physical point of view this difference can be easily understood. A neutral particle is incapable of interacting with an electromagnetic background. Since it is effectively a free particle, it has the symmetries of a free particle: the whole Poincaré group. In the string case, even if it is neutral, it has charges attached at its ends. These charges feel the electromagnetic background, interacting with it. Obviously, such an interaction reduces the global symmetries of the theory. The correct analogy for the neutral particle would be a noncharged string $(q_1=q_2=0)$.

Let us find the mass of the string in the electromagnetic background. It would be a mistake to use for the interacting string mass the same expression as in the free string case:

$$M^2=P^2 . \qquad (2.32)$$

Since P_μ is nonconserved, such a definition would have no physical sense. There exists some confusion in the literature about this point [10].

We can find a meaningful definition of the string mass if we take into account that the eigenvalues of the string mass operator in the quantized theory represent the masses of the particles that form the string spectrum. Each one of these charged relativistic particles is interacting with a constant and homogeneous electromagnetic background field. The mass of such a kind of particle is a conserved quantity and an invariant of the subgroup of Poincaré transformations that characterize the space-time symmetries of the problem. Therefore, in order to obtain the correct expression for the string mass we must find a conserved quantity which is invariant under the group of transformations (2.31). These two criteria, however, are not sufficient to determine M. In addition, the mass must coincide in the pointlike limit of the string theory (2.12) with the correspondent particle mass, and also it must be reduced to the neutral string mass when $q_1=-q_2=q$ and to the free-string mass when $q_1=q_2=0$.

Taking into account the above arguments we arrive at the square mass definition

$$M^2=\pi^2+2Q\sum_n M_{2n\,2n+1}F^{2n\,2n+1} . \qquad (2.33)$$

It is easy to prove that M^2 is an invariant of the group (2.33):

$$\{M^2,\pi_\mu\}=\{M^2,M_{2n\,2n+1}\}=0 . \qquad (2.34)$$

Moreover, if $q_1=-q_2=q$ $(Q=0)$, it reduces to

$$M^2=\pi^2 , \qquad (2.35)$$

where the conserved momentum π_μ can be written in this case as

$$\pi_\mu=\int d\sigma(T\dot{X}_\mu+qF_{\mu\nu}X'^\nu) . \qquad (2.36)$$

Expressions (2.35) and (2.36) coincide with the results in papers [8,10,18]. Note also that, in particular, if $q=0$, then M^2 becomes the free-string square mass.

III. NEUTRAL STRING

For the sake of understanding, we will review in this section the steps that yield an explicit expression for the mass of an open neutral string. This mass has been used in paper [18] to obtain the free energy of a neutral string gas in electromagnetic background.

The neutrality condition $q_1=-q_2=q$ can be used in (2.21) in order to write it in the convenient way [10]

$$S=\frac{T}{2}\int d\tau\, d\sigma(\dot{X}_\mu\dot{X}^\mu-X'_\mu X'^\mu-2f_{\mu\nu}X'^\mu\dot{X}^\nu) , \qquad (3.1)$$

where we have introduced the rescaled field strength

$$f_{\mu\nu}=\frac{qF_{\mu\nu}}{T} . \qquad (3.2)$$

The solution of the boundary problem originating from (3.1) is

$$X^\mu = a^\mu + b^\mu \tau - f^{\mu\nu} b_\nu \sigma + \frac{i}{\sqrt{\pi T}} \sum_{n=-\infty}^{\infty}{}' \left[\eta^{\mu\nu} \frac{\alpha_{n\nu}}{n} e^{-in\tau} \cos n\sigma + i f^{\mu\nu} \frac{\alpha_{n\nu}}{n} e^{-in\tau} \sin n\sigma \right] . \tag{3.3}$$

Substituting this solution in the density of the canonical momentum, $p_\mu = T(X_\mu + f_{\mu\nu} X'^\nu)$, we obtain

$$p_\mu = T(b_\mu - f_{\mu\nu} f^{\nu\rho} b_\rho) + \left[\frac{T}{\pi} \right]^{1/2} \sum_{n=-\infty}^{\infty}{}' (\eta_{\mu\nu} \alpha_n^\nu - f_{\mu\nu} f^{\nu\rho} \alpha_{n\rho}) e^{-in\tau} \cos n\sigma . \tag{3.4}$$

Note that the total canonical momentum obtained from (3.1),

$$P_\mu = \int d\sigma \, p_\mu = \int d\sigma \, T(\dot{X}_\mu + f_{\mu\nu} X'^\nu) , \tag{3.5}$$

is just the same as the neutral string conserved total momentum π_μ, defined in Eq. (2.36); hence, P_μ is a conserved quantity in the neutral case.

Using (3.3)–(3.5) it can be shown that

$$P_\mu = \int p_\mu d\sigma = \pi T (b_\mu - f_{\mu\nu} f^{\nu\rho} b_\rho) . \tag{3.6}$$

In terms of P_μ, Eqs. (3.3) and (3.4) take the form

$$X^\mu(\sigma,\tau) = a^\mu + \frac{\tau}{\pi T} [(1-f^2)^{-1}]^\mu_\nu P^\nu - \frac{\sigma}{\pi T} f^{\mu\nu} [(1-f^2)^{-1}]^\rho_\nu P_\rho + \frac{i}{\sqrt{\pi T}} \sum_{n=-\infty}^{\infty}{}' \left[\eta^{\mu\nu} \frac{\alpha_{n\nu}}{n} e^{-in\tau} \cos n\sigma + i f^{\mu\nu} \frac{\alpha_{n\nu}}{n} e^{-in\tau} \sin n\sigma \right] , \tag{3.7}$$

$$p_\mu(\sigma,\tau) = \frac{P_\mu}{\pi} + \left[\frac{T}{\pi} \right]^{1/2} \sum_{n=-\infty}^{\infty}{}' (\eta_{\mu\nu} \alpha_n^\nu - f_{\mu\nu} f^{\nu\rho} \alpha_{n\rho}) e^{-in\tau} \cos n\sigma . \tag{3.8}$$

A short computation involving (3.7), (3.8), and the Poisson brackets (2.32) shows that the zero and Fourier modes satisfy

$$\{\alpha^\mu, P^\nu\} = -\eta^{\mu\nu} , \quad \{\alpha_m^\mu, \alpha_n^\nu\} = i m \delta_{m+n} \eta^{\mu\rho} [(1-f^2)^{-1}]_{\rho\nu} . \tag{3.9}$$

Our aim is to find an expression for the string mass in which we have already eliminated the contribution of the spurious degrees of freedom. To this purpose it is convenient to introduce the variables

$$X^\pm = \frac{X^0 \pm X^1}{\sqrt{2}} , \tag{3.10a}$$

$$P^\pm = \frac{P^0 \pm P^1}{\sqrt{2}} , \tag{3.10b}$$

$$\alpha_n^\pm = \frac{\alpha_n^0 \pm \alpha_n^1}{\sqrt{2}} , \tag{3.10c}$$

and the notation

$$\alpha_0^\pm = \frac{P^\pm}{\sqrt{\pi T}(1-e^2)} , \quad \alpha_0^{\substack{2i \\ 2i+1}} = \frac{P^{\substack{2i \\ 2i+1}}}{\sqrt{\pi T}(1+h_i^2)} , \quad i = 1, \ldots, \frac{D}{2} - 1 , \tag{3.11}$$

where

$$e = f_{01} , \quad h_i = -f_{2i\,2i+1} , \quad i = 1, \ldots, \frac{D}{2} - 1 . \tag{3.12}$$

In terms of (3.10) and (3.11) the constraints (2.7) take the form

$$2(1-e^2) \sum_{m=-\infty}^{\infty} \alpha_{n-m}^+ \alpha_m^- - \sum_{m=-\infty}^{\infty} \sum_{i=1}^{(D/2)-1} (1+h_i^2)(\alpha_{n-m}^{2i} \alpha_m^{2i} + \alpha_{n-m}^{2i+1} \alpha_m^{2i+1}) = 0 . \tag{3.13}$$

Let us consider the light-cone gauge [10]

$$X'^- = e \dot{X}^- , \quad \dot{X}^- = e X'^- + \frac{P^-}{\pi T} . \tag{3.14}$$

It gives rise to the condition

$$\alpha_m^- = 0 \quad \text{for } m \neq 0 . \tag{3.15}$$

Then, evaluating (3.15) in (3.13), one obtains, for the $n=0$ constraint,

$$P^+ P^- = \frac{\pi T}{2} (1-e^2) \times \sum_{i=1}^{(D/2)-1} (1+h_i^2) \sum_{m=-\infty}^{\infty} (\alpha_{-m}^{2i} \alpha_m^{2i} + \alpha_{-m}^{2i+1} \alpha_m^{2i+1}) . \tag{3.16}$$

As we already observed, the square mass of the neural string is

$$M^2 = \pi^2 . \tag{3.17}$$

Then we can use (3.16) to obtain an expression for the mass that depends on the transversal degrees of freedom only:

$$M^2=-\sum_{i=1}^{D/2-1}\frac{e^2+h_i^2}{1+h_i^2}[(P^{2i})^2+(P^{2i+1})^2]$$
$$+2\pi T(1-e^2)\sum_{i=1}^{D/2-1}\sum_{m=1}^{\infty}m(a_m^{\dagger 2i}a_m^{2i}+a_m^{\dagger 2i+1}a_m^{2i+1}) . \tag{3.18}$$

In the above expression the following notation for the normalized oscillators,

$$a_n^{\substack{2i\\2i+1}}=\left[\frac{1+h_i^2}{n}\right]^{1/2}\alpha_n^{\substack{2i\\2i+1}},\quad n>0 , \tag{3.19a}$$

$$a_n^{\dagger\substack{2i\\2i+1}}=\left[\frac{1+h_i^2}{n}\right]^{1/2}\alpha_{-n}^{\substack{2i\\2i+1}},\quad n>0 \tag{3.19b}$$

was used. Upon canonical quantization, their nonzero commutators are

$$[a_m^{2i},a_n^{\dagger 2j}]=[a_m^{2i+1},a_n^{\dagger 2j+1}]=\delta_{mn}\delta_{ij} . \tag{3.20}$$

The square mass operator corresponding to the classical expression (3.18) contains an additional c-number term, due to order problems, that produces a tachyonic contribution. Thus, the square mass operator is given by

$$M^2=-\sum_{i=1}^{D/2-1}\frac{e^2+h_i^2}{1+h_i^2}[(P^{2i})^2+(P^{2i+1})^2]+2\pi T(1-e^2)\left[\sum_{i=1}^{D/2-1}\sum_{m=1}^{\infty}m(a_m^{\dagger 2i}a_m^{2i}+a_m^{\dagger 2i+1}a_m^{2i+1})-\frac{D-2}{24}\right] . \tag{3.21}$$

There has existed some misunderstanding in the literature in relation to result (3.18). For example, in Refs. [20,21], the authors attempted to redefine the square mass of the neutral string with the aim of removing the tachyonic contribution in the transverse direction [first term in (3.18)]. Later on, in Ref. [10], the expression (3.18) was again accepted, but no physical reason was given to justify this decision. Throughout the present paper we have seen that a deep physical reason does exist to justify the correctness of Eq. (3.18) and, in general, of Eq. (2.33).

IV. CONCLUDING REMARKS

In this paper, we have studied the global symmetries of open strings in electromagnetic background. We derived the algebra of the conserved quantities corresponding to the global symmetries of an open bosonic string coupled to a constant and homogeneous electromagnetic field. It resulted in a subalgebra of the Poincaré algebra, because the background breaks some of the symmetries normally present in the free string case. A remarkable feature is that this subalgebra coincides with the one obtained for a relativistic charged particle. This property helped identify the correct expression for the string mass, since each string single mode is associated with a relativistic charged particle interacting with the background field. Therefore, the string mass must be just one of the group invariants and it must be reduced to the usual string mass in the zero field limit. We hope this result helps to solve some of the misunderstandings that exist in the literature with respect to the correct definition of the string mass for open, neutral, and charged strings.

In the neutral string case, the above results were used in the study of the thermodynamical properties of the system and in the computation of the Hagedorn temperature of the neutral string gas in electromagnetic background [18]. This Hagedorn temperature is different from the one of the free-string case because it contains a factor depending on the background electric field. The modification of the Hagedorn temperature due to the background is a nonperturbative effect. The interaction with the background is an effective, nonperturbative way to represent interactions among strings. In contrast with this, perturbative string interactions cannot modify the critical temperature as it has been recently shown by Bytsenko *et al.* [11], by calculating the genus-g free energy of the open string gas.

Although there are still many formal questions in the thermodynamics of strings that should be better understood, it is undoubtedly an amazing research field. We expect in the near future to go a step forward and use our present results to explore possible effects of finite densities in strings coupled to background fields.

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