

Covariant effective action and one-loop renormalization of two-dimensional dilaton gravity with fermionic matter

E. Elizalde*

*Division of Applied Mechanics, The Norwegian Institute of Technology,
and Department of Theoretical Physics, University of Trondheim, N-7034 Trondheim, Norway*

S. Naftulin

Institute for Single Crystals, 60 Lenin Ave., 310141 Kharkov, Ukraine

S. D. Odintsov†

*Departamento de Estructura y Constituyentes de la Materia, Facultad de Física, Universidad de Barcelona,
Diagonal 647, E-08028, Barcelona, Spain
and Tomsk Pedagogical Institute, 634041 Tomsk, Russia*

(Received 2 July 1993; revised manuscript received 15 October 1993)

Two-dimensional dilaton gravity interacting with a four-fermion model and scalars is investigated, all the coefficients of the Lagrangian being arbitrary functions of the dilaton field. The one-loop covariant effective action for two-dimensional (2D) dilaton gravity with Majorana spinors (including the four-fermion interaction) is obtained, and the technical problems which appear in an attempt at generalizing such calculations to the case of the most general four-fermion model described by Dirac fermions are discussed. A solution to these problems is found, based on its reduction to the Majorana spinor case. The general covariant effective action for 2D dilaton gravity with the four-fermion model described by Dirac spinors is given. The one-loop renormalization of dilaton gravity with Majorana spinors is carried out and the specific conditions for multiplicative renormalizability are found. A comparison with the same theory but with a classical gravitational field is done.

PACS number(s): 04.50.+h, 04.60.Kz, 11.10.Gh

I. INTRODUCTION

There is an increasing interest in the study of two-dimensional (2D) dilaton gravity for different reasons. First, the insurmountable difficulties involved in dealing with 4D quantum gravity make 2D dilaton gravity a very interesting laboratory, which may presumably lead to the understanding of general properties of true quantum gravity. In fact, it is much easier to work with 2D gravity, because there the by now well-known methods of conformal field theory can be successfully applied. Second, 2D dilaton gravity with matter may well serve as a good toy model to study very important features of black hole evaporation and thereby connected issues (see [1] for a review and list of references).

Different approaches to the quantization of 2D dilaton gravities (mainly, string-inspired models) have been discussed in papers [2–9] (see also the references therein). Specifically, the covariant effective action and the one-loop renormalization of some specific models have been studied in Refs. [3,5,7].

In the present paper we shall obtain the covariant ef-

fective action corresponding to a very general, multiplicatively renormalizable [3,4] (in a generalized sense) model of 2D gravity with matter. Its action has the form

$$S = - \int d^2x \sqrt{g} \left[\frac{1}{2} Z(\Phi) g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + C(\Phi) R - \frac{i}{2} q(\Phi) \bar{\psi}_a \gamma^\lambda \partial_\lambda \psi_a + b(\Phi) (\bar{\psi}_a N_{ab} \psi_b)^2 - \frac{1}{2} f(\Phi) g^{\mu\nu} \partial_\mu \chi_i \partial_\nu \chi_i + V(\Phi, \chi) \right]. \quad (1)$$

It includes a dilaton field Φ , n Majorana fermions ψ_a interacting quartically via a symmetric constant matrix N_{ab} , and m real scalars χ_i . We shall also consider the (much more difficult) case in which this action contains 2D Dirac fermions. Notice that we have chosen the matter to interact with the dilaton via arbitrary functions.

This action describes and generalizes many well-known dilaton models. For instance, the celebrated bosonic string effective action corresponds to

$$Z(\Phi) = 8e^{-2\Phi}, \quad C(\Phi) = e^{-2\Phi}, \quad V(\Phi) = 4\lambda^2 e^{-2\Phi}, \quad (2)$$

$$q(\Phi) = b(\Phi) = 0, \quad f(\Phi) = 1.$$

On the other hand, in the absence of matter our action for

$$Z = 0, \quad C(\Phi) = \Phi, \quad V(\Phi) = \Lambda\Phi, \quad (3)$$

coincides with the Jackiw-Teitelboim action [9]. But

*On leave of absence from and permanent address: Department E.C.M., Faculty of Physics, Barcelona University, Diagonal 647, 08028 Barcelona, Spain. Electronic address: eli @ ebubecm1.bitnet

†Electronic address: odintsov @ ebubecm1.bitnet

other dilaton models are parametrized by the set $\{Z, C, q, b, f, V\}$. It goes without saying that one can also add gauge fields to the matter sector.

In principle, at the classical level the theory defined by the action (1) can be transformed into an equivalent theory whose corresponding action is more simple. Indeed, this can be done by choosing the field φ_1 as defined through the equation

$$Z^{1/2}(\Phi)\partial_\mu\Phi = \partial_\mu\varphi_1, \quad (4)$$

and by expressing Φ as $\Phi = \Phi(\varphi_1)$. Next, let us introduce a new field φ_2 via

$$c\varphi_2 = C(\Phi(\varphi_1)), \quad (5)$$

and write then $\Phi = \Phi(\varphi_2)$. After having done this, by making the transformation [6]

$$g_{\mu\nu} \longrightarrow e^{2\rho(\varphi_2)}\bar{g}_{\mu\nu}, \quad (6)$$

with a properly chosen $\rho(\varphi_2)$ [6], one can see that the theory (1) with the transformed metric (6) is classically equivalent to the more simple, particular case

$$Z = 1, \quad C = c\varphi_2, \quad V = e^{2\rho(\varphi_2)}V(\varphi_2, \chi), \quad f(\Phi) = f(\varphi_2), \quad (7)$$

$$b(\Phi) = b(\varphi_2)e^{2\rho(\varphi_2)}, \quad q(\Phi) = q(\varphi_2)e^{\rho(\varphi_2)}.$$

However, the model (7) [which, generally speaking, may be considered as a representative of the general class (1)] is still complicated enough. Moreover, it still includes arbitrary functions of the dilaton (now φ_2), as (1) does. Finally, the classical equivalence may be lost at the quantum level. For all these reasons, we choose to consider the quantum effective action corresponding to the more general theory (1).

As is well known, there exists some equivalence between the calculation of the effective action in 2D gravity and string calculations in the σ -model approach [10–12], because 2D gravity can also be presented as some particular $D = 2$ σ model. An approach based on the σ -model interpretation has been recently developed in Ref. [8], where it has been shown that in the conformal gauge the coupling constants of 2D dilaton gravity may be considered as initial data for covariant β -function equations which correspond to the σ -model interpretation. Some choices of these coupling constants may lead to new dilaton gravities. Our approach is in fact similar to the standard renormalization of quantum gravity and, as we will see, may also lead to some new models of 2D dilaton gravity defined by different conditions when compared with [8].

Being more specific, in this paper we shall construct the covariant effective action of the theory (1), study its one-loop renormalization, and discuss some thereby connected issues. The work is organized as follows. In the next section we describe in full detail the calculation of the one-loop covariant effective action in 2D dilaton gravity with Majorana spinors. This is, to our knowledge, the first example of such a kind of calculation in two dimensions. The inclusion of scalars is also discussed in that section. Section III is devoted to the computation of the

covariant effective action of dilaton, scalars, and Majorana spinors for quantum systems in classical spacetime. In Sec. IV we discuss the technical problems that appear in the derivation of the covariant effective action in 2D dilaton gravity with Dirac fermions. The solution of these problems is found, via reduction of the system to the previously discussed case of the theory of quantum dilaton gravity with Majorana spinors. In Sec. V the one-loop renormalization of quantum dilaton gravity with Majorana spinors is discussed. The conditions of multiplicative renormalizability are specified and some examples of multiplicatively renormalizable dilaton potentials are obtained. Finally, the conclusions of the paper are presented in Sec. VI. There is also a short appendix on the sine-Gordon model.

II. THE ONE-LOOP EFFECTIVE ACTION OF 2D QUANTUM GRAVITY WITH MATTER

In this section we will calculate the covariant effective action for the theory given by the action (1). For simplicity, we first set the scalars $\chi_i = 0$ and discuss dilaton-Majorana gravity only, i.e., the action

$$S = - \int d^2x \sqrt{g} \left[\frac{1}{2} Z(\Phi) g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + C(\Phi) R - \frac{i}{2} q(\Phi) \bar{\psi}_a \gamma^\lambda \partial_\lambda \psi_a + b(\Phi) (\bar{\psi}_a N_{ab} \psi_b)^2 + V(\Phi) \right]. \quad (8)$$

Our purpose is to calculate first the effective action for the theory (8), before proceeding with the general case.

Let us introduce some notation:

$$T^{\mu\nu} = -\frac{i}{4} (\bar{\psi}_a \gamma^\mu \nabla^\nu \psi_a + \bar{\psi}_a \gamma^\nu \nabla^\mu \psi_a), \quad T = T^\mu_\mu, \quad J = \bar{\psi}_a N_{ab} \psi_b. \quad (9)$$

Notice that $T_{\mu\nu}$ is *not* the stress tensor. We are going to use the formalism of the background field method, representing

$$\psi \longrightarrow \psi + \eta, \quad \Phi \longrightarrow \Phi + \varphi, \quad e_a^\mu \longrightarrow e_a^\mu + h_a^\mu, \quad (10)$$

where ψ , Φ , and e_a^μ are background fields. The action (8) will be expanded to second order in quantum fields. The Lorentz symmetry of the zweibein (e_a^μ) is “frustrated” by imposing the no-torsion condition $e_{a\mu} h_b^\mu - e_{b\mu} h_a^\mu = 0$ as a constraint: we insert the corresponding δ function into the path integral but do not exponentiate it. The corresponding ghost contribution, which is proportional to $\delta(0)$, may be discarded. This procedure fixes one out of four gauge parameters and leaves three to be undone by the metric variations. Thus, the variation of the zweibein h_a^μ is solved for the metric variation $h_{\mu\nu}$:

$$h_a^\mu = \frac{1}{2} h_\lambda^\mu e_a^\lambda + \frac{3}{8} h^{\mu\lambda} h_{\lambda\nu} e_a^\nu + \dots \quad (11)$$

That is, we will work with the variations of the metric rather than the zweibein. The quantum fields are arranged into a vector $\phi^i = \{\varphi; \bar{h}_{\mu\nu}; \eta_a\}$, where $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} g_{\mu\nu}$. The equations of motion with respect to the

background fields are

$$-\frac{\delta_R S}{\delta \psi} = 0 = iq\gamma^\lambda \nabla_\lambda \psi + i\frac{q'}{2}(\nabla_\lambda \Phi)(\gamma^\lambda \psi) - 4bJ(N\psi), \quad (12)$$

$$-\frac{\delta S}{\delta \Phi} = 0 = -Z(\Delta \Phi) - \frac{1}{2}Z'(\nabla^\lambda \Phi)(\nabla_\lambda \Phi) + C'R + V' + qT + bJ^2, \quad (13)$$

$$-g_{\mu\nu} \frac{\delta S}{\delta g_{\mu\nu}} = 0 = -\Delta C + V + \frac{1}{2}qT + bJ^2. \quad (14)$$

Here δ_R denotes the right functional derivative.

The covariant gauge-fixing condition is

$$\begin{aligned} S_{\text{gf}} &= -\frac{1}{2} \int c_{\mu\nu} \chi^\mu \chi^\nu, \\ \chi^\mu &= -\nabla_\nu \bar{h}^{\mu\nu} + \frac{C'}{C} \nabla^\mu \varphi + \frac{iq}{4C} \bar{\psi} \gamma^\mu \eta, \\ c_{\mu\nu} &= -C \sqrt{g} g_{\mu\nu}, \end{aligned} \quad (15)$$

and the total quadratic contribution to the action takes the form

$$S_{\text{tot}}^{(2)} = -\frac{1}{2} \int d^2 x \sqrt{g} \phi^i \hat{H}_{ij} \phi^j, \quad (16)$$

where \hat{H} is the second-order minimal operator (though the coefficient matrix multiplying the Laplacian is not invertible, since the fermionic fields are of first order). Following Ref. [13], we introduce another operator

$$\hat{\Omega}_{ij} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & P_{\mu\nu, \mu'\nu'} & 0 \\ 0 & 0 & 0 & -i\delta_{ab} \gamma^\sigma \nabla_\sigma \end{pmatrix} \quad (17)$$

and define $\hat{\mathcal{H}} = \hat{H}\hat{\Omega}$ so that the one-loop effective action in terms of supertraces becomes

$$\Gamma = \frac{i}{2} \text{Tr} \ln \hat{\mathcal{H}} - \frac{i}{4} \text{Tr} \ln \hat{\Omega}^2 - i \text{Tr} \ln \hat{\mathcal{M}}, \quad (18)$$

$$\begin{aligned} \Gamma_{2-\text{div}} = \frac{i}{2} \text{Tr} \ln \hat{\mathcal{H}} &= -\frac{1}{2\epsilon} \int d^2 x \sqrt{g} \left\{ \frac{8-n}{6} R + \frac{2}{C} V + \frac{2}{C'} V' + \left(\frac{2C'}{C} - \frac{Z}{C'} \right) (\Delta \Phi) - \left(\frac{3C'^2}{2C^2} + \frac{C''Z}{C'^2} \right) (\nabla^\lambda \Phi)(\nabla_\lambda \Phi) \right. \\ &\quad \left. + \left(\frac{3qZ}{4C'^2} - \frac{q}{4C} - \frac{q'}{C'} \right) T + \left(\frac{4bZ}{C'^2} - \frac{4b}{C} - \frac{2b'}{C'} - \frac{8bq'}{qC'} + \frac{16b^2}{q^2} (N_{ab} N_{ba}) \right) J^2 - \frac{32b^2}{q^2} J(\bar{\psi} N^3 \psi) \right\}, \end{aligned} \quad (22)$$

where we have used the Majorana identity $\bar{\psi} \gamma^\lambda N^k \psi = 0$ for all k .

The next step is to subtract the squared contribution, i.e., the second term in Eq. (18):

$$\begin{aligned} -\frac{i}{4} \text{Tr} \ln \hat{\Omega}^2 \Big|_{\text{div}} &= \frac{i}{4} \text{Tr} \ln \left(-\Delta + \frac{R}{4} \hat{1} \right) \Big|_{\text{div}} \\ &= -\frac{1}{2\epsilon} \int d^2 x \sqrt{g} \frac{n}{12} R. \end{aligned} \quad (23)$$

To complete the calculation one has to consider the

where, as usual, $\hat{\mathcal{H}} = -\hat{K}\Delta + \hat{L}^\lambda \nabla_\lambda + \hat{P}$, and the last term in (18) is the ghost operator corresponding via (15) to diffeomorphisms.

Now,

$$\hat{K}_{ij} = \begin{pmatrix} Z - C'^2/C & C'/2 & 0 & q'\bar{\psi} \\ C'/2 & 0 & 0 & (q/4)\bar{\psi} \\ 0 & 0 & -\frac{C}{2} P^{\mu\nu, \alpha\beta} & 0 \\ 0 & 0 & 0 & q\hat{1} \end{pmatrix} \quad (19)$$

and the other (essential) matrix elements¹ are defined by the second variation of the classical action. To calculate the divergent part we use the same technique as in Ref. [3], representing

$$\hat{\mathcal{H}} = -\hat{K} \left(\hat{1}\Delta + 2\hat{E}^\lambda \nabla_\lambda + \hat{\Pi} \right), \quad (20)$$

$$\hat{E}^\lambda = -\frac{1}{2} \hat{K}^{-1} \hat{L}^\lambda, \quad \hat{\Pi} = -\hat{K}^{-1} \hat{P}.$$

After that, the standard algorithm can be used: namely,

$$\begin{aligned} \frac{i}{2} \text{Tr} \ln \hat{\mathcal{H}} &= \frac{i}{2} \text{Tr} \ln \left(\hat{1}\Delta + 2\hat{E}^\lambda \nabla_\lambda + \hat{\Pi} \right) \Big|_{\text{div}} \\ &= \frac{1}{2\epsilon} \int d^2 x \sqrt{g} \text{Tr} \left(\hat{\Pi} + \frac{R}{6} \hat{1} - \hat{E}^\lambda \hat{E}_\lambda \right. \\ &\quad \left. - \nabla_\lambda \hat{E}^\lambda \right), \end{aligned} \quad (21)$$

where $\epsilon = 2\pi(n-2)$. The matrices \hat{E}^λ and $\hat{\Pi}$ can be defined in the same way as in Refs. [5]. We do not write them explicitly, since some of the components have very long expressions. To obtain the divergent part, $\Gamma_{2-\text{div}}$, we have to evaluate the functional (super)traces of the matrices \hat{E}^λ and $\hat{\Pi}$ according to (21). After some tedious algebra we arrive at

ghost operator

$$\hat{\mathcal{M}}_\nu^\mu \equiv \frac{\delta_R \chi^\mu}{\delta \phi^j} \nabla_\nu^j, \quad (24)$$

where again δ_R stands for the right functional derivative and the generators of the diffeomorphisms are

¹In the nonspinor sectors the 't Hooft-Veltman doubling procedure [14] is assumed.

$$\nabla_\nu^\varphi = -(\nabla_\nu \Phi), \quad \nabla_\nu^{\bar{h}\rho\sigma} = g_{\rho\sigma} \nabla_\nu - g_{\nu\rho} \nabla_\sigma - g_{\nu\sigma} \nabla_\rho, \quad (25)$$

$$\nabla_\nu^\eta = -(\nabla_\nu \psi) + \frac{1}{8} [\gamma_\nu, \gamma_\lambda] \psi \nabla^\lambda.$$

Explicitly,

$$\begin{aligned} \widehat{\mathcal{M}}_\nu^\mu &= g_\nu^\mu \Delta - \frac{C'}{C} (\nabla_\nu \Phi) \nabla^\mu - \frac{C'}{C} (\nabla^\mu \nabla_\nu \Phi) \\ &\quad + R_\nu^\mu - i \frac{q}{4C} (\bar{\psi} \gamma^\mu \nabla_\nu \psi), \end{aligned} \quad (26)$$

which leads to

$$\begin{aligned} \Gamma_{\text{gh-div}} &= -\frac{1}{2\epsilon} \int d^2x \sqrt{g} \left\{ \frac{8}{3} R - \frac{C'}{C} (\Delta \Phi) \right. \\ &\quad \left. + \left(\frac{C''}{C} - \frac{3C'^2}{2C^2} \right) (\nabla^\lambda \Phi) (\nabla_\lambda \Phi) + \frac{q}{C} T \right\}. \end{aligned} \quad (27)$$

The final answer is

$$\begin{aligned} \Gamma_{\text{div}} &= -\frac{1}{2\epsilon} \int d^2x \sqrt{g} \left\{ \frac{48-n}{12} R + \frac{2}{C} V + \frac{2}{C'} V' + \left(\frac{C'}{C} - \frac{Z}{C'} \right) (\Delta \Phi) + \left(\frac{C''}{C} - 3 \frac{C'^2}{C^2} - \frac{C''Z}{C'^2} \right) (\nabla^\lambda \Phi) (\nabla_\lambda \Phi) \right. \\ &\quad \left. + \left(\frac{3qZ}{4C'^2} + \frac{3q}{4C} - \frac{q'}{C'} \right) T + \left(\frac{4bZ}{C'^2} - \frac{4b}{C} - \frac{2b'}{C'} - \frac{8bq'}{qC'} + \frac{16b^2}{q^2} (N_{ab} N_{ba}) \right) J^2 - \frac{32b^2}{q^2} J(\bar{\psi} N^3 \psi) \right\}. \end{aligned} \quad (28)$$

Notice that all surface terms have been kept in Eq. (28).

Having performed the above calculation, it is not difficult now to take into account the scalar fields χ_i , and to repeat it for the effective action (1). In fact, the introduction of the scalars leads to minor changes. All matrices become 5×5 (rather than 4×4): in the background field notation $\chi_i \rightarrow \chi_i + \sigma_i$ the quantum fields are $\phi^i = \{\varphi; h; \bar{h}_{\mu\nu}; \eta_a; \sigma_i\}$. Scalars do not spoil the minimality of the second functional derivative operator, so that the gauge condition may be left untouched. As a consequence, neither ghost terms nor the squared contributions change, and only a few matrix elements of \widehat{L}^λ and \widehat{P} do. The 't Hooft and Veltman procedure [14] is

assumed to have been implemented whenever necessary, and we use the notation

$$V' \equiv \frac{\delta V}{\delta \Phi}, \quad V_{,i} \equiv \frac{\delta V}{\delta \chi_i}. \quad (29)$$

The matrix \widehat{K} becomes

$$\widehat{K} = \begin{pmatrix} Z - C'^2/C & C'/2 & 0 & q'\bar{\psi} & 0 \\ C'/2 & 0 & 0 & (q/4)\bar{\psi} & 0 \\ 0 & 0 & -(C/2)P^{\mu\nu,\alpha\beta} & 0 & 0 \\ 0 & 0 & 0 & q\hat{1} & 0 \\ 0 & 0 & 0 & 0 & -f\hat{1} \end{pmatrix}. \quad (30)$$

Repeating the above procedure we may calculate the extra terms which appear in the effective action as a result of the scalar fields contribution:

$$\Gamma_{2\text{-div extra terms}} = -\frac{1}{2\epsilon} \int d^2x \sqrt{g} \left\{ -\frac{m}{6} R - \frac{1}{f} V_{,ii} + \frac{mf'}{2f} \Delta \Phi + \left(\frac{mf''}{2f} - \frac{mf'^2}{4f^2} \right) (\nabla^\lambda \Phi) (\nabla_\lambda \Phi) \right\}. \quad (31)$$

Thus, the complete one-loop divergences for the theory (1) become

$$\begin{aligned} \Gamma_{\text{div}} &= -\frac{1}{2\epsilon} \int d^2x \sqrt{g} \left\{ \frac{48-n+2m}{12} R + \frac{2}{C} V + \frac{2}{C'} V' - \frac{V_{,ii}}{f} + \left(\frac{C''}{C} - \frac{3C'^2}{C^2} - \frac{C''Z}{C'^2} - \frac{mf'^2}{4f^2} + \frac{mf''}{2f} \right) (\nabla^\lambda \Phi) (\nabla_\lambda \Phi) \right. \\ &\quad \left. + \left(\frac{C'}{C} - \frac{Z}{C'} + \frac{mf'}{2f} \right) \Delta \Phi + \left(\frac{3qZ}{4C'^2} + \frac{3q}{4C} - \frac{q'}{C'} \right) T + \left(\frac{4bZ}{C'^2} - \frac{4b}{C} - \frac{2b'}{C'} - \frac{8bq'}{qC'} + \frac{16b^2}{q^2} (N_{ab} N_{ba}) \right) J^2 \right. \\ &\quad \left. - \frac{32b^2}{q^2} J(\bar{\psi} N^3 \psi) \right\}. \end{aligned} \quad (32)$$

This is the main result of the present section — the one-loop effective action for 2D dilaton gravity with matter. Furthermore, one can easily generalize this expression to the case when Maxwell fields are added, namely, when one considers action (1) plus the Maxwell action:

$$S = - \int d^2x \sqrt{g} \left[\dots + \frac{1}{4} f_1(\Phi) F_{\mu\nu}^2 \right]. \quad (33)$$

With the background vs quantum field separation $A_\mu \rightarrow$

$A_\mu + Q_\mu$, in the Lorentz gauge,

$$S_{\text{Lorentz}} = - \int d^2x \sqrt{g} f_1(\Phi)(\nabla_\mu Q_\mu)^2, \tag{34}$$

the extra contributions are known to split into the F^2 terms and total divergences (see the second and third references [5]), which may be calculated independently, to yield

$$\Gamma_{\text{max,div}}^{\text{extra}} = -\frac{1}{2\epsilon} \int d^2x \sqrt{g} \left[R + \left(\frac{f_1'}{2C'} - \frac{f_1}{2C} \right) F_{\mu\nu}^2 + \frac{f_1'}{f_1} \Delta\Phi + \left(\frac{f_1''}{f_1} - \frac{f_1'^2}{f_1^2} \right) (\nabla^\lambda\Phi)(\nabla_\lambda\Phi) \right]. \tag{35}$$

Thus, the total divergent contribution to the one-loop effective action of the theory (1) plus the Maxwell terms (33) is given by

$$\Gamma_{\text{div}} = -\frac{1}{2\epsilon} \int d^2x \sqrt{g} \left\{ \frac{-n-2m+60}{12} R + \frac{2}{C} V + \frac{2}{C'} V' - \frac{V_{,ii}}{f} + \left(\frac{f_1'}{2C'} - \frac{f_1}{2C} \right) F_{\mu\nu}^2 + \left(\frac{C''}{C} - \frac{3C'^2}{C^2} - \frac{C''Z}{C'^2} - \frac{mf'^2}{4f^2} + \frac{mf''}{2f} + \frac{f_1''}{f_1} - \frac{f_1'^2}{f_1^2} \right) (\nabla^\lambda\Phi)(\nabla_\lambda\Phi) + \left(\frac{C'}{C} - \frac{Z}{C'} + \frac{mf'}{2f} + \frac{f_1'}{f_1} \right) \Delta\Phi + \left(\frac{3qZ}{4C'^2} + \frac{3q}{4C} - \frac{q'}{C'} \right) T + \left(\frac{4bC}{C'^2} - \frac{4b}{C} - \frac{2b'}{C'} - \frac{8bq'}{qC'} + \frac{16b^2}{q^2} (N_{ab}N_{ba}) \right) J^2 - \frac{32b^2}{q^2} J(\bar{\psi}N^3\psi) \right\}. \tag{36}$$

This gives the divergences of the covariant effective action (for a recent discussion of the covariant effective action formalism, see [15]; a general review is given in [16]). The renormalization of the theory (1) using the one-loop effective action (32) will be discussed in the next sections.

III. THE ONE-LOOP EFFECTIVE ACTION IN CURVED SPACETIME

In this section we will calculate the one-loop effective action for the theory (1) in the case when the gravitational field is a classical one, but the dilaton and the rest of the matter fields are quantized. Such a calculation is much simpler than the one carried out in the previous section, since there are no gauge-fixing terms and corresponding ghosts.

The effective action has the form

$$\Gamma = \frac{i}{2} \text{Tr} \ln \hat{\mathcal{H}} - \frac{i}{4} \text{Tr} \ln \hat{\Omega}^2, \tag{37}$$

where

$$\hat{\mathcal{H}} = -\hat{K}\Delta + \hat{L}^\lambda\nabla_\lambda + \hat{P}, \tag{38}$$

and the quantum fields are arranged in the vector form

$$\{\varphi'; \eta'_a; \sigma_i\}.$$

The basic matrices which enter in the operator of small disturbances are

$$\hat{\Omega} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -i\gamma^\lambda\nabla_\lambda\hat{1} & 0 \\ 0 & 0 & \hat{1} \end{pmatrix}, \tag{39}$$

$$\hat{K} = \begin{pmatrix} Z & q'\bar{\psi} & 0 \\ 0 & q\hat{1} & 0 \\ 0 & 0 & -f\hat{1} \end{pmatrix},$$

and the \hat{L}^λ and \hat{P} can be calculated easily. The divergent part of the effective action is expressed in terms of the traces of matrices similar to the ones discussed in Sec. II. We then get

$$\Gamma_{\text{div}} = -\frac{1}{2\epsilon} \int d^2x \sqrt{g} \left[\frac{n}{12} R + \text{Tr}(\hat{E}^\lambda\hat{E}_\lambda) + \text{Tr}(\nabla^\lambda\hat{E}_\lambda) - \text{Tr} \left(\hat{\Pi} + \frac{R}{6}\hat{1} \right) \right], \tag{40}$$

where the first term comes from the matrix $\hat{\Omega}$ squared. Calculating these functional traces, we arrive at

$$\Gamma_{\text{div}} = -\frac{1}{2\epsilon} \int d^2x \sqrt{g} \left\{ \left(\frac{C''}{Z} - \frac{n+2m+2}{12} \right) R + \frac{V''}{Z} - \frac{V_{,ii}}{f} + \left(\frac{f'^2}{2fZ} - \frac{f''}{2Z} \right) (\nabla^\lambda\chi)(\nabla_\lambda\chi) + \left(\frac{mf''}{2f} - \frac{mf'^2}{4f^2} - \frac{Z'^2}{4Z^2} \right) (\nabla^\lambda\Phi)(\nabla_\lambda\Phi) + \left(\frac{mf'}{2f} - \frac{Z'}{2Z} \right) \Delta\Phi + \left(\frac{q''}{Z} - 2\frac{q'^2}{qZ} \right) T + \left(\frac{b''}{Z} - \frac{8bq'^2}{qZ} + \frac{16b^2}{q^2} (N_{ab}N_{ba}) \right) J^2 - \frac{32b^2}{q^2} J(\bar{\psi}N^3\psi) \right\}. \tag{41}$$

Expression (41) gives the one-loop effective action of the system composed of quantized dilaton, scalars, and Majorana fermions in an external gravitational field. It is interesting to note that, contrary to what happens when the gravitational field itself is quantized (Sec. II), we get a nontrivial renormalization of the $C(\Phi)R$ term and $f(\Phi)$ in the kinetic term of scalars.

IV. THE ONE-LOOP EFFECTIVE ACTION FOR DILATON GRAVITY WITH DIRAC FERMIONS

In this section we will be interested in the covariant effective action of dilaton gravity (DG) with Dirac fermions. We will show that some technical problems of the covariant formalism may actually be solved in this case, after which the calculation will be performed precisely as in Sec. II for Majorana fermions with dilaton gravity.

Let us start from the Gross-Neveu model of n Dirac fermions Ψ . The action of such a model interacting with dilaton gravity is

$$S = \int d^2x \sqrt{g} \left[\frac{i}{2} q(\Phi) \bar{\Psi} \gamma^\lambda \overleftrightarrow{\nabla}_\lambda \Psi - b(\Phi) (\bar{\Psi} \Psi)^2 \right] + S^{\text{DG}}, \quad (42)$$

where

$$S^{\text{DG}} = \int d^2x \sqrt{g} \left[\frac{1}{2} Z(\Phi) g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + C(\Phi) R + V(\Phi) \right]. \quad (43)$$

The functions $b(\Phi)$ and $q(\Phi)$ are supposed to be smooth enough. (Notice that recently a semiclassical approach to the Gross-Neveu model with Jackiw-Teitelboim dilaton gravity has been discussed in Ref. [17].)

The equations of motion corresponding to the action yield

$$\begin{aligned} -iq \bar{\Psi} \gamma^\lambda \nabla_\lambda \Psi + \frac{i}{2} q (\nabla_\lambda J^\lambda) + 2bJ^2 - 2g_{\mu\nu} \frac{\delta S^{\text{DG}}}{\delta g_{\mu\nu}} &= 0, \\ -iq \bar{\Psi} \gamma^\lambda \nabla_\lambda \Psi - \frac{i}{2} q' (\nabla_\lambda \Phi) J^\lambda + 2bJ^2 &= 0, \\ iq' \bar{\Psi} \gamma^\lambda \nabla_\lambda \Psi + \frac{i}{2} q'' (\nabla_\lambda \Phi) J^\lambda - b' J^2 + \frac{\delta S^{\text{DG}}}{\delta \Phi} &= 0. \end{aligned} \quad (44)$$

Here we denoted the currents $J = \bar{\Psi} \Psi$ and $J^\lambda = \bar{\Psi} \gamma^\lambda \Psi$.

To compute the one-loop effective action we find the terms quadratic in the quantum fields ($\Psi \rightarrow \Psi + \chi$, etc.). We write down only a few of the most important terms:

$$S^{(2)} = \frac{1}{2} \int d^2x \sqrt{g} \left[iq \bar{\chi} \gamma^\lambda \overleftrightarrow{\nabla}_\lambda \chi - iq \bar{h}_{\mu\nu} \bar{\Psi} \gamma^\mu \nabla^\nu \chi - 2b(\bar{\Psi} \chi)(\bar{\Psi} \chi) + \dots \right]. \quad (45)$$

The last two terms give rise to a nonminimal operator after the fermionic squaring $\bar{\chi} \rightarrow \bar{\chi}$, $\chi \rightarrow -i\gamma^\lambda \nabla_\lambda \chi$. One can choose the gauge fixing condition to cancel the second term in Eq. (45), as was done in [13], but the last term remains and there is obviously no chance of getting rid of it. This problem always appears in the four-Fermi theories but so far it has not been made public since the four-Fermi terms are not renormalizable on index in the spaces with $d \geq 3$.

The problem we tackled never arises in the Majorana

case since with the help of the Majorana transposition rules one gets

$$(\bar{\Psi} \chi)(\bar{\Psi} \chi) = (\bar{\chi} \Psi)(\bar{\Psi} \chi). \quad (46)$$

This could hardly solve all the problems one encounters in higher spacetime dimensions, but in $d = 2$ it allows us to surmount the problem of the nonminimality.

Let us introduce two subsets of the Majorana real-valued fields $\psi_{1,2}$ according to the rule

$$\Psi = \frac{\psi_1 + i\psi_2}{\sqrt{2}}, \quad \bar{\Psi} = \frac{\bar{\psi}_1 - i\bar{\psi}_2}{\sqrt{2}}. \quad (47)$$

The existence of the Majorana representation in two spacetime dimensions ensures that the fields may be taken as real, so that $\bar{\psi}$ is the charge conjugate of ψ , as it should be. Using the Majorana properties, one has

$$S = \int d^2x \sqrt{g} \left[\frac{i}{2} q(\Phi) \bar{\psi}_1 \gamma^\lambda \nabla_\lambda \psi_1 + \frac{i}{2} q(\Phi) \bar{\psi}_2 \gamma^\lambda \nabla_\lambda \psi_2 - \frac{1}{4} b(\Phi) (\bar{\psi}_1 \psi_1 + \bar{\psi}_2 \psi_2)^2 \right] + \text{dilaton gravity}, \quad (48)$$

where no surface term has been dropped. Notice also that the covariant derivatives here may be substituted with the ordinary ones, cf. Eq. (8) above, since the spin connection drops out of the Majorana bilinears in two dimensions. The latter circumstance makes the calculation of the one-loop effective action much easier since it is not necessary to vary the spin connection.

Further, it is profitable to arrange the two Majorana fields into a larger multiplet $\{\psi_a\}_{a=1}^{2n}$ defined as

$$\psi \equiv \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}. \quad (49)$$

Clearly, this is a Majorana field as well, with all its inherent properties. Thus we have

$$S = \int d^2x \sqrt{g} \left[\frac{i}{2} q(\Phi) \bar{\psi} \gamma^\lambda \partial_\lambda \psi - b(\Phi) (\bar{\psi} N \psi)^2 \right] + \text{dilaton gravity}, \quad (50)$$

where $N_{ab} = \frac{1}{2} \delta_{ab}$.

The scalar current may be recast in the Majorana form as

$$J \equiv \bar{\Psi} \Psi = \bar{\psi} N \psi. \quad (51)$$

Also, let us introduce a quantity

$$\begin{aligned} T^{\mu\nu} &\equiv -\frac{i}{4} \left(\bar{\Psi} \gamma^\mu \overleftrightarrow{\nabla}^\nu \Psi + \bar{\Psi} \gamma^\nu \overleftrightarrow{\nabla}^\mu \Psi \right), \\ T &\equiv T^\nu_\nu = -\frac{i}{2} \bar{\Psi} \gamma^\lambda \overleftrightarrow{\nabla}_\lambda \Psi, \end{aligned} \quad (52)$$

then by the same reasoning as before we get

$$\begin{aligned} T^{\mu\nu} &= -\frac{i}{2} (\bar{\psi}\gamma^\mu\nabla^\nu\psi + \bar{\psi}\gamma^\nu\nabla^\mu\psi) , \\ T &= -i\bar{\psi}\gamma^\lambda\nabla_\lambda\psi . \end{aligned} \quad (53)$$

$$\begin{aligned} \Gamma_{\text{div}} &= -\frac{1}{2\epsilon} \int d^2x \sqrt{g} \left\{ \frac{24-n}{6} R + \frac{2}{C} V + \frac{2}{C'} V' + \left(\frac{C'}{C} - \frac{Z}{C'} \right) (\Delta\Phi) + \left(\frac{C''}{C} - 3\frac{C'^2}{C^2} - \frac{C''Z}{C'^2} \right) (\nabla^\lambda\Phi)(\nabla_\lambda\Phi) \right. \\ &\quad \left. + \left(\frac{3qZ}{4C'^2} + \frac{3q}{4C} - \frac{q'}{C'} \right) T + \left(\frac{8b^2}{q^2} (n-1) + \frac{4bZ}{C'^2} - \frac{4b}{C} - \frac{2b'}{C'} - \frac{8bq'}{qC'} \right) J^2 \right\} . \end{aligned} \quad (54)$$

Let us now try to generalize our result to the most general case of the four-Fermi interaction $(\bar{\Psi}A\Psi)^2$, the matrix A being an arbitrary combination of the Dirac algebra elements (some examples are: $\gamma^\nu\gamma_5$, $\gamma^\mu\gamma^\nu\gamma^\lambda$, and so on). The fact is that all these A 's fall into three cases since the two-dimensional Dirac algebra basis consists of just three elements: 1 , γ^μ , and γ_5 . Thus the most general situation is

$$\begin{aligned} S_{\text{quartic}} &= -\int d^2x \sqrt{g} \left[b_1(\Phi)J^2 + b_2(\Phi)J_5^2 \right. \\ &\quad \left. + b_3(\Phi)J^\mu J_\mu \right] , \end{aligned} \quad (55)$$

with arbitrary smooth functions $b_1(\Phi)$, $b_2(\Phi)$, and $b_3(\Phi)$. The currents here are defined as usual:

$$J = \bar{\Psi}\Psi , \quad J_5 = \bar{\Psi}\gamma_5\Psi , \quad J^\mu = \bar{\Psi}\gamma^\mu\Psi . \quad (56)$$

However, things may be simplified even further, with the use of the two-dimensional Fierz identity which yields

$$J^\mu J_\mu = J_5^2 - J^2 . \quad (57)$$

Hence we can eliminate one of the three structures and, technically, the simplest choice is to set $b_3 = 0$. So, what one has to do is to complete the action (42) with the axial term

$$S_{\text{ax}} = -\int d^2x \sqrt{g} \left[a(\Phi)(\bar{\Psi}\gamma_5\Psi)^2 \right] . \quad (58)$$

Let us turn our attention to Eq. (57). At first glance, it may seem a little strange. The reason is that its left-hand side is chiral invariant while both terms on the right are not—however, the noninvariant contributions cancel in pairs. Thus we note that the total action $S + S_{\text{ax}}$ is only chiral invariant if $a(\Phi) \equiv -b(\Phi)$, and so are the one-loop divergences. Hence the extra divergences to arise [cf. Eq. (54) above] are

$$\begin{aligned} \Gamma_{\text{ax,div}} &= -\frac{1}{2\epsilon} \int d^2x \sqrt{g} \left(\frac{8a^2}{q^2} (1-n) + \frac{4aZ}{C'^2} \right. \\ &\quad \left. - \frac{4a}{C} - \frac{2a'}{C'} - \frac{8aq'}{qC'} \right) J_5^2 + \dots , \end{aligned} \quad (59)$$

Now the problem reduces to the old one: that for the Majorana case, with only minor changes. The divergent part of the one-loop effective action for the Gross-Neveu model with the Dirac fields becomes (we use for this calculation the results of Sec. II)

where the dots stand for possible mixed terms proportional to ab . We will compute these terms at the end of the section.

First, go to the Majorana field multiplet ψ so that

$$J_5 = i\bar{\psi}\gamma_5 M\psi , \quad M_{ab} = \begin{pmatrix} 0 & 1/2 \\ -1/2 & 0 \end{pmatrix} , \quad (60)$$

and

$$J^\mu = i\bar{\psi}\gamma^\mu M\psi . \quad (61)$$

Note also that

$$\bar{\psi}M\psi = 0 , \quad \bar{\psi}M^2\psi = -\frac{1}{2}J^2 . \quad (62)$$

Second, expand the action

$$S_{\text{ax}} = \int d^2x \sqrt{g} a(\Phi) (\bar{\psi}\gamma_5 M\psi)^2$$

in quantum fields $\psi \rightarrow \psi + \eta$ and note that the gauge-fixing terms (and thus the ghosts) acquire no extra terms. After the fermion operator squaring we obtain some *additive* corrections to elements of the matrices \hat{E}^λ and $\hat{\Pi}$. Therefore, it is the $\text{Tr}(\hat{E}^\lambda\hat{E}_\lambda)$ that gives the desired mixed contribution.

A little thought suggests that the only correction that leads to ab terms is

$$\delta(\hat{E}^\lambda)_4^4 = -\frac{2a}{q} J_5(M\gamma_5\gamma^\lambda) - \frac{4ia}{q} (\gamma_5 M\psi)(\bar{\psi}M\gamma_5\gamma^\lambda) \quad (63)$$

so that the desired mixed term contribution is found to be

$$\Gamma_{\text{ax,div}}^{\text{mixed terms}} = -\frac{1}{2\epsilon} \int d^2x \sqrt{g} \frac{8ab}{q^2} (J_5^2 - J^2 - J^\mu J_\mu) , \quad (64)$$

which is zero by virtue of Eq. (57). The final answer is

$$\begin{aligned} \Gamma_{\text{div}} = & -\frac{1}{2\epsilon} \int d^2x \sqrt{g} \left\{ \frac{24-n}{6} R + \frac{2}{C} V + \frac{2}{C'} V' + \left(\frac{C'}{C} - \frac{Z}{C'} \right) (\Delta\Phi) + \left(\frac{C''}{C} - 3\frac{C'^2}{C^2} - \frac{C''Z}{C'^2} \right) (\nabla^\lambda\Phi)(\nabla_\lambda\Phi) \right. \\ & + \left(\frac{3qZ}{4C'^2} + \frac{3q}{4C} - \frac{q'}{C'} \right) T + \left(\frac{8b^2}{q^2} (n-1) + \frac{4bZ}{C'^2} - \frac{4b}{C} - \frac{2b'}{C'} - \frac{8bq'}{qC'} \right) J^2 \\ & \left. + \left(\frac{8a^2}{q^2} (1-n) + \frac{4aZ}{C'^2} - \frac{4a}{C} - \frac{2a'}{C'} - \frac{8aq'}{qC'} \right) J_5^2 \right\}. \end{aligned} \quad (65)$$

Thus, we have calculated the one-loop divergences of the covariant effective action in 2D dilaton gravity within the most general four-fermionic theory described by Dirac fermions. An interesting remark is that the renormalization of all fermionic terms in the action is given by the same term (and the same generalized coupling constant). For example, if $b(\Phi) = 0$ in (42), then the term J^2 is absent in (65).

V. THE ONE-LOOP RENORMALIZATION

In the previous sections we have calculated the divergences of the one-loop covariant effective action for 2D dilaton gravity interacting via various kinds of fermionic matter. Let us here discuss the issue of renormalization, to one-loop order. Without loss of generality, we will restrict ourselves to the case of 2D dilaton gravity with Majorana spinors, viz. Eq. (28). By adding to the classical action (8) the corresponding counterterms (Γ_{div} with

opposite sign), one obtains the one-loop renormalized effective action.

Choosing the renormalization of the metric tensor in the form

$$g_{\mu\nu} = \exp \left\{ \frac{1}{\epsilon} \left[\frac{1}{C(\Phi)} + \frac{Z(\Phi)}{2C'^2(\Phi)} \right] \right\} g_{\mu\nu}^R \quad (66)$$

(this choice absorbs all the divergences of the dilaton kinetic term), one can obtain the renormalized effective action as (for simplicity, we drop the superscript “R” off $g_{\mu\nu}$ and R)

$$\begin{aligned} S_R = & - \int d^2x \sqrt{g} \left\{ \frac{1}{2} Z g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + CR + V + \frac{1}{\epsilon} \left(-\frac{V'}{C'} + \frac{ZV}{2C'^2} \right) + T \left[q + \frac{1}{\epsilon} \left(\frac{q}{8C} - \frac{Zq}{8C'^2} + \frac{q'}{2C'} \right) \right] \right. \\ & \left. + J^2 \left[b + \frac{1}{\epsilon} \left(\frac{3b}{C} - \frac{3bZ}{4C'^2} + \frac{b'}{C'} + \frac{4bq'}{qC'} - \frac{8b^2(n-2)}{q^2} \right) \right] \right\}, \end{aligned} \quad (67)$$

where we choose $N_{ab} = \delta_{ab}$.

Now, the conditions of multiplicative renormalizability of the theory in the usual sense have the form

$$\begin{aligned} -\frac{V'}{C'} + \frac{ZV}{2C'^2} &= a_1 V, \\ \frac{q}{8C} - \frac{Zq}{8C'^2} + \frac{q'}{2C'} &= a_2 q, \quad (68) \\ \frac{3b}{C} - \frac{3bZ}{4C'^2} + \frac{b'}{C'} + \frac{4bq'}{qC'} - \frac{8b^2(n-2)}{q^2} &= a_3 b, \end{aligned}$$

where a_1, a_2 and a_3 are arbitrary constants. These conditions restrict the form of the functions under discussion.

Some sets of solutions of Eqs. (68) can be obtained explicitly. The simplest choice in the gravity sector is

$$Z = 1, \quad C(\Phi) = C_1 \Phi, \quad V = 0, \quad (69)$$

where C_1 is an arbitrary constant. Choosing also $n = 2$, we get the following family of renormalizable potentials:

$$q(\Phi) = q_1 \Phi^{-1/4} e^{\alpha_1 \Phi}, \quad b(\Phi) = b_1 \Phi^{-2} e^{\alpha_2 \Phi}, \quad (70)$$

where q_1, α_1 and b_1, α_2 are coupling constants. The renormalization of these coupling constants follows as:

$$q_1^R = q_1 \left[1 + \frac{1}{\epsilon} \left(\frac{\alpha_1}{2C_1} - \frac{1}{8C_1^2} \right) \right], \quad (71)$$

$$b_1^R = b_1 \left[1 + \frac{1}{\epsilon} \left(\frac{\alpha_2}{C_1} + \frac{4\alpha_1}{C_1} - \frac{3}{4C_1^2} \right) \right],$$

and α_1 and α_2 do not get renormalized in the one-loop approximation. It is interesting to notice that for $\alpha_1 = 1/(4C_1)$ and $\alpha_2 = -1/(4C_1)$, the coupling constants q_1 and b_1 do not get renormalized in the one-loop approach either.

For $n \geq 2$ these multiplicatively renormalizable potentials look like

$$q(\Phi) = q_1 \Phi^{-1/4} e^{\alpha_1 \Phi}, \quad b(\Phi) = b_1 \Phi^{-3/2} e^{2\alpha_1 \Phi}, \quad (72)$$

but now C_1 is fixed by $(12a_2 - a_3)C_1 + 3/(4C_1) = 0$ and $q_1/b_1 = 16(n-2)C_1$. Another interesting choice is

$$Z = e^{d_1 \Phi}, \quad C = e^{d_1 \Phi}, \quad V = 0. \quad (73)$$

We find, in this case (for $n = 2$),

$$q = q_1 \exp \left[2a_2 e^{d_1 \Phi} + \frac{1}{4} \left(\frac{1}{d_1} - d_1 \right) \Phi \right],$$

$$b = b_1 \exp \left[(a_3 - 8a_2) e^{d_1 \Phi} - \left(\frac{1}{4d_1} + 2d_1 \right) \Phi \right]. \quad (74)$$

$$S_R = - \int d^2x \sqrt{g} \left\{ \frac{1}{2} Z g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + CR + V - \frac{1}{2\epsilon} \left(\frac{V''}{Z} + \frac{Z''}{2Z} - \frac{3Z'^2}{4Z^2} \right) \right. \\ \left. + T \left[q - \frac{1}{2\epsilon} \left(\frac{q''}{Z} - \frac{2q'^2}{qZ} \right) \right] + J^2 \left[b - \frac{1}{2\epsilon} \left(\frac{b''}{Z} - \frac{bq'^2}{qZ} + \frac{16b^2(n-2)}{q^2} \right) \right] \right\}. \quad (75)$$

As we see, the conditions of multiplicative renormalizability look completely different from (68):

$$\frac{V''}{Z} = a_1 V, \quad \frac{Z''}{2Z} - \frac{3Z'^2}{4Z^2} = a_2 Z, \quad (76)$$

$$\frac{q''}{Z} - \frac{2q'^2}{qZ} = a_3 q, \quad \frac{b''}{Z} - \frac{bq'^2}{qZ} + \frac{16b^2(n-2)}{q^2} = a_4 b. \quad (77)$$

In the same way one can study the renormalization of Dirac spinorial matter with 2D dilaton gravity. Notice also that one can also investigate the renormalization in the $1/n$ approximation; however, then only the four-fermion term is renormalized.

VI. CONCLUSIONS

In summary, we have studied in this paper the covariant effective action approach in 2D quantum dilaton gravity with four-fermion models described by Majorana or Dirac spinors. The one-loop renormalization of the theory has been considered and the (rather involved) conditions for multiplicative renormalizability have been obtained. The solution of these conditions gives explicit families of multiplicatively renormalizable dilaton potentials. These potentials may be the starting points to discuss 2D quantum dilaton-fermion cosmology along the ideas expressed in Refs. [18,19].

One can also investigate the generalized renormalization group flow in the models under discussion. To be more specific, let us consider again the theory (8), and let $T \equiv \{Z, C, q, b, V\}$ be the set of generalized effective couplings. The general structure of the renormalization is now

$$T_0 = \mu^{2\epsilon} \left[T + \sum_{k=1}^{\infty} \frac{a_{kT}(Z, C, q, b, V)}{\epsilon^k} \right], \quad (78)$$

where, as follows from (32),

$$a_{1Z} = -\frac{Z'}{C'} + \frac{2C'^2}{C^2} + \frac{2ZC''}{C'^2},$$

$$a_{1C} = 0,$$

$$a_{1V} = -\frac{V}{C} - \frac{V'}{C'}, \quad (79)$$

Now it is interesting to compare the conditions of multiplicative renormalizability (68) with the case when the gravitational field is a purely classical one. Using expression (41) we easily get the renormalized effective action for the dilaton interacting with Majorana spinors in curved spacetime:

$$a_{1q} = -\frac{3qZ}{8C'^2} - \frac{3q}{8C} + \frac{q'}{2C'},$$

$$a_{1b} = -\frac{2bZ}{C'^2} + \frac{2b}{C} + \frac{b'}{C'} + \frac{4bq'}{qC'} - \frac{8b^2(n-2)}{q^2}.$$

Now, the generalized β functions can be defined according to

$$\beta_T = -a_{1T} + Z \frac{\delta a_{1T}}{\delta Z} + C \frac{\delta a_{1T}}{\delta C} + V \frac{\delta a_{1T}}{\delta V} \\ + b \frac{\delta a_{1T}}{\delta b} + q \frac{\delta a_{1T}}{\delta q}. \quad (80)$$

Notice that these β functions are different from the ones appearing in the σ -model approach. Applying this rule to the above functions, we get

$$\beta_Z = \frac{Z'}{C'} + \frac{2C'^2}{C^2} - \frac{ZC''}{C'^2} - \frac{2CZ'C''}{C'^3} - \frac{4C''}{C} + \frac{3CZ''}{C'^2},$$

$$\beta_C = 0, \quad (81)$$

$$\beta_V = \frac{V}{C} + \frac{V'}{C'} - \frac{VC''}{C'^2} - \frac{CV''}{C'^2} + \frac{2CV'C''}{C'^3},$$

$$\beta_q = \frac{3q}{8C} - \frac{q'}{2C'} - \frac{3CZq'}{4C'^3} - \frac{3CqZ'}{4C'^3} + \frac{9CqZC''}{C'^4} + \frac{Cq''}{2C'^2} \\ - \frac{Cq'q''}{C'^3} - \frac{3qZ}{8C'^2} + \frac{qC''}{2C'^2},$$

$$\beta_b = -\frac{2b}{C} - \frac{5b'}{C'} - \frac{2bZ}{C'^2} + \frac{5bC''}{C'^2} \\ + \frac{Cb''}{C'^2} + \frac{8(n-2)b^2}{q^2} \\ - \frac{2Cb'C''}{C'^3} - \frac{4Cb'Z}{C'^3} - \frac{4CbZ'}{C'^3} + \frac{12CbZC''}{C'^4} \\ + \frac{4Cb'q'}{qC'^2} + \frac{4Cbq''}{qC'^2} - \frac{4Cbq'^2}{q^2C'^2} - \frac{8Cbq'C''}{qC'^3}. \quad (82)$$

The renormalization group fixed points of the system under discussion are defined by the zeros of the above β functions. What we obtain is the following. As is not difficult to see, the same structure of fixed points that we analyzed in detail in our paper [7] is maintained here. In fact, the first three of the β functions are exactly the same as the ones for that restricted case, and it suffices to impose $q(\Phi) \equiv 0$ and $b(\Phi) \equiv 0$ in order to obtain cor-

responding fixed points in the present, generalized case. However, one has to take care of the limits; that is, we must actually set $q(\Phi) = \eta = \text{const}$, where η is arbitrarily small, in order that the families of fixed points obtained in [7] also give corresponding families here, which are approached as $\eta \rightarrow 0$. We shall not repeat this construction here and simply refer the reader to this paper.

In the same way, the case of the more general four-fermion theory can also be discussed.

ACKNOWLEDGMENTS

S.D.O. wishes to thank H. Osborn and A. Tseytlin for useful discussions on related problems, and T. Banks for a helpful remark. E.E. is grateful to Professor I. Brevik and Professor K. Olaussen, and to Professor L. Brink for the hospitality extended to him at the Universities of Trondheim and Göteborg, respectively. This work has been supported by DGICYT (Spain) and by CIRIT (Generalitat de Catalunya).

APPENDIX

In this short appendix we shall see how our results give the one-loop renormalization of such a well-known model as the sine-Gordon one. Let us start from the action

$$S = - \int d^2x \sqrt{g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + \sqrt{2} \Phi R + \mu e^{-\sqrt{2}\Phi} + m \cos(p\chi) e^{\alpha\Phi} + \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi \right]. \quad (\text{A1})$$

Here p is some number and m , α , and μ are coupling constants. The one-loop effective action of this theory has been calculated in Sec. II, Eq. (28). Using this result and making the renormalization of the metric according to

$$g_{\mu\nu} = \exp \left[\frac{1}{\epsilon \sqrt{2}\Phi} \right] g_{\mu\nu}^R, \quad (\text{A2})$$

one obtains the renormalization of the coupling constants in the following way:

$$\mu_R = \mu \left(1 + \frac{1}{\epsilon} \right), \quad m_R = m \left[1 + \frac{1}{\epsilon} \left(\frac{p^2}{2} - \frac{\alpha}{\sqrt{2}} \right) \right]. \quad (\text{A3})$$

The coupling constant α is not renormalized at one-loop order. Hence, there is no interesting renormalization group dynamics at the one-loop approximation. (It is well known that, in fact, interesting dynamics appear in the nonperturbative approach, as in the matrix models [20].)

-
- [1] C.G. Callan, S.B. Giddings, J.A. Harvey, and A. Strominger, *Phys. Rev. D* **45**, R1005 (1992); E. Witten, *ibid.* **44**, 314 (1991); J.A. Harvey and A. Strominger, Report No. EFI-92-41, 1992 (unpublished); L. Susskind, L.Thorlacius, and J. Uglum, *Phys. Rev. D* **48**, 3743 (1993).
- [2] S. de Alwis, *Phys. Rev. D* **46**, 5429 (1992); K. Hamada and A. Tsuchiya, *Int. J. Mod. Phys. A* **8**, 4897 (1993). A. Bilal and C. Callan, *Nucl. Phys.* **B394**, 73 (1993).
- [3] S.D. Odintsov and I.L. Shapiro, *Phys. Lett. B* **263**, 183 (1991); *Mod. Phys. Lett. A* **7**, 437 (1992); *Int. J. Mod. Phys. D* **1**, 571 (1993).
- [4] T. Banks and M. O'Loughlin, *Nucl. Phys.* **B362**, 649 (1991); *Phys. Rev. D* **48**, 698 (1993).
- [5] R. Kantowski and C. Marzban, *Phys. Rev. D* **46**, 5449 (1992); E. Elizalde and S.D. Odintsov, *Nucl. Phys.* **B399**, 581 (1993); E. Elizalde, S. Naftulin, and S.D. Odintsov, *Z. Phys. C* **60**, 327 (1993); F.D. Mazzitelli and N. Mohammedi, *Nucl. Phys. B* (to be published).
- [6] J.G. Russo and A. Tseytlin, *Nucl. Phys.* **B382**, 259 (1992).
- [7] E. Elizalde, S. Naftulin, and S.D. Odintsov, *Phys. Lett. B* **313**, 347 (1993).
- [8] S. Giddings and A. Strominger, *Phys. Rev. D* **47**, 2454 (1993).
- [9] R. Jackiw, *Nucl. Phys.* **B252**, 343 (1985); C. Teitelboim, *Phys. Lett.* **126B**, 41 (1983); R. Mann, in *Proceedings of the 4th Canadian Conference on General Relativity and Relativistic Astrophysics*, edited by G. Kunstatter, D. Vincent, and J. Williams (World Scientific, Singapore, 1993); I.M. Lichtzier and S.D. Odintsov, *Mod. Phys. Lett. A* **6**, 1953 (1991); D. Cangemi and R. Jackiw, *Phys. Rev. Lett.* **69**, 233 (1992); *Ann. Phys.* (to be published); G. Grignani and G. Nardelli, Report No. UTF-292-1993 (unpublished); A. Chamseddine, *Phys. Lett. B* **256**, 379 (1991).
- [10] D. Friedan, *Ann. Phys. (N.Y.)* **163**, 318 (1985).
- [11] C. Callan, D. Friedan, E. Martinec, and M. Perry, *Nucl. Phys.* **B262**, 593 (1985); E. Fradkin and A. Tseytlin, *ibid.* **B261**, 1 (1985).
- [12] T. Banks and J. Lykken, *Nucl. Phys.* **B331**, 173 (1990).
- [13] A.O. Barvinski and G.A. Vilkovisky, *Nucl. Phys.* **B191**, 237 (1981).
- [14] G. 't Hooft and M. Veltman, *Ann. Inst. H. Poincaré* **20**, 69 (1974).
- [15] I.G. Avramidi, *Phys. Lett. B* **305**, 27 (1993).
- [16] I.L. Buchbinder, S.D. Odintsov, and I.L. Shapiro, *Effective Action in Quantum Gravity* (IOP, Bristol, 1992).
- [17] T. Muta and S.D. Odintsov, *Prog. Theor. Phys.* **90**, 247 (1993).
- [18] J. Polchinski, *Nucl. Phys.* **B394**, 123 (1989).
- [19] A. Cooper, L. Susskind, and L. Thorlacius, *Nucl. Phys.* **B263**, 132 (1991).
- [20] G. Moore, Report No. hep-th 9203061 (unpublished); E. Hsu and D. Kutasov, *Nucl. Phys.* **B396**, 693 (1993).