Constraints on $|V_{td}|$ from radiative decays of the B mesons

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The possibility of using a future measurement of the ratio $B(B\to\rho\gamma)/B(B\to K^*\gamma)$ to improve the constraints on the CKM parameter $|V_{td}|$ is investigated. Special attention is paid to the two main sources of theoretical uncertainties: the SU(3) flavor symmetry-breaking effects and the contribution from final-state interactions.

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I. INTRODUCTION

The present constraints on $|V_{td}|$ [1] are derived from the experimental value of the B_d^0-$\bar{B}_d^0$ mixing parameter $x_d \equiv \Delta M/\Gamma$. The mass difference ΔM is assumed to be that given by the standard model, and it is calculated from the usual box diagrams. However, even if the value of m_t is known, this calculation is severely hindered by the uncertainty in the decay constant f_B [2]. Moreover, it is possible that new physics contributes significantly to ΔM which would invalidate this method of measuring $|V_{td}|$ [3]. One would then like to find an alternative process. Within a scenario where new physics does not affect significantly quark decays, $|V_{td}|$ would be measured, optimally, in t-quark decays; but this is unrealistic. Instead, one can consider decays that are dominated by virtual t quarks. It has been suggested [4, 5] to look at the rare decay $K^+\to\pi^+\nu\bar{\nu}$: the branching ratio is predicted to be around 10^{-10} [6], and the present experimental upper bound is 5.2×10^{-9} [7] from E787 at BNL. I wish to discuss another possibility that has been suggested by Ali and Greub [8], namely the radiative decay $B\to\rho\gamma$, whose branching ratio, in a first approximation, is proportional to $|V_{td}|^2$. The first observations of a radiative B-meson decay, $B\to K^*\gamma$, were recently reported by the CLEO Collaboration, and give $B(B\to K^*\gamma) = (4.5\pm1.5\pm0.9)\times10^{-5}$ [9]. Up to small corrections that are discussed in what follows, one expects $B(B\to\rho\gamma)/B(B\to K^*\gamma)\simeq |V_{td}/V_{ts}|^2$. Then, given the CLEO result and the present constraints on the Cabibbo-Kobayashi-Maskawa (CKM) parameters, the decay $B\to\rho\gamma$ has a branching ratio of the order 10^{-6}, which is marginally within reach of the ongoing experiments and might be measured with considerable accuracy at a future B factory.

The major uncertainty in determining the branching ratios of the exclusive B-meson radiative decays, in terms of the CKM parameters, is in the evaluation of the hadronic matrix elements of the quark-level operators in the Hamiltonian. For example, the present estimates for $B(B\to K^*\gamma)/B(b\to s\gamma)$ range between $\sim 5\%$ [10–12], from constituent quark models for the meson states, to $\sim 30\%$ [13, 14], from QCD sum rules. This difficulty is overcome in part when considering the ratio [8]

$$\Omega \equiv \frac{B(B\to\rho\gamma)}{B(B\to K^*\gamma)}\,; \tag{1}$$

since the decays are related by the SU(3) flavor symmetry, the ratio of the hadronic matrix elements is exactly one, in the symmetric limit. The deviation from this limit can only be evaluated in the context of a specific model, and it is a source of theoretical uncertainty. I will show that the Isgur-Scora-Grinstein-Wise (ISGW) [15] and Bauer-Stech-Wirbel (BSW) [16] models give very different predictions. The latter model seems to be more reliable for this type of decays (with a large recoil), and its prediction for $B(B\to K^*\gamma)$ is in very good agreement with the recent result from CLEO.

Another delicate problem is the effect on Ω of the final state interactions (FSI's) in $B\to\rho\gamma$ and $B\to K^*\gamma$: they can affect the branching ratios significantly, and differently for the two decays. The decay $B\to K^*\gamma$ is dominated by the short-distance contribution from the electromagnetic penguin diagram with a virtual t quark. On the other hand, for $B\to\rho\gamma$ this is strongly Cabibbo suppressed, and so the decay amplitude may have significant contributions from both real and virtual intermediate hadronic states that scatter into $\rho\gamma$ through FSI's. Because such low energy effects do not involve virtual t quarks, they introduce corrections to the branching ratio that are not proportional to $|V_{td}|^2$. The real intermediate states give an absorptive part to the decay amplitude, that generates a CP-violating asymmetry. In Ref. [17], it was shown that this asymmetry can be quite sizeable for the $b\to d\gamma$ decays. Here, I also need to estimate the dispersive part of the FSI corrections, i.e., the contribution of the virtual intermediate states. I use the result for the absorptive part, and a convenient dispersion relation to obtain their ratio.

The expression for the ratio Ω, in terms of the CKM parameters, is then determined. As an illustration, I show the constraints that are imposed on the latter by a particular value of Ω.

II. THE HADRONIC MATRIX ELEMENTS

In the spectator approximation, the Hamiltonian for the $\Delta B = 1$ radiative decays follows from the quark decay process $b\to q\gamma$ ($q = s,\, d$), which occurs through the one-loop electromagnetic penguin diagram shown in Fig. 1. It is given by

$$H = \xi(x_tF + x_uF')m_b\bar{q}_L\sigma^{\mu\nu}b_RF_{\mu\nu}, \tag{2}$$

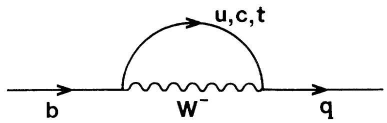

FIG. 1. The electromagnetic penguin diagram that dominates the decays $B \to V\gamma$; the photon is to be attached to every possible location.

where the CKM factors are $x_i \equiv V_{iq}^* V_{ib}$ and $\xi \equiv G_F e/(4\sqrt{2}\pi^2)$. The amplitude for the exclusive decay $B \to V\gamma$ ($V = K^*, \rho$) is then

$$A = \frac{G_F e}{2\sqrt{2}\pi^2}(x_t F + x_u F') m_b \mathcal{M}, \tag{3}$$

where

$$\mathcal{M} = \varepsilon_\mu^* \langle V(\mathbf{P}_V, \varepsilon_V) | \bar{q}_L i\sigma^{\mu\nu} k_\nu b_R | B(\mathbf{P}_B)\rangle \tag{4}$$

is the hadronic matrix element to be dealt with.

The dominant contribution to the Hamiltonian in Eq. (2) is from the t quark loop, and it gives the form factor $F = F_2^t - F_2^c \simeq F_2^t$ [18]. The QCD corrections are known to be important [19], and they have been calculated in a leading-log approximation (LLA) to all orders in α_s [20]: for m_t =(100–180) GeV and $\Lambda_{\rm QCD}^{n_f=5}$ =(100–300) MeV, the QCD-corrected form factor is F =0.26–0.35; it is the same for both the $b \to s\gamma$ and the $b \to d\gamma$ decays [8]. The term proportional to x_u in the Hamiltonian is due to the contribution from the diagrams with light quarks in the loop. It can be neglected for the $b \to s\gamma$ case, because of the double Cabibbo supression in $|x_u/x_t|$, but it can be significant for $b \to d\gamma$. This is further discussed in Sec. III.

The hadronic matrix elements that are relevant for the transitions of the type $B \to V$ are parametrized in the following way:

$$\langle V|\bar{q}\gamma_\mu b|B\rangle = 2T_1(k^2) i\varepsilon_{\mu\nu\alpha\beta}\varepsilon_V^{*\nu} P_B^\alpha P_V^\beta\,, \tag{5}$$

$$\langle V|\bar{q}\gamma_\mu\gamma_5 b|B\rangle = -2T_2(k^2)(M_B^2 - M_V^2)\varepsilon_{V\mu}^* - 2T_3(k^2)(\varepsilon_V^*\cdot k)(P_B+P_V)_\mu - 2T_4(k^2)(\varepsilon_V^*\cdot k)(P_B - P_V)_\mu\,, \tag{6}$$

$$\begin{aligned}\langle V|\bar{q}_L i\sigma_{\mu\nu}k^\nu b_R|B\rangle = {} & f_1(k^2) i\varepsilon_{\mu\nu\alpha\beta}\varepsilon_V^{*\nu} P_B^\alpha P_V^\beta + f_2(k^2)[(M_B^2 - M_V^2)\varepsilon_{V\mu}^* - (\varepsilon_V^*\cdot k)(P_B+P_V)_\mu] \\ & + f_3(k^2)(\varepsilon_V^*\cdot k)[(P_B - P_V)_\mu - \frac{k^2}{M_B^2 - M_V^2}(P_B + P_V)_\mu]\,,\end{aligned} \tag{7}$$

where $k \equiv P_B - P_V$ (for the radiative decays, $k^2 = 0$, which corresponds to the maximum recoil for the vector meson V). The form factors $T_{1-4}(k^2)$ and $f_{1-3}(k^2)$ are not independent; in particular, they satisfy the relations [21, 22]

$$f_2(0) = -\tfrac{1}{2} f_1(0), \tag{8}$$

and, in the approximation of a static b quark in the B rest frame,

$$f_1(0) = -\frac{M_B^2 - M_V^2}{2M_B}[T_1(0) - 2T_2(0)]. \tag{9}$$

From angular momentum conservation, both the vector meson and the photon in $B \to V\gamma$ must have the same helicity $\lambda = \pm 1$; the corresponding expressions for $\mathcal{M}$ are

$$\begin{aligned}\mathcal{M}_{\lambda=\pm1} &= \tfrac{1}{2}(M_B^2 - M_V^2)[\pm f_1(0) + 2f_2(0)] \\ &= \begin{cases} 0 & \text{for } \lambda = +1\,, \\ -(M_B^2 - M_V^2) f_1(0) & \text{for } \lambda = -1\,. \end{cases}\end{aligned} \tag{10}$$

That the amplitude for the positive helicity final state vanishes can be easily understood from the quark decay process [given that the operator in the Hamiltonian of Eq. (2) produces a left-handed quark q]. The value of $f_1(0)$ is obtained from an overlap of the initial and final meson wave functions. I proceed to estimate this form factor, for the cases $V = K^*$ and $V = \rho$, using two different constituent quark models for the mesons.

A. $f_1(0)$ in the ISGW model

The meson wave functions in the ISGW model [15] are approximate nonrelativistic solutions of the usual Coulomb+linear potential that binds the constituent quarks. Using the variational method with harmonic oscillator wave functions gives

$$\phi(\mathbf{p}) = (\pi\beta^2)^{-3/4}\exp\left[-\frac{\mathbf{p}^2}{2\beta^2}\right] \tag{11}$$

for the $1S$ states. For the B, K^*, and ρ mesons, the variational parameter β is $\beta_B = 0.41$ GeV, $\beta_{K^*} = 0.34$ GeV, and $\beta_\rho = 0.31$ GeV. The expression for the form factor $f_1(0)$ in this model was derived in Ref. [12] and it is

$$\begin{aligned} f_1(0) = {} & -\sqrt{\frac{E_V}{M_B}}\int d^3p\, \phi_V^*(\mathbf{p} + \frac{m_{\rm sp}}{m_q + m_{\rm sp}}\mathbf{P}_V)\phi_B(\mathbf{p}) \\ & \times\sqrt{\frac{E_b + m_b}{2E_b}}\sqrt{\frac{E_q + m_q}{2E_q}}\left(1 + \frac{p_z}{E_b + m_b}\right) \\ & \times\left(1 + \frac{p_z + |\mathbf{P}_V|}{E_q + m_q}\right). \end{aligned} \tag{12}$$

For the constituent quark masses $m_b = 5.0$ GeV, $m_s = 0.55$ GeV, and $m_d = m_{\rm sp} = 0.33$ GeV, a numerical integration yields $f_1(0) = -0.18$ and -0.037 for $B \to K^*$ and $B \to \rho$, respectively. This gives

$$\frac{B(B\to K^*\gamma)}{B(b\to s\gamma)}=\left(\frac{M_B}{m_b}\right)^3\left(\frac{M_B^2-M_{K^*}^2}{M_B^2}\right)^3 f_1(0)^2$$
$$=3.5\%. \tag{13}$$

Since $B(B \to e^{\pm}\nu_e \text{ hadrons})\simeq 11\%$ [23] and $B(b \to s\gamma)/B(b\to ce^-\bar{\nu}_e) = (3.17\times10^{-2})F^2$ [20] (for $m_c/m_b = 0.3$), it follows that

$$B(B\to K^*\gamma)=(0.82-1.5)\times10^{-5}, \tag{14}$$

which is lower than the recent experimental observations. As for the $B\to\rho$ transition, the model gives

$$\frac{B(B\to\rho\gamma)}{B(b\to d\gamma)}=0.15\%. \tag{15}$$

This is very different from the $B\to K^*$ case, and it would indicate a very large breaking of the SU(3) flavor symmetry; but I will argue that these values are not very reliable.

The fact that a nonrelativistic wave function is used makes this model most suitable for the region of low recoil momenta, where the internal momentum of the recoiling meson is nonrelativistic. For the extreme case of maximum recoil that occurs in here, the validity of the ISGW description is questionable. Nevertheless, it has been used in the literature [10–12] to predict the $B\to K^*\gamma$ rate, and here I have extended those calculations to the case of the $B\to\rho\gamma$ decay. The overlap of the two meson wave functions that gives $f_1(0)$ is now smaller: this is mostly due to the difference in the mass of the constituent quarks d and s. A lower mass of the nonspectator quark q, that carries the recoil momentum, means that a larger internal momentum $|\mathbf{p}|=|\mathbf{P}_V|m_{\rm sp}/(m_q+m_{\rm sp})$ is needed for the ρ than for the K^*. That this is the dominant feature in the breaking of the SU(3) flavor symmetry can be better seen from an analytical expression for $f_1(0)$. Using the approximation of a static b quark ($E_b\simeq m_b$) and an ultrarelativistic q quark ($E_q\simeq|\mathbf{P}_V|\gg m_q$), Eq. (12) gives

$$f_1(0)=-2\sqrt{\frac{E_V}{M_B}}\left(\frac{\beta_B\beta_V}{\beta_B^2+\beta_V^2}\right)^{3/2}\left[2-\frac{\beta_B^2}{\beta_B^2+\beta_V^2}\frac{m_{\rm sp}}{m_q+m_{\rm sp}}\frac{|\mathbf{P}_V|+m_b}{m_b}\right.$$
$$\left.+\frac{\beta_B^2\beta_V^2}{\beta_B^2+\beta_V^2}\frac{1}{2|\mathbf{P}_V|m_b}+\left(\frac{\beta_B^2}{\beta_B^2+\beta_V^2}\right)^2\left(\frac{m_{\rm sp}}{m_q+m_{\rm sp}}\right)^2\frac{|\mathbf{P}_V|}{2m_b}\right]\exp\left[-\frac{1}{2}\frac{|\mathbf{P}_V|^2}{\beta_B^2+\beta_V^2}\left(\frac{m_{\rm sp}}{m_q+m_{\rm sp}}\right)^2\right]. \tag{16}$$

The ratio $m_q/m_{\rm sp}$ indeed dominates the difference between the exponential suppression of the overlap in the K^* and the ρ cases. The form of this mass dependence stems from the wave function in Eq. (11), and the error in the non-relativistic approximation is amplified when probing the tail of the exponential, as it happens here due to the high recoil momentum. The amount of SU(3) flavor symmetry breaking predicted by this model is therefore quite unreliable and I will not use it.

B. $f_1(0)$ in the BSW model

The BSW model [16] attempts to provide a relativistic picture of the mesons as bound states of their constituent quarks. The meson states are described in the infinite momentum frame by a wave function

$$\psi(\mathbf{p}_T,x)=N\sqrt{x(1-x)}\exp\left(-\frac{\mathbf{p}_T^2}{2\omega^2}\right)$$
$$\times\exp\left[-\frac{(m+m_{\rm sp})^2}{2\omega^2}(x-\bar{x})^2\right], \tag{17}$$

that is chosen as the solution of a relativistic harmonic oscillator; $\mathbf{p}_T$ is the transverse internal momentum and $x=p_z/|\mathbf{P}|$ is the fraction of the longitudinal momentum carried by the nonspectator quark. The central values $\mathbf{p}_T=0$ and $x=\bar{x}\equiv m/(m+m_{\rm sp})$ (m is the mass of the nonspectator quark) correspond to zero internal momentum of the meson. The transverse momentum distribution is assumed to be approximately the same for the different mesons; the value $\omega\simeq0.40$ GeV is chosen, as it gives results for the D-meson decays in good agreement with the data. On the other hand, the peak of the x distribution varies substantially according to the mass of the constituents. The normalization factor N is such that

$$\int d^2p_T dx|\psi(\mathbf{p}_T,x)|^2=1. \tag{18}$$

The expressions for the form factors $T_1(0)$ and $T_2(0)$ that appear in the case of the semileptonic decays were derived in Ref. [16], and they are

$$\frac{M_B^2-M_V^2}{m_b-m_q}T_1(0)=-\frac{M_B^2-M_V^2}{m_b+m_q}2T_2(0)=I, \tag{19}$$

with

$$I=\int d^2p_T dx\frac{1}{x}\psi_V^*(\mathbf{p}_T,x)\psi_B(\mathbf{p}_T,x). \tag{20}$$

According to Eq. (9) this gives

$$f_1(0)=-\frac{m_b}{M_B}I, \tag{21}$$

in the BSW model. The exponential suppression of $f_1(0)$ appears from the separation in the peaks of the x distributions in the overlap integral I. Because the b quark is much heavier than the spectator, the value of $\bar{x}$ is near one for the B meson; whereas for the ρ meson $\bar{x}=1/2$, since both constituents have the same mass. For K^*, because $m_s>m_{\rm sp}$, $\bar{x}$ is slightly larger than $1/2$, and this implies a larger overlap with the B wave function than for the ρ case. This is the kinematical effect that was already described in the previous section, but here one hopes that the relativistic description will give a better approximation for the constituent mass dependence in the exponents.

For the same set of constituent quark masses that was used before, a numerical integration gives $f_1(0) = -0.35$ and -0.31 for $B \to K^*$ and $B \to \rho$, respectively. This corresponds to

$$\frac{B(B \to K^*\gamma)}{B(b \to s\gamma)} = 13\% \quad \text{and} \quad \frac{B(B \to \rho\gamma)}{B(b \to d\gamma)} = 11\%, \tag{22}$$

which are significantly larger than the predictions of the ISGW model. In particular, the prediction for the $B \to K^*\gamma$ branching ratio is now

$$B(B \to K^*\gamma) = (3.1 - 5.6) \times 10^{-5}, \tag{23}$$

in good agreement with the CLEO result. This suggests that the BSW model gives indeed a better description of the radiative B decays. Here, I am interested in the quantity $T \equiv [f_1(0)]_\rho/[f_1(0)]_{K^*}$ that equals one in the SU(3) flavor symmetric limit. In this model $T \simeq 0.90$. The error due to the uncertainty in the constituent quark masses is negligible, but to take into account the model dependence of this estimate I will assume the symmetry breaking to be somewhere between 5 and 15%.

III. THE CONTRIBUTION FROM FINAL STATE INTERACTIONS

In order to estimate the effect of the final state interactions in the decays $B \to V\gamma$, I follow the perturbative approach that was advocated in Refs. [24, 25] for the calculation of the absorptive part of the decay amplitudes. It relies on the quark-hadron duality to describe the multiple intermediate hadronic states that scatter into the final state, in terms of a few free quark states. The strong scattering is treated perturbatively, and a crude estimate of the FSI contribution (which is truly a long distance effect) is then obtained from the appropriate quark level diagrams. For the case of the radiative decays, these are the u- and c-quark loops in the electromagnetic penguin diagram [17]. They give the term proportional to x_u, in the $B \to V\gamma$ amplitude of Eq. (3). Because of the unitarity of the CKM matrix, $F' = F_2^u - F_2^c$ vanishes if the u- and c-quark mass difference can be neglected. Although this is a good approximation for the lowest order result from Fig. 1 [26], it is no longer valid for the order α_s diagrams in Fig. 2. Namely, it was shown in Ref. [17] that the imaginary part of F' that appears from the absorptive part of those diagrams is quite significant. Here, I give a more complete evaluation of F', including its real part.

The form factors F_2^l, for $l = u, c$, are functions of both $y_l \equiv m_l^2/M_W^2$ and $z_l \equiv m_b^2/(4m_l^2)$, and they can be decomposed as [27]

$$F_2^l(y_l, z_l) = F_2^l(y_l, 0) + \Delta F_2^l(z_l). \tag{24}$$

Since z_u and z_c are larger than one, the real intermediate states $du\bar{u}$ and $dc\bar{c}$ (I take $m_d = 0$) give an absorptive contribution to $\Delta F_2^l(z_l)$ (the contribution from the intermediate state dg occurs at the same order, but it is the same for both F_2^u and F_2^c and so it cancels in F'). On the other hand, $F_2^l(y_l, 0)$ must be real, and so $\mathrm{Im}\{F_2^l\} = \mathrm{Im}\{\Delta F_2^l\}$. In Ref. [17] the calculation of the absorptive part was performed analytically in the limit $m_l \to 0$, and

$$\lim_{z_l \to \infty} \mathrm{Im}\{\Delta F_2^l(z_l)\} \simeq -\tfrac{1}{4}\alpha_s. \tag{25}$$

The calculation for an arbitrary value of z_l is more complicated, as one must keep track of the m_l dependence. I find that

$$\mathrm{Im}\{\Delta F_2^l(z_l)\} = -\alpha_s \frac{2}{27}\int_{1/z_l}^{1} dx (1-x)^2 \sqrt{1 - \frac{1}{x z_l}} \left(1 - \frac{13}{16}\frac{1}{x z_l}\right) - \alpha_s \frac{2}{9}\int_{1/z_l}^{1} dx \int_{-y_0}^{y_0} dy$$
$$\times \left\{ \int_{-1}^{z_0} dz \left[\frac{1-x}{2Az_l} + \frac{y + (1-x)/2}{1-z}\left(1 + \frac{|B|}{A}\right)\right] + \int_{z_0}^{1} dz \left[\frac{1-x}{2Az_l} + \frac{y + (1-x)/2}{1-z}\left(1 - \frac{|B|}{A}\right)\right] \right\}, \tag{26}$$

with

$$y_0 \equiv \frac{1}{2}(1-x)\sqrt{1 - \frac{1}{x z_l}}\,,$$
$$z_0 \equiv \frac{\frac{1}{2}(1-x)^2 + y(1+x)}{(1-x)[y + \frac{1}{2}(1+x)]}\,,$$
$$B \equiv z(1-x)[y + \tfrac{1}{2}(1+x)] - \tfrac{1}{2}(1-x)^2 - y(1+x)\,,$$
$$A \equiv \sqrt{\Delta^2 + \frac{1}{z_l}(1-x)^2(1-z^2)}. \tag{27}$$

The integrations are performed numerically, and give the result shown in Fig. 3. Notice that the extreme right of the curve tends to the limit given in Eq. (25).

Using the result for $\mathrm{Im}\{\Delta F_2^l(z_l)\}$, the real part can be obtained from the dispersion relation [22]

$$\mathrm{Re}\{\Delta F_2^l(z_l)\} = \frac{1}{\pi} P \int_1^\infty dz z_l \frac{\mathrm{Im}\{\Delta F_2^l(z)\}}{z(z - z_l)}. \tag{28}$$

The integration is performed numerically to yield $\mathrm{Re}\{\Delta F_2^l\}$ for any chosen value of z_l. This completes the derivation of ΔF_2^l.

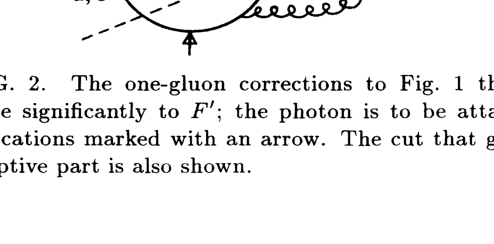

FIG. 2. The one-gluon corrections to Fig. 1 that contribute significantly to F'; the photon is to be attached to the locations marked with an arrow. The cut that gives the absorptive part is also shown.

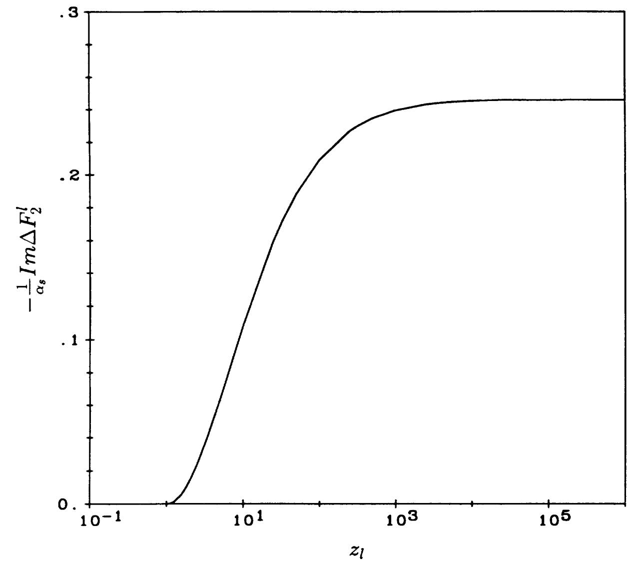

FIG. 3. $\mathrm{Im}\{\Delta F_2^l\}$ as a function of $z_l \equiv m_b^2/4m_l^2$.

As for the remaining term in Eq. (24), $F_2^l(y_l, 0)$, because $y_l \ll 1$ it can be approximated by

$$F_2^l(y_l, 0) = a + b \ln y_l. \tag{29}$$

The constants a and b do not depend on y_l, and so only b is relevant for the form factor F'. It can be determined by requiring that $F_2^l(y_l, z_l)$ be finite when $m_l \to 0$ [25]; then, the coefficient b must be such that $b \ln y_l$ cancels exactly the logarithmic behavior of $\mathrm{Re}\{\Delta F_2^l(z_l)\}$ in the limit $m_l \to 0$. The origin of such behavior is easy to see replacing the limit of Eq. (25) in the dispersion relation. It gives

$$\lim_{z_l \to \infty} \mathrm{Re}\{\Delta F_2^l(z_l)\} = \alpha_s \frac{1}{4\pi} \ln z_l, \tag{30}$$

and so $b = \alpha_s/(4\pi)$. Finally, the form factor F' in the Hamiltonian of Eq. (2) is given by

$$\begin{aligned} \mathrm{Im}\{F'\} &= \mathrm{Im}\{\Delta F_2^u\} - \mathrm{Im}\{\Delta F_2^c\} \simeq -0.164 \times \alpha_s, \\ \mathrm{Re}\{F'\} &= \alpha_s \frac{1}{4\pi} \ln\left(\frac{m_u^2}{m_c^2}\right) + \mathrm{Re}\{\Delta F_2^u\} - \mathrm{Re}\{\Delta F_2^c\} \\ &\simeq -0.114 \times \alpha_s, \end{aligned} \tag{31}$$

where the numerical values correspond to $m_b = 5$ GeV, $m_c = 1.5$ GeV, and $m_u = 330$ MeV (the dependence on the rather uncertain value of m_u is not very significant). Experimental evidence for F' can be obtained in the future, by measuring the CP-violating asymmetry in the $B \to \rho\gamma$ decay, which is proportional to $\mathrm{Im}\{F'\}$.

IV. RESULTS

The decay rate for $B \to V\gamma$ is

$$\Gamma = \frac{G_F^2 \alpha}{32\pi^4} |x_t F + x_u F'|^2 m_b^2 M_B^3 \left(\frac{M_B^2 - M_V^2}{M_B^2}\right)^3 f_1(0)^2, \tag{32}$$

and so the ratio in Eq. (1) is given by

$$\Omega = T^2 z^3 \frac{1}{|V_{ts}^* V_{tb}|^2} \left|V_{td}^* V_{tb} + r V_{ud}^* V_{ub}\right|^2, \tag{33}$$

where $T = 0.85 - 0.95$ is the model-dependent evaluation of the SU(3) flavor symmetry breaking, and $r \equiv F'/F$ is the rough estimate of the FSI effects based on a quark level description. For $m_t \simeq 140$ GeV [28] and $\Lambda_{\mathrm{QCD}}^{n_f=5} \simeq 175$ MeV [23], $F = 0.31$ and so

$$r \simeq -\alpha_s(0.37 + i0.53) \tag{34}$$

[I will take $\alpha_s = \alpha_s(m_b) = 0.24$], with a large theoretical uncertainty that is hard to estimate. Notice that for the decay $B \to K^*\gamma$, $|x_u| \ll |x_t|$ and the effect of r is negligible. The factor z is due to the difference in phase space in the two decays: $z \equiv (M_B^2 - M_\rho^2)/(M_B^2 - M_{K^*}^2) \simeq 1.0$ and it can be ignored.

To improve the statistics, it is convenient to add the branching ratios of the CP-conjugated processes, and consider instead the ratio

$$\Omega' \equiv \frac{B(B \to \rho\gamma) + B(\bar{B} \to \bar{\rho}\gamma)}{B(B \to K^*\gamma) + B(\bar{B} \to \bar{K}^*\gamma)}. \tag{35}$$

The term linear in $\mathrm{Im}\{r\}$ that appears in $B(B \to \rho\gamma)$, and that gives the CP-violating asymmetry in Ref. [17], cancels in the summation $B(B \to \rho\gamma) + B(\bar{B} \to \bar{\rho}\gamma)$. This diminishes the effect of r, since only the smaller real part remains in the linear terms. Then, the constraint equation for the CKM parameters that follows is (using Wolfenstein's parametrization [29] for the CKM matrix)

$$\begin{aligned} (1-\rho)^2 + \eta^2 = &\frac{\Omega'}{T^2 z^3 \lambda^2} - 2\mathrm{Re}\{r\}[\rho(1-\rho) - \eta^2] \\ &- |r|^2(\rho^2 + \eta^2). \end{aligned} \tag{36}$$

As an illustration, I plot this constraint in Fig. 4 for an hypothetical value: $\Omega' = 0.08$. The full line corresponds to the value of r in Eq. (34), and the central value for T. The constraints for $r = 0$ are circles centered at the point $\rho = 1$, $\eta = 0$, and they can be used as a lower bound for the effect of the FSI. On the other hand, the future limits on the CP-violating asymmetry will constrain $\mathrm{Im}\{r\}$;

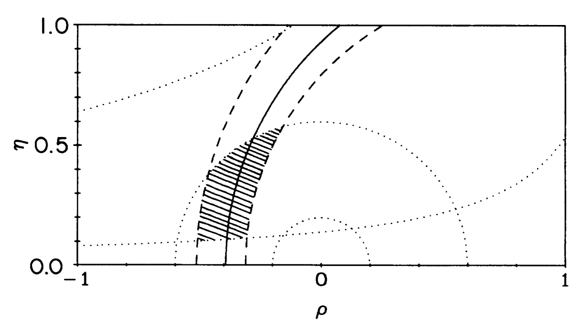

FIG. 4. Constraints on the CKM parameters for $\Omega' = 0.08$: with $T = 0.90$ and $\mathrm{Re}\{r\} = -0.088$ (full line), $T = 0.85$ and $\mathrm{Re}\{r\} = 0$, $T = 0.95$ and $\mathrm{Re}\{r\} = -0.132$ (dashed lines). The dotted lines show the constraints from the $|V_{ub}|$ and $|\varepsilon_K|$ measurements. The dashed area is the allowed region.

using the dispersion relation, an upper bound can then be obtained for r with little theoretical uncertainty. The two dashed lines in Fig. 4 correspond to neglecting the effect of r (and using $T = 0.85$), and to setting r higher than what was found here by 50% (with $T = 0.95$).

Also shown in Fig. 4 are the constraints obtained from the $|V_{ub}|$ and $|\varepsilon_K|$ measurements [1]:

$$\rho^2 + \eta^2 = (0.4 \pm 0.2)^2 \tag{37}$$

and, for $m_t = 140$ GeV,

$$\frac{1}{\eta} = (0.8^{+0.7}_{-0.4}) + (2.1^{+3.2}_{-1.5}) \times (1 - \rho). \tag{38}$$

The constraint from the measurement of x_d is very uncertain due to the poorly known value of f_B; for $m_t = 140$ GeV, $f_B\sqrt{B_B \eta_{\rm QCD}} = (150 \pm 50)$ MeV ($\eta_{\rm QCD}$ is a QCD correction factor) and the CKM parameter $A = 0.85 \pm 0.15$:

$$(1 - \rho)^2 + \eta^2 = 1.4^{+3.5}_{-0.9}. \tag{39}$$

It is not shown in Fig. 4; alternatively, and despite the theoretical uncertainties in the quantities r and T, the ratio Ω' may provide a much tighter constraint on $|V_{td}|$ than that extracted from x_d. Given the CLEO result for $B(B \to K^*\gamma)$, the predictions from the BSW model for the rates of the exclusive decays [Eq. (22)], and the CKM constraints in Eqs. (37)–(39):

$$B(B \to \rho\gamma) \simeq (0.4 - 7) \times 10^{-6} \tag{40}$$

(for m_t larger than 140 GeV, the lower bound is 0.2×10^{-6}). A measurement of this branching ratio might then be achieved with a reasonable precision at a future B factory.

Note added in proof. It was pointed out that nonspectator contributions are another source of uncertainty in Eq. (1). Indeed, I estimate their effect in $B(B \to e\delta)$ to be of order 10% (i.e., of a similar size as the corrections discussed in here). These contributions are absent, however, if one considers the ratio $B(B_s^0 \to \bar{K}^{*0}\delta)/B(B_d^0 \to K^{*0}\delta)$ instead.

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[1] For a detailed discussion of the constraints on the CKM parameters, see, for example, Y. Nir, in *Proceedings of the 1991 Theoretical Advanced Study Institute*, Boulder, Colorado, edited by R. K. Ellis, C. T. Hill, and J. D. Lykken (World Scientific, Singapore, 1992), p. 339; in here, I use updated results that can be found in B. Winstein and L. Wolfenstein, University of Chicago Report No. EFI 92-55, 1992 (unpublished), or in [22].

[2] C. Q. Geng and P. Turcotte, University of Montreal Report No. UdeM-LPN-TH-78, 1991 (unpublished).

[3] J. M. Soares and L. Wolfenstein, Phys. Rev. D **47**, 1021 (1993).

[4] I. I. Bigi and F. Gabbiani, Nucl. Phys. **B367**, 3 (1991).

[5] C. O. Dib, Phys. Lett. B **282**, 201 (1992); G. Harris and J. Rosner, Phys. Rev. D **45**, 946 (1992).

[6] G. Bélanger and C. Q. Geng, Phys. Rev. D **43**, 140 (1991).

[7] E787 Collaboration, M. S. Atiya *et al.*, Phys. Rev. D **48**, 1 (1993).

[8] A. Ali and C. Greub, Phys. Lett. B **287**, 191 (1992).

[9] M. Procario (private communication).

[10] T. Altomari, Phys. Rev. D **37**, 677 (1988).

[11] N. G. Deshpande, P. Lo, and J. Trampetic, Z. Phys. C **40**, 369 (1988) (however, see comments in [12]).

[12] P. J. O'Donnell and H. K. K. Tung, Phys. Rev. D **44**, 741 (1991).

[13] C. A. Dominguez, N. Paver, and Riazuddin, Phys. Lett. B **214**, 459 (1988).

[14] T. M. Aliev, A. A. Ovchinnikov, and V. A. Slobodenyuk, Phys. Lett. B **237**, 569 (1990).

[15] B. Grinstein, M. B. Wise, and N. Isgur, Phys. Rev. Lett. **56**, 258 (1986); N. Isgur *et al.*, Phys. Rev. D **39**, 799 (1989).

[16] M. Wirbel, B. Stech, and M. Bauer, Z. Phys. C **29**, 637 (1985).

[17] J. M. Soares, Nucl. Phys. **B367**, 575 (1991).

[18] T. Inami and C. S. Lim, Prog. Theor. Phys. **65**, 297 (1981); **65**, 772(E) (1982).

[19] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Phys. Rev. D **18**, 2583 (1978); Ya. I. Kogan and M. A. Shifman, Yad. Fiz. **38**, 1045 (1983) [Sov. J. Nucl. Phys. **38**, 628 (1983)]; S. Bertolini, F. Borzumati, and A. Masiero, Phys. Rev. Lett. **59**, 180 (1987); N. G. Deshpande *et al.*, *ibid.* **59**, 183 (1987).

[20] I use the results of M. Misiak, Phys. Lett. B **269**, 161 (1991); a thorough discussion can be found in B. Grinstein, R. Springer, and M. B. Wise, Nucl. Phys. **B339**, 269 (1990) [see also Phys. Lett. B **202**, 138 (1988); B. Grinstein, M. J. Savage, and M. B. Wise, Nucl. Phys. **B319**, 271 (1989); R. Grigjanis *et al.*, Phys. Lett. B **213**, 355 (1988); **223**, 239 (1989)]. The next-to-leading order has been calculated in part by M. Misiak, Nucl. Phys. **B393**, 23 (1993), and the effects of a t quark much heavier than the W boson have been investigated by P. Cho and B. Grinstein, Nucl. Phys. **B365**, 279 (1991); only small corrections have been found in either case.

[21] P. J. O'Donnell and H. K. K. Tung, Phys. Rev. D **48**, 2145 (1993).

[22] J. M. Soares, Ph.D. thesis, Carnegie Mellon University, 1993.

[23] Particle Data Group, K. Hikasa *et al.*, Phys. Rev. D **45**, S1 (1992).

[24] M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. **43**, 242 (1979); Yu. L. Dokshitser and N. G. Uraltsev, Pis'ma Zh. Eksp. Teor. Fiz. **52**, 1109 (1990) [JETP Lett. **52**, 509 (1990)]; H. Simma, G. Eilam, and D. Wyler, Nucl. Phys. **B352**, 367 (1991).

[25] J.-M. Gérard and W.-S. Hou, Phys. Rev. D **43**, 2909 (1991).

[26] Also in the calculation of the hard QCD corrections that were included in F, the term in x_u does not appear since m_u and m_c are neglected (as they are below the renormalization point $\mu \simeq m_b$).

[27] H. Simma and D. Wyler, Nucl. Phys. **B344**, 283 (1990).

[28] P. Langacker and M. Luo, Phys. Rev. D **44**, 817 (1991).

[29] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).