Self-consistently improved finite temperature effective potential for gauge theories

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The finite temperature effective potential of the Abelian Higgs model is studied using the self-consistent composite operator method, which can be used to sum up the contributions of daisy and superdaisy diagrams. The effect of the momentum dependence of the effective masses is estimated by using a Rayleigh-Ritz variational approximation.

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I. INTRODUCTION

It has been recently conjectured that the observed baryon asymmetry might have been generated at the electroweak scale if the phase transition is strongly first order [1]. Unfortunately, when the temperature T is close to the critical temperature T_c the finite temperature effective potential $V_T(\phi)$, which is an important mathematical tool in the study of the phase transitions and can be used to determine their order, cannot be evaluated reliably using the ordinary perturbative approach [2–4]; in fact, at these temperatures certain multiloop diagrams become non-negligible even when the coupling constants are very small. In particular, by using power counting it has been argued [4,5] that when $T \sim T_c$ there are important contributions from the infinite classes of *daisy* and *superdaisy* diagrams (represented in Fig. 1), which render the ordinary one-loop approximation of $V_T(\phi)$ unreliable for all $\phi < T$. Therefore the contributions for these diagrams must be added to the one-loop result. The corresponding improved approximation of $V_T(\phi)$ is expected [5,6] to be reliable (even when $T \sim T_c$) for all $\phi > gT$ and up to order g^3, where g is the biggest coupling constant of the theory.

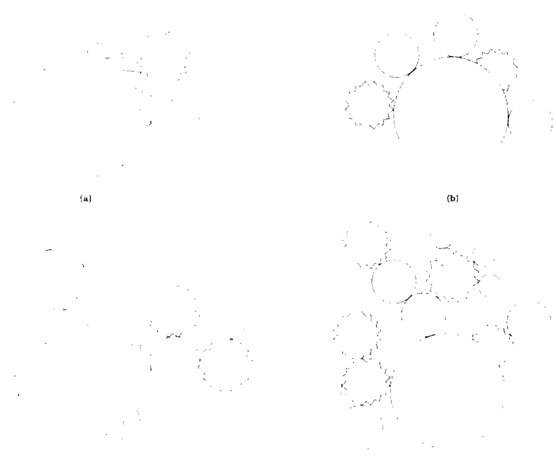

FIG. 1. Examples of (a,b) daisy and (c,d) superdaisy diagrams.

Because of the recent interest in the electroweak baryogenesis, several techniques of evaluation of the contribution of daisy (and superdaisy) diagrams to the finite temperature effective potential for various theories have been presented in the literature [5–21]. In Ref. [19], in a study of the $\lambda\Phi^4$ scalar theory, Pi and I used a method of resummation of the daisy and superdaisy diagrams which is based on the generalization at finite temperature of the Cornwall-Jackiw-Tomboulis [22] effective potential for composite operators $V_T(\phi, G)$. For bosonic quantum fields $\Phi(x)$, $V_T(\phi, G)$ is given by

$$V_T(\phi,G)=V_{\rm cl}(\phi)+\tfrac{1}{2}\,{\rm Tr}\ln D_0 G^{-1}+\tfrac{1}{2}\,{\rm Tr}[D^{-1}G-1] + V_{T(2)}(\phi,G)\ , \qquad (1.1)$$

where [19,21] G is a possible expectation value of $\mathcal{T}\Phi(x)\Phi(y)$ (here $\mathcal{T}$ indicates time ordering), D_0 is the free propagator, D is the tree-level propagator, $V_{\rm cl}(\phi)$ is the classical potential, and $V_{T(2)}(\phi,G)$ is given by all the two-particle irreducible vacuum-to-vacuum graphs with two or more loops in the theory with vertices given by the interaction part of the shifted $(\Phi \rightarrow \Phi + \phi)$ Lagrangian and propagators set equal to G.

The ordinary effective potential $V_T(\phi)$ is related to $V_T(\phi, G)$ by the relation

$$V_T(\phi)=V_T(\phi,G_0)\ , \qquad (1.2a)$$

$$\left. \frac{\delta V_T(\phi,G)}{\delta G} \right|_{G=G_0} = 0\ . \qquad (1.2b)$$

Using this formalism, it is easy to see [19,20] that the resummation of the daisy and superdaisy contributions corresponds to the following approximation of the finite temperature effective potential:

$$V_T(\phi) \simeq V_t^{\rm res}(\phi,G_0)\ , \qquad (1.3a)$$

$$V_T^{\rm res}(\phi,G) \equiv V_{\rm cl}(\phi)+\tfrac{1}{2}\,{\rm Tr}\ln D_0 G^{-1}+\tfrac{1}{2}\,{\rm Tr}[D^{-1}G-1] + [V_{T(2)}(\phi,G)]_{O(g^2)}\ , \qquad (1.3b)$$

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$$\left[\frac{\delta V_T^{\rm res}(\phi,G)}{\delta G} \right]_{G=G_0} = 0 \ , \tag{1.3c}$$

i.e., the expression for $V_T(\phi)$ given by (1.1) and (1.2) but with $V_{T(2)}(\phi,G)$ approximated by the leading two-loop contributions in G.

The primary purpose of this paper is to address some technical issues which often arise in the application of this composite operator method, but are absent in the leading order calculation for the $\lambda\Phi^4$ theory presented in Ref. [19].

The most important technical complication is the momentum dependence of the effective masses (defined in the following section in analogy with Ref. [19]), which appears when diagrams of the type in Fig. 2(b) are included in the approximation of $V_{T(2)}(\phi,G)$. In the leading order calculation for the $\lambda\Phi^4$ scalar theory, such contributions can be neglected. However, unless one assumes that the gauge coupling is much smaller than the scalar self-coupling, in the study of gauge theories diagrams of the type in Fig. 2(b) contribute in the leading order. In the recent literature the importance of the problem of the momentum dependence of the effective masses has been often pointed out [11,17,20], but no consistent solution has been yet developed. As a consequence, in most studies the finite temperature effective potential has been calculated using the *ad hoc* substitution of the effective masses by their value at zero moment. I show that the structure of Eq. (1.3), in which the effective potential is obtained as the solution of a variational problem, naturally leads to a variational approximation which allows one to estimate analytically the effect of the momentum dependence of the effective masses. For definiteness and simplicity, I illustrate this technique by studying the high-temperature effective potential of the Abelian Higgs model, which has been the subject of numerous recent investigations [13–18], especially as a toy model of the standard electroweak model. My variational approximation of the daisy and superdaisy resummed effective potential of the Abelian Higgs model agrees to order e^3 with the result of the "improvement of the one loop" performed in Ref. [9]. Moreover, I use the terms of order e^4 and higher in my result to estimate how important the full higher order correction can be expected to be and to show that an accurate evaluation of the effective potential at the critical temperature also requires appropriate handling of the momentum dependence of the effective masses.

The paper is organized as follows. In Sec. II, I derive

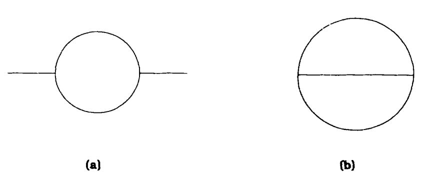

FIG. 2. When diagrams of shape (b) are included in the approximation of $V_{T(2)}(\phi,G)$, $\delta V_T^{\rm res}(\phi,G)/\delta G$ contains contributions from self-energy diagrams of shape (a), which lead to momentum-dependent effective masses.

for the Abelian Higgs model the approximation of the finite temperature effective potential for composite operators defined in Eq. (1.3b) and the *gap* equations (1.3c). Because of the momentum dependence of the effective masses, the daisy and superdaisy resummed effective potential $V_T^{\rm res}$ cannot be evaluated analytically. In Sec. III, I make a Rayleigh-Ritz variational approximation and evaluate analytically the corresponding effective potential in the high-temperature limit. Section IV is devoted to the discussion of the results and the conclusions.

II. DAISY AND SUPERDAISY RESUMMED EFFECTIVE POTENTIAL

In this section, using Eq. (1.3), I study the exact daisy and superdaisy resummed finite temperature effective potential for the Abelian Higgs model. Here and in the following sections, the analysis is performed in the Landau gauge. Field theory at finite temperature T is described using imaginary (Euclidean) time τ, which is restricted to the interval $0 \le \tau \le 1/T$. The Feynman rules are the same as those of ordinary field theory, except that the momentum space integral over the time component k_4 is replaced by a sum over discrete frequencies $k_4 = i\pi n T$:

$$\int \frac{d^4k}{(2\pi)^4} \to T \sum_n \int \frac{d^3k}{(2\pi)^3} \equiv \sum\!\!\!\!\!\!\int_k \ , \tag{2.1}$$

where n is even (odd) for bosons (fermions).

The Lagrangian which describes the theory is

$$\begin{aligned} L = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu\Phi_a\partial^\mu\Phi^a + \frac{1}{2}m^2\Phi^2 - \frac{\lambda}{4!}\Phi^4 \\ & - e\epsilon_{ab}\partial_\mu\Phi_a\Phi_b A^\mu + \frac{1}{2}e^2\Phi^2A^2 - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 \\ & - \eta^+\partial_\mu\partial^\mu\eta \ , \end{aligned} \tag{2.2}$$

where

$$\Phi^2 \equiv \Phi_a\Phi^a \ , \quad \Phi^4 \equiv (\Phi^2)^2 \ , \quad a = 1,2 \ , \quad b = 1,2 \ ,$$

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \ , \quad \mu = 1,2,3,4 \ , \quad \nu = 1,2,3,4 \ . \tag{2.3}$$

A_μ, Φ_a, and η are the gauge, Higgs, and ghost fields, respectively. Note that the ghost η completely decouples from the rest of the theory. Because of the singularity of the inverse of the gauge boson propagator in the Landau gauge, it is appropriate to keep track of the gauge parameter ξ and take the limit $\xi \to 0$ only at the end.

After shifting the Higgs field Φ (i.e., $\Phi_1 \to \Phi_1 + \phi_1$ and $\Phi_2 \to \Phi_2 + \phi_2$) from (2.2), one obtains the tree-level mass matrices

$$\begin{aligned} (m_\gamma^2)_{\mu\nu} &= e^2\phi^2 g_{\mu\nu} \ , \\ (m_\Phi^2)_{ab} &= \left[-m^2 + \frac{\lambda}{6}\phi^2 \right]\delta_{ab} + \frac{\lambda}{3}\phi_a\phi_b \ , \end{aligned} \tag{2.4}$$

for the gauge and the Higgs fields, respectively, and the classical potential

$$V_{\rm cl}(\phi) = -\frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 \ . \tag{2.5}$$

The propagator in momentum space is given by

$$[D^{-1}(\phi;k)]_{\mu\nu}=(e^2\phi^2-k^2)\left[\frac{k_\mu k_\nu}{k^2}-g_{\mu\nu}\right]+\left[\frac{k^2}{\xi}-e^2\phi^2\right]\frac{k_\mu k_\nu}{k^2}\ ,$$
$$[D^{-1}(\phi;k)]_{ab}=(m^2_\Phi)_{ab}-\delta_{ab}k^2\ ,\qquad (2.6)$$
$$[D^{-1}(\phi;k)]_{a\mu}=-iek_\mu\epsilon_{ab}\phi_b\ ,$$

where $k^2\equiv k_\mu k^\mu$.

In the application of the composite operator method, it is first necessary to identify, based on the magnitude of the coupling constants, the diagrams which give the leading contribution to $V_{T(2)}(\phi,G_0)$. As argued in Ref. [17], in order to justify the high temperature approximation and to have a well-defined loop expansion, one needs $e^4\ll\lambda\ll e^2$; however, the scenario is different depending on whether $e^3\ll\lambda\ll e^2$ or $e^4\ll\lambda\leq e^3$. If $e^3\ll\lambda\ll e^2$, $V_{T(2)}$ is approximated by the sum of the four diagrams in Fig. 3, whereas if $e^4\ll\lambda\leq e^3$, the diagram in Fig. 3(a) is neglected. I study the richer scenario $e^3\ll\lambda\ll e^2$ (the results for the other case can be obtained as the appropriate limit of the results for $e^3\ll\lambda\ll e^2$); in these hypotheses, $V_T^{\rm res}$ can be written as

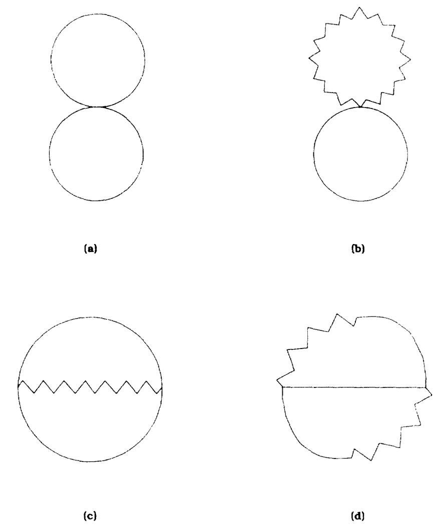

FIG. 3. Diagrams of order (a) λ or (b,c,d) e^2 which contribute to $V_{T(2)}$.

$$V_T^{\rm res}=V_{\rm cl}(\phi)+\tfrac{1}{2}\,{\rm Tr}\ln D_0G_0^{-1}+\tfrac{1}{2}{\rm Tr}[D^{-1}G_0-1]+V_2^{(a)}(G_0)+V_2^{(b)}(G_0)+V_2^{(c)}(G_0)+V_2^{(d)}(G_0)\ ,\qquad (2.7a)$$

$$V_2^{(a)}(G)\equiv-\frac{\lambda}{4!}\sum\!\!\!\!\!\!\int_p\sum\!\!\!\!\!\!\int_q[G_{aa}(p)G_{bb}(q)+2G_{ab}(p)G_{ba}(q)]\ ,\qquad (2.7b)$$

$$V_2^{(b)}(G)\equiv-\frac{e^2}{2}g^{\mu\nu}\sum\!\!\!\!\!\!\int_p\sum\!\!\!\!\!\!\int_q G_{\mu\nu}(q)G_{aa}(p)\ ,\qquad (2.7c)$$

$$V_2^{(c)}(G)\equiv\frac{e^2}{4}\sum\!\!\!\!\!\!\int_p\sum\!\!\!\!\!\!\int_q\epsilon_{ab}\epsilon_{cd}(2p+q)^\mu(2p+q)^\nu G_{\mu\nu}(q)G_{ad}(p)G_{bc}(p+q)\ ,\qquad (2.7d)$$

$$V_2^{(d)}(G)\equiv\frac{e^2}{2}\sum\!\!\!\!\!\!\int_p\sum\!\!\!\!\!\!\int_q\epsilon_{ac}\epsilon_{db}(2q+p)^\mu(2q+p)^\nu G_{ab}(p+q)G_{c\nu}(p)G_{\mu d}(q)\ ,\qquad (2.7e)$$

where [see (1.3)] G_0 is the solution of

$$\frac{\delta V_T^{\rm res}}{\delta G}=0\ .\qquad (2.8)$$

Using (2.7), (2.8) can be rewritten as the following system of gap equations:

$$\begin{aligned}G_{ab}^{-1}(k)&=D_{ab}^{-1}(k)-2\frac{\delta V_2}{\delta G_{ba}(k)}\\&=D_{ab}^{-1}(k)+\frac{\lambda}{6}\sum\!\!\!\!\!\!\int_p[2G_{ab}(p)+\delta_{ab}G_{cc}(p)]+e^2\delta_{ab}\sum\!\!\!\!\!\!\int_p g^{\mu\nu}G_{\mu\nu}(p)\\&\quad+e^2\sum\!\!\!\!\!\!\int_p\epsilon_{ad}\epsilon_{bc}(2k+p)^\mu(2k+p)^\nu G_{\mu\nu}(p)G_{cd}(p+k)\\&\quad+e^2\sum\!\!\!\!\!\!\int_p\epsilon_{ad}\epsilon_{bc}(p+k)^\mu(2k-p)^\nu G_{\mu d}(k-p)G_{c\nu}(p)\ ,\end{aligned}\qquad (2.9a)$$

$$\begin{aligned}G_{\mu\nu}^{-1}(k)&=D_{\mu\nu}^{-1}(k)-2\frac{\delta V_2}{\delta G_{\nu\mu}(k)}\\&=D_{\mu\nu}^{-1}(k)+e^2\sum\!\!\!\!\!\!\int_p g_{\mu\nu}G_{aa}(p)+\frac{e^2}{2}\sum\!\!\!\!\!\!\int_p\epsilon_{ab}\epsilon_{dc}(2p+k)_\mu(2p+k)_\nu G_{ad}(p)G_{bc}(p+k)\ ,\end{aligned}\qquad (2.9b)$$

$$G_{a\mu}^{-1}(k)=D_{a\mu}^{-1}(k)-2\frac{\delta V_2}{\delta G_{\mu a}(k)}$$
$$=D_{a\mu}^{-1}(k)+e^2 \sum\!\!\!\!\!\!\int_p \epsilon_{ab}\epsilon_{cd}(2p+k)_\mu(2k+p)^\nu G_{db}(p+k)G_{c\nu}(p) , \quad (2.9c)$$

where $V_2 \equiv V_2^{(a)}+V_2^{(b)}+V_2^{(c)}+V_2^{(d)}$. The diagrammatic representation of the various $\delta V_2/\delta G_{xy}$ is given in Fig. 4.

In order to study the system of equations (2.9), I introduce effective masses by taking an ansatz for the propagator:

$$G_{\mu\nu}^{-1}=[\mathcal{M}_t^2(k)-k^2]t_{\mu\nu}(k)+[\mathcal{M}_l^2(k)-k^2]l_{\mu\nu}(k)+\left[\frac{k^2}{\xi}-e^2\phi^2\right]\frac{k_\mu k_\nu}{k^2} , \quad (2.10a)$$

$$G_{ab}^{-1}=[\mathcal{M}_\Phi^2(k)]_{ab}-\delta_{ab}k^2 , \quad (2.10b)$$

$$G_{a\mu}^{-1}=-iek_\mu\epsilon_{ab}\phi_b , \quad (2.10c)$$

where $t_{\mu\nu}$ and $l_{\mu\nu}$ are defined by (N.B. $\{\mu,\nu\}=1,2,3,4$; $\{i,j\}=1,2,3$; $\mathbf{k}^2\equiv k_i k^i$)

$$t_{\mu\nu}(k)\equiv\delta_{\mu i}\delta_{\nu j}\left[\delta^{ij}-\frac{k^i k^j}{\mathbf{k}^2}\right] , \quad l_{\mu\nu}(k)\equiv\frac{k_\mu k_\nu}{k^2}-g_{\mu\nu}-t_{\mu\nu} . \quad (2.11)$$

Equations (2.10) express the propagator in terms of effective masses $\mathcal{M}_x(k)$ and are consistent in the Landau gauge with Eqs. (2.9). The two transverse modes of the gauge boson acquire the same effective mass $\mathcal{M}_t(k)$, whereas the longitudinal mode has an independent effective mass $\mathcal{M}_l(k)$, as required by the way Lorentz invariance is broken at finite temperature [23]. Also note the symmetry of Eq. (2.9) under exchange of ϕ_1 and ϕ_2 (a consequence of the analogue property of the propagator D) indicates that $[\mathcal{M}_\Phi^2(k)]_{12}=[\mathcal{M}_\Phi^2(k)]_{21}$.

Using (2.10) and the following properties of $t_{\mu\nu}$ and $l_{\mu\nu}$,

$$t_{\mu\sigma}(k)t^{\sigma\nu}(k)=-t_\mu{}^\nu(k) , \quad t_\mu{}^\mu(k)=-2 , \quad l_{\mu\sigma}(k)l^{\sigma\nu}(k)=-l_\mu{}^\nu(k) , \quad l_\mu{}^\mu(k)=-1 , \quad (2.12)$$

Eqs. (2.9) can be rewritten in terms of the effective masses as

$$[\mathcal{M}_\Phi^2(k)]_{ab}=\left[-m^2+\frac{\lambda}{6}\phi^2\right]\delta_{ab}+\frac{\lambda}{3}\phi_a\phi_b+\frac{\lambda}{6}\sum\!\!\!\!\!\!\int_p \frac{2\epsilon_{ac}\epsilon_{bd}[\mathcal{M}_\Phi^2(p)]_{cd}+\delta_{ab}\{[\mathcal{M}_\Phi^2(p)]_{cc}-4p^2\}}{\{[\mathcal{M}_\Phi^2(p)]_{11}-p^2\}\{[\mathcal{M}_\Phi^2(p)]_{22}-p^2\}-\{[\mathcal{M}_\Phi^2(p)]_{12}\}^2}$$
$$-e^2\delta_{ab}\sum\!\!\!\!\!\!\int_p\left[\frac{1}{\mathcal{M}_t^2(p)-p^2}+\frac{1}{\mathcal{M}_l^2(p)-p^2}\right]$$
$$+e^2\sum\!\!\!\!\!\!\int_p \epsilon_{ad}\epsilon_{bc}(2k+p)^\mu(2k+p)^\nu\left[\frac{t_{\mu\nu}(p)}{\mathcal{M}_t^2(p)-p^2}+\frac{l_{\mu\nu}(p)}{\mathcal{M}_l^2(p)-p^2}\right]$$
$$\times\frac{\epsilon_{cm}\epsilon_{dn}[\mathcal{M}_\Phi^2(p+k)]_{mn}-\delta_{cd}(p+k)^2}{\{[\mathcal{M}_\Phi^2(p+k)]_{11}-p^2\}\{[\mathcal{M}_\Phi^2(p+k)]_{22}-p^2\}-\{[\mathcal{M}_\Phi^2(p+k)]_{12}\}^2} , \quad (2.13a)$$

$$\mathcal{M}_s^2(k)=e^2\phi^2-e^2\sum\!\!\!\!\!\!\int_p\frac{[\mathcal{M}_\Phi^2(p)]_{cc}-2p^2}{\{[\mathcal{M}_\Phi^2(p)]_{11}-p^2\}\{[\mathcal{M}_\Phi^2(p)]_{22}-p^2\}-\{[\mathcal{M}_\Phi^2(p)]_{12}\}^2}$$
$$+\frac{(1+\rho_s)e^2}{4}\sum\!\!\!\!\!\!\int_p \epsilon_{ab}\epsilon_{dc}(2p+k)_\mu(2p+k)_\nu W_s^{\mu\nu}(k)$$
$$\times\frac{\epsilon_{am}\epsilon_{dn}[\mathcal{M}_\Phi^2(p)]_{mn}-\delta_{ad}p^2}{\{[\mathcal{M}_\Phi^2(p)]_{11}-p^2\}\{[\mathcal{M}_\Phi^2(p)]_{22}-p^2\}-\{[\mathcal{M}_\Phi^2(p)]_{12}\}^2}$$
$$\times\frac{\epsilon_{bh}\epsilon_{ci}[\mathcal{M}_\Phi^2(p+k)]_{hi}-\delta_{bc}(p+k)^2}{\{[\mathcal{M}_\Phi^2(p+k)]_{11}-p^2\}\{[\mathcal{M}_\Phi^2(p+k)]_{22}-p^2\}-\{[\mathcal{M}_\Phi^2(p+k)]_{12}\}^2} , \quad (2.13b)$$

where $s=t,l$, $\rho_t=0$, $\rho_l=1$, $W_t^{\mu\nu}\equiv t^{\mu\nu}$, and $W_l^{\mu\nu}\equiv l_{\mu\nu}$. In (2.13) the Landau gauge limit $\xi\to 0$ has been taken; no divergence occurs because of a cancellation between the $1/\xi$ terms which appear in $G_{\mu\nu}^{-1}$ [as given in (2.10a)] and $D_{\mu\nu}^{-1}$.

Equations (2.13) show that, because of the contributions corresponding to the nonlocal self-energy diagrams in Figs. 4(c) and 4(g) [the last terms on the right-hand side (RHS) of Eqs. (2.13a) and (2.13b)], the effective masses must have a highly nontrivial dependence on the

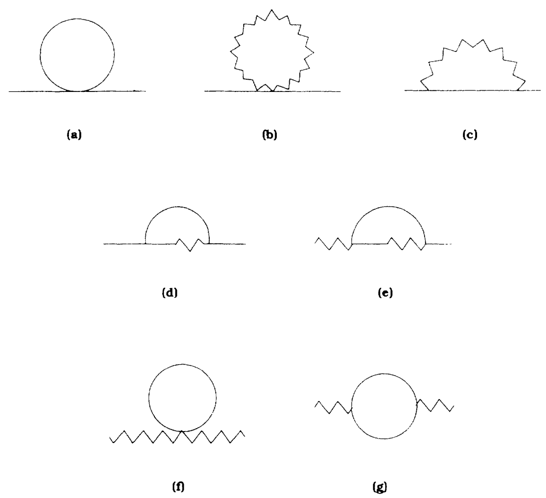

FIG. 4. Diagrammatic representation of the integrals which contribute to (a,b,c,d) $\delta V_2/\delta G_{ab}$, (e) $\delta V_2/\delta G_{a\mu}$, and (f,g) $\delta V_2/\delta G_{\mu\nu}$.

momentum k, and this makes the search for solutions very difficult. (Note that in the leading order calculation for the $\lambda\Phi^4$ theory, no nonlocal self-energy diagram contributes to the gap equation for the effective mass [19]. As a result, the gap equation can be solved analytically in the high-temperature approximation.)

However, $V_T^{\rm res}$ could be evaluated from Eqs. (2.7) and (2.9) (after renormalization) using numerical methods. The fact that the exact daisy and superdaisy resummed effective potential is given by Eqs. (2.7) and (2.9) is a first important result of the composite operator method.

III. RAYLEIGH-RITZ APPROXIMATION

In this section I study $V_T^{\rm res}$ analytically using the observation that an approximate solution of the variational problem (1.3) can be obtained by evaluating $V_T^{\rm res}(\phi,G)$ with specific parameter-dependent expressions for the propagator $G(k)$ and then varying these parameters. This type of procedure is known [22] as the "Rayleigh-Ritz variational approximation." I shall take as the parameter-dependent $G(k)$ the expression (2.10) with all the momentum-dependent "exact" effective masses $\mathcal{M}_x(k)$ replaced by constant "Rayleigh-Ritz effective masses" M_x. This consistently leads, as we will see, to momentum-independent gap equations for the M_x.

The present approximation is quite different from the approximation which has been frequently used in the recent literature, where one makes the *ad hoc* assumption $\mathcal{M}_x(k)\simeq\mathcal{M}_x(0)$ in the gap equations for the effective masses. The differences are twofold. First, in my approximation the self-consistency between the effective potential and the gap equations is preserved, whereas in the "$\mathcal{M}_x(k)\simeq\mathcal{M}_x(0)$ approximation" this self-consistency is lost. Second, as I shall argue in Sec. IV based on the analysis of the results obtained in the following, some average effect of the dependence of the diagrams in Figs. 4(c) and 4(g) on the external momentum is reflected in my approximation of $V_T^{\rm res}$, even though the Rayleigh-Ritz effective masses are constant. In the $\mathcal{M}_x(k)\simeq\mathcal{M}_x(0)$ approximation, this momentum dependence is completely neglected, leading to an uncontrolled (and probably large, as argued in Refs. [5,15,17]) error in the evaluation of the daisy and superdaisy diagrams. [The spirit of performing the approximation at the level of the expectation value of the Hamiltonian rather than at the level of the equations that follow from varying the exact Hamiltonian is very similar to the difference between the Kohn-Sham approximation and the Slater approximation in Hartree-Fock many-body theory: The former is an approximation to the Hamiltonian expectation value, which is then varied; the latter is an approximation to the variational equations. It is known that the Kohn-Sham method is better [24]. This result should encourage the use of the Rayleigh-Ritz approximation; however, because of the differences between the problem at hand and Hartree-Fock many-body theory, it does not necessarily imply that the $\mathcal{M}_x(k)\simeq\mathcal{M}_x(0)$ approximation be inadequate.]

The number of independent Rayleigh-Ritz effective masses to be varied can be reduced by using the fact that the symmetry of the Lagrangian suggests that $V_T^{\rm res}$ depends on $\phi^2\equiv\phi_1^2+\phi_2^2$ rather than separately on ϕ_1 and ϕ_2. This invariance of $V_T^{\rm res}$ under rotations in ϕ space allows us to perform the calculations starting from a diagonal tree-level mass matrix for the Higgs boson

$$(m_\Phi^2)_{ab}=\left[-m^2+\frac{\lambda}{2}\phi^2\right]\delta_{a1}\delta_{b1}+\left[-m^2+\frac{\lambda}{6}\phi^2\right]\delta_{a2}\delta_{b2}\ ,$$

and in this hypothesis one can consistently assume that also the Rayleigh-Ritz mass matrix for the Higgs boson is diagonal. This completes the definition of the variational problem which is studied in this section; the corresponding approximation of the daisy and superdaisy resummed finite temperature effective potential can be formally written as

$$V_T^{\rm res}\simeq V_T^{\rm res}(\phi,G(\{M_0\}))\ , \tag{3.1a}$$

$$\left[\frac{\delta V_T^{\rm res}(\phi,G(\{M\}))}{\delta M^n}\right]_{\{M\}=\{M_0\}}=0\ , \tag{3.1b}$$

where

$$\{M\}\equiv\{M^1,M^2,M^3,M^4\}\equiv\{M_\phi,M_\chi,M_t,M_l\}\ ,$$

$V_T^{\rm res}(\phi,G)$ is defined in Eq. (2.7), and $G(\{M\})$ is given by

$$G_{\mu\nu}^{-1}=(M_t^2-k^2)t_{\mu\nu}(k)+(M_l^2-k^2)l_{\mu\nu}(k)+\left[\frac{k^2}{\xi}-e^2\phi^2\right]\frac{k_\mu k_\nu}{k^2}\ , \tag{3.2a}$$

$$G_{ab}^{-1}=\delta_{a1}\delta_{b1}(M_\phi^2-k^2)+\delta_{a2}\delta_{b2}(M_\chi^2-k^2)\ , \tag{3.2b}$$

$$G_{a\mu}^{-1}=-iek_\mu\epsilon_{ab}\phi_b\ . \tag{3.2c}$$

The effective potential $V_T^{\rm res}(\phi,G(\{M\}))$ in Eq. (3.1) includes divergent integrals; therefore, a regularization and renormalization procedure is necessary. The self-consistency of Eqs. (1.1) and (1.2) implies [22] that the effective potential $V_T(\phi,G)$ and the gap equations are renormalizable. Such a renormalization has been discussed in detail in the analogue study of the daisy and superdaisy resummed effective potential of the $\lambda\Phi^4$ scalar theory presented in Ref. [19]. In that case it has been shown that the only effect of renormalization on the high-temperature part of the effective potential is the substitution of the bare parameters with renormalized ones. This is due to the fact that the dominant high-temperature contributions to the effective potential come from the infrared and are not sensitive to the ultraviolet behavior [25]. In the following I shall assume that the same applies in the case of the Abelian Higgs model, and therefore, rather than performing renormalization explicitly, I will simply omit the (zero-temperature) ultraviolet contributions and substitute in my high-temperature effective potential the bare parameters with renormalized ones.

The gap equation (3.1b) can be put in simple form by performing the high-temperature approximation of $V_T^{\rm res}(\phi,G(\{M\}))$. Using the well-known results [3]

$$\begin{aligned}
&\tfrac{1}{2}\,\mathrm{Tr}\ln[k^2-y^2]\equiv\tfrac{1}{2}\sum\!\!\!\!\!\!\int_k \ln[-(n\pi T)^2-\mathbf{k}^2-y^2]\simeq-\frac{\pi^2T^4}{90}+\frac{y^2T^2}{24}-\frac{y^3T}{12\pi}+\frac{c_\Omega y^4}{32\pi^2}\,,\\
&\Omega(y)\equiv\sum\!\!\!\!\!\!\int_k \frac{1}{(n\pi T)^2+\mathbf{k}^2+y^2}=\frac{1}{y}\frac{\partial}{\partial y}\left\{\frac{1}{2}\,\mathrm{Tr}\ln[k^2-y^2]\right\}\simeq\frac{T^2}{12}-\frac{Ty}{4\pi}+\frac{c_\Omega}{8\pi^2}y^2\,,\\
&c_\Omega\equiv\frac{1}{2}\ln\left[\frac{T^2}{\sigma^2}\right]+\frac{1}{2}+\ln(4\pi)-\gamma_{\rm Euler}
\end{aligned}\tag{3.3}$$

[where σ is a renormalization scale and as planned the (zero-temperature) ultraviolet divergent contributions have been omitted], the high-temperature approximations of $(\mathrm{Tr}\ln G^{-1})/2$ and $(\mathrm{Tr}[D^{-1}G-1])/2$ in the Landau gauge are easily obtained:

$$\begin{aligned}
\tfrac{1}{2}\,\mathrm{Tr}\ln G^{-1}&\simeq\tfrac{1}{2}\,\mathrm{Tr}\ln[(k^2-M_\phi^2)(k^2-M_\chi^2)(k^2-M_t^2)^2(k^2-M_l^2)]\\
&\simeq-\frac{\pi^2T^4}{18}+\frac{T^2}{24}(M_\phi^2+M_\chi^2+2M_t^2+M_l^2)-\frac{T}{12\pi}(M_\phi^3+M_\chi^3+2M_t^3+M_l^3)+\frac{c_\Omega}{32\pi^2}(M_\phi^4+M_\chi^4+2M_t^4+M_l^4)\,,
\end{aligned}\tag{3.4a}$$

$$\begin{aligned}
\tfrac{1}{2}\,\mathrm{Tr}[D^{-1}G-1]&=\tfrac{1}{2}\,\mathrm{Tr}[(D^{-1}-G^{-1})G]\\
&=\tfrac{1}{2}(m_\phi^2-M_\phi^2)\Omega(M_\phi)+\tfrac{1}{2}(m_\chi^2-M_\chi^2)\Omega(M_\chi)+(e^2\phi^2-M_t^2)\Omega(M_t)+\tfrac{1}{2}(e^2\phi^2-M_l^2)\Omega(M_l)\\
&\simeq(m_\phi^2-M_\phi^2)\left[\frac{T^2}{24}-\frac{TM_\phi}{8\pi}+\frac{c_\Omega}{16\pi^2}M_\phi^2\right]+(m_\chi^2-M_\chi^2)\left[\frac{T^2}{24}-\frac{TM_\chi}{8\pi}+\frac{c_\Omega}{16\pi^2}M_\chi^2\right]\\
&\quad+2(e^2\phi^2-M_t^2)\left[\frac{T^2}{24}-\frac{TM_t}{8\pi}+\frac{c_\Omega}{16\pi^2}M_t^2\right]+(e^2\phi^2-M_l^2)\left[\frac{T^2}{24}-\frac{TM_l}{8\pi}+\frac{c_\Omega}{16\pi^2}M_l^2\right]\,,
\end{aligned}\tag{3.4b}$$

where $m_\phi^2\equiv-m^2+\lambda\phi^2/2$ and $m_\chi^2\equiv-m^2+\lambda\phi^2/6$. Note that in (3.4a) I neglected an unimportant infinite constant (i.e., independent of the effective masses) which appears as a consequence of the $1/\xi$ pole in the determinant of the $G_{\mu\nu}^{-1}$ of Eq. (3.2a). In obtaining (3.4b) the limit $\xi\to0$ presented no complication because of the cancellation between the $1/\xi$ terms which appear in $D_{\mu\nu}^{-1}-G_{\mu\nu}^{-1}$ [see (2.6) and (3.2)]. In the following evaluations of $V_2^{(a)}$, $V_2^{(b)}$, $V_2^{(c)}$, and $V_2^{(d)}$, the limit $\xi\to0$ is also taken and no divergences appear.

$V_2^{(a)}(G(\{M\}))$ and $V_2^{(b)}(G(\{M\}))$ have the same structure as the two-loop contribution, which is discussed in detail in Ref. [19]; their high-temperature approximations are given by

$$\begin{aligned}
V_2^{(a)}(G(\{M\}))&=\frac{\lambda}{24}[3\Omega^2(M_\phi)+3\Omega^2(M_\chi)+2\Omega(M_\phi)\Omega(M_\chi)]\\
&\simeq\frac{\lambda T^2}{432}-\frac{\lambda T^3(M_\phi+M_\chi)}{144\pi}+\frac{\lambda T^2M_\phi M_\chi}{192\pi^2}+\frac{\lambda T^2(M_\phi^2+M_\chi^2)}{\pi^2}\left[\frac{c_\Omega}{288}+\frac{1}{128}\right]\,,
\end{aligned}\tag{3.5a}$$

$$V_2^{(b)}(G(\{M\}))=\frac{e^2}{2}[2\Omega(M_t)+\Omega(M_l)][\Omega(M_\phi)+\Omega(M_\chi)]$$
$$\simeq\frac{e^2T^2}{48}-\frac{e^2T^3(4M_t+2M_l+3M_\phi+3M_\chi)}{96\pi}+\frac{e^2T^2(2M_t+M_l)(M_\phi+M_\chi)}{32\pi^2}$$
$$+\frac{e^2T^2c_\Omega(4M_t^2+2M_l^2+3M_\phi^2+3M_\chi^2)}{192\pi^2}\ . \quad (3.5b)$$

The corresponding approximation of $V_2^{(c)}(G(\{M\}))$, based on results obtained in Refs. [17,26], is discussed in the Appendix; the result is

$$V_2^{(c)}(G(\{M\}))\simeq-\frac{e^2T^2}{288}+\frac{e^2T^3(2M_t-M_l)}{48\pi}+\frac{e^2T^2(M_t^2-2M_\phi^2-2M_\chi^2)}{32\pi^2}\ln\left[\frac{M_t+M_\phi+M_\chi}{3T}\right]$$
$$+\frac{e^2T^2(M_\phi M_\chi-M_tM_\phi-M_tM_\chi)}{32\pi^2}+\frac{e^2T^2(c_{\Theta l}+c_{\Theta l})(M_\phi^2+M_\chi^2)}{64\pi^2}$$
$$-\frac{e^2T^2[(3c_{\Theta t}+8c_\Omega)M_t^2+(3c_{\Theta l}-4c_\Omega)M_l^2]}{384\pi^2}\ . \quad (3.6)$$

$c_{\Theta t}$ and $c_{\Theta l}$ are defined in the Appendix.

Finally, the contribution to $V_T^{\rm res}$ from the diagram in Fig. 3(d) can be easily calculated by noting that Eq. (3.2) implies that in the Landau gauge $G_{a\mu}=0$:

$$V_2^{(d)}(G(\{M\}))=0\ . \quad (3.7)$$

[Note that (3.7) is consistent with Eqs. (2.6), (2.9c), and (3.2).]

Using (3.4)–(3.7), it is now possible to express the gap equations (3.1b) in the form

$$M_{\phi(\chi)}^2\simeq m_{\phi(\chi)}^2+\left[\frac{\lambda}{18}+\frac{e^2}{4}\right]T^2-\left[a-\frac{1}{\pi}\ln\left[\frac{M_t+M_\phi+M_\chi}{3T}\right]\right]e^2TM_{\phi(\chi)}$$
$$-\left[\frac{\lambda}{24\pi}+\frac{e^2}{4\pi}\right]TM_{\chi(\phi)}-\frac{e^2}{4\pi}T(M_t+M_l)+\frac{e^2T}{4\pi}\frac{2M_\phi^2+2M_\chi^2-M_t^2}{M_\phi+M_\chi+M_t}+\frac{c_\Omega}{\pi}\frac{M_{\phi(\chi)}^3}{T}\ , \quad (3.8a)$$

$$M_t^2\simeq e^2\phi^2+\left[\frac{c_{\Theta t}}{16\pi}-\frac{1}{4\pi}\ln\left[\frac{M_t+M_\phi+M_\chi}{3T}\right]\right]e^2TM_t-\frac{e^2T}{8\pi}\left[M_\phi+M_\chi+\frac{M_t^2-2M_\phi^2-2M_\chi^2}{M_\phi+M_\chi+M_t}\right]+\frac{c_\Omega}{\pi}\frac{M_t^3}{T}\ , \quad (3.8b)$$

$$M_l^2\simeq e^2\phi^2+\frac{e^2}{3}T^2-e^2\left[\frac{c_\Omega}{3\pi}-\frac{c_{\Theta l}}{8\pi}\right]TM_l-\frac{e^2T}{4\pi}(M_\phi+M_\chi)+\frac{c_\Omega}{\pi}\frac{M_l^3}{T}\ , \quad (3.8c)$$

where

$$a\equiv[(c_\Omega+c_{\Theta t}+c_{\Theta l})/(4\pi)]+\lambda[(4c_\Omega+9)/(72\pi e^2)]\ .$$

Note that in these calculations one can assume that $eT<\phi<T$ because, as already discussed in the Introduction, the daisy and superdaisy resummed effective potential is expected to give a meaningful approximation of the full effective potential only for $\phi>eT$, and the daisy and superdaisy diagrams should be negligible when $\phi>T$. This observation combined with Eqs. (3.8) justifies *a posteriori* the high-temperature approximations which I performed; in fact, the effective masses in (3.8) are of order eT or $e\phi$, so that $M_{\phi,\chi,t,l}/T<1$ when $eT<\phi<T$.

Equations (3.8) also indicate that $M_t<M_{\phi,\chi,l}$ because only (3.8b) does not contain a contribution of order e^2T^2; moreover, (3.8a) indicates that $M_\phi^2\simeq M_\chi^2$ (they differ only at order $\lambda\phi^2$). Therefore one can make the approximations

$$\ln\left[\frac{M_\phi+M_\chi+M_t}{3T}\right]\simeq\ln\left[\frac{M_\phi+M_\chi}{3T}\right]+\frac{M_t}{M_\phi+M_\chi}\ ,\quad \frac{2M_\phi^2+2M_\chi^2-M_t^2}{M_\phi+M_\chi+M_t}\simeq M_\phi+M_\chi-M_t\ , \quad (3.9)$$

which allows one to rewrite the gap equations (3.8) as

$$M_{\phi(\chi)}^2\simeq m_{\phi(\chi)}^2+\left[\frac{\lambda}{18}+\frac{e^2}{4}\right]T^2-\left[a-\frac{1}{4\pi}-\frac{1}{\pi}\ln\left[\frac{M_\phi+M_\chi}{3T}\right]\right]e^2TM_{\phi(\chi)}$$
$$-\frac{\lambda}{24\pi}TM_{\chi(\phi)}-\frac{e^2}{4\pi}TM_l+\frac{c_\Omega}{\pi}\frac{M_{\phi(\chi)}^3}{T}\ , \quad (3.10a)$$

$$M_t^2 \simeq e^2\phi^2 + \left[\frac{c_{\Theta t}}{16\pi} - \frac{1}{8\pi} - \frac{1}{4\pi}\ln\left[\frac{M_\phi+M_\chi}{3T}\right]\right] e^2 T M_t + \frac{c_\Omega}{\pi}\frac{M_t^3}{T} \;, \tag{3.10b}$$

$$M_l^2 \simeq e^2\phi^2 + \frac{e^2}{3}T^2 - e^2\left[\frac{c_\Omega}{3\pi} - \frac{c_{\Theta l}}{8\pi}\right] T M_l - \frac{e^2 T}{4\pi}(M_\phi+M_\chi) + \frac{c_\Omega}{\pi}\frac{M_l^3}{T} \;. \tag{3.10c}$$

Using (2.5), (3.4)–(3.7), (3.9), and (3.10), one finds that the desired Rayleigh-Ritz and high-temperature approximation of the daisy and superdaisy resummed finite temperature effective potential for the Abelian Higgs model is given by (omitting unimportant ϕ-independent contributions)

$$\begin{aligned} V_T^{\rm res}(\phi,\{M_0\}) \simeq & -\frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{T^2}{24}(m_\phi^2 + m_\chi^2 + 3e^2\phi^2) - \frac{T}{12\pi}(M_{\phi,0}^3 + M_{\chi,0}^3 + 2M_{t,0}^3 + M_{l,0}^3) \\ & + \frac{e^2T^2(2M_{\phi,0}^2 + 2M_{\chi,0}^2 - M_{t,0}^2)}{32\pi^2}\ln\left[\frac{M_{\phi,0}+M_{\chi,0}}{3T}\right] - \frac{e^2T^2}{32\pi^2}M_{l,0}(M_{\phi,0}+M_{\chi,0}) \\ & + \left[\frac{e^2}{32} - \frac{\lambda}{192}\right]\frac{T^2}{\pi^2}M_{\phi,0}M_{\chi,0} - \frac{e^2T^2}{\pi^2}\left[\frac{c_\Omega}{48} + \frac{c_{\Theta t}}{128}\right]M_{l,0}^2 + \tilde{a}\frac{e^2T^2}{\pi^2}(M_{\phi,0}^2 + M_{\chi,0}^2) \\ & + \frac{c_{\Theta t}}{128\pi^2}e^2T^2M_{t,0}^2 + \frac{3c_\Omega}{32\pi^2}(M_{\phi,0}^4 + M_{\chi,0}^4 + 2M_{t,0}^4 + M_{l,0}^4) \;, \end{aligned} \tag{3.11}$$

where $M_{\phi,0}$, $M_{\chi,0}$, $M_{t,0}$, and $M_{l,0}$ are the solutions of the gap equations (3.10) and $\tilde{a} \equiv \frac{1}{32} - (c_{\Theta t} + c_{\Theta l})/64 + (\lambda/e^2)(c_\Omega/288 - \frac{1}{128})$.

Note that in (3.11) all terms linear in the effective masses have canceled out [27]. In the literature there has been an extensive debate on the possibility that the resummation of the daisy and superdaisy diagrams might induce contributions to the finite temperature effective potential which are linear in the effective masses. Using the general form of the Rayleigh-Ritz approximation with momentum-independent effective masses, one can show [28] that a cancellation of the linear terms always occurs.

The Rayleigh-Ritz variational approximation and the high-temperature expansion have led to a great simplification of the very complicated expression of $V_T^{\rm res}$ obtained in Eqs. (2.7) and (2.9). Still, as given by (3.10) and (3.11), $V_T^{\rm res}$ cannot be calculated analytically, but now the required numerical evaluations are very simple. A first estimate of the effective potential in (3.11) can be obtained analytically by using the approximations

$$M_{\phi,0} \simeq M_{\chi,0} \;, \quad \ln(M_{\phi,0}^2/T^2) \simeq \ln(\lambda/18 + e^2/4) \;,$$
$$M_{\phi,0} \to (e^2T^2/4 + \lambda T^2/18)^{1/2}$$

in (3.10c) and $M_{l,0} \to (e^2T^2/3)^{1/2}$ in (3.10a). These approximations are reliable when $\phi \ll T$ [see (3.10)].

IV. DISCUSSION AND CONCLUSIONS

As discussed in the Introduction, the daisy and superdaisy improved effective potential $V_T^{\rm res}(\phi)$ should give a reliable approximation of the full effective potential $V_T(\phi)$ in the high-temperature limit, but when $T \sim T_c$ one expects $V_T^{\rm res}(\phi) \simeq V_T(\phi)$ only for $\phi > eT_c$. This implies that the evaluation of $V_T^{\rm res}(\phi)$ is sufficient in the investigation of strongly first order phase transitions [if at the critical temperature the asymmetric minimum ϕ_c is greater than eT_c, $V_T^{\rm res}(\phi)$ should reliably determine the existence and the position of ϕ_c], but $V_T^{\rm res}(\phi)$ cannot be used to discriminate between a second order and a very weakly first order phase transition. Recent models of baryogenesis at the electroweak phase transition require that this transition be strongly first order, and therefore the evaluation of $V_T^{\rm res}(\phi)$ for the standard electroweak model should lead to a reliable test of these models.

Also note that, because I neglected the contributions to $V_{T(2)}$ from diagrams of order e^4 and higher (see Fig. 5), the result (3.11) gives a reliable approximation of the full effective potential up to order e^3. The terms in (3.11) of order e^4 and higher can be used to estimate how important the complete higher order correction should be expected to be and to verify whether or not the differences between the Rayleigh-Ritz approximation performed in the preceding section and the $\mathcal{M}_x(k) \simeq \mathcal{M}_x(0)$ approximation used in the literature can result in relevantly different physical predictions.

The comparison of the Rayleigh-Ritz approximation with the $\mathcal{M}_x(k) \simeq \mathcal{M}_x(0)$ approximation is indeed the next topic that I want to comment on. In addition to the conceptual issues which I already discussed, probably the clearest shortcoming of the $\mathcal{M}_x(k) \simeq \mathcal{M}_x(0)$ approximation is that it completely ignores the contribution of the self-energy diagram in Fig. 4(c); in fact, this diagram vanishes in the zero external momentum limit. In the Rayleigh-Ritz approximation, the contributions to the

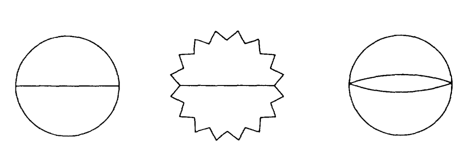

FIG. 5. Examples of diagrams which are neglected in the present lowest order in the couplings approximation of $V_{T(2)}$.

gap equations which correspond to the diagram in Fig. 4(c) are the ones coming from $\delta V_2^{(c)}/\delta M_{\phi(\chi)}$; Eqs. (3.6) and (3.8a) show that these contributions are certainly non-negligible, and, in particular, for small e the term of order

$$e^2 T M_\phi \ln(M_{\phi(\chi)}/T) \sim e^2 \ln(e) T M_{\phi(\chi)}$$

is the dominant term linear in $M_{\phi(\chi)}$ in (3.8a). Even for the self-energy diagram in Fig. 4(g), which does not vanish in the zero external momentum limit, there are significant quantitative differences between the Rayleigh-Ritz approximation and the $\mathcal{M}_x(k) \simeq \mathcal{M}_x(0)$ approximation. Clearly, the Rayleigh-Ritz approximation accounts for some average effect of the dependence on the external momentum of the diagrams in Figs. 4(c) and 4(g), which is the origin of the momentum dependence of the exact effective masses $\mathcal{M}_x(k)$.

These differences in the structure of the gap equations between the Rayleigh-Ritz approximation and the $\mathcal{M}_x(k) \simeq \mathcal{M}_x(0)$ approximation result in important quantitative differences at the level of the effective potential. In Fig. 6 it is shown that the two approximations lead to relevantly different predictions for the shape of the effective potential at the critical temperature; in particular, the position of the asymmetric minimum ϕ_c differs by 20–30%. Some physical phenomena at the phase transitions depend critically on the value of ϕ_c (for example, the rate of baryon number violation at the electroweak phase transition is exponentially sensitive to ϕ_c), and therefore Fig. 6 shows that an accurate analysis of these phenomena requires a proper handling of the momentum dependence of the effective masses.

Concerning the nature of the phase transition of the Abelian Higgs model, it is useful to note (again using $eT \ll \phi \ll T$) that Eqs. (3.10) and (3.11) imply that (i) in addition to the expected contributions involving even powers of ϕ, there is a negative contribution of order $e^3 T\phi^3$ to the effective potential, which comes from the TM_t^3 term, and (ii) there is no contribution of order $e^3 T^3 \phi$. These observations indicate [5,6] that there is a critical temperature T_c at which $V_T^{\text{res}}(\phi)$ has two degenerate minima. From Eq. (3.11) it is also easy to realize that $\phi_c > eT_c$ when $e^2/\lambda \gg 1$, which indicates that at least in these hypotheses the Abelian Higgs model has a first order phase transition. This is the same qualitative conclusion that one reaches using the one-loop improved effective potential evaluated in Ref. [9]; indeed, it is easy to verify that Eq. (3.11) agrees with the improved one-loop result to order e^3. However, it is important to observe that at the critical temperature the complete V_T^{res}, including the contributions of order e^4 and higher, is relevantly different from the one-loop improved effective potential (see Fig. 7); in particular, the position of the asymmetric minimum receives a significant correction. This should suggest some caution concerning the accuracy of recent predictions which have been obtained using the improved one-loop approximation, such as the lower limit on the Higgs boson mass for successful baryogenesis at the one-Higgs-boson-doublet electroweak phase transition.

I conclude by emphasizing that the techniques discussed in this analysis of the Abelian Higgs model clearly apply to any gauge theory. Apart from the obvious complication of having to deal with a richer particle content, even in the study of more complex gauge theories the major elements of difficulty will still be (i) the momentum dependence of the effective masses which is introduced by nonlocal self-energy diagrams of the type generally represented in Fig. 2(a) and (ii) the evaluation of diagrams of the type generally represented in Fig. 2(b). Using the composite operator method, one can do better than the daisy and superdaisy resummation by going beyond the present lowest order in the coupling approximation of $V_T(2)$. Also, my Rayleigh-Ritz approximation can be improved [so that the effect of the momentum dependence of the exact effective masses $\mathcal{M}_x(k)$ is estimated even more accurately] by using more elaborate versions of the parameter-dependent expression for G; for example, one can make the substitutions $M_x^2 \to M_x^2 + Y_x \mathbf{k}^2$ in Eq. (3.2) and vary not only the M_x's, but also the additional parameters Y_x.

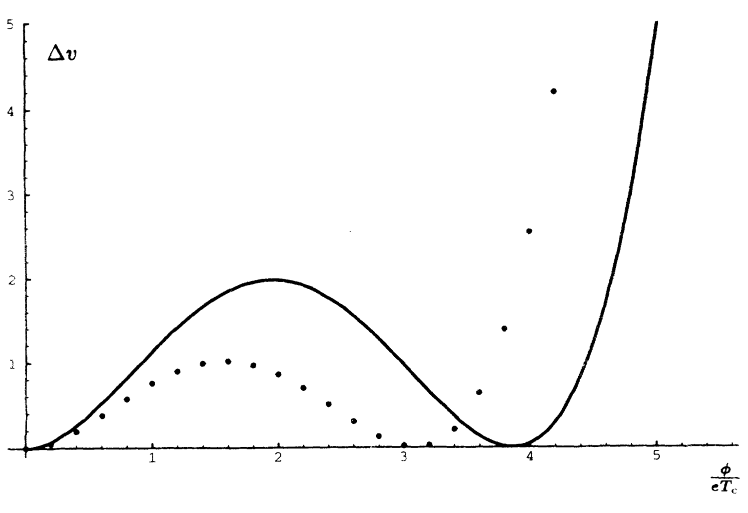

FIG. 6. Plot at $T = T_c$, $e = 0.2$, and $\lambda = 0.01$ of $\Delta v(\phi) \equiv 10^7 \,\text{Re}[V_T(\phi) - V_T(0)]/T_c^4$ vs ϕ/eT_c for the Rayleigh-Ritz approximation (solid curve) and for the $\mathcal{M}_x(k) \simeq \mathcal{M}_x(0)$ approximation (dotted curve).

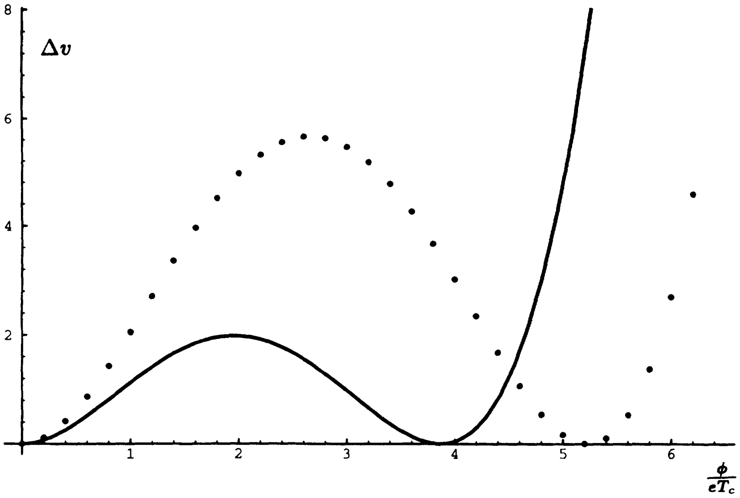

FIG. 7. Plot at $T=T_c$, $e=0.2$, and $\lambda=0.01$ of $\Delta v(\phi)\equiv 10^7\,\mathrm{Re}[V_T(\phi)-V_T(0)]/T_c^4$ vs ϕ/eT_c for the Rayleigh-Ritz approximation of the effective potential, described in Eq. (3.11) (solid curve) and for the effective potential which is obtained by neglecting all the contributions of order e^4 and higher in Eq. (3.11) (dotted curve).

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APPENDIX: HIGH-TEMPERATURE APPROXIMATION OF $V_2^{(c)}$

This appendix is devoted to the high-temperature approximation of $V_2^{(c)}(G(\{M\}))\equiv V_2^{(c)}(M_\phi,M_\chi,M_t,M_l)$, which is used in the calculations of Sec. III. As can be inferred from Secs. II and III, $V_2^{(c)}(M_\phi,M_\chi,M_t,M_l)$ is given by

$$V_2^{(c)}(M_\phi,M_\chi,M_t,M_l)=\frac{e^2}{2}\sum\!\!\!\!\!\!\!\int_p \sum\!\!\!\!\!\!\!\int_q \frac{(2p+q)^\mu(2p+q)^\nu}{[M_\phi^2-p^2][M_\chi^2-(p+q)^2]}\left[\frac{t_{\mu\nu}(q)}{M_t^2-q^2}+\frac{l_{\mu\nu}(q)}{M_l^2-q^2}\right]. \tag{A1}$$

The high-temperature approximation of $V_2^{(c)}(M_\phi,M_\chi,M_t,M_l)$ has been evalauted in Ref. [17] in the limit $M_t=M_l$. The result of [17] is based on the observation that

$$\begin{aligned}V_2^{(c)}(M_\phi,M_\chi,M_\gamma,M_\gamma)=-\frac{e^2}{2}\Bigg\{&\Omega(M_\gamma)[\Omega(M_\phi)+\Omega(M_\chi)]-\Omega(M_\phi)\Omega(M_\chi)+(M_\gamma^2-2M_\phi^2-2M_\chi^2)\Theta(M_\phi,M_\chi,M_\gamma)\\&+\frac{M_\phi^2-M_\chi^2}{M_\gamma^2}[\Omega(M_\gamma)-\Omega(0)][\Omega(M_\phi)-\Omega(M_\chi)]+\frac{(M_\phi^2-M_\chi^2)^2}{M_\gamma^2}[\Theta(M_\phi,M_\chi,M_\gamma)\\&-\Theta(M_\phi,M_\chi,0)]\Bigg\}, \end{aligned}\tag{A2}$$

where $\Omega(y)$ has been defined in (3.3) and

$$\Theta(x,y,z)\equiv\sum\!\!\!\!\!\!\!\int_p \sum\!\!\!\!\!\!\!\int_q \frac{1}{[x^2-q^2][y^2-p^2][z^2-(p+q)^2]}. \tag{A3}$$

Equation (A2), combined with the high-temperature approximation of $\Theta(x,y,z)$ which was obtained in Refs. [17,26] and the corresponding approximation of $\Omega(y)$ which is reported in (3.3), leads to the high-temperature approximation of $V_2^{(c)}(M_\phi,M_\chi,M_\gamma,M_\gamma)$. For the analysis of Sec. III, it is sufficient to consider the limit $(M_\phi-M_\chi)^2\ll(M_\phi+M_\chi)^2$; in this hypothesis one finds

$$V_2^{(c)}(M_\phi,M_\chi,M_\gamma,M_\gamma)\simeq e^2\left\{-\frac{T^4}{288}+\frac{T^3M_\gamma}{48\pi}+\frac{T^2}{32\pi^2}(M_\phi M_\chi-M_\gamma M_\phi-M_\gamma M_\chi)+\frac{T^2}{32\pi^2}(M_\gamma^2-2M_\phi^2-2M_\chi^2)\ln\left[\frac{M_\gamma+M_\phi+M_\chi}{3T}\right]-\left[\frac{c_\Omega}{96\pi^2}+\frac{c_\Theta}{128\pi^2}\right]T^2M_\gamma^2+\frac{c_\Theta}{64\pi^2}T^2(M_\phi^2+M_\chi^2)\right\}, \tag{A4}$$

where the ϕ-independent quantity c_Θ is the analogue of the c_Ω of Eq. (3.3) and is expressed in Ref. [26] as a combination of integrals which can be evalauted numerically. [Note that in (A4), following the strategy outlined in Sec. III, I omitted some ultraviolet divergent contributions.]

The calculation of $V_2^{(c)}(M_\phi,M_\chi,M_t,M_l)$ is more difficult than the one of $V_2^{(c)}(M_\phi,M_\chi,M_\gamma,M_\gamma)$ because only when $M_t=M_l$ the integrand in (A1) takes the simple form which leads to the relation (A2). Rather than proceeding to its explicit evaluation, I shall obtain the high-temperature approximation of $V_2^{(c)}(M_\phi,M_\chi,M_t,M_l)$ by using as a guide the result (A4) and by exploiting the fact that $V_2^{(c)}(M_\phi,M_\chi,M_\gamma,0)$ and $V_2^{(c)}(M_\phi,M_\chi,0,M_\gamma)$ have very different analytic properties. [N.B. Eq. (A1) implies that $V_2^{(c)}(M_\phi,M_\chi,M_t,M_l)=V_2^{(c)}(M_\phi,M_\chi,M_t,0)+V_2^{(c)}(M_\phi,M_\chi,0,M_l)$.]

The first important observation is that terms of the type $T^2M_\gamma M_{\phi(\chi)}$ can only be present in $V_2^{(c)}(M_\phi,M_\chi,M_\gamma,0)$. In fact, this type of nonanalytic term [29] originates from the peculiar infrared properties of the diagram in Fig. 3(c) and it gets a contribution only from the $p_4=0$, $q_4=0$ term of the sums in (A1). By looking at the structure of $l_{\mu\nu}$, one sees that $V_2^{(c)}(M_\phi,M_\chi,0,M_\gamma)$ could not include such contributions.

Similarly, it is easy to realize that terms of the type of the one in Eq. (A4) which is proportional to $\ln(M_\gamma+M_\phi+M_\chi)$ (which is also nonanalytic) can only contribute to $V_2^{(c)}(M_\phi,M_\chi,M_\gamma,0)$. This is best seen by tracing back the calculation of Refs. [17,26] and noting that also this term comes from the part of the gauge boson propagator which is proportional to $t_{\mu\nu}$.

Finally, one can note that the terms $e^2T^3M_\gamma/48\pi$ and $-e^2c_\Omega T^2M_\gamma^2/96\pi^2$ in (A4) originate from the following contribution to $V_2^{(c)}(M_\phi,M_\chi,M_\gamma,M_\gamma)$:

$$-\frac{e^2}{2}\sum\!\!\!\!\!\!\int_q\frac{1}{M_\gamma^2-q^2}\left[\sum\!\!\!\!\!\!\int_p 4\frac{p^2-(pq)^2/q^2}{[M_\phi^2-p^2][M_\chi^2-(p+q)^2]}\right]_{q=0;M_{\phi(\chi)}=0}\simeq e^2\left[-\frac{T^4}{144}+\frac{T^3M_\gamma}{48\pi}-\frac{c_\Omega T^2M_\gamma^2}{96\pi^2}\right]. \tag{A5}$$

In the expression of $V_2^{(c)}(M_\phi,M_\chi,M_t,M_l)$, the contribution (A5) should be substituted by

$$-\frac{e^2}{2}\left\{\sum\!\!\!\!\!\!\int_q\frac{1}{M_t^2-q^2}\left[\sum\!\!\!\!\!\!\int_p\frac{4p^\mu p^\nu t_{\mu\nu}(q)}{[M_\phi^2-p^2][M_\chi^2-(p+q)^2]}\right]_{q=0;M_{\phi(\chi)}=0}\times\sum\!\!\!\!\!\!\int_q\frac{1}{M_l^2-q^2}\left[\sum\!\!\!\!\!\!\int_p\frac{4p^\mu p^\nu l_{\mu\nu}(q)}{[M_\phi^2-p^2][M_\chi^2-(p+q)^2]}\right]_{q=0;M_{\phi(\chi)}=0}\right\}$$
$$\simeq e^2\left[-\frac{T^4}{144}+\frac{T^3(2M_t-M_l)}{48\pi}-\frac{c_\Omega T^2(2M_t^2-M_l^2)}{96\pi^2}\right]. \tag{A6}$$

These observations, combined with (A4), lead to the conclusion that

$$V_2^{(c)}(M_\phi,M_\chi,M_t,M_l)\simeq-\frac{e^2T^2}{288}+\frac{e^2T^3(2M_t-M_l)}{48\pi}+e^2T^2\frac{(M_t^2-2M_\phi^2-2M_\chi^2)}{32\pi^2}\ln\left[\frac{M_t+M_\phi+M_\chi}{3T}\right]+\frac{e^2T^2(M_\phi M_\chi-M_tM_\phi-M_tM_\chi)}{32\pi^2}+\frac{e^2T^2(c_{\Theta t}+c_{\Theta l})(M_\phi^2+M_\chi^2)}{64\pi^2}-\frac{e^2T^2[(3c_{\Theta t}+8c_\Omega)M_t^2+(3c_{\Theta l}-4c_\Omega)M_l^2]}{384\pi^2}, \tag{A7}$$

where the ϕ-independent quantities $c_{\Theta t}$ and $c_{\Theta l}$ verify $c_{\Theta t}+c_{\Theta l}=c_\Theta$ and, like c_Θ, are given by combinations of integrals which can be evaluated numerically [26].

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