PHYSICAL REVIEW D VOLUME 49, NUMBER 6 15 MARCH 1994

# Gravity and the domain-wall problem

Balram Rai* and Goran Senjanović†
*International Centre for Theoretical Physics, Trieste 34100, Italy*
(Received 22 January 1993)

It is well known that the spontaneous breaking of discrete symmetries may lead to a conflict with big-bang cosmology. This is due to the formation of domain walls which give an unacceptable contribution to the energy density of the Universe. On the other hand it is expected that gravity breaks global symmetries explicitly. In this work we propose that this could provide a natural solution to the domain-wall problem.

PACS number(s): 98.80.Cq, 04.60.+n, 11.30.Er, 11.30.Qc

## I. INTRODUCTION

The idea that the discrete symmetries, especially the fundamental ones such as time reversal and parity invariance, could be broken spontaneously is old and appealing. Twenty years ago, in a pioneering work, Lee [1] suggested that $CP$ (or $T$ reversal) may be broken spontaneously at the cost of adding another Higgs doublet to the standard model. It was shown later that parity can also be broken spontaneously [2]. Yet another interesting example is a discrete symmetry needed in the two Higgs doublet model to ensure natural flavor conservation [3]. However, in a beautiful paper Zel'dovich, Kobzarev, and Okun [4] investigated the cosmological consequences of spontaneous breaking of a discrete symmetry with the conclusion that this would be in conflict with cosmology. Kibble [5] and other authors [6] concluded the same, although with a slightly different analysis. Since then the particle physics models with spontaneously broken discrete symmetry have been considered unacceptable (see below for some possible exceptions). In this paper we propose that the possibility of gravity leading to violation of global (discrete) symmetries may provide an attractive way out of this problem. We find it only natural that the space-time dynamical effects of gravity would play this role for the space-time discrete symmetries. Although the arguments for violation of global discrete symmetry, as we will describe in the following, are speculative, the point we wish to emphasize is that even expectedly tiny effects of gravity may suffice. In what follows we first describe the above mentioned problem and then discuss how gravity may possibly provide a solution.

## II. THE PROBLEM

The spontaneous breaking of a discrete symmetry leads to the existence of domain walls, i.e., kinklike classical solutions separating different degenerate vacua. This can be illustrated by a simple example of a real scalar field $\phi$ with a symmetry $D$, $\phi \to -\phi$, and a Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\lambda}{4}(\phi^2 - v^2)^2, \tag{1}$$

where $\lambda$ is taken to be positive in order to ensure that the energy is bounded from below. The symmetry $D$ is spontaneously broken, since the minimum of the potential is given by $\phi_{\rm vac} = \pm v$. It is easy to show that this theory has a static domain-wall-like classical solution, say, for a wall lying in the $x$-$y$ plane,

$$\phi_{\rm cl}(z) = v\,\tanh(\sqrt{\lambda}vz), \tag{2}$$

which clearly connects vacua $-v$ and $+v$ as $z$ traverses from $-\infty$ to $\infty$. The field is different from its vacuum values in a region of width $\delta_W \approx (\sqrt{\lambda}v)^{-1}$, determined by the scale of symmetry breaking $v$. The scale of symmetry breaking also determines the energy density per unit area $\sigma \approx v^3$.

Now, at high temperatures the potential $V(\phi)$ receives an additional contribution

$$\delta V = \frac{\lambda}{12}T^2\phi^2. \tag{3}$$

Since $\delta V$ is necessarily positive, for sufficiently high temperature $T > T_c \approx v$ the symmetry is restored [7]. In the standard big-bang cosmological scenario, the field $\phi$ is expected to undergo a phase transition as the Universe cools down from $T > T_c$ to $T < T_c$. For separations larger than the correlation length or horizon size around the time of phase transition, the field $\phi$ will independently take either of its vacuum values, giving rise to corresponding domains and domain walls. To understand the generic features of this system of domain walls one may consider the following idealized problem. Imagine splitting space into cubes of the size of the correlation length. And, say, the probability for the field to take a particular vacuum value in a given cube is $p$ $(= \frac{1}{N}$, where $N$ is the number of degenerate vacua). The nature of the domain structure obtained, a domain being a set of connected cubes carrying the same vacuum value of the field,

*Present address: Theory Group, Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 400005, India. E-mail: balram@theory.tifr.res.in
†Electronic address: goran@itsictp.bitnet, vxicp1::gorans

 

is a basic question in percolation theory [8]. The main result we will need here is that if $p$ is greater than a certain critical value $p_c$, then apart from finite size domains there will be one and only one domain of "infinite" size formed. For $p$ less than $p_c$ there will not be any infinite size domain. Generically, i.e., considering even other lattices than just cubic, $p_c$ happens to be less than 0.5. In the example of the real scalar field we have been considering, $p$ equals 0.5. Thus there would be an "infinite" domain corresponding to each vacuum and therefore an infinite wall of a very complicated topology. Of course, there will also be a network of finite size walls. The question one is interested in is the energy density contribution of this domain wall system as it evolves. The following crude analysis addresses this question [6].

The dynamics of the wall is mainly decided by the force per unit area $f_T$ due to tension and frictional force $f_F$ with the surrounding medium. Since the tension in the wall is proportional to the energy per unit area $\sigma$, we get $f_T \sim \frac{\sigma}{R}$ for a radius of curvature scale $R$. Moreover, $f_F \sim sT^4$ where $s$ is the speed of the wall and $T$ the temperature of the system [9]. When the speed of the wall has stabilized we have

$$sT^4 = \frac{\sigma}{R}. \qquad (4)$$

Thus, the typical time $t_R \sim \frac{R}{s}$ taken by a wall portion of radius scale $R$ to straighten out would be

$$t_R \sim \frac{R^2T^4}{\sigma} \approx \frac{R^2}{G\sigma t^2}. \qquad (5)$$

Making the plausible assumption that if $t_R < t$, the wall curvature on the scale $R$ would be smoothed out by time $t$, we get that the scale on which the wall is smooth grows as

$$R(t) \approx (G\sigma)^{\frac{1}{2}} t^{\frac{3}{2}}. \qquad (6)$$

The energy density contribution $\rho_W$ to the Universe by walls goes as

$$\rho_W \sim \frac{\sigma R^2}{R^3} \sim \left(\frac{\sigma}{Gt^3}\right)^{\frac{1}{2}}. \qquad (7)$$

Therefore $\rho_W$ becomes comparable to the energy density $\rho \sim \frac{1}{Gt^2}$ of the Universe in the radiation-dominated era around $t_0 \sim \frac{1}{G\sigma}$. Thus domain walls would significantly alter the evolution of the Universe after $t_0$.

Now, the discrete symmetries relevant for particle physics typically tend to be broken at mass scales above the weak scale $M_W \approx 100$ Gev, giving $t_0 \leq 10^8$ sec. This would be certainly true of $P$ and $T$ $(CP)$, the examples we are most interested in. Hence from above considerations one would conclude that discrete symmetries cannot be broken spontaneously.

There are two possible ways out of this impasse. One possibility is that, even for low scales of symmetry breaking, the phase transition that would have restored the symmetry does not take place, at least not until high enough temperatures to allow inflation to dilute the energy density in the domain walls. This in general requires a more complicated Higgs structure than the minimal one and realistic examples have been discussed in the literature [10].

Another way out [6], the one of interest to us in this paper, is the possibility that a spontaneously broken discrete symmetry is also explicitly broken by a small amount, which lifts the degeneracy of the two vacua $+v$ and $-v$. For instance, in our example we could imagine adding to the Lagrangian a small $\phi^3$ term which, obviously, breaks $\phi \to -\phi$ symmetry. It should not come as a surprise that this effect may provide a mechanism for the decay of domain walls; after all now there is a unique vacuum. Crudely, the way it works is as follows [6]. Lifting of the degeneracy of the two vacua by an amount $\epsilon$ gives a pressure difference of the same amount between the two sides of the wall, with a tendency to push the wall into false vacuum region. Thus the dynamics of the wall is now going to be decided by a combination of the pressure $\epsilon$, forces $f_T$ due to tension, and $f_F$ due to friction mentioned before. Clearly at some point the forces due to friction and tension become small, compared to the pressure difference $\epsilon$, because they are proportional to $T^4 \sim \frac{1}{Gt^2}$ and $\frac{\sigma}{R} \sim (\frac{\sigma}{Gt^3})^{\frac{1}{2}}$, respectively. At that stage the pressure difference will dominate and cause shrinking of the false vacuum. Actually it is difficult to find out precisely when the false vacuum region, and hence the domain walls, disappears. However, it may be crudely estimated to be the time when the pressure $\epsilon$ exceeds the force due to tension, or when it exceeds the force due to friction for a relativistically moving wall so as to dominate the dynamics. For either requirement to be satisfied before $t_0 \sim \frac{1}{G\sigma}$, the time when the wall contribution $\rho_W$ would have become comparable to the energy density of the Universe, one obtains

$$\epsilon \geq G\sigma^2 \sim \frac{v^6}{M_{\rm Pl}^2}. \qquad (8)$$

Of course, it is not very appealing to introduce *ad hoc* the symmetry-breaking terms just in order to eliminate the domain-wall problem. Ideally, we would prefer these effects to be a natural consequence of underlying theory. An interesting example recently discussed in the literature [11] is that of a discrete symmetry explicitly broken due to instanton induced effects.

## III. ROLE OF GRAVITY

In this paper we invoke the possibility that the needed mechanism for explicit breaking may be naturally provided by gravity. One expects that gravity, because of black-hole physics, may not respect global symmetries, both continuous and discrete ones. This expectation is motivated by two important points: First, the "no-hair" theorems of black-hole physics that state that stationary black holes are completely characterized by quantum numbers associated with long-range gauge fields, and second, that the Hawking radiation in evaporation of black hole is thermal [12]. Now, consider a process in which a certain amount of normal matter, which is in a state that is "odd" under the discrete symmetry in consideration, collapses under gravity to form a black hole. Because of no hair being associated with the global discrete symmetry, any information regarding it is lost to observers outside the black hole. Hawking radiation from the black hole being thermal in nature does not carry any information about the internal state of the black hole either. Of course, it is not certain what the properties of evaporation would be at late stages when the semiclassical approximation breaks down. Unless for some reason the processes at late stages cause the final system to have same global discrete charges as those of the initial normal matter that collapsed, the symmetry will stand violated.

We wish to note that from very different viewpoints there have been discussions in the literature regarding the possibility of $CP$ or $T$ violation in the context of gravity. Ashtekar, Balachandran, and Jo have discussed [13] the $CP$ "problem" in the framework of the canonical quantization of gravity. In the Ashtekar variables reformulation of general relativity, the canonical variables of the theory resemble those of Yang-Mills theory. This allows for the discussion of $\theta$ sectors in a canonical quantization framework for Yang-Mills theory to be taken over to the gravity case. Moreover, an analogue of the $\theta F F^d$ term in the action can also be given.

Another set of observations that interest us particularly was made by Penrose about $T$ asymmetry [14]. He contends, based on arguments related to the Bekenstein-Hawking formula, that there must be some as yet unknown theory of quantum gravity that is time asymmetric. We recall here only an easy to state, interesting point from his discussion. Corresponding to a solution of Einstein's equation describing the collapse of normal matter to form a black hole that stays forever (classically), there would be a time-reversed solution, a white hole, describing an explosion of a singularity into normal matter. Now, according to the Bekenstein-Hawking formula the surface area $A$ of a black hole's horizon is proportional to its intrinsic entropy $S$:

$$S = \frac{kc^3}{4\hbar G} A. \tag{9}$$

In classical processes, the area is nondecreasing with time and hence entropy. If an intrinsic entropy is associated with a white hole, it is again expected to be proportional to the area of its horizon. Time reversal of the area principle would give that this area, and hence the corresponding entropy, can never increase, an antithermodynamic behavior. Especially it would be a strongly antithermodynamic behavior by the white hole when it ejects a substantial amount of matter. This is among the reasons that led Penrose to consider the possibility that there may be a general principle that rules out the existence of white holes and would therefore be time asymmetric.

With the premise, in view of the preceding discussion, that gravity may violate a global discrete symmetry we wish to explore its consequences for the domain-wall problem.

The crucial issue one faces in implementing this kind of approach is the determination of the precise form of these symmetry-breaking terms. In the present-day understanding of gravity it does not seem possible to give a satisfactory answer to this question. The strategy followed in the literature [15], which we also adopt here, in analogous discussions has been to write all the higher dimensional effective operators allowed by gauge invariance of an underlying theory. Of course, one could take a point of view that the dimension four and lower terms may also break the discrete symmetry. We take no stand on this point. In any case, even if this happens it can only help in destabilizing the domain walls due to increased symmetry breaking. Our point is that even the tiny higher dimensional symmetry-breaking terms, cut off by powers of the Planck mass, may be sufficient in solving the domain-wall problem.

To illustrate how this works, we turn again to our simple example of a real scalar field. The effective higher dimensional operators would take the form

$$V_{\text{eff}} = \frac{C_5}{M_{\text{Pl}}}\phi^5 + \frac{C_6}{M_{\text{Pl}}^2}\phi^6 + \cdots . \tag{10}$$

Obviously, all the terms with odd powers of $\phi$ break the discrete symmetry $\phi \to -\phi$. (We should mention that while discussing difficulties with a certain compactification scheme in superstring theory, it was remarked by Ellis *et al.* [16] that a specific discrete symmetry in their model may be broken by terms inversely proportional to $M_{\text{Pl}}$. But they note further that massless modes of string theory would not induce such terms and that the massive modes could be the only possible source of such effects. However, as we have been pursuing here, the nonperturbative effects are expected to be a natural source of breaking of global discrete symmetries, independent of whether string theory turns out to be a correct theory of gravity. Furthermore, as we have emphasized before, we feel that these effects should be taken seriously as a possible solution to the domain-wall problem associated with the fundamental discrete symmetries of nature such as parity or time-reversal invariance.)

In estimating precisely the amount of symmetry breaking we would need to know the values of coefficients $C_n$. Barring some unexpected conspiracy, in the following we will assume that $C_n$ will be of order 1 as they are dimensionless. Moreover, it is understood that the scale of spontaneous symmetry breaking $v$ lies below the Planck scale. With all this in mind, the energy-density split between the two vacua would be $\approx \frac{C_5}{M_{\text{Pl}}}v^5$. This is obviously much bigger than the amount $\frac{v^6}{M_{\text{Pl}}^2}$ needed to make the domain walls disappear. This holds true as long as $C_5 \gg \frac{v}{M_{\text{Pl}}}$, which for lower scales of symmetry breaking gets to be more and more plausible. For example, if $v = M_{\text{GUT}} \approx 10^{15}$ GeV, we need $C_5 > 10^{-4}$, whereas for $v = M_W \approx 100$ GeV, we only need $C_5 > 10^{-17}$. In general, if a leading operator in Eq. (10) is of dimension $n$, the condition for disappearance of domain walls is $C_n > (\frac{M_{\text{Pl}}}{v})^{n-6}$. Clearly $n = 6$ is the critical value, since for $n > 6$ $C_n$ would have to be unreasonably large whereas for $n = 6$ $C_6{\sim}1$ may suffice.

## IV. EXAMPLES OF $T$ AND $P$

An important issue that remains to be discussed is what happens in realistic examples of discrete symmetries associated with gauge theories of strong and electroweak interactions. This clearly must be case dependent. Here we study two discrete symmetries of central importance: $CP(T)$ and $P$.

$CP$ or $T$ invariance: The simplest and most popular example of the spontaneous breaking of $CP$ is the two Higgs doublet version of the standard model. One simply imagines all the couplings in the Lagrangian to be real and looks for minimum of energy with nontrivial phases. By an $\mathrm{SU}(2)\times\mathrm{U}(1)$ gauge transformation we can write the most general solution in the form

$$\langle\phi_1\rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle\phi_2\rangle = \begin{pmatrix} 0 \\ v_2 e^{i\delta} \end{pmatrix}. \tag{11}$$

where $v_i, \delta$ are real numbers. Through the terms of the type $\phi_1^\dagger\phi_2$ and $(\phi_1^\dagger\phi_2)^2$ in the potential, the theory knows of the phase $\delta$ and in general we can write

$$V = A + B\cos\delta + C\cos 2\delta. \tag{12}$$

For a range of parameters there is a solution with nonvanishing and nontrivial phase $\delta$, giving a spontaneous breaking of $CP$ and hence the domain-wall problem.

If we now follow our logic of expanding the effective higher degree potential induced by gravity in inverse powers of $M_{\mathrm{Pl}}$, keeping gauge invariance intact, the leading term is of dimension six:

$$\Delta V = \frac{c_{ij}}{M_{\mathrm{Pl}}^2}(\phi_i^\dagger\phi_j)^3 + \mathrm{H.c.} + \text{higher-order terms}. \tag{13}$$

Again, the possibility of breaking of $CP$ by gravity would be reflected in complex coefficients $c_{ij}$. As we observed before, only for the dimensionless coefficient $C_6 \gtrsim 1$ the domain walls would be unstable. Thus, the understanding of the detailed consequences of gravity turns out to be crucial in such a low energy issue as $CP$ violation and we believe that any hint in this direction is extremely important. The situation would change dramatically if one is willing to introduce a complex singlet into the theory and attribute to its complex vacuum expectation value the source of spontaneous $CP$ violation. Now clearly the leading operator, analogous to our simple example of a real scalar field, is of dimension five.

Parity: The simplest models which describe spontaneous breaking of parity are based on $\mathrm{SU}(2)_L\times \mathrm{SU}(2)_R\times\mathrm{U}(1)$ gauge group with $g_L = g_R$ gauge couplings [17]. The breaking of parity is attributed to the large mass for the right-handed gauge boson: $M_{W_R} \gg M_{W_L}$. By introducing Higgs multiplets $\phi_L$ and $\phi_R$ which are nontrivial representations (doublets or triplets) under $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$, respectively, the above can be achieved through spontaneous breaking of parity: $|\langle\phi_R\rangle| \gg |\langle\phi_L\rangle|$ (where $P : \phi_L \leftrightarrow \phi_R$). The analysis of gravitational effects parallels completely our discussion on $CP$. If $P$ is broken through the vacuum expectation value of the nonsinglet then the leading $P$-breaking operators must be of dimension six or larger, and the fate of domain walls will depend on the dimensionless parameters which characterize breaking terms. Another possibility is the existence of parity odd singlet $\sigma$ ($P : \sigma \to -\sigma$), and $P$ being broken through $\langle\sigma\rangle \neq 0$. In this case, the leading $P$-breaking gravity-induced term could be of dimension five.

## V. CONCLUSION

In this paper we have taken seriously the possibility that gravity breaks global discrete symmetries. If so, this could be a natural source of instability of domain walls. We find it curious that such a mechanism to solve the domain-wall problem does not involve any other scales than $M_{\mathrm{Pl}}$ and the scale of spontaneous symmetry breaking $v$, already present in the theory. Although our main examples were of $CP(T)$ and $P$, by no means do we wish to imply that the effects of gravity have to stop there. As it emerged from our discussion, whether gravity can play the desired role depends on the lowest degree induced effective operator.

We are well aware of the speculative nature of our suggestion so that one cannot yet be certain that it is actually realized. However, we hope to have conveyed the necessity of further understanding of gravity before one can claim the existence of the domain-wall problem.

## ACKNOWLEDGMENTS

It is a pleasure to thank G. Dvali, R. Mohapatra, and V. Rubakov for discussions and encouragement.


[1] T.D. Lee, Phys. Rev. D **8**, 1226 (1973).

[2] R.N. Mohapatra and G. Senjanović, Phys. Rev. D **12**, 1502 (1975).

[3] S.L. Glashow and S. Weinberg, Phys. Rev. D **15**, 1958 (1977); E.A. Paschos, *ibid.* **15**, 1964 (1977).

[4] Ya.B. Zel'dovich, I. Yu. Kobzarev, and L.B. Okun, Zh. Eksp. Teor. Fiz. **67**, 3 (1974) [Sov. Phys. JETP **40**, 1 (1974)].

[5] T.W.B. Kibble, J. Phys. A **9**, 1387 (1976).

[6] For a review and further references, see A. Vilenkin, Phys. Rep. **121**, 263 (1985).

[7] D.A. Kirzhnitz and A.D. Linde, Phys. Lett. **72B**, 471 (1972); L. Dolan and R. Jackiw, Phys. Rev. D **9**, 3320 (1974); S. Weinberg, *ibid.* **9**, 3357 (1974).

[8] D. Stauffer, Phys. Rep. **54**, 1 (1979).

[9] A.E. Everett, Phys. Rev. D **10**, 316 (1974).

[10] R.N. Mohapatra and G. Senjanović, Phys. Lett. **89B**, 57 (1979); Phys. Rev. D **20**, 3390 (1979). The possibility that symmetries are not necessarily restored at higher temperature was also known to S. Weinberg; see his paper

in Ref. [7].

[11] J. Preskill, S.P. Trivedi, F. Wilczek, and M. Wise, Nucl. Phys. **B363**, 207 (1991).

[12] See, e.g., R.M. Wald, *General Relativity* (University of Chicago Press, Chicago, 1984), and references therein.

[13] A. Ashtekar, A.P. Balachandran, and S. Jo, Int. J. Mod. Phys. A **4**, 1493 (1989).

[14] R. Penrose, in *General Relativity: An Einstein Centenary Survey*, edited by S.W. Hawking and W. Israel (Cambridge University Press, Cambridge, England, 1979), p. 581.

[15] There are many papers on this subject. See, for example, an early work of R. Barbieri, J. Ellis, and M.K. Gaillard, Phys. Lett. **90B**, 249 (1980). Also, for a recent revival of interest in the Planck scale effects on global symmetries, see M. Kamionkowski and J. March-Russell, Phys. Lett. B **282**, 137 (1992); Phys. Rev. Lett. **69**, 1485 (1992); R. Holman *et al.*, Phys. Lett. B **282**, 132 (1992); Phys. Rev. Lett. **69**, 1489 (1992); S. Barr and D. Seckel, Phys. Rev. D **46**, 539 (1992); S. Ghigna, M. Lusiquoli, and M. Roncadelli, Phys. Lett. B **283**, 1978 (1992).

[16] J. Ellis, K. Enqvist, D.V. Nanopoulos, K. Olive, M. Quiros, and F. Zwirner, Phys. Lett. B **176**, 403 (1986). We became aware of this reference after completion of our work and wish to thank Dennis Comelli and Massimo Pietroni for bringing it to our attention.

[17] J.C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974); R.N. Mohapatra and J.C. Pati, *ibid.* **11**, 566 (1975); **11**, 2558 (1975); see also Ref. [2].