

Nucleation of quark matter bubbles in neutron stars

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The thermal nucleation of quark matter bubbles inside neutron stars is examined for various temperatures which the star may realistically encounter during its lifetime. It is found that for a bag constant less than a critical value a very large part of the star will be converted into the quark phase within a fraction of a second. Depending on the equation of state for neutron star matter and strange quark matter, all or some of the outer parts of the star may subsequently be converted by a slower burning or a detonation.

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I. INTRODUCTION

If pulsars or the central parts of these can be made of quark matter rather than neutrons [1], does this then apply to all or just some of them, and when and how does the phase transformation take place?

According to some investigations [2], the transformation occurs during the supernova explosion. In this scenario, the released binding energy is what makes the supernova succeed in the first place, supplying the final "push" which seems to lack in most of the computer simulations of the events.

Another model for strange star formation (in the context of absolutely stable strange quark matter) was introduced by Baym *et al.* [3], describing the transformation as a slow burning (combustion) rather than a violent event connected with a supernova detonation.

Regardless of the way in which the transformation occurs, an initial seed of quark matter is needed to start it. Alcock *et al.* [4] suggested a variety of possibilities ranging from pressure induced conversion via two-flavor quark matter to collision with either highly energetic neutrinos or smaller lumps of strange quark matter. However, they did not provide a rate for conversion of neutron stars and thus left it as an open question, whether every compact object is a strange star, or whether they are rare objects, even if quark matter formation is energetically favorable. It has also been suggested [5] that strange matter seeds (in the case of quark matter stability) from strange star collisions or of cosmological origin would trigger the transformation of all neutron stars, in which case the thermal nucleation would be of relevance to the case of unstable quark matter only [6].

Other possibilities are that shock waves in the supernova trigger the conversion; a seed could be produced by nonthermal quantum fluctuations, or a phase transition could be started around impurities. We are not able to estimate the probability of either method, but would expect at least quantum fluctuations to be less likely than the thermal nucleation process discussed below.

An estimate for quark matter formed via thermally induced fluctuations was given by Horvath *et al.* [7], using typical numbers for various physical quantities. It was

found that all neutron stars are converted into strange stars (assuming stable strange quark matter) if the temperature at some time during the stars lifetime has exceeded 2–3 MeV.

In the following, we choose an approach similar to the one in Ref. [7], but with an extra term in the expression for the surface energy of quark matter, and with the two phases treated in a more self-consistent way. Unlike most of the approaches mentioned above [1–5, 7], we will be considering the formation of both strange stars (for absolutely stable strange matter) and hybrid stars, where strange matter is formed only in the central regions due to the high pressure. First, we will deal with some general aspects of nucleation (Sec. II). For pedagogical purposes, Sec. III treats the problem using a simplified model with the hadron phase being a free degenerate neutron gas and the quark phase a bag model with only u and d quarks. A more detailed model for the neutron star is presented in Sec. IV, followed by some concluding remarks (Sec. V).

II. BUBBLE FORMATION

The free energy involved in the formation of a spherical quark bubble of radius R is given by

$$F = -\frac{4\pi}{3}R^3\Delta P + 4\pi\sigma R^2 + 8\pi\gamma R + N_q\Delta\mu, \quad (1)$$

where $\Delta P = P_q - P_h$ is the pressure difference, $\sigma = \sigma_q + \sigma_h$ the surface tension, $\gamma = \gamma_q - \gamma_h$ the curvature energy density, and $\Delta\mu = \mu_q - \mu_h$ the difference in chemical potential. N_q is the total baryon number in the quark bubble. The indices h and q denote the hadron and quark phase, respectively. (We set $T = 0$ in the thermodynamical expressions; since $T \ll \mu$ throughout, this only leads to minor errors.) Defining $C = C(\mu_q) \equiv \Delta P - n_q\Delta\mu$ and $b \equiv 2\gamma C/\sigma^2$ the free energy has a maximum at the critical radius

$$r_c = \frac{\sigma}{C} \left(1 + \sqrt{1+b}\right), \quad (2)$$

and the corresponding free energy

$$W_c \equiv F(r_c) = \frac{4\pi}{3} \frac{\sigma^3}{C^2} [2 + 2(1+b)^{3/2} + 3b] \quad (3)$$

is the work required to form a bubble of this radius which is the smallest bubble capable of growing.

It is a standard assumption in the theory of bubble nucleation in first-order phase transitions that bubbles form at this particular radius at a rate given by [8]

$$\mathcal{R} \approx T^4 \exp(-W_c/T). \quad (4)$$

It is possible to show that W_c has a minimum as a function of μ_q at $\mu_q = \mu_h$. This gives a maximum in the rate for bubble formation, and because of the exponential in Eq. (4) one may safely assume that nucleation happens in chemical equilibrium. Thus, C reduces to $C = \Delta P$.

Throughout this paper, we will consider the strange quark to be massless, in which case chemical equilibrium would give $\mu_u = \mu_d = \mu_s = \frac{1}{3}\mu_h$. However, equilibrium is established only on a weak interaction time scale, whereas the formation of bubbles is governed by the strong interaction and is many orders of magnitude faster. So, although we have chemical equilibrium between the two phases this is not so between the three quark flavors. Instead, flavor must be conserved during the phase transition.

A consequence of having massless quarks is that $\sigma_q \equiv 0$ [9], and since σ_h is negligible compared to γ , Eqs. (2) and (3) reduce to

$$r_c = \sqrt{\frac{2\gamma}{\Delta P}} \quad (5)$$

and

$$W_c = \frac{16\pi}{3} \sqrt{\frac{2\gamma^3}{\Delta P}}. \quad (6)$$

III. PURE NEUTRON GAS

Before considering a more realistic equation of state it is instructive to study the nucleation of a pure neutron gas into quarks. The quark bubbles formed consist of u and d quarks in the ratio 1:2; only later weak interactions may change the composition to an energetically more favorable state. Thus quark chemical potentials are related by $\mu_d = 2^{1/3}\mu_u$, and $\mu_n = \mu_u + 2\mu_d = (1 + 2^{4/3})\mu_u$, assuming chemical equilibrium across the phase boundary.

The pressure difference is given by

$$\Delta P = P_{ud} - P_n = \frac{\mu_u^4 + \mu_d^4}{4\pi^2} - B - P_n \quad (7)$$

and the curvature energy coefficient [10]

$$\gamma = \frac{\mu_u^2 + \mu_d^2}{8\pi^2}. \quad (8)$$

For the question in hand we choose the simplest possible equation of state for the neutron gas, namely, that of a zero temperature, nonrelativistic degenerate Fermi gas, where

$$P_n = \frac{(\mu_n^2 - m_n^2)^{5/2}}{15\pi^2 m_n} \quad (9)$$

and the baryon density

$$n_B = \frac{(\mu_n^2 - m_n^2)^{3/2}}{3\pi^2}. \quad (10)$$

A necessary condition for bubble nucleation is that $\Delta P > 0$. This leads to an upper limit on the bag constant, B_{\max} , from Eq. (7) as illustrated in Fig. 1 (the corresponding limit for the Bethe-Johnson equation of state is shown for comparison; it is seen to be very similar).

Also shown in Fig. 1 is the limit on the bag constant below which bubble nucleation takes place at rates exceeding $1 \text{ km}^{-3} \text{ Gyr}^{-1}$ and $1 \text{ m}^{-3} \text{ s}^{-1}$, respectively, for temperatures of 1, 2, 3, and 10 MeV (B_{\max} can be considered as the limit for infinite temperature). One notes that the possibility of bubble nucleation is fairly insensitive to the temperature as soon as T exceeds a few MeV, whereas thermally induced bubble nucleation is impossible for $T < 2$ MeV [it is known from the stability of ordinary nuclei against decay into quark matter that $B \geq (145 \text{ MeV})^4$]. This confirms the estimate in [7]. The range of bag constants for which a hot neutron star may transform into quark matter is thus roughly $145 \text{ MeV} \leq B^{1/4} \leq 152 \text{ MeV}$.

An interesting feature of the solution is the existence of a maximum in B as a function of n_B . This indicates that

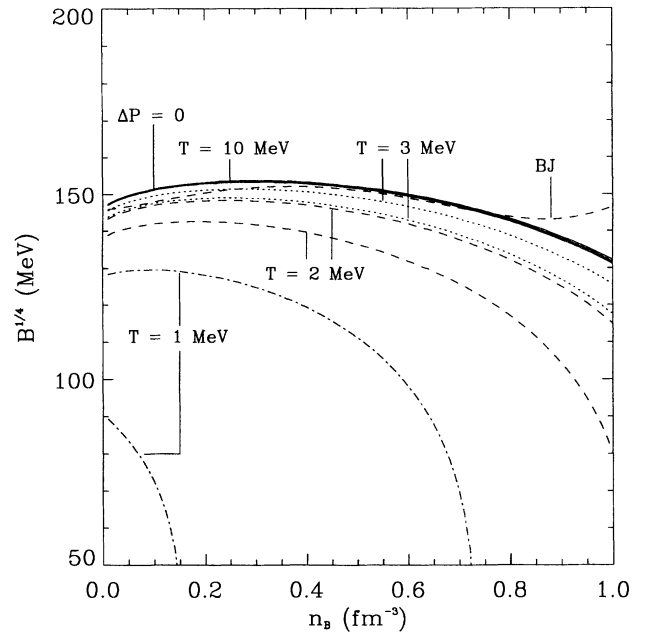


FIG. 1. The upper limits on the bag constant allowing boiling of a nonrelativistic neutron gas into ud quark matter as a function of the baryon density in the hadron phase is shown for different nucleation rates and temperatures. The upper curve for each temperature corresponds to a rate of one nucleation per km^3 per Gyr; the lower to one per m^3 per s. As comparison is shown the $\Delta P = 0$ line for a Bethe-Johnson (BJ) equation of state. This is seen to deviate only for very large densities.

there is a *range in densities* for which boiling can take place for a fixed value of B . If the central density of a neutron star exceeds the upper limit of n_B permitting boiling, it may therefore happen that boiling is initiated off center, with potentially interesting consequences for supernova energetics, neutrino fluxes, γ bursters, etc. This may be explained as follows: With increasing density one has an increase in μ , and since $P_{ud} \sim \mu^4$ while $P_n \sim \mu^5$, higher densities must imply still lower B_{\max} in order to satisfy the condition, $\Delta P = 0$. That the maximum does not occur at the same density for all temperatures is due to the μ dependence of γ . This effect, however, occurs at much higher n_B and B for the more realistic equation of state discussed below, so we shall not pursue the issue further.

IV. MEAN FIELD APPROXIMATION

In the following, a mean field model is used to describe the equation of state in the hadron phase. The model includes the light hadron octet and is described in detail in Refs. [11–13]. For the quark phase, the only difference from Sec. III is that strange quarks are introduced in accordance with Eq. (11) below. This leads to additional contributions to Eqs. (7) and (8).

Integrating the Oppenheimer-Volkoff equation gives the structure of the initial neutron star: pressure, chemical potential, and number density of each hadron species as a function of radius.

For a flavor-conserving phase transformation, the relative number densities $r_i \equiv n_i/n_B$ are given by

$$\begin{pmatrix} r_u \\ r_d \\ r_s \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 & 2 & 1 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 2 & 2 \end{pmatrix} \begin{pmatrix} r_p \\ r_n \\ r_\Lambda \\ r_{\Sigma^+} \\ r_{\Sigma^0} \\ r_{\Sigma^-} \\ r_{\Xi^0} \\ r_{\Xi^-} \end{pmatrix}. \quad (11)$$

The absolute number densities are then given from equality between the chemical potential per baryon in the two phases.

Doing a calculation similar to the one already done for a pure neutron star, one obtains a qualitatively similar result with the limits being independent of the nucleation rate considered for $T \geq 10$ MeV but with slightly higher values of B_{\max} , and with the maximum in B at higher densities (Fig. 2).

Again, the temperature variation of the B limits show that there is some critical temperature $T_{\text{crit}} \approx 3$ MeV below which the conversion takes place only for extremely low values of the bag constant. This corresponds to the limit found by Horvath *et al.* [7] although the method for obtaining it is quite different. As mentioned in the previous section, increasing the value of T brings the limits closer to the $\Delta P = 0$ curve.

A typical temperature for a newborn neutron star is about 10 MeV at the center with off-center temperatures

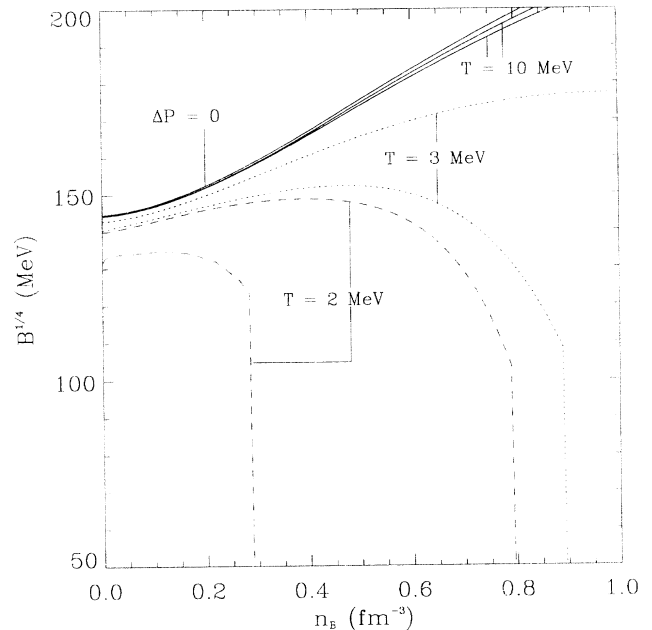


FIG. 2. Limits on the bag constant in the mean field approximation. Notation as in Fig. 1. (The model used corresponds to $x=0.6$ in Ref. [13].)

up to 20–30 MeV, but since we have just seen that the exact value is unimportant in the high- T limit, it is enough to consider only $T = 10$ MeV (this agrees well with the choice of $\tau = 1$ s, since a typical cooling time for newborn neutron stars is of this order [14]). The constraints on a $1.4 M_\odot$ neutron star is shown in Fig. 3. For $B^{1/4} \leq 165$ MeV the center of the star is converted, and for lower values of the bag constant still larger fractions of the

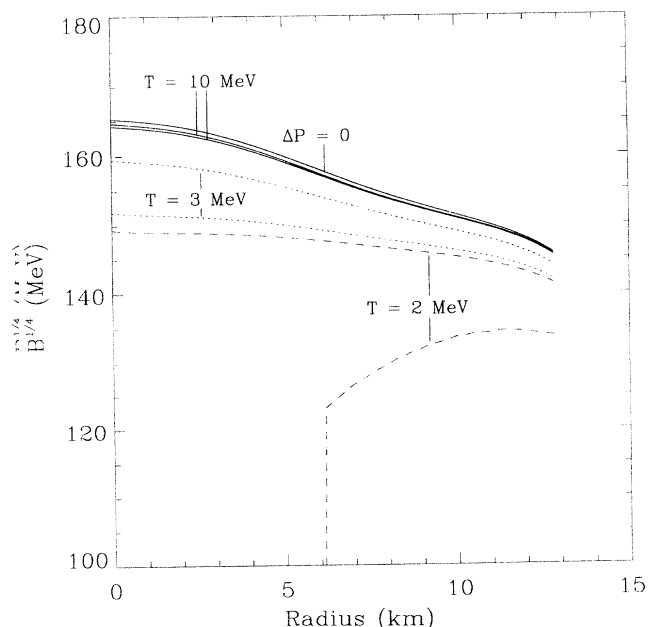


FIG. 3. Limits on the bag constant for a $1.4 M_\odot$ neutron star. Notation as in Fig. 1.

star will undergo the phase transition in the initial stage of its existence. At $B^{1/4} = 145$ MeV the entire star is transformed but for this value ordinary nuclei (e.g., ^{56}Fe) would probably be unstable [15]. For different choices of the hyperon coupling constants, the limits on B_{max} at the center vary from $(162 \text{ MeV})^4$ to $(170 \text{ MeV})^4$, with high B_{max} corresponding to low values of the hyperon-to-hadron coupling constant. Thus a weaker coupling of the hyperons tends to destabilize hadronic matter. (The coupling constants mentioned above are chosen to fit the Λ binding energy, as described in Ref. [13].) At larger radii, the relative density of strange baryons decreases and the limits converge toward the ones shown in Fig. 3.

So far we have only considered $M = 1.4M_{\odot}$ since observational data seem to suggest this as the most typical value. The effect of varying the mass is displayed in Fig. 4. For low masses, the star consists almost only of neutrons, even at the center, and thus the deviation from $(145 \text{ MeV})^4$ seen here is due only to a nonzero pressure, and it appears that in this case, a conversion via thermal bubble nucleation is less likely to take place. At masses near the maximum mass $B_{\text{max}}^{1/4} \rightarrow 200$ MeV so here all but unrealistically high B gives a transition into quark matter.

Turning again to the case $M = 1.4M_{\odot}$, it could be interesting to examine the effects of having a nonzero strong coupling constant α_s . For massless quarks, this gives corrections to the number density and pressure which to first order is given by $n_q = n_{q,0}(1 - \frac{2\alpha_s}{\pi})$ and $P_q = P_{q,0}(1 - \frac{2\alpha_s}{\pi})$, where 0 denotes the values for $\alpha_s = 0$.

Unfortunately, the corresponding expression for $\gamma(\alpha_s)$ [and in the case of massive quarks also $\sigma(\alpha_s)$] is presently unknown and thus we cannot correctly estimate the effect on the nucleation rate. What we can do, however, is to examine the $\Delta P = 0$ curve, and thereby obtain limits

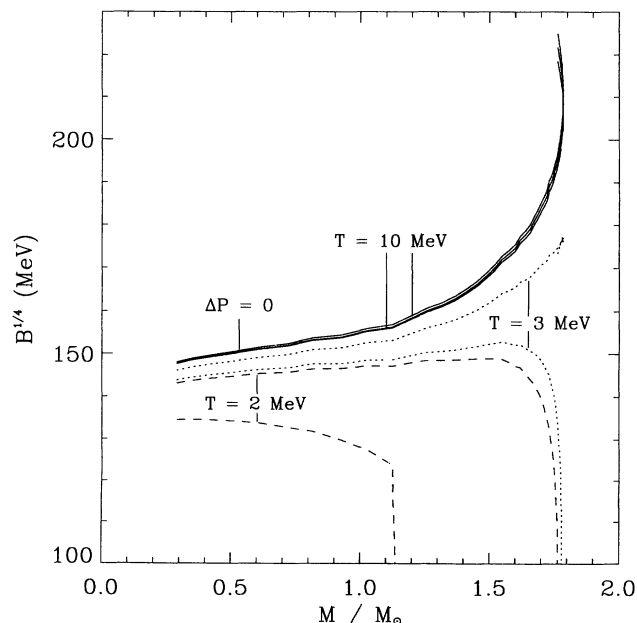


FIG. 4. Critical bag constants in the center of neutron stars as a function of stellar mass. Notation as in Fig. 1.

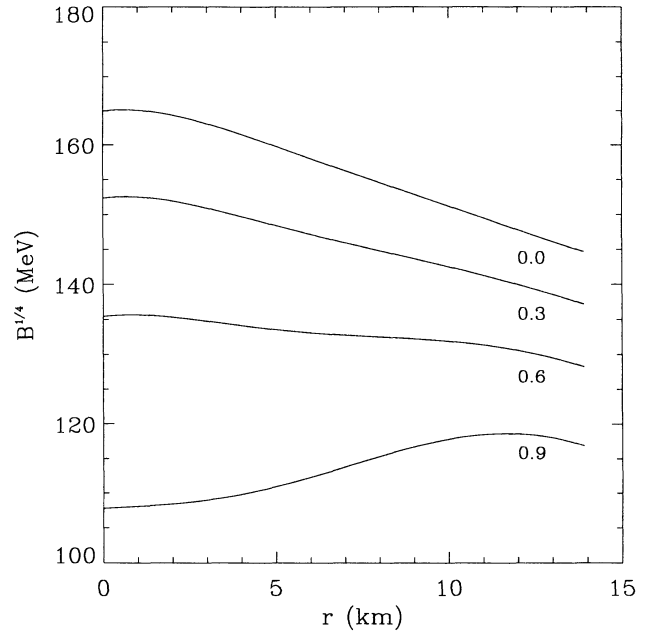


FIG. 5. Curves representing pressure equilibrium between the two phases for various values of the strong coupling constant, α_s .

on \mathcal{R} , since this is bound to be below the curve of equal pressure.

It is seen that although both B_{crit} and the limiting bag constant for ud quark matter stability (B_{crit} taken at $r = R$ corresponding to zero external pressure [16]) are decreasing functions of α_s , one has a narrowing of the relevant interval in B (Fig. 5), and above $\alpha_s \approx 0.6$ conversion of neutron stars seems to be ruled out. (Similar results were obtained in Ref. [17]).

The effect on T_{crit} can only be guessed at, but assuming that $W_c \sim (1 - \frac{2\alpha_s}{\pi})^a$, where $|a| \leq 1 - 2$ and $\alpha_s \leq 0.6$, T_{crit} should be correct within a factor of 2, and thus the temperatures accompanying supernova explosions should still be enough to ensure conversion into strange stars (or hybrid stars) provided that the bag constant is below B_{crit} .

V. DISCUSSION AND CONCLUSION

What we have seen is that if the bag constant lies in the interval where three-flavor but not two-flavor quark matter is stable at zero pressure and temperature ($145 \text{ MeV} \leq B^{1/4} \leq 163 \text{ MeV}$, see Ref. [15]) then all or parts of a neutron star will be converted into strange matter during the first seconds of its existence (but note the cautionary remark in [14]). The rest will then be transformed either by a slow burning on a time scale of a few seconds to a few minutes [3] or by a detonation [2]. For bag constants above the stability interval, we have seen that a partial transformation is still possible, but since this seems to depend heavily on the exact equation of state, one should be careful before drawing any definite conclusions.

Since a large fraction of the star is converted on a rel-

atively short time scale, the released energy may well provide a significant contribution to the total energy of a supernova (cf. Ref. [2]).

Another investigation by Krivoruchenko and Martemyanov [17], taking $\Delta P = 0$ as a criterion for a possible transformation into strange stars by a flavor conserving phase transition, have found similar results. This is an effect not as much of equivalence of the methods used, but rather of the fact that the high temperatures of newborn neutron stars together with the exponential in Eq. (4) causes the rate to be insensitive to T .

Another interesting feature is that if a star is born with a mass that for a given bag constant is too small for the conversion to take place even in the center, then accretion from a neighboring star, leading to a higher mass for the neutron star and thus in principle a larger transition probability, will only lead to a phase transition via thermal nucleation *if* at the same time the neutron matter is heated to at least 2–5 MeV by the energy released by the accretion process or by other mechanisms, such as capture of high-energy neutrinos. (It is very unlikely that a significant mass can be transferred during the first second or so.) Thus, one may conclude that if no mechanism for a significant heating of the star can be found, the initial mass uniquely determines the future of the star

if one has to rely solely on thermal nucleation. (Other possible mechanisms that may lead to a transformation were mentioned in Sec. I.)

By introducing a nonzero α_s , a narrowing in the interesting range for B was seen; both as an absolute measure and in terms of the fraction of the interval where uds quark matter is stable at zero pressure.

In this work we have ignored the effect of the mass of the strange quark, which corresponds to a somewhat inadequate treatment of the equation of state, surface and curvature effects. However, even in the center of the star no more than 2–4% of the quarks in the hadrons are s quarks, and thus the effects during bubble nucleation are very small. As far as the surface energy is concerned, a more important effect comes from taking $\sigma_{\text{hadron}} \simeq (30 \text{ MeV})^3$ (from typical nuclear mass formulas), but even here it turns out that $4\pi r_c^2 \sigma \ll 8\pi r_c \gamma$, so that inclusion of such a term would not change our conclusions.

ACKNOWLEDGMENTS

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