

## Two-body nonleptonic decays of charmed mesons

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Two-body nonleptonic decays of charmed mesons are studied on the basis of a simple pole-dominance model involving the vector, pseudoscalar, and axial-vector meson poles.

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### I. INTRODUCTION

In this paper, we calculate the two-body nonleptonic decays of charmed mesons using the pole-dominance model. A preliminary analysis of this type was first attempted [1] several years ago, but we believe that a systematic investigation of the type presented here has not been carried out. The nonleptonic weak decay of a pseudoscalar meson into two pseudoscalar mesons is parity violating, so in such decays the pole-dominance model would involve only vector-meson poles. It is well known [2] that this idea of vector dominance provides a successful description of the  $K \rightarrow \pi\pi$  decays. Recently we have also discussed [3] the decay of  $D$  mesons into two pseudoscalars ( $PP$ ) through vector dominance and there, too, we find reasonable agreement with experiments. In the present work we extend the analysis to the general two-body nonleptonic decays of  $D$  mesons to include the decay to a pseudoscalar and a vector ( $PV$ ) meson and to two vector ( $VV$ ) mesons. Since we now deal with parity-conserving decays as well, we have to go beyond vector dominance. In this work, we extend our consideration to include also the pseudoscalar and the axial-vector-meson poles. For simplicity, we consider only the lowest lying poles [4].

Our work has no direct relationship to the extensively studied factorization model [5] which has generally been quite successful in describing the decays except for those that proceed only through what are known as the annihilation diagrams. The pole-dominance model, on the other hand, has enjoyed many successes in low-energy hadron phenomenology, and it would be interesting to see how it describes charm decay [6].

### II. PRELIMINARIES

For nonleptonic decay of charm, the effective weak Hamiltonian may be written in the current-current form as [7]

$$H_W = \frac{G_F}{\sqrt{2}} [a_1 (\bar{u}d')_\mu (\bar{s}'c)_\mu + a_2 (\bar{s}'d')_\mu (\bar{u}c)_\mu], \quad (1)$$

where  $(\bar{q}^\alpha q_\beta)_\mu$  are color-singlet  $V - A$  currents

$$(\bar{q}^\alpha q_\beta)_\mu = i\bar{q}^\alpha \gamma_\mu (1 + \gamma_5) q_\beta = (V_\mu)_\beta^\alpha + (A_\mu)_\beta^\alpha. \quad (2)$$

The indices  $\alpha, \beta = 1, 2, 3, 4$  represent the flavor  $SU(4)$  in-

trices and  $a_1, a_2$  are real coefficients which we treat as phenomenological parameters. The primed quark fields are related to the unprimed ones by the usual Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. For the nonleptonic decays of  $D$  mesons into two mesons, the Hamiltonian (1) leads to two main classes of quark-model diagrams, the spectator and the annihilation diagrams shown in Figs. 1(a) and 1(b), respectively. We ignore the Penguin-type contributions. It is well known that the annihilation-diagram contribution in the quark model is helicity suppressed.

In the pole-dominance model, we take the currents in  $H_W$  to be the hadronic currents given by the field current identities ( $\alpha, \beta = 1, 2, \dots, 4$ )

$$\begin{aligned} (V_\mu)_\beta^\alpha &= \sqrt{2} g_V (\phi_\mu)_\beta^\alpha, \\ (A_\mu)_\beta^\alpha &= \sqrt{2} f_P \partial_\mu P_\beta^\alpha + \sqrt{2} g_A (a_\mu)_\beta^\alpha, \end{aligned} \quad (3)$$

where  $(\phi_\mu)_\beta^\alpha$ ,  $P_\beta^\alpha$ , and  $(a_\mu)_\beta^\alpha$  are the field operators of the vector, the pseudoscalar and the axial-vector mesons of  $SU(4)$ , respectively, and  $g_V$ ,  $f_P$ , and  $g_A$  are the corresponding decay constants. The nonleptonic weak interaction can then be represented by a two-meson vertex which can be read off from (1) upon substituting (3).

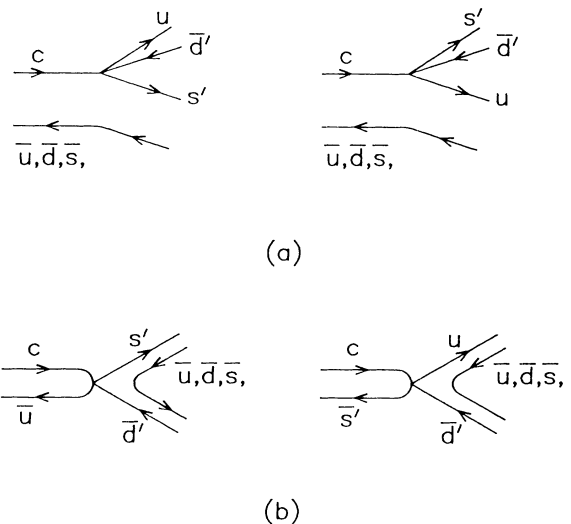


FIG. 1. Quark-model diagrams for the nonleptonic decay of  $D$  into two mesons. 1(a) describe the spectator diagrams and 1(b) the annihilation diagrams.

### III. CALCULATION OF THE DECAY AMPLITUDE

The Feynman diagrams for the two-body decays  $D \rightarrow PP$ ,  $PV$ , and  $VV$  are displayed in Figs. 2–4. In these figures, the dark dot represents a weak interaction vertex and the open circle a strong vertex. Also the dotted lines, solid lines, and wavy lines represent pseudoscalar, vector, and axial-vector mesons respectively. Note that the figures 2(b), 3(c), 3(e), 4(b), and 4(c) are the analogues of the annihilation diagrams in the quark model, the remaining figures corresponding to the spectator diagrams. Only the vector-meson pole contributes to the parity-violating decay  $D \rightarrow PP$  as shown in Fig. 2. Also, since  $D \rightarrow PV$  is a parity conserving decay, a diagram such as Fig. 3(b) where the pseudoscalar pole is replaced by a vector pole cannot contribute.

We have several types of strong vertices appearing in the diagrams. These include the  $VPP$ ,  $VVP$ ,  $VVV$ , and the  $VPA$  vertices. Unfortunately, many of these couplings are not known. For numerical work, we choose to relate these couplings by a suitable flavor symmetry. In fact the  $VVP$  and  $VVV$  couplings can be related to the  $VPP$  couplings through an extended spin-SU(4) symmetry. Accordingly, we take the strong Hamiltonian as

$$H_{\text{str}} = ig \text{Tr} \left[ \phi_\mu P \vec{\partial}_\mu P - \frac{2}{M} \epsilon_{\mu\nu\lambda\rho} P \partial_\mu \phi_\nu \partial_\lambda \phi_\rho + \frac{2}{3} F_{\mu\nu} \phi_\mu \phi_\nu - \frac{2}{9M^2} F_{\mu\nu} F_{\nu\lambda} F_{\lambda\mu} \right]. \quad (4)$$

This is an obvious generalization of the Sakita-Wali interaction Hamiltonian [8] which is relevant to flavor SU(3). In (4), the trace is over the SU(4) multiplets,  $g$  is the coupling constant and  $M$  represents a mass scale. We shall identify  $M$  with the mass of the decaying particle, and take  $g$  to be the  $\rho\pi\pi$  coupling determined from the  $\rho$  width. The  $VPA$  interaction is not contained in (4) and will be taken to be [9]

$$H_{\text{str}}(VPA) = ig_s M \text{Tr} \{ \phi_\mu [P, a_\mu] \} + \frac{ig_d}{M} \text{Tr} \{ \partial_\mu \phi_\nu [P, \partial_\nu a_\mu] \}. \quad (5)$$

Despite some attempts in the past [9,10], the coupling constants  $g_s$  and  $g_d$  have not been determined successfully. In our present analysis, we will treat these as unknown parameters.

It should be mentioned that experience based on flavor SU(3) symmetry indicates that coupling constants are

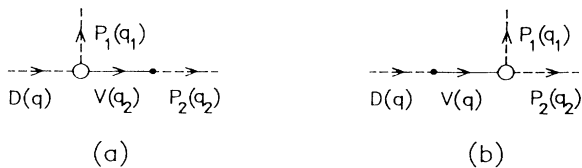


FIG. 2. Feynman diagrams for the decay  $D \rightarrow P_1 P_2$ .

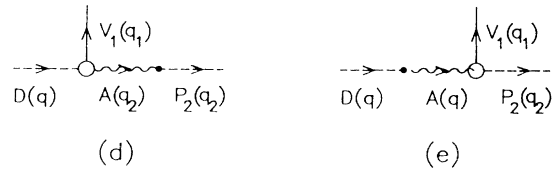
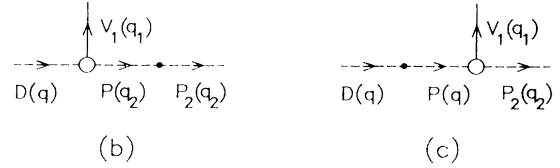
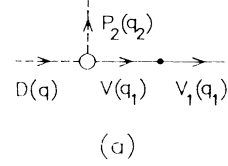


FIG. 3. Feynman diagrams for the decay  $D \rightarrow V_1 P_2$ .

better described by group symmetry than the masses. Furthermore, the vector-meson couplings may be expected to have a somewhat special status embodied through universality. At the present time, however, we can only hope that couplings related by the larger SU(4) symmetry are not unreasonable. Future experiments involving the strong decays of charmed vector or axial vector mesons would shed light on this issue.

The weak vertices in Figs. 2–4 involve the decay constants  $g_V$ ,  $f_P$ , and  $g_A$  for various particles. Once again only a few of these are known from experiments. Now, spectral function sum rules [11,12] based on asymptotic flavor symmetries have been quite successful in the past, and we may use them here to obtain suitable relations. From asymptotic SU(4) symmetry, it is easy to derive [13] the result that  $g_V/m_V$  would be the same for all vector

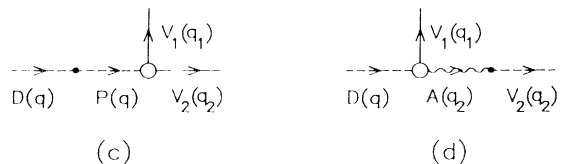
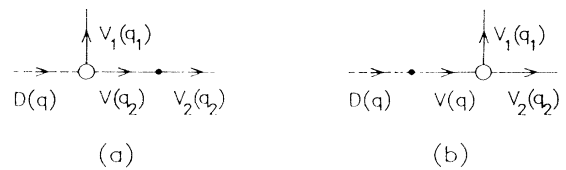


FIG. 4. Feynman diagrams for the decay  $D \rightarrow V_1 V_2$ .

TABLE I. Numerical values of the coupling constants and other parameters used in the paper.

Quantity	Value
$g$	4.28
$g_V/m_V$	152 MeV
$f_\pi$	92.6 MeV
$f_K$	$1.2 f_\pi$
$f_\eta$	$f_\pi$
$f_{\eta'}$	$f_\pi$
$f_D$	$1.46 f_\pi$
$f_{D_S}$	$f_D$
$g_{A_1}/m_{A_1}$	121 MeV
$g_{K_1}/m_{K_1}$	104 MeV
$g_{f_1}/m_{f_1}$	121 MeV
$g_{D_1}/m_{D_1}$	69.5 MeV
$g_{D_{S1}}/m_{D_{S1}}$	69.5 MeV

mesons. Since we can extract  $g_\rho$  from the data on the decay  $\rho \rightarrow \bar{l}l$ , this result leads to a determination of all  $g_V$ . Also from asymptotic chiral SU(4) symmetry, we can derive the spectral function sum rule [14]

$$g_A^2/m_A^2 + f_P^2 = g_V^2/m_V^2, \quad (6)$$

where  $A$ ,  $P$ , and  $V$  are particles with the same internal quantum numbers. Now experimentally, only  $f_\pi$  and  $f_K$  are known. However,  $f_D$  has been determined [15] from QCD sum rules [16]. With the  $g_V$ 's and  $f_P$ 's known, the

TABLE II. Amplitude  $A$  defined in Eq. (7) for the decay  $D \rightarrow P_1 P_2$ .

Decay	Amplitude $A$ ( $10^6 \text{ GeV}^{-1}$ )
$D^0 \rightarrow K^- \pi^+$	$1.45(a_1 - 0.24a_2)$
$D^0 \rightarrow \bar{K}^0 \pi^0$	$1.63a_2$
$D^0 \rightarrow \bar{K}^0 \eta$	$0.50a_2$
$D^+ \rightarrow \bar{K}^0 \pi^+$	$1.45(a_1 + 1.35a_2)$
$D_S^+ \rightarrow \bar{K}^0 K^+$	$2.05a_2$
$D_S^+ \rightarrow \eta \pi^+$	$-0.99a_1$
$D_S^+ \rightarrow \eta' \pi^+$	$1.75a_1$
$D^0 \rightarrow K^+ K^-$	$-0.39a_1$
$D^0 \rightarrow K^0 \bar{K}^0$	0
$D^0 \rightarrow \pi^+ \pi^-$	$-0.37a_1$
$D^0 \rightarrow \pi^0 \pi^0$	$-0.37a_2$
$D^+ \rightarrow K^+ \bar{K}^0$	$-0.39a_1$
$D^+ \rightarrow \pi^+ \pi^0$	$-0.26(a_1 + a_2)$
$D^+ \rightarrow \eta \pi^+$	$0.17a_2 + 0.21a_1$
$D^+ \rightarrow \eta' \pi^+$	$0.08a_2 + 0.36a_1$
$D_S^+ \rightarrow K^0 \pi^+$	$0.47a_1$

sum rule (6) then serves to determine the  $g_A$ 's.

The numerical values of the various couplings used in our analysis are displayed in Table I. For the axial-vector-meson multiplet, we have taken the particles to be [17]  $A_1(1260)$ ,  $K_1(1400)$ ,  $f_1(1285)$ ,  $D_1(2420)$ , and  $D_{S1}(2536)$ .

The decay amplitudes for the  $PP$ ,  $VP$ , and  $VV$  modes are defined as

$$M(D(q) \rightarrow P_1(q_1)P_2(q_2)) = \frac{-i(2\pi)^4 \delta^{(4)}(q - q_1 - q_2)}{\sqrt{2q_0 V 2q_{10} V 2q_{20} V}} iA, \quad (7)$$

$$M(D(q) \rightarrow V_1(q_1, \lambda_1)P_2(q_2)) = \frac{-i(2\pi)^4 \delta^{(4)}(q - q_1 - q_2)}{\sqrt{2q_0 V 2q_{10} V 2q_{20} V}} q_2 \cdot \varepsilon^{(\lambda_1)}(q_1) B, \quad (8)$$

$$M(D(q) \rightarrow V_1(q_1, \lambda_1)V_2(q_2, \lambda_2)) = \frac{-i(2\pi)^4 \delta^{(4)}(q - q_1 - q_2)}{\sqrt{2q_0 V 2q_{10} V 2q_{20} V}} [iC \delta_{\alpha\beta} + iD \varepsilon_{\mu\alpha\nu\beta} q_{1\mu} q_{2\nu} + iE q_{1\beta} q_{2\alpha}] \varepsilon_a^{(\lambda_1)}(q_1) \varepsilon_\beta^{(\lambda_2)}(q_2), \quad (9)$$

where  $A, B, \dots, E$  are constants representing the invariant amplitudes. The decay widths are easily calculated in terms of these amplitudes to be

$$\Gamma(D \rightarrow P_1 P_2) = \frac{1}{8\pi} \frac{k}{M^2} |A|^2, \quad (10)$$

$$\Gamma(D \rightarrow V_1 P_2) = \frac{1}{8\pi} \frac{k^3}{m_1^2} |B|^2, \quad (11)$$

$$\Gamma(D \rightarrow V_1 V_2) = \frac{1}{8\pi} \frac{k}{M^2} \left[ |C|^2 \left[ 3 + \frac{M^2 k^2}{m_1^2 m_2^2} \right] - (CE^* + C^*E) \frac{M^2(M^2 - m_1^2 - m_2^2)}{2m_1^2 m_2^2} k^2 + |D|^2 2M^2 k^2 + |E|^2 \frac{M^4 k^4}{m_1^2 m_2^2} \right], \quad (12)$$

TABLE III. Amplitude  $B$  defined in Eq. (8) for the decay  $D \rightarrow V_1 P_2$ .

Decay	Amplitude $B$ ( $10^6$ )
$D^0 \rightarrow K^{*-} \pi^+$	$2.16a_2 + 0.008a_1 + 0.323a_2(g_s - 0.611g_d) - 0.074a_1(g_s + 0.382g_d)$
$D^0 \rightarrow \bar{K}^{*0} \pi^0$	$-0.320a_2 - 0.230a_2(g_s - 0.611g_d)$
$D^0 \rightarrow \rho^+ K^-$	$-2.16a_2 + 1.30a_1 - 0.323a_2(g_s - 0.548g_d)$
$D^0 \rightarrow \rho^0 \bar{K}^0$	$1.64a_2 + 0.230a_2(g_s - 0.548g_d) - 0.066a_2(g_s + 0.379g_d)$
$D^0 \rightarrow \phi \bar{K}^0$	$2.16a_2 + 0.323a_2(g_s - 0.614g_d)$
$D^0 \rightarrow \omega \bar{K}^0$	$-1.43a_2 - 0.230a_2(g_s - 0.554g_d) - 0.066a_2(g_s + 0.376g_d)$
$D^0 \rightarrow \bar{K}^{*0} \eta$	$3.67a_2 + 0.388a_2(g_s - 0.571g_d)$
$D^+ \rightarrow \bar{K}^{*0} \pi^+$	$1.72a_2 + 0.008a_1 - 0.074a_1(g_s + 0.382g_d)$
$D^+ \rightarrow \rho^+ \bar{K}^0$	$0.153a_2 + 1.30a_1 - 0.093a_2(g_s + 0.379g_d)$
$D_s^+ \rightarrow \rho^+ \pi^0$	$-2.38a_1 - 0.565a_1(g_s - 0.573g_d)$
$D_s^+ \rightarrow K^{*+} \bar{K}^0$	$1.69a_1 + 0.153a_2 + 0.400a_1(g_s - 0.570g_d) - 0.098a_2(g_s + 0.366g_d)$
$D_s^+ \rightarrow \bar{K}^{*0} K^+$	$-1.56a_1 + 1.53a_2 - 0.400a_1(g_s - 0.570g_d)$
$D_s^+ \rightarrow \phi \pi^+$	$0.008a_1 - 0.078a_1(g_s + 0.364g_d)$
$D_s^+ \rightarrow \omega \pi^+$	0
$D_s^+ \rightarrow \rho^+ \eta$	$-0.756a_1$
$D_s^+ \rightarrow \rho^+ \eta'$	$1.68a_1$
$D^0 \rightarrow K^{*-} K^+$	$0.355a_2 + 0.031a_1 + 0.085a_2(g_s - 0.580g_d) - 0.020a_1(g_s + 0.350g_d)$
$D^0 \rightarrow K^{*+} K^-$	$-0.355a_2 + 0.361a_1 - 0.085a_2(g_s - 0.580g_d)$
$D^0 \rightarrow \bar{K}^{*0} K^0$	$-0.029a_2 - 0.001a_2(g_s - 0.580g_d)$
$D^0 \rightarrow K^{*0} \bar{K}^0$	$0.029a_2 + 0.001a_2(g_s - 0.580g_d)$
$D^+ \rightarrow \bar{K}^{*0} K^+$	$-0.353a_1 - 0.086a_1(g_s - 0.580g_d) - 0.020a_1(g_s + 0.350g_d)$
$D^+ \rightarrow \phi \pi^+$	$0.479a_2$
$D^+ \rightarrow \omega \pi^+$	$0.001a_1 + 0.228a_2 - 0.012a_1(g_s + 0.409g_d)$
$D^+ \rightarrow \rho^0 \pi^+$	$0.541a_1 - 0.223a_2 + 0.122a_1(g_s - 0.582g_d) + 0.012a_1(g_s + 0.413g_d)$
$D^+ \rightarrow \rho^+ \eta$	$-0.010a_2 + 0.196a_1 + 0.005a_2(g_s + 0.373g_d)$
$D^+ \rightarrow \rho^+ \eta'$	$0.087_2 + 0.334a_1 - 0.011a_2(g_s + 0.284g_d)$

where

$$k = \frac{1}{2M} [(M^2 - m_1^2 - m_2^2)^2 - 4m_1^2 m_2^2]^{1/2}, \quad (13)$$

$M$  is the mass of the decaying particle and  $m_1, m_2$  are the masses of the particles in the decay product. The amplitudes  $A, B, \dots, E$  can be written down from the Feynman diagrams in Figs. 2–4. Using the numerical values of the couplings in Table I, these can be expressed in terms of the four parameters:  $a_1, a_2, g_s$ , and  $g_d$ . The results for the Cabibbo allowed and once-suppressed decays are listed [18] in Tables II–IV.

#### IV. RESULTS

The decay amplitudes for  $D \rightarrow PP$  listed in Table II, do not involve the parameters  $g_s$  and  $g_d$ . Thus it is best to obtain a fit for  $a_1$  and  $a_2$  from these decays. For this purpose we shall use the data on  $D \rightarrow K\pi$  decays.

So far we have ignored the final state interactions. We shall take these into account by considering only elastic scattering in the final state. In  $D \rightarrow K\pi$  decays, we have, in terms of isospin amplitudes,

TABLE IV. Amplitudes  $C, D$ , and  $E$  defined in Eq. (9) for the decay  $D \rightarrow V_1 V_2$ .

Decay	Amplitudes		
	$C$ ( $10^6 \text{ GeV}^{-1}$ )	$D$ ( $10^6 \text{ GeV}$ )	$E$ ( $10^6 \text{ GeV}$ )
$D^0 \rightarrow K^{*-} \rho^+$	$-0.215a_2 - 0.103a_1g_s$	$-1.16a_2 - 0.696a_1$	$0.030a_1g_d$
$D^0 \rightarrow \bar{K}^{*0} \rho^0$	$0.152a_2 - 0.093a_2g_s$	$0.171a_2$	$0.027a_2g_d$
$D^0 \rightarrow \bar{K}^{*0} \omega$	$-0.136a_2 - 0.093a_2g_s$	$-1.47a_2$	$0.027a_2g_d$
$D^+ \rightarrow \bar{K}^{*0} \rho^+$	$-0.103a_1g_s - 0.131a_2g_s$	$-0.924a_2 - 0.696a_1$	$0.030a_1g_d + 0.038a_2g_d$
$D_s^+ \rightarrow \phi \rho^+$	$-0.108a_1g_s$	$-0.660a_1$	$0.028a_1g_d$
$D_s^+ \rightarrow K^{*+} \bar{K}^{*0}$	$-0.139a_2g_s$	$-0.873a_2 - 0.853a_1$	$0.036a_2g_d$
$D^0 \rightarrow K^{*0} \bar{K}^{*0}$	0	$-0.241a_2$	0
$D^0 \rightarrow \phi \rho^0$	$-0.025a_2g_s$	$-0.181a_2$	$0.007a_2g_d$
$D^+ \rightarrow K^{*+} \bar{K}^{*0}$	$-0.028a_1g_s$	$-0.400a_1$	$0.008a_1g_d$
$D^+ \rightarrow \phi \rho^+$	$-0.036a_2g_s$	$-0.256a_2$	$0.010a_2g_d$

$$A(D^0 \rightarrow K^- \pi^+) = \frac{1}{\sqrt{3}} A_{3/2} + \left(\frac{2}{3}\right)^{1/2} A_{1/2},$$

$$A(D^0 \rightarrow \bar{K}^0 \pi^0) = \left(\frac{2}{3}\right)^{1/2} A_{3/2} - \frac{1}{\sqrt{3}} A_{1/2}, \quad (14)$$

$$A(D^+ \rightarrow \bar{K}^0 \pi^+) = \sqrt{3} A_{3/2},$$

where

$$A_I = |A_I| e^{i\delta_I}$$

is the amplitude in the isospin state  $I$  and  $\delta_I$  is the phase shift in that channel. Using the data from the particle properties data booklet [19], it is easy to obtain

$$|A_{1/2}| = 2.94 \times 10^{-6} \text{ GeV},$$

$$|A_{3/2}| = 7.37 \times 10^{-7} \text{ GeV}, \quad (15)$$

$$\delta_{1/2} - \delta_{3/2} = 93.4^\circ.$$

Now, in our model, the isospin amplitudes can be constructed if we invert the relations in Eq. (14) and use our results for the  $D \rightarrow K\pi$  amplitudes from Table II. This gives

$$|A_{1/2}| = 1.18 \times 10^{-6} (a_1 - 1.04a_2) \text{ GeV},$$

$$|A_{3/2}| = 8.37 \times 10^{-7} (a_1 + 1.35a_2) \text{ GeV}. \quad (16)$$

Using the values for these amplitudes given by (15), we obtain the solution [20]

$$a_1 = 1.79, \quad a_2 = -0.67. \quad (17)$$

There is another solution where  $|a_2/a_1| > 1$ , which we discard, as discussed in Ref. [3]. We also note that the result (17) is not very far from the values of  $a_1$  and  $a_2$  obtained by Bauer *et al.* [5].

With  $a_1$  and  $a_2$  determined, we fit  $g_s$  and  $g_d$  from the

TABLE V. Branching ratio for the decay  $D \rightarrow P_1 P_2$ .

Decay	Branching ratio	
	Theory	Experiment [19]
$D^0 \rightarrow K^- \pi^+$	$3.6 \times 10^{-2}$	$(3.65 \pm 0.21) \times 10^{-2}$
$D^0 \rightarrow \bar{K}^0 \pi^0$	$2.1 \times 10^{-2}$	$(2.1 \pm 0.5) \times 10^{-2}$
$D^0 \rightarrow \bar{K}^0 \eta$	$6.4 \times 10^{-4}$	$< 2.3 \times 10^{-2}$
$D^+ \rightarrow \bar{K}^0 \pi^+$	$2.6 \times 10^{-2}$	$(2.6 \pm 0.4) \times 10^{-2}$
$D_S^+ \rightarrow \bar{K}^0 K^+$	$1.1 \times 10^{-2}$	$(2.8 \pm 0.7) \times 10^{-2}$
$D_S^+ \rightarrow \eta \pi^+$	$2.0 \times 10^{-2}$	$(1.5 \pm 0.4) \times 10^{-2}$
$D_S^+ \rightarrow \eta' \pi^+$	$5.1 \times 10^{-2}$	$(3.7 \pm 1.2) \times 10^{-2}$
$D^0 \rightarrow K^+ K^-$	$2.4 \times 10^{-3}$	$(4.1 \pm 0.4) \times 10^{-3}$
$D^0 \rightarrow K^0 \bar{K}^0$	$5.0 \times 10^{-4}$	$(1.1 \pm 0.4) \times 10^{-3}$
$D^0 \rightarrow \pi^+ \pi^-$	$1.6 \times 10^{-3}$	$(1.63 \pm 0.19) \times 10^{-3}$
$D^0 \rightarrow \pi^0 \pi^0$	$3.9 \times 10^{-5}$	$< 4.6 \times 10^{-3}$
$D^+ \rightarrow K^+ \bar{K}^0$	$7.3 \times 10^{-3}$	$(7.3 \pm 1.8) \times 10^{-3}$
$D^+ \rightarrow \pi^+ \pi^0$	$1.5 \times 10^{-3}$	$< 5.3 \times 10^{-3}$
$D^+ \rightarrow \eta \pi^+$	$1.1 \times 10^{-3}$	$(6.6 \pm 2.2) \times 10^{-3}$
$D^+ \rightarrow \eta' \pi^+$	$4.4 \times 10^{-3}$	$< 8 \times 10^{-3}$
$D_S^+ \rightarrow K^0 \pi^+$	$4.5 \times 10^{-3}$	$< 6 \times 10^{-3}$

TABLE VI. Branching ratio for the decay  $D \rightarrow V_1 P_2$ .

Decay	Branching ratio	
	Theory	Experiment [19]
$D^0 \rightarrow K^{*-} \pi^+$	$1.4 \times 10^{-2}$	$(4.5 \pm 0.6) \times 10^{-2}$
$D^0 \rightarrow \bar{K}^{*0} \pi^0$	$6.8 \times 10^{-3}$	$(2.1 \pm 1.0) \times 10^{-2}$
$D^0 \rightarrow \rho^+ K^-$	$10.2 \times 10^{-2}$	$(7.3 \pm 1.1) \times 10^{-2}$
$D^0 \rightarrow \rho^0 \bar{K}^0$	$1.3 \times 10^{-2}$	$(6.1 \pm 3.0) \times 10^{-3}$
$D^0 \rightarrow \phi \bar{K}^0$	$1.1 \times 10^{-3}$	$(8.8 \pm 1.2) \times 10^{-3}$
$D^0 \rightarrow \omega \bar{K}^0$	$1.8 \times 10^{-3}$	$(2.5 \pm 0.5) \times 10^{-2}$
$D^0 \rightarrow \bar{K}^{*0} \eta$	$1.0 \times 10^{-2}$	$(2.1 \pm 1.2) \times 10^{-2}$
$D^+ \rightarrow \bar{K}^{*0} \pi^+$	$1.4 \times 10^{-2}$	$(1.9 \pm 0.7) \times 10^{-2}$
$D^+ \rightarrow \rho^+ \bar{K}^0$	$6.4 \times 10^{-2}$	$(6.6 \pm 1.7) \times 10^{-2}$
$D_S^+ \rightarrow \rho^+ \pi^0$	$2.2 \times 10^{-3}$	$< 2.2 \times 10^{-3}$
$D_S^+ \rightarrow K^{*+} \bar{K}^0$	$1.6 \times 10^{-2}$	$(3.3 \pm 0.9) \times 10^{-2}$
$D_S^+ \rightarrow \bar{K}^{*0} K^+$	$3.6 \times 10^{-3}$	$(2.6 \pm 0.5) \times 10^{-2}$
$D_S^+ \rightarrow \phi \pi^+$	$3.5 \times 10^{-2}$	$(2.8 \pm 0.5) \times 10^{-2}$
$D_S^+ \rightarrow \omega \pi^+$	0	$< 1.4 \times 10^{-2}$
$D_S^+ \rightarrow \rho^+ \eta$	$3.3 \times 10^{-2}$	$(7.9 \pm 2.1) \times 10^{-2}$
$D_S^+ \rightarrow \rho^+ \eta'$	$4.4 \times 10^{-2}$	$(9.5 \pm 2.7) \times 10^{-2}$
$D^0 \rightarrow K^{*-} K^+$	$2.2 \times 10^{-3}$	$(2.0 \pm 1.0) \times 10^{-3}$
$D^0 \rightarrow K^{*+} K^-$	$2.9 \times 10^{-3}$	$(3.5 \pm 0.8) \times 10^{-3}$
$D^0 \rightarrow \bar{K}^{*0} K^0$	$2.0 \times 10^{-6}$	$< 1.6 \times 10^{-3}$
$D^0 \rightarrow K^{*0} \bar{K}^0$	$2.0 \times 10^{-6}$	$< 8 \times 10^{-4}$
$D^+ \rightarrow \bar{K}^{*0} K^+$	$5.3 \times 10^{-3}$	$(4.7 \pm 0.9) \times 10^{-3}$
$D^+ \rightarrow \phi \pi^+$	$1.7 \times 10^{-3}$	$(6.0 \pm 0.8) \times 10^{-3}$
$D^+ \rightarrow \omega \pi^+$	$1.1 \times 10^{-3}$	$< 6 \times 10^{-3}$
$D^+ \rightarrow \rho^0 \pi^+$	$1.1 \times 10^{-3}$	$< 1.2 \times 10^{-3}$
$D^+ \rightarrow \rho^+ \eta$	$5.1 \times 10^{-3}$	$< 1.0 \times 10^{-2}$
$D^+ \rightarrow \rho^+ \eta'$	$9.6 \times 10^{-4}$	$< 1.4 \times 10^{-2}$

data on the decay modes  $D \rightarrow PV$  and  $D \rightarrow VV$ . We find that the best fit corresponds to the values

$$g_s = -10.14, \quad g_d = -9.85. \quad (18)$$

The branching ratios for various decay modes determined by our values of the parameters in (17) and (18) are exhibited in Tables V–VII, together with the experimental data. In preparing these tables, we have used, wherever possible, phase shifts for elastic scattering in the final state as determined from the data. Considering the uncertainties in our choice of the coupling constants, it is remarkable that most of our results are in reasonable agreement with the data.

TABLE VII. Branching ratio for the decay  $D \rightarrow V_1 V_2$ .

Decay	Branching ratio	
	Theory	Experiment [19]
$D^0 \rightarrow K^{*-} \rho^+$	$6.5 \times 10^{-2}$	$(6.2 \pm 2.5) \times 10^{-2}$
$D^0 \rightarrow \bar{K}^{*0} \rho^0$	$8.5 \times 10^{-3}$	$(1.5 \pm 0.6) \times 10^{-2}$
$D^0 \rightarrow \bar{K}^{*0} \omega$	$7.9 \times 10^{-3}$	$< 1.5 \times 10^{-2}$
$D^+ \rightarrow \bar{K}^{*0} \rho^+$	$4.3 \times 10^{-2}$	$(4.1 \pm 1.5) \times 10^{-2}$
$D_S^+ \rightarrow \phi \rho^+$	$5.7 \times 10^{-2}$	$(5.2 \pm 1.4) \times 10^{-2}$
$D_S^+ \rightarrow K^{*+} \bar{K}^{*0}$	$1.5 \times 10^{-2}$	$(5.0 \pm 1.7) \times 10^{-2}$
$D^0 \rightarrow K^{*0} \bar{K}^{*0}$	$2.6 \times 10^{-5}$	$(2.7 \pm 1.5) \times 10^{-3}$
$D^0 \rightarrow \phi \rho^0$	$2.2 \times 10^{-4}$	$(1.8 \pm 0.5) \times 10^{-3}$
$D^+ \rightarrow K^{*+} \bar{K}^{*0}$	$5.9 \times 10^{-3}$	$(2.6 \pm 1.1) \times 10^{-2}$
$D^+ \rightarrow \phi \rho^+$	$1.2 \times 10^{-3}$	$< 1.3 \times 10^{-2}$

It should be remarked that our parameter fit in (17) and (18) leads to a destructive interference between the pseudoscalar and axial-vector meson poles in the annihilation diagrams in  $D \rightarrow PV$ . Thus decays such as  $D^0 \rightarrow \phi \bar{K}^0$  and  $D_S^+ \rightarrow \rho^+ \pi^0$  which proceed only through the annihilation diagrams lead to tiny branching ratios due to large cancellations in the contributions of Figs. 3(c) and 3(e). Unfortunately, because of this, small changes in our choice of the couplings can get magnified in the prediction of these branching ratios [21]. We would also like to point out that in  $D \rightarrow VV$  decays, we did not consider the diagram analogous to 4(c) where the

pseudoscalar meson pole is replaced by an axial-vector meson. There is no straightforward way of estimating the  $AVV$  strong coupling. The general agreement of our results with experiments in fact shows that the axial-vector pole diagram in  $D \rightarrow VV$  may be neglected. It also shows that the simple pole model considered in this paper generally works quite well.

#### ACKNOWLEDGMENTS

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- [20] These values of  $a_1$  and  $a_2$  are slightly different from those in Ref. [3] since we have used a slightly different value of the strong coupling constant  $g$ .
- [21] In fact the fit (18) for  $g_s$  and  $g_d$  was fine-tuned to reproduce the experimental bound for the decay  $D_S^+ \rightarrow \rho^+ \pi^0$ .