

Phenomenological quark mass matrix model with two adjustable parameters

Yoshio Koide

Department of Physics, University of Shizuoka, 395 Yada, Shizuoka 422, Japan

(Received 7 June 1993)

A phenomenological quark mass matrix model which includes only two adjustable parameters is proposed from the point of view of the unification of quark and lepton mass matrices. The model can provide reasonable values of quark mass ratios and Kobayashi-Maskawa matrix parameters.

PACS number(s): 12.15.Ff

It is widely accepted that the family number of ordinary quarks and leptons is three. (This does not rule out a possibility that there are some extraordinary families, e.g., a family with an extremely heavy neutrino, and so on.) Then, we have ten observable quantities related to up- and down-quark mass matrices, M_u and M_d , i.e., six up- and down-quark masses and four parameters of Kobayashi-Maskawa (KM) [1] matrix. On the other hand, most of quark mass matrix models currently proposed include adjustable parameters more than five [two parameters for each quark mass matrix M_q ($q = u, d$) and one relative phase parameter between up- and down-quark mass matrix phase parameters]. At present, every model is comparably plausible, and is in agreement with the present experimental data. Nevertheless, we cannot resist the temptation to investigate a further new type of mass matrix form of (M_u, M_d) with parameters less than four, because we expect that the quark and lepton families are governed by a more fundamental law of nature.

In the present paper, we propose the following model of quark and lepton mass matrices inspired by an extended technicolorlike model:

$$M_f = m_0 G O_f G, \tag{1}$$

$$G = \text{diag}(g_1, g_2, g_3), \tag{2}$$

$$O_f = \mathbf{1} + 3a_f X(\phi_f), \tag{3}$$

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad X(\phi) = \frac{1}{3} \begin{pmatrix} 1 & e^{i\phi} & 1 \\ e^{-i\phi} & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \tag{4}$$

where $f = \nu, e, u$, and d are indices for neutrinos, charged leptons, up, and down quarks, respectively. Here, the diagonal matrix G denotes a coupling constant matrix of a hypercolored boson ϕ_α with ordinary fermions f_i and hypercolored fermions $F_{i\alpha}$ (α and i are hypercolor and family indices, respectively), and the matrix O_f denotes the condensation of the hypercolored fermions $\langle(\bar{F}F)\rangle$. Since we consider the so-called seesaw mechanism [2] for the neutrino mass matrix, the matrix M_ν given in (1) should be taken as the Dirac mass matrix part of the neutrino mass matrix.

As we discuss below, since we take $a_e = 0$ in the charged lepton mass matrix M_e , the parameters g_i are fixed as $\sqrt{m_0}G = \text{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$, so that the mass matrix M_f is effectively given by

$$M_f = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} + a_f \begin{pmatrix} m_e & e^{i\phi_f} \sqrt{m_e m_\mu} & \sqrt{m_e m_\tau} \\ e^{-i\phi_f} \sqrt{m_e m_\mu} & m_\mu & \sqrt{m_\mu m_\tau} \\ \sqrt{m_e m_\tau} & \sqrt{m_\mu m_\tau} & m_\tau \end{pmatrix}. \tag{5}$$

In the present paper, we put an ansatz for ϕ_f ($\phi_u = 0$, $\phi_d = \pi/2$), so that adjustable parameters in the quark mass matrices M_u and M_d are only two, a_u and a_d . As we demonstrate later, a suitable choice of the parameters a_u and a_d will provide not only reasonable values of up- and down-quark mass ratios m_i^u/m_j^u and m_i^d/m_j^d ($i, j = 1, 2, 3$), respectively, but also reasonable values of the ratios m_i^u/m_i^d as well as reasonable values of KM matrix parameters.

The mass matrix forms M_e and M_ν were proposed by the author [3,4] from the phenomenological point of view. In fact, the present quark mass matrix model was inspired by the phenomenological success of the charged and neutrino mass matrices as we review below.

Ten years ago, the author [3] proposed a charged lepton mass matrix model, in which charged lepton masses $m_i^e = (m_e, m_\mu, m_\tau)$ are generated through the condensations of hypercolored fermions $E_{i\alpha}$, $\langle(\bar{E}E)\rangle$, and the

exchanges of a hypercolored vector boson ϕ_α which is coupled with $\sum_i g_i \bar{e}_i E_{i\alpha}$, i.e., the masses m_i^e are given by $m_i^e \simeq g_i^2 \langle(\bar{E}E)\rangle / m_\phi^2$. (The model is similar to the extended technicolor model [5], but we consider that the vector boson ϕ_α is not a gauge boson.) Here, the hypercolored boson ϕ_α (hereafter we drop the index α) is a particularly mixed state among SU(3)-family octet bosons ϕ_3 and ϕ_8 , and singlet boson ϕ_0 , which are the λ_3 , λ_8 , and λ_0 components of SU(3). We consider that the octet bosons acquire large masses at an energy scale Λ_H except for one component $\phi^{(8)}$ which is a linear combination of ϕ_3 and ϕ_8 , while $\phi^{(8)}$ has exactly the same mass as ϕ_0 . Then, if there is a mixing term between $\phi^{(8)}$ and ϕ_0 , the 45° mixing between $\phi^{(8)}$ and ϕ_0 is inevitably caused. We assume that only one of the two states can contribute to the mass matrix M_e , so that the coupling constant g_i is given by $g_i = (g_i^{(8)} + g_0) / \sqrt{2}$, where $g_i^{(8)}$ and g_0 are

coupling constants of $\phi^{(8)}$ and ϕ_0 with $\bar{e}_i E_i$, respectively, and they satisfy the relations $g_1^{(8)} + g_2^{(8)} + g_3^{(8)} = 0$ and $(g_1^{(8)})^2 + (g_2^{(8)})^2 + (g_3^{(8)})^2 = 3g_0^2$, because we consider that $\phi^{(8)}$ and ϕ_0 belong to the nonet of the U(3) family. Then, the coupling constants g_i satisfy the relation

$$g_1^2 + g_2^2 + g_3^2 = \sum_{i=1}^3 \left(\frac{g_i^{(8)} + g_0}{\sqrt{2}} \right)^2 = 3g_0^2 = \frac{2}{3} \left(\sum_{i=1}^3 \frac{g_i^{(8)} + g_0}{\sqrt{2}} \right)^2 = \frac{2}{3} (g_1 + g_2 + g_3)^2, \quad (6)$$

which leads to a charged lepton mass sum rule [3]

$$m_e + m_\mu + m_\tau = \frac{2}{3} (\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2. \quad (7)$$

The sum rule (7) predicts $m_\tau = 1776.97$ MeV from the input values of m_e and m_μ . The predicted value 1777 MeV is in excellent agreement with the observed values of m_τ which have recently been reported by the ARGUS [6], BES [7], and CLEO [8] Collaborations. Thus, the phenomenological success of the charged lepton mass matrix $M_e = m_0 G1G$ is our main motivation to consider the mass matrix form of the type $m_0 G O_f G$.

In Ref. [3], the boson state ϕ is more explicitly given:

$$\phi = -\frac{1}{\sqrt{2}} \left[\cos\left(\frac{\pi}{4} - \epsilon\right) \phi_3 - \sin\left(\frac{\pi}{4} - \epsilon\right) \phi_8 \right] + \frac{1}{\sqrt{2}} \phi_0. \quad (8)$$

As a result, the matrix G is given by

$$G = \frac{1 + \epsilon}{2\sqrt{2}\sqrt{1 + \epsilon^2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{1 - \epsilon}{2\sqrt{6}\sqrt{1 + \epsilon^2}} \begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (9)$$

where $\cos(\pi/4 - \epsilon)$ and $\sin(\pi/4 - \epsilon)$ are replaced by $(1 + \epsilon)/\sqrt{2(1 + \epsilon^2)}$ and $(1 - \epsilon)/\sqrt{2(1 + \epsilon^2)}$, respectively. In the limit of ‘‘ideal mixing,’’ i.e., $\epsilon = 0$, the model leads to a massless electron. This explains why electron mass is very small compared with other charged lepton masses.

The motivation to consider the matrix form O_f of the type $1 + 3a_f X$ is as follows: Recently, in order to explain a neutrino mixing value $\sin\theta_{e\mu} \simeq 0.04$ ($\sin^2 2\theta_{e\mu} \simeq 7 \times 10^{-3}$) suggested by GALLEX [9], the author [4] has proposed a neutrino mass matrix model, in which the neutrino mass matrix M_ν is given by $M_\nu \simeq M_\nu^D M_M^{-1} M_\nu^D = (M_\nu^D)^2 / m_M$ (m_M is a Majorana neutrino mass) on the basis of the conventional seesaw mechanism scenario [2], and the Dirac mass matrix M_ν^D is given by the form $M_\nu^D = m_\nu^D G[1 + 3a_\nu X(0)]G$, where a_ν is a numerical parameter with $a_\nu \gg 1$. Here we have supposed that the hypercolored neutrino condensation $\langle (\bar{N}N) \rangle$ takes a democratic term [$X(0)$ term] dominance form, $1 + 3a_\nu X(0)$, differently from the case of $\langle (\bar{E}E) \rangle \propto 1$. The model can lead to a desirable prediction [4] $\sin\theta_{e\mu} \simeq (1/2)\sqrt{m_e/m_\mu} \simeq 0.035$ for $a_\nu \gg 1$.

In general, the mass matrix (5) with $\phi_f = 0$ provides the relation [4]

$$m_1^f / m_2^f \simeq 3m_e / 4m_\mu = 0.00363, \quad (10)$$

in the limit of $1/a_f \rightarrow 0$, where m_i are eigenvalues of the mass matrix (5) and are defined as $|m_1| < |m_2| <$

$|m_3|$. The conventional values [10] of the running quark masses at 1 GeV, $m_u \simeq 5.1$ MeV and $m_c \simeq 1.35$ GeV, provide $m_u/m_c \simeq 0.0038$, which is in agreement with the prediction (10). This is just the motivation to consider the mass matrix M_u given by (1) with $\phi_u = 0$, i.e.,

$$M_u = m_0^u G[1 + 3a_u X(0)]G. \quad (11)$$

Hereafter, for convenience, we will refer the Gasser-Leutwyler values [10] for $\Lambda_{\overline{\text{MS}}}^{(3)} = 0.150$ GeV [11] (where $\overline{\text{MS}}$ denotes the modified minimal subtraction scheme) as running quark mass values (in units of GeV) at an energy scale 1 GeV:

$$\begin{aligned} m_u &= 0.0051 \pm 0.0015, & m_d &= 0.0089 \pm 0.0026, \\ m_c &= 1.35 \pm 0.05, & m_s &= 0.175 \pm 0.055, \\ m_t &= 226_{-49}^{+43}, & m_b &= 5.58 \pm 0.13. \end{aligned} \quad (12)$$

The value of $m_t(1 \text{ GeV})$, which is not listed in the original paper by Gasser and Leutwyler, has been estimated by using the standard model parameter fitting value $m_t^{\text{phys}} = 130_{-28}^{+25}$ GeV [12] and $\Lambda_{\overline{\text{MS}}}^{(3)} = 0.150$ GeV ($\Lambda_{\overline{\text{MS}}}^{(4)} = 0.114$ GeV, $\Lambda_{\overline{\text{MS}}}^{(5)} = 0.0699$ GeV). However, the values in (12) should not be taken rigidly, because the estimates are highly dependent on the value of $\Lambda_{\overline{\text{MS}}}$ and models (prescriptions) at present [13].

The mass matrix given by (11) actually can predict reasonable up-quark mass ratios: for example, $m_u/m_c = 0.00389$ (0.00379) and $m_c/m_t = 0.00597$ (−0.00598) for $a_u = 16.45$ ($a_u = -19.02$). Here, since the quark mass ratio m_u/m_c is insensitive to the value of a_u , we have determined the value of a_u from the value of m_c/m_t in (12). The prediction of m_u/m_c is in excellent agreement with the value of m_u/m_c provided by (12).

Next, we seek for a mass matrix form for down quarks. We cannot choose the same mass matrix form as that for up quarks, i.e., $M_d = m_0^d G[1 + 3a_d X(0)]G$, because it leads to a wrong down-quark mass ratio $m_1^d/m_2^d \simeq 3m_e/4m_\mu$. In addition, we must introduce a CP violation phase to the model. We assume a down-quark matrix form M_d which is similar to M_u , but which has a phase factor $\phi_d \neq 0$ as given by (4).

In general, the eigenvalues m_i^f of the mass matrix (5) are given by

$$\frac{m_1^f}{m_\tau} \simeq \frac{\kappa_f(3 + \kappa_f) - 4\sin^2(\phi_f/2)}{\kappa_f^2(2 + \kappa_f)} \varepsilon_1, \quad (13)$$

$$\frac{m_2^f}{m_\tau} \simeq \frac{2 + \kappa_f}{1 + \kappa_f} \varepsilon_2, \quad \frac{m_3^f}{m_\tau} \simeq \frac{1 + \kappa_f}{\kappa_f},$$

where $\kappa_f = 1/a_f$, $\varepsilon_1 = m_e/m_\tau$, and $\varepsilon_2 = m_\mu/m_\tau$. Note that, differently from the case of M_u with $\phi_u = 0$, we

cannot take a limit of $\kappa_d \rightarrow 0$ in the down-quark mass matrix M_d , because the mass ratio m_1^d/m_2^d includes a factor $1/\kappa_d^2$. The relation

$$\frac{m_s}{m_b} \simeq \frac{(2 + \kappa_d)\kappa_d}{1 + \kappa_d} \frac{m_\mu}{m_\tau} \quad (14)$$

suggests a small but visible value $\kappa_d \simeq -0.2$ because $|m_s/m_b| \simeq 0.03$ and $m_\mu/m_\tau \simeq 0.06$. Then, the relation

$$\frac{m_d m_s}{m_b^2} \simeq -\frac{4}{(1 + \kappa_d)^3} \frac{m_e m_\mu}{m_\tau^2} \sin^2 \frac{\phi_d}{2} \quad (15)$$

suggests $|\phi_d| \simeq \pi/2$. For simplicity, we fix ϕ_d to be $\phi_d = \pi/2$, which leads to a maximal CP violation.

In conclusion, we assume that the down-quark mass matrix M_d is given by

$$M_d = m_0^d G[1 + 3a_d X(\pi/2)]G. \quad (16)$$

Then, a suitable choice of a_d can provide excellent predictions of m_d/m_s and m_s/m_b : for example, $m_d/m_s = -0.0507$ and $m_s/m_b = -0.0313$ for $a_d = -4.81$. It is noted that, in the mass matrix M_f with $\phi_f = \pi/2$, in general, two values of a_f , $a_f = a_f^{(1)}$, and $a_f = a_f^{(2)}$, which satisfy the relation $(1/a_f^{(1)}) + (1/a_f^{(2)}) = -2$, can yield the same mass ratios m_1^f/m_2^f and m_2^f/m_3^f . Therefore, the alternative choice $a_d = a_d^{(2)} = -0.558$ provides the same predictions of the down-quark mass ratios as the case of $a_d = a_d^{(1)} = -4.81$.

The quark mass matrix model (M_u, M_d) given in (11) and (16) predicts the KM matrix elements V_{ij} in the limit of $1 \ll |\kappa_d| \ll |\kappa_u| \rightarrow 0$ as follows:

$$|V_{us}|^2 \simeq 2 \frac{1 + \kappa_d}{(2 + \kappa_d)^2 \kappa_d^2} \frac{m_e}{m_\mu}, \quad (17)$$

$$|V_{cb}|^2 \simeq \frac{\kappa_d^2}{(1 + \kappa_d)^2} \frac{m_\mu}{m_\tau} \simeq \frac{m_e/m_\tau}{|V_{us}|^2}, \quad (18)$$

$$|V_{ub}|^2 \simeq m_e/m_\tau. \quad (19)$$

The relation (17) leads to the well-known Weinberg-Fritzsch empirical relation [14] $|V_{us}| \simeq \sqrt{-m_d/m_s}$, because the mass ratio m_d/m_s is given by

$$\frac{m_d}{m_s} \simeq -\frac{(1 + \kappa_d)^2(2 - 2\kappa_d - \kappa_d^2)}{(2 + \kappa_d)^2 \kappa_d^2} \frac{m_e}{m_\mu}. \quad (20)$$

The predicted values of $|V_{cb}|$ and $|V_{ub}|$ from (18) and (19) are somewhat large compared with the observed values. This disagreement comes from the approximation in which we took $\kappa_u = 0$. The values of $|V_{ij}|$ are sensitive to κ_u and κ_d as well as to ε_1 and ε_2 . As we demonstrate below, a suitable choice of $\kappa_u = 1/a_u$ can predict reasonable values of $|V_{cb}|$ and $|V_{ub}|$ numerically.

In Table I, we show predictions on the KM matrix

parameters for the values of a_u and a_d which provide reasonable quark mass ratios. We also list the prediction of the rephasing invariant quantity J [15]. The case $a_d = a_d^{(1)} = -4.81$ can provide reasonable values of the KM matrix parameters except that the value of $|V_{ub}|$ is somewhat small. The value of $|V_{ub}|$ is highly sensitive to the value of the phase parameter ϕ_d when M_d is given by $M_d = m_0^d G[1 + 3a_d X(\phi_d)]G$, and a choice of ϕ_d slightly different from $\phi_d = \pi/2$ predicts a fairly large value of $|V_{ub}|$ compared with the case of exact $\phi_d = \pi/2$. It is likely that the prediction of $|V_{ub}|$ becomes reasonable value by renormalization effects for M_q .

On the other hand, the second case $a_d = a_d^{(2)} = -0.558$ cannot provide reasonable values of $|V_{cb}|$ and $|V_{ub}|$ as seen in Table I. However, it should be noted that the case $a_d = -0.558$ can provide not only the excellent predictions of m_d/m_s and m_s/m_b but also the excellent prediction of m_d/m_u if we consider $m_0^d = m_0^u$. When we put $m_2^d = 0.175$ GeV (i.e., $m_0^u/m_0^e = m_0^d/m_0^e = 6.52$) in order to compare our prediction with the Gasser-Leutwyler values (12), we obtain the following quark mass values at energy scale 1 GeV for the case of $(a_u, a_d) = (-19.02, -0.558)$:

$$\begin{aligned} m_1^u &= 0.00504 \text{ GeV}, & m_1^d &= 0.00887 \text{ GeV}, \\ m_2^u &= +1.33 \text{ GeV}, & m_2^d &= -0.175 \text{ GeV}, \\ m_3^u &= -223 \text{ GeV}, & m_3^d &= +5.59 \text{ GeV}. \end{aligned} \quad (21)$$

The values (21) are in excellent agreement with the Gasser-Leutwyler values (12). In most of the conventional quark mass matrix models, if we want to explain the fact $m_t \gg m_b$, then we must be contented with saying that the fact $m_u \sim m_d$ is an accidental coincidence in the model. In the case of $a_d = a_d^{(2)}$, we can obtain the reasonable ratio of m_u/m_d together with the reasonable ratios m_i^u/m_j^u and m_i^d/m_j^d . Therefore, the case of $a_d = -0.558$ is worth being taken into consideration as well as the case $a_d = -4.81$.

It should also be noted that predictions of $|V_{ij}|$ in the case of $a_d^{(2)}$ are, in general, exactly the same as those in the case of $a_d^{(1)}$ if we take

$$V = U_u P U_d^\dagger, \quad (22)$$

$$P = \text{diag}(1, 1, -1), \quad (23)$$

instead of $V = U_u U_d^\dagger$. The modification (22) means that the mass matrices ($M_u; M_d$) given by (11) and (16) are not those for the weak eigenstate quark basis $(u_0, c_0, t_0; d_0, s_0, b_0)$, but those for the quark basis $(u_0, c_0, \pm t_0; d_0, s_0, \mp b_0)$. Although the origin of such phase inversion is not clear, if we accept the scenario, we can provide not only the reasonable values (21) of quark masses but also reasonable values of the KM ma-

TABLE I. Prediction on the KM matrix parameters.

a_u	a_d	$ V_{us} $	$ V_{cb} $	$ V_{ub} $	$ V_{td} $	$ V_{ub}/V_{cb} $	J
+16.45	-4.81	0.204	0.0624	0.00186	0.01291	0.0298	2.31×10^{-5}
-19.02	-4.81	0.203	0.0393	0.00139	0.00882	0.0353	0.891×10^{-5}
+16.45	-0.558	0.201	0.495	0.0170	0.0907	0.0344	1.10×10^{-3}
-19.02	-0.558	0.199	0.515	0.0169	0.0942	0.0328	1.12×10^{-3}

trix parameters:

$$|V_{us}| = 0.203, \quad |V_{cb}| = 0.0393, \quad (24)$$

$$|V_{ub}| = 0.00139, \quad |V_{td}| = 0.00882; \quad J = 0.891 \times 10^{-5},$$

by fixing $(a_u, a_d) = (-19.02, -0.558)$.

So far, we have neglected the energy scale dependence of the quark masses and KM parameters. We consider that the mass matrix form (1) is given at an energy scale $\mu = M_X$. We expect that fine tuning of our parameters in consideration of the renormalization group equations can provide further excellent predictions of quark masses and KM mixing parameters.

We consider that m_0^q and m_0^e satisfy $m_0^u = m_0^d = m_0^e$ at the energy scale M_X and the value $(m_0^q/m_0^e)_{1 \text{ GeV}} = 6.52$ will be explained by evolving m_0^q and m_0^e down from M_X to 1 GeV. In the present model, the energy scale M_X need not be identical with the weak boson mass scale $v \simeq 250$ GeV. In order to give a rough estimate of M_X , we neglect electroweak interaction and use, for convenience, the equation for QCD running quark mass (for example, see Ref. [10]) (not the renormalization group equation for the Yukawa couplings). The value of M_X estimated is highly sensitive to the choice of $\Lambda_{\text{QCD}} (\Lambda_{\overline{\text{MS}}}^{(n)})$. If we adopt a recent experimental value $\Lambda_{\overline{\text{MS}}}^{(4)} = 0.260$ GeV [16] ($\Lambda_{\overline{\text{MS}}}^{(3)} = 0.311$ GeV, $\Lambda_{\overline{\text{MS}}}^{(5)} = 0.175$ GeV, $\Lambda_{\overline{\text{MS}}}^{(6)} = 0.0709$ GeV), we obtain $M_X \sim 10^{18}$ GeV. The value of M_X is somewhat large. However, the present estimate of M_X is only a trial and it should not be taken seriously. The estimate is also highly dependent on the models. In order to give more accurate estimate of M_X , we must build the model more concretely.

In conclusion, we have proposed a phenomenological quark and lepton mass matrix model (1). The matrix form (1) has a possibility of unified description of quark and lepton masses and their mixings. The mass matrix form $m_0 G O_f G$ can be understood from an extended technicolorlike scenario (but our boson ϕ_α is not a gauge boson). However, such a mass matrix form (1) can also be understood from a Higgs-boson scenario with some additional U(1) charges. In both scenarios, it is essential that there are heavy fermions which behave as intermediate states in the mass generation mechanism of the light fermions. In the derivation of the sum rule (7), it is essential that the 45° mixing between octet and singlet parts in the U(3)-family nonet scheme. In Ref. [17], the sum rule (7) has been rederived from a Higgs potential model with a mixing term between SU(3)-family octet and singlet. However, Ref. [17] did not discuss clearly on the additional U(1) charges which should be introduced in the scenario. Recently, a detailed study of the U(1) charges related to the horizontal symmetry has been given by Leurer, Nir, and Seiberg [18]. We will find a clue to the justification of the present scenario in their paper, in which we can see relations of $|V_{ij}|$ similar to our relations (17)–(19), although in our model the parameters κ_u and κ_d are not negligible. However, the purpose of the present paper is to propose a new-type mass matrix form (1), and not to give a reasonable mass generation mechanism for the mass matrix form (1). Theoretical justification of the model (1) will be given elsewhere.

The author would like to thank Professor M. Tanimoto for a careful reading of the manuscript and helpful comments. This work was supported in part by a Shizuoka Prefectural Government Grant.

-
- [1] M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49**, 652 (1973).
- [2] M. Gell-Mann, P. Rammond, and R. Slansky, in *Supergravity*, Proceedings of the Workshop, Stony Brook, New York, 1979, edited by P. van Nieuwenhuizen and D. Z. Freedman (North-Holland, Amsterdam, 1979); T. Yanagida, in *Proceedings of the Workshop on Unified Theory and Baryon Number in the Universe*, Tsukuba, Japan, 1979, edited by A. Sawada and A. Sugamoto (KEK Report No. 79-18, Tsukuba, 1979).
- [3] Y. Koide, *Phys. Rev. D* **28**, 252 (1983).
- [4] Y. Koide, *Mod. Phys. Lett.* **8**, 2071 (1993).
- [5] S. Dimopoulos and L. Susskind, *Nucl. Phys.* **B155**, 237 (1979); E. Eichten and K. D. Lane, *Phys. Lett.* **90B**, 125 (1980).
- [6] ARGUS Collaboration, H. Albrecht *et al.*, *Phys. Lett. B* **292**, 221 (1992).
- [7] BES Collaboration, J. Z. Bai *et al.*, *Phys. Rev. Lett.* **69**, 3021 (1992).
- [8] CLEO Collaboration, R. Balest *et al.*, *Phys. Rev. D* **47**, R3671 (1993).
- [9] GALLEX Collaboration, P. Anselman *et al.*, *Phys. Lett. B* **285**, 390 (1992).
- [10] J. Gasser and H. Leutwyler, *Phys. Rep.* **87**, 77 (1983).
- [11] Usually the value $m_b = 5.3 \pm 0.1$ GeV is referred to as the Gasser-Leutwyler value of m_b . However, the value is not that for $\Lambda_{\overline{\text{MS}}}^{(3)} = 0.150$ GeV.
- [12] J. Ellis, G. L. Fogli, and E. Lisi, *Phys. Lett. B* **274**, 456 (1992); Z. Hioki, *Mod. Phys. Lett.* **6**, 2129 (1991).
- [13] For a recent study of light quark masses, for example, see J. F. Donoghue and B. E. Holstein, *Phys. Rev. Lett.* **69**, 3444 (1992).
- [14] S. Weinberg, *Ann. N.Y. Acad. Sci.* **38**, 185 (1977); H. Fritzsch, *Phys. Lett.* **73B**, 317 (1978); *Nucl. Phys.* **B155**, 189 (1979).
- [15] C. Jarlskog, *Phys. Rev. Lett.* **55**, 1839 (1985); O. W. Greenberg, *Phys. Rev. D* **32**, 1841 (1985); I. Dunitz, O. W. Greenberg, and D.-d. Wu, *Phys. Rev. Lett.* **55**, 2935 (1985); C. Hamzaoui and A. Barroso, *Phys. Lett.* **154B**, 202 (1985); D.-d. Wu, *Phys. Rev. D* **33**, 860 (1986).
- [16] Particle Data Group, K. Hikasa *et al.*, *Phys. Rev. D* **45**, S1 (1992).
- [17] Y. Koide, *Mod. Phys. Lett. A* **5**, 2319 (1990).
- [18] M. Leurer, Y. Nir, and N. Seiberg, *Nucl. Phys.* **B398**, 319 (1993).