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t expansion of heavy-light mesons

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We use the t expansion method to calculate the heavy-light spectrum of Hamiltonian lattice QCD with two massless dynamical quarks within the Kogut-Susskind formulation. Restricting ourselves to the static approximation, i.e., infinite heavy quark mass, we construct all lattice heavy-light mesons which are contained in an elementary cube. We calculate the bound-state energies of the $1S$, $1P_{1/2}$, and $1P_{3/2}$ states. We obtain an estimate for the $1P_{1/2}$-$1P_{3/2}$ mass splitting in the range of 220–260 MeV and 240–280 MeV for the $1P_{1/2}$-$1S$ splitting.

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I. INTRODUCTION

Over the past few years, the idea of heavy-quark symmetry has been under extensive study. The notion behind this idea [1,2] is that in the limit of infinite quark mass, the dynamics of QCD is invariant under spin-flavor symmetries of the heavy-quark degrees of freedom. In particular, one can investigate mesons containing a single heavy quark Q. In the limit $M_Q \to \infty$, the spectrum of such mesons will be independent of M_Q. Therefore, if we line up the ground states, corresponding to subtracting the mass of the heavy quark, then the spectra built on different flavors of heavy quarks (b or c) should be the same [3].

In this paper we employ the t expansion method to calculate the spectrum of heavy-light mesons. We work within the Kogut-Susskind formulation, using a Hamiltonian of two massless dynamical quarks. In this method, the incorporation of infinitely heavy quarks is straightforward, and does not impose any new ingredients in the calculation process.

This work is a continuation of a recent publication [4], where we attempted to calculate baryons heavier than the nucleon. For this purpose we repeated the analysis of finding the particle content of higher lattice states which was carried out by Golterman and Smit [5] for the case of four flavors. We have calculated the masses of the Δ and the N^* ($\frac{1}{2}^-$ nucleon), and found them to be about 25% heavier than their observed values. We associated this fact with the extended nature of these states, which prevents them to propagate on the lattice within the H^7 order expansion that we have carried out.

Here we repeat the group-theoretical analysis for the heavy-light spectrum. The analysis is essentially the same as for the baryonic spectrum, with the baryons replaced by a single light quark. We construct all heavy-light mesonic states which are contained in an elementary cube. We then calculate the bound-state energies of all the one-link states and find the corresponding mass splittings.

This paper is organized as follows. In Sec. II we analyze the relation between lattice and continuum heavy-light states and construct all the lattice states which are contained in an elementary cube. In Sec. III we briefly describe the t expansion of the Kogut-Susskind Hamiltonian. In Sec. IV we calculate the bound-state energies of heavy-light mesons. Section V contains discussion of results.

II. HEAVY-LIGHT MESON STATES

As we already stated, in the limit $M_Q \to \infty$, the spectrum of heavy-light mesons will be independent of M_Q, and mass splittings for different flavors of heavy quarks (b or c) should be the same. Moreover, since the spin of the heavy quark decouples, we should find states occurring in doublets corresponding to the two possible orientations of the heavy-quark spin. These states are characterized by their total angular momentum $J = l + S_q$, where l is the relative orbital angular momentum, and S_q the spin of the light quark.

In the presence of a static quark, all the shift symmetries of the lattice are broken, including the π rotations in isospace. The smaller symmetry, which remains for the heavy-light states, is the octahedral group O, i.e., $\pi/2$ simultaneous rotations in space and isospace.

Since both the total angular momentum and the nuclear isospin are half-integers, we should address ourselves to the baryonic irreducible representations (irreps) of the group G. The reduction of the irreps of O to the continuum $SU(2)_{spin} \times SU(2)_{flavor}$ states was given in Refs. [4,5]. Here we should only consider the $I=\frac{1}{2}$ states. Looking at $J=\frac{1}{2},\frac{3}{2},\frac{5}{2}$ states we quote the results of Refs. [4,5] in Table I. The rightmost column in Table I relates O irreps to the total angular momentum of continuum heavy-light states, which are doubly degenerate.

In analogy to light-light lattice states, we can formulate the heavy-light mesons in two distinct classes. The one-link states are of the form

$$\sum_i \sum_{\hat{\mathbf{n}}} \alpha(\hat{\mathbf{n}})[U_{ij}(\mathbf{0},\hat{\mathbf{n}})\chi_j(\hat{\mathbf{n}}) \pm U_{ij}(\mathbf{0},-\hat{\mathbf{n}})\chi_j(-\hat{\mathbf{n}})] \;. \tag{2.1}$$

This operator corresponds to a heavy-light meson where we have localized a static quark at the origin. The pres-

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TABLE I. Reduction of O irreps with respect to continuum J of heavy-light states.

O		$J=l+S_q$	$J=l+S_q+S_Q$
$\mathbf{A}_1$	$\to$	$\frac{1}{2}$	0,1
$\mathbf{A}_2$	$\to$	$\frac{5}{2}$	2,3
$\mathbf{E}$	$\to$	$\frac{3}{2},\frac{5}{2}$	1,2
$\mathbf{F}_1$	$\to$	$\frac{1}{2},\frac{3}{2},\frac{5}{2}$	0,1
$\mathbf{F}_2$	$\to$	$\frac{3}{2},\frac{5}{2}$	1,2

ence of an external source is indicated by the violation of Gauss law at the origin. This state is clearly not gauge invariant and we fix the gauge by summing over the color at that site, a choice directed by computational convenience.

Similarly, the second class of three-link operators reads

$$\sum_i \sum_{\{U\}} \sum_{\hat{\mathbf{n}}} \sum_{\epsilon_1,\epsilon_2,\epsilon_3} \alpha(\epsilon_1,\epsilon_2,\epsilon_3) U_{ij}(\mathbf{0},\epsilon_1\hat{\mathbf{x}}+\epsilon_2\hat{\mathbf{y}}+\epsilon_3\hat{\mathbf{z}}) \times \chi_j(\mathbf{0}+\epsilon_1\hat{\mathbf{x}}+\epsilon_2\hat{\mathbf{y}}+\epsilon_3\hat{\mathbf{z}}) , \quad (2.2)$$

where $\epsilon_1,\epsilon_2,\epsilon_3=\pm1$, and $\sum_{\{U\}}$ means the summation over all possible shortest paths of gauge links connecting sites $\mathbf{0}$ and $\epsilon_1\hat{\mathbf{x}}+\epsilon_2\hat{\mathbf{y}}+\epsilon_3\hat{\mathbf{z}}$.

Clearly, the above states possess definite parity and total angular momentum, but they do not have zero total linear momentum, since they were constructed in the rest frame of the heavy quark. Nevertheless, we still hope to extract the continuum ground states from our lattice operators by means of the t expansion.

The explicit form of the lattice heavy-light states is found by applying the projection operator

$$\Psi_i^{(\mu)} = \frac{n_\mu}{g} \sum_R D_{ii}^{*(\mu)}(R) O_R \Psi , \quad (2.3)$$

where n_μ is the dimension of the representation μ, g is the group size, $D_{ii}^{(\mu)}(R)$ the matrix element of $R\in O$ in the representation μ, and O_R is the operator corresponding to R. We construct all possible one- and three-link operators. From the one-link states only the $\mathbf{A}_1^+$, $\mathbf{E}^+$, and $\mathbf{F}_1^-$ multiplets can be constructed, while $\mathbf{A}_1^-$, $\mathbf{A}_2^+$, $\mathbf{F}_1^+$, and $\mathbf{F}_2^-$ multiplets are the only ones that can be realized by three-link states. The results of the construction are summarized in Tables II and III.

III. t EXPANSION OF THE KOGUT-SUSSKIND HAMILTONIAN

The SU(3) pure gauge theory as defined by the Kogut-Susskind Hamiltonian is [6]

$$H_G = \frac{g^2}{2}\left[\sum_l \mathbf{E}_l^2 + x\sum_p (6-\mathrm{tr}U_p - \mathrm{tr}U_p^\dagger)\right] , \quad (3.1)$$

where g is the coupling constant and $x=2/g^4$. The link operators $\mathbf{E}_l$ and U_l, which appear in Eq. (3.1), are conjugate quantum variables satisfying the commutation relations

$$[U_l^a, U_{l'}] = \frac{\lambda^a}{2} U_l \delta_{ll'} , \quad (3.2)$$

where λ^a are the eight Gell-Mann matrices of SU(3). E_l^a is the color electric flux operator associated with the link l, and $\mathrm{tr}U_p$ is the color magnetic flux operator associated with the plaquette p.

For dynamical fermions we employ the Kogut-Susskind scheme [7–9], in which the fermions are represented by a single degree of freedom per site:

$$\{\chi_i^\dagger(\mathbf{r}), \chi_j(\mathbf{r}')\} = \delta_{\mathbf{r},\mathbf{r}'}\delta_{ij} , \quad (3.3)$$

where i,j are color indices. The fermionic part of the Hamiltonian is

$$H_F = \frac{i}{2}\sum_{\mathbf{r},\mu} \eta_\mu(\mathbf{r})[\chi^\dagger(\mathbf{r})U(\mathbf{r},\mu)\chi(\mathbf{r}+\mu) - \chi^\dagger(\mathbf{r}+\mu)U^\dagger(\mathbf{r},\mu)\chi(\mathbf{r})] , \quad (3.4)$$

where

$$\eta_x(\mathbf{r})=(-1)^z, \quad \eta_y(\mathbf{r})=(-1)^x , \quad \eta_z(\mathbf{r})=(-1)^y , \quad (3.5)$$

$\mathbf{r}$ varies over the lattice sites, and μ over the three directions of space. H_F describes QCD with two massless quarks (u and d).

TABLE II. One-link heavy-light mesonic states. $|v\rangle$ is the strong-coupling vacuum.

$O^\pm$	One-link lattice heavy-light state
$\mathbf{A}_1^+$	$\{[U(\mathbf{0},\hat{\mathbf{z}})\chi(\mathbf{0}+\hat{\mathbf{z}}) - U(\mathbf{0},-\hat{\mathbf{z}})\chi(\mathbf{0}-\hat{\mathbf{z}})] + \text{cyclic permutations}\}\|v\rangle$
$\mathbf{E}^+$	$\{[U(\mathbf{0},\hat{\mathbf{z}})\chi(\mathbf{0}+\hat{\mathbf{z}}) - U(\mathbf{0},-\hat{\mathbf{z}})\chi(\mathbf{0}-\hat{\mathbf{z}})] - [U(\mathbf{0},\hat{\mathbf{x}})\chi(\mathbf{0}+\hat{\mathbf{x}}) - U(\mathbf{0},-\hat{\mathbf{x}})\chi(\mathbf{0}-\hat{\mathbf{x}})]\}\|v\rangle$ $\{[U(\mathbf{0},\hat{\mathbf{z}})\chi(\mathbf{0}+\hat{\mathbf{z}}) - U(\mathbf{0},-\hat{\mathbf{z}})\chi(\mathbf{0}-\hat{\mathbf{z}})] + [U(\mathbf{0},\hat{\mathbf{x}})\chi(\mathbf{0}+\hat{\mathbf{x}}) - U(\mathbf{0},-\hat{\mathbf{x}})\chi(\mathbf{0}-\hat{\mathbf{x}})] - 2[U(\mathbf{0},\hat{\mathbf{y}})\chi(\mathbf{0}+\hat{\mathbf{y}}) - U(\mathbf{0},-\hat{\mathbf{y}})\chi(\mathbf{0}-\hat{\mathbf{y}})]\}\|v\rangle$
$\mathbf{F}_1^-$	$[U(\mathbf{0},\hat{\mathbf{x}})\chi(\mathbf{0}+\hat{\mathbf{x}}) + U(\mathbf{0},-\hat{\mathbf{x}})\chi(\mathbf{0}-\hat{\mathbf{x}})]\|v\rangle$ $[U(\mathbf{0},\hat{\mathbf{y}})\chi(\mathbf{0}+\hat{\mathbf{y}}) + U(\mathbf{0},-\hat{\mathbf{y}})\chi(\mathbf{0}-\hat{\mathbf{y}})]\|v\rangle$ $[U(\mathbf{0},\hat{\mathbf{z}})\chi(\mathbf{0}+\hat{\mathbf{z}}) + U(\mathbf{0},-\hat{\mathbf{z}})\chi(\mathbf{0}-\hat{\mathbf{z}})]\|v\rangle$

TABLE III. Three-link heavy-light mesonic states. Summation over all possible shortest paths of gauge links is omitted. $\epsilon_1,\epsilon_2,\epsilon_3=\pm 1$.

$O^{\pm}$	Three-link lattice heavy-light state
$\mathbf{A}_1^-$	$\sum_{\epsilon_1,\epsilon_2,\epsilon_3} U(0,\epsilon_1\hat{\mathbf{x}}+\epsilon_2\hat{\mathbf{y}}+\epsilon_3\hat{\mathbf{z}})\chi(0+\epsilon_1\hat{\mathbf{x}}+\epsilon_2\hat{\mathbf{y}}+\epsilon_3\hat{\mathbf{z}})\|v\rangle$
$\mathbf{A}_2^+$	$\sum_{\epsilon_1,\epsilon_2,\epsilon_3} \epsilon_1\epsilon_2\epsilon_3 U(0,\epsilon_1\hat{\mathbf{x}}+\epsilon_2\hat{\mathbf{y}}+\epsilon_3\hat{\mathbf{z}})\chi(0+\epsilon_1\hat{\mathbf{x}}+\epsilon_2\hat{\mathbf{y}}+\epsilon_3\hat{\mathbf{z}})\|v\rangle$
$\mathbf{F}_1^+$	$\sum_{\epsilon_1,\epsilon_2,\epsilon_3} \epsilon_1 U(0,\epsilon_1\hat{\mathbf{x}}+\epsilon_2\hat{\mathbf{y}}+\epsilon_3\hat{\mathbf{z}})\chi(0+\epsilon_1\hat{\mathbf{x}}+\epsilon_2\hat{\mathbf{y}}+\epsilon_3\hat{\mathbf{z}})\|v\rangle$
	$\sum_{\epsilon_1,\epsilon_2,\epsilon_3} \epsilon_2 U(0,\epsilon_1\hat{\mathbf{x}}+\epsilon_2\hat{\mathbf{y}}+\epsilon_3\hat{\mathbf{z}})\chi(0+\epsilon_1\hat{\mathbf{x}}+\epsilon_2\hat{\mathbf{y}}+\epsilon_3\hat{\mathbf{z}})\|v\rangle$
	$\sum_{\epsilon_1,\epsilon_2,\epsilon_3} \epsilon_3 U(0,\epsilon_1\hat{\mathbf{x}}+\epsilon_2\hat{\mathbf{y}}+\epsilon_3\hat{\mathbf{z}})\chi(0+\epsilon_1\hat{\mathbf{x}}+\epsilon_2\hat{\mathbf{y}}+\epsilon_3\hat{\mathbf{z}})\|v\rangle$
F_2^-	$\sum_{\epsilon_1,\epsilon_2,\epsilon_3} \epsilon_2\epsilon_3 U(0,\epsilon_1\hat{\mathbf{x}}+\epsilon_2\hat{\mathbf{y}}+\epsilon_3\hat{\mathbf{z}})\chi(0+\epsilon_1\hat{\mathbf{x}}+\epsilon_2\hat{\mathbf{y}}+\epsilon_3\hat{\mathbf{z}})\|v\rangle$
	$\sum_{\epsilon_1,\epsilon_2,\epsilon_3} \epsilon_3\epsilon_1 U(0,\epsilon_1\hat{\mathbf{x}}+\epsilon_2\hat{\mathbf{y}}+\epsilon_3\hat{\mathbf{z}})\chi(0+\epsilon_1\hat{\mathbf{x}}+\epsilon_2\hat{\mathbf{y}}+\epsilon_3\hat{\mathbf{z}})\|v\rangle$
	$\sum_{\epsilon_1,\epsilon_2,\epsilon_3} \epsilon_1\epsilon_2 U(0,\epsilon_1\hat{\mathbf{x}}+\epsilon_2\hat{\mathbf{y}}+\epsilon_3\hat{\mathbf{z}})\chi(0+\epsilon_1\hat{\mathbf{x}}+\epsilon_2\hat{\mathbf{y}}+\epsilon_3\hat{\mathbf{z}})\|v\rangle$

Starting with the strong-coupling vacuum $|0\rangle$, which is the state annihilated by the color electric field

$$\mathbf{E}_l|0\rangle=0\ , \tag{3.6}$$

the full vacuum is chosen in a staggered form which divides the lattice into two sublattices, that of even r (i.e., even $x+y+z$) and that of odd r:

$$|v\rangle=\Pi_{\text{odd }r}|+\rangle\Pi_{\text{even }r}|-\rangle|0\rangle\ . \tag{3.7}$$

The t expansion method has been reviewed extensively in the context of pure gauge theories [10–13]. Its application to lattice theories with dynamical quarks was described in Refs. [14,15]. Its main idea is that the energy function

$$E_\phi(t)=\frac{\langle\phi|He^{-tH}|\phi\rangle}{\langle\phi|e^{-tH}|\phi\rangle}\ , \tag{3.8}$$

where $|\phi\rangle$ is some hadronic state, tends to the mass of the lightest particle with the same quantum numbers as $|\phi\rangle$ in the same limit. Moreover, $E_\phi(t)$ can be written as

$$E_\phi(t)=\sum_{n=0}^{\infty}\frac{(-t)^n}{n!}\langle H^{n+1}\rangle_\phi^c\ , \tag{3.9}$$

where the connected matrix elements $\langle H^{n+1}\rangle_\phi^c$ are calculated as a sum of connected diagrams. After evaluating $E_\phi(t)$, we subtract from it the vacuum energy $E_0(t)$ to obtain a prediction for the mass function, $M_\phi(t)$. All our physical predictions will be presented in the form of mass ratios, which, beyond the crossover region [15], represent the lattice approximation of the continuum theory.

Exploiting such Taylor series of any t-expansion of an operator $O(t)$ is done by forming D-Padé approximants [11] to the series in t and using them to obtain the asymptotic value of $O(t)$. This means approximating the t derivative of the expression by nondiagonal L/M Padé approximants which are integrated out to infinity. In order for the integration to work the degrees of the polynomials have to be chosen so that $M\geq L+2$. The number of approximants generated this way is quite larger than that of the diagonal approximants that one can generate for the finite series.

In general, all our mass ratios are of order t^6, and thus we can plot only the 0/2, 0/3, 0/4, 0/5, 1/3, and 1/4 D-Padé approximants. Usually, some of the approximants have singularities or vary strongly. In all figures that follow, we will display only those D-Padé approximants that are not plagued by these features, and that are more or less consistent with one another. In all our ratios, we expect to find the physical result just beyond the crossover region [15], i.e., near $y\sim 2$.

IV. HEAVY-LIGHT MESON MASS SPLITTINGS

We have calculated all the one-link heavy-light states, namely, the states corresponding to the $\mathbf{A}_1^+$, $\mathbf{E}^+$, and $\mathbf{F}_1^-$ irreps. Converting to standard spectroscopic notations, these states will be denoted as $1P_{1/2}$, $1P_{3/2}$, and $1S$, respectively, where the subscripts stand for the light quark total angular momentum, $l+S_q$. The series for the bound-state energies of these states, up to the t^6 order, read

$$E(1P_{1/2})=\frac{4}{3y}+\frac{11y}{4}t+\frac{1}{12}(-3y^3-26y)t^2+\frac{1}{864}(-72y^5-3303y^3+832y)t^3+\frac{1}{1296}(795y^5+8914y^3-416y)t^4$$
$$+\frac{1}{622\,080}(96\,120y^9+96\,120y^7+343\,0047y^5-4239\,028y^3+53\,248y)t^5$$
$$+\frac{1}{5598\,720}(20\,475y^{11}-228\,393y^9-8139\,852y^7-788\,249\,548y^5+30\,881\,536y^3-106\,496y)t^6+O(t^7)\ , \tag{4.1}$$

$$E(1P_{3/2})=\frac{4}{3y}+\frac{5y}{4}t+\frac{1}{24}(3y^3-28y)t^2+\frac{1}{864}(36y^5-1959y^3+448y)t^3+\frac{1}{5184}(-735y^5+22\,600y^3-896y)t^4$$
$$+\frac{1}{4976\,640}(-32\,400y^9+62\,640y^7+17\,156\,049y^5-23\,737\,856y^3+229\,376)t^5$$
$$+\frac{1}{22\,394\,880}(-40\,950y^{11}+863\,730y^9-518\,7924y^7-192\,596\,205y^5$$
$$+98\,464\,768y^3-229\,376)t^6+O(t^7)\ , \tag{4.2}$$

$$E(1S)=\frac{4}{3y}+\frac{5y}{4}t-\frac{7}{6}yt^2+\frac{1}{864}(-1831y^3+448y)t^4+\frac{1}{2592}(435y^5+11\,028y^3-448y)t^4$$
$$+\frac{1}{622\,080}(48\,600y^7+2337\,477y^5-2932\,416y^3+28\,672y)t^5$$
$$+\frac{1}{5598\,720}(75\,600y^9-5373\,081y^7-44\,306\,382y^5+24\,478\,976y^3-57\,344y)t^6+O(t^7)\ . \tag{4.3}$$

These states are, in principle, easier to calculate than the light-light mesons due to the static nature of one of the quarks. Nonetheless, they require the evaluation of a few hundred connected diagrams already in the t^6 order. Taking the difference between any two series for the bound-state energies, we obtain the corresponding mass splitting. Hence, we shall look at ratios between these mass splittings and the hadrons which we have previously calculated, and obtain an estimate for the mass splittings by using the observed values for the hadron masses appearing in these ratios. All the series (4.1)–(4.3) start at 4/3 in the strong-coupling limit, while the series of their differences start at zero. In addition, the $1P_{3/2}$ and the $1S$ series are identical also in order $O(t)$. As a result, the series for their mass splitting has a vanishing coefficient in t, and is not accessible for the D-Padé analysis. Hence we restrict ourselves to study the $1P_{1/2}$-$1P_{3/2}$ and $1P_{1/2}$-$1S$ mass splittings.

The first hadron we have at hand is the nucleon, studied in Ref. [15]. Unfortunately, the nucleon state also has a vanishing energy in the strong-coupling limit. Therefore, we cannot form the desired ratio with the nucleon. We have tried, however, to plot the ratios between the nucleon and the bound-state energies themselves. The only ratio which turned out to be devoid of singularities is the $N/E(1P_{1/2})$ ratio, depicted in Fig. 1. The ratio scales more or less after the crossover region and near $y\sim 2$ we read the ratio to be in the range of 1.15–1.3. This gives us an estimate of 725–750 MeV for the $1P_{1/2}$ bound-state energy.

Next, we plot the ratios between these mass splittings and the ω meson. In Fig. 2 we plot the ratio between

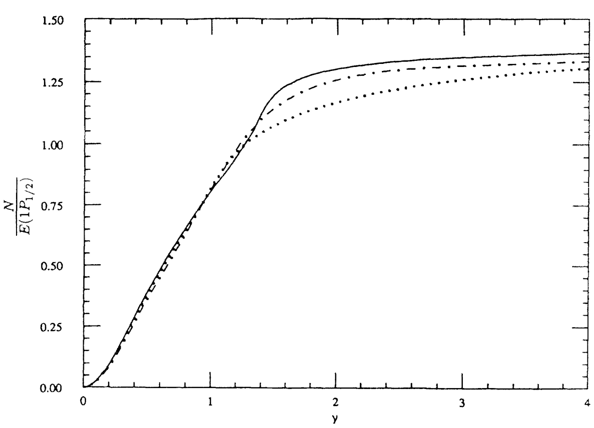

FIG. 1. Ratio between the nucleon and the bound-state energy of $1P_{1/2}$. The solid dotted, and dot-dashed lines are the 0/4, 0/5, and 1/4 approximants, respectively.

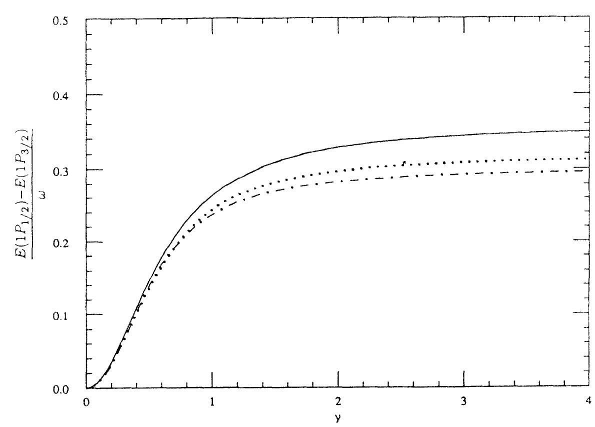

FIG. 2. Ratio between $1P_{1/2}$-$1P_{3/2}$ mass splitting and the ω-meson mass. The solid, dotted, and dot-dashed lines are the 0/3, 0/4, and 0/5 approximants, respectively.

$1P_{1/2}$-$1P_{3/2}$ and the ω mass. The solid, dotted, and dot-dashed curves correspond to the 0/3, 0/4, and 0/5 *D*-Padé approximant, respectively. In all figures that follow, except the one involving the 0^{++} scalar mass, we shall plot the same three approximants. The 0/2 approximant is always very different from the other ones, while the 1/3 and 1/4 *D*-Padé are plagued with singularities. It is also true that the 0/4 and 0/5 approximants coincide better with one another than with the 0/3 one, and they show a better trend for scaling. In Fig. 2 we read near $y \sim 2$ a ratio in the range of 0.28–0.33, which yields a $1P_{1/2}$-$1P_{3/2}$ splitting of 220–255 MeV. Figure 3 displays the ratio between the $1P_{1/2}$-$1S$ splitting and the ω mass, and provides a mass splitting in the range of 240–280 MeV. This splitting is larger than the $1P_{1/2}$-$1P_{3/2}$ one.

Next, we have plotted the ratios between the heavy-light mass differences and the 0^{++} state. Only the ratio that involves the $1P_{1/2}$-$1P_{3/2}$ splitting turned out to have a nonsingular behavior, although it suffers from the features which are characteristic of 0^{++} plots; namely, it has a peak in the crossover region and it breaks down right after $y \sim 2$. In this case, we estimate the ratio from the peak in the $y \sim 1.5$–2 region. Taking the known value for the 0^{++} mass to be of order 1300 MeV [12], we read from Fig. 4 a ratio of 0.17–0.20, or a $1P_{1/2}$-$1P_{3/2}$ splitting of 220–260 MeV, in agreement with Fig. 2.

Finally, we examine the ratio between our mass differences and the heavier baryons, the Δ and the N^*. These states have been calculated in Ref. [15] and their calculated masses turned out to be about 25% higher than their observed values. This is not surprising once we understand that because they are extended objects on the lattice, they cannot propagate on the lattice within our order of expansion. Another feature of these states is that mass ratios between them and the lowest-lying hadrons usually fail to scale, especially in the case of the Δ.

In Figs. 5 and 6, we display the $(1P_{1/2}-1P_{3/2})/\Delta$ and $(1P_{1/2}-1S)/\Delta$ ratios, respectively. These approximates do not show any scaling tendency; nonetheless, we can read near $y \sim 2$ values for these splittings: 160–195 and

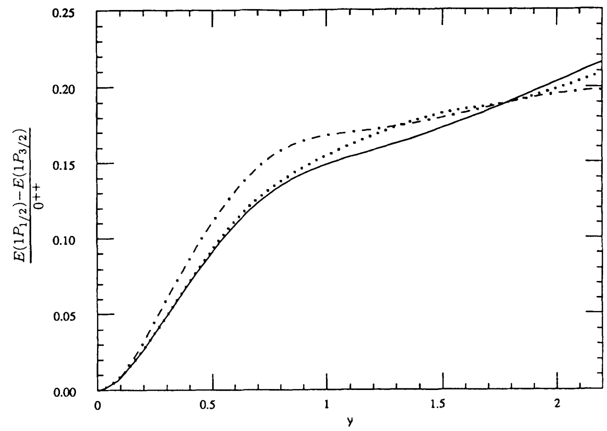

FIG. 4. The ratio between $1P_{1/2}$-$1P_{3/2}$ splitting and scalar mass. The solid, dotted, and dot-dashed lines are ratios of the 0/4, 0/5, and 1/4 approximants, respectively.

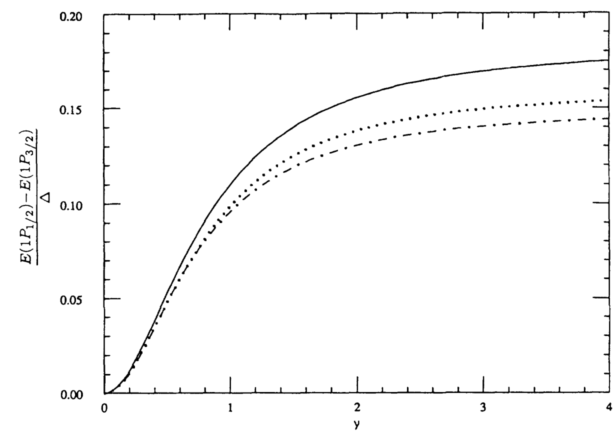

FIG. 5. $1P_{1/2}$-$1P_{3/2}$ mass splitting to Δ mass ratio. Shown are the 0/3 (solid line), 0/4 (dotted line), and 0/5 (dot-dashed line) approximants.

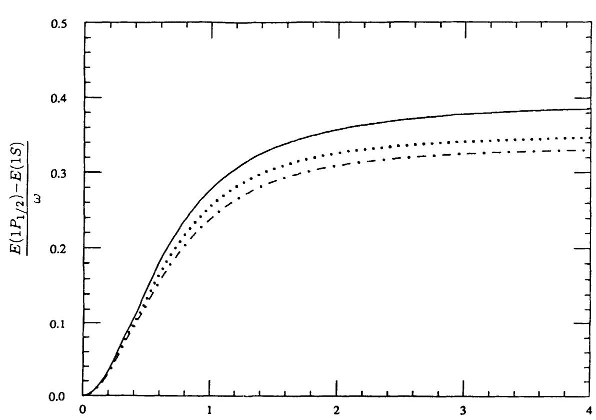

FIG. 3. Ratio between $1P_{1/2}$-$1S$ mass splitting and the ω-meson mass. Shown are the 0/3 (solid line), 0/4 (dotted line), and 0/5 (dot-dashed line) approximants.

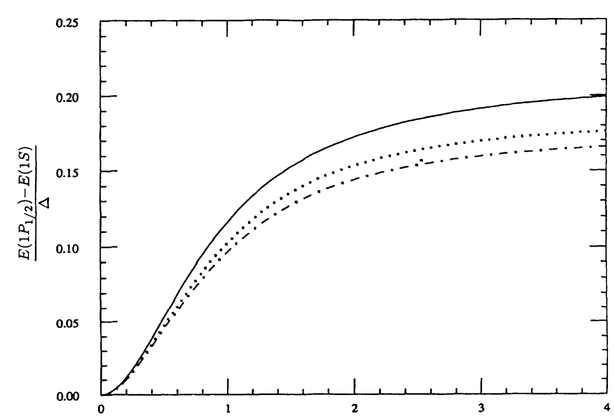

FIG. 6. Ratio between $1P_{1/2}-1S$ mass splitting and the Δ mass. The solid, dotted, and dot-dashed lines are the 0/3, 0/4, and 0/5 approximants, respectively.

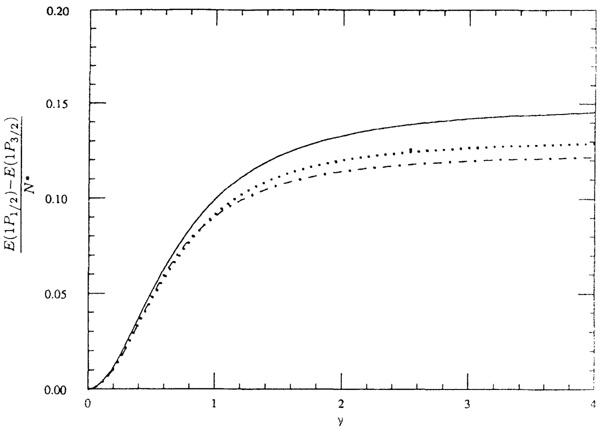

FIG. 7. $1P_{1/2}-1P_{3/2}$ mass splitting to N^* mass ratio. Shown are the 0/3 (solid line), 0/4 (dotted line), and 0/5 (dot-dashed line) approximants.

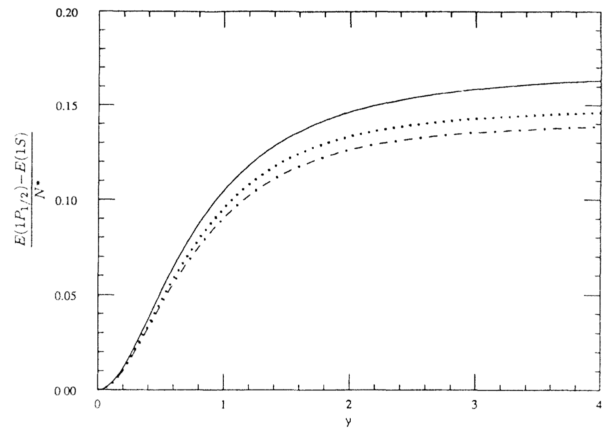

FIG. 8. Ratio between $1P_{1/2}$-$1S$ mass splitting and the N^* mass. The solid, dotted, and dot-dashed lines are the 0/3, 0/4, and 0/5 approximants, respectively.

180–215 MeV, respectively. The same ratios for the N^* case show a similar, though somewhat better, behavior. In Figs. 7 and 8 we find a $1P_{1/2}-1P_{3/2}$ splitting of 175–205 MeV and a $1P_{1/2}-1S$ splitting of 190–225 MeV. These numbers are consistent with the baryon masses being about 25% higher than their observed values.

V. DISCUSSION

In our lattice QCD calculations, we use a diagrammatic Hamiltonian approach, the t expansion. We apply this method to the theory with two massless dynamical quarks, within the Kogut-Susskind formulation. The inclusion of infinitely heavy quarks is natural in our language, and the calculation of heavy-light states is essentially the same as in the light-light case. We have calculated all the three states that are one-link heavy-light states. Relying on Figs. 2 and 3, which gave the most stable results, we obtained the following estimates: The $1P_{1/2}$-$1P_{3/2}$ mass difference is of order 220-255 MeV, while the $1P_{1/2}$-$1S$ splitting is of order 240-280 MeV. These level splittings are too small compared to the charmed spectrum. In the case of the b quark, there is no sufficient data. The double degeneracy that we have due to the spin of the heavy quark should be lifted by the $O(1/M_Q)$ correction.

We note that no such calculation was carried out by Monte Carlo (MC) simulations with fully dynamical quarks. MC calculations were done in the quenched approximation with Wilson fermions, where multistate smearing functions were used [16]. The result reported there for the $1P_{1/2}$-$1S$ mass splitting is of order 400 MeV, while for the $1P_{3/2}$-$1P_{1/2}$ the reported value is of order 60 MeV with a large error. This value for the $1P_{1/2}$-$1S$ splitting is more plausible than ours compared to the charmed data. But in other MC simulations of heavy-light masses, e.g. [17], mass splittings have turned out usually to be too low. This behavior is not surprising since the more energetic the states one calculates, the deeper one should reach towards the weak-coupling region in order to obtain sound results. We note that we have obtained a different line up of the heavy-light levels compared to the MC results; namely, the order of the $1P_{1/2}$ and $1P_{3/2}$ levels is interchanged.

The usage of multistate smearing in the MC calculation is done in order to isolate better the ground state. It is also possible to use such smearing in our formulation by taking a trial state which is a superposition of degenerate states and minimizing the results by a variational method. Such a method could be useful also in the case of light states, but has not been employed due to its computational complexity.

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