

Isospin multiplet structure in ultraheavy fermion bound states

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The coupled Bethe-Salpeter bound state equations for a $Q\bar{Q}$ system, where $Q=(U,D)$ is a degenerate, fourth generation, superheavy quark doublet, are solved in several ladder approximation models. The exchanges of gluon, Higgs, and Goldstone modes in the standard model are calculated in the ultraheavy quark limit where weak γ , W^\pm , and Z^0 contributions are negligible. A natural $I=0$ and $I=1$ multiplet pattern is found, with large splittings occurring between the different weak isospin states when M_Q , the quark masses, are larger than values in the range $0.4 \text{ TeV} < M_Q < 0.8 \text{ TeV}$, depending on which model is used. Consideration of ultraheavy quark lifetime constraints and U - D mass splitting constraints are reviewed to establish the plausibility of lifetime and mass degeneracy requirements assumed for this paper.

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I. INTRODUCTION

The prospect of Superconducting Super Collider (SSC) and CERN Large Hadron Collider (LHC) experiments to probe TeV phenomena has stimulated a wide variety of ideas about the physics at these energies. The standard model itself, expanded to a superheavy fourth generation, could display some striking new effects. There is a variety of "heavy-neutrino" fourth generation models in the literature [1], and we adopt the attitude that some model of this type with the Higgs mechanism providing the fermion and gauge boson mass generation represents nature at 0.5–1.0 TeV. The effect that we explore in this paper is the strong Yukawa coupling of the Higgs sector to the fourth generation and the bound state spectrum in the heavy quark-antiquark system ($Q\bar{Q}$) that is generated by this strong coupling [2,3].

There have been a number of heavy fermion physics studies since the Higgs mechanism and the standard model were proposed [4,5]. Like the heavy Higgs boson limit studies, the indications are that perturbative calculations break down and some new, strong coupling features of the standard model and/or totally new physics should emerge at the 0.5–1.0 TeV energy ranges. In particular Chanowitz, Furman, and Hinchliffe [4] showed that the tree approximation amplitudes of the type which serve as the kernels of our ladder Bethe-Salpeter integrals violate partial wave unitarity when the ultraheavy fermion mass is larger than about 500 GeV for a degenerate ultraheavy doublet. The ladder approximation Bethe-Salpeter equation used in the present work formally unitarizes the tree approximation kernel. We therefore expect our calculations to be reliable in their qualitative feature, but not quantitative detail, even above 500 GeV.

In a previous work [3], we described some new, deep-

binding features of the 0^- ground state of a single, superheavy $Q\bar{Q}$ system that were driven by the dominant, attractive Higgs scalar exchange plus weaker, but in some cases still significant, gluon exchange. The principal feature was that a very strong binding compared to QCD alone was generally exhibited for $M_Q > 0.4$ – 0.5 TeV in all of the model calculations that we tested.

In this paper, we expand significantly on this theme by doing a coupled channel analysis of a heavy quark doublet (U,D) including the Goldstone boson exchanges as well as the Higgs boson exchanges, and we discuss the s -channel exchange as well as the t -channel exchange contributions to the Bethe-Salpeter equation kernel. Here we include the results of calculations of the 0^+ and 1^- masses in addition to the 0^- ground state case analyzed previously.

Because of the weak isodoublet structure of the superheavy (U,D) system the bound states decompose naturally into weak isotriplet and weak isosinglet states when $M_U \simeq M_D$ and $M_Z \simeq M_W$. We find that the interplay between the scalar Higgs boson exchange and the (opposite sign) pseudoscalar Goldstone boson (longitudinal W and Z) exchange results in a clear isotriplet-isosinglet mass splitting in all of the channels in all model calculations we used. This signature is quite distinct for the fourth generation fermion doublet, since it must be approximately degenerate according to present constraints from W/Z physics.

In the following section, we outline the coupled-channel Bethe-Salpeter formalism used to calculate bound state masses. In Sec. III we present results of the calculations in the different approximation schemes used and comment on the main features. In Sec. IV we show that requiring metastability of fourth generation quarks puts only weak constraints on mixing with a lighter generation. We also review the evidence that a fourth generation of quarks, if they exist, would have to be approximately degenerate in mass. In Sec. V we summarize and conclude. In Appendix A, we describe the subtraction used to regulate the Bethe-Salpeter equations. In Appen-

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dix B, we give a short analysis of the running of the Yukawa coupling and its relevance to the calculations presented.

II. STRONG YUKAWA BINDING OF ULTRAHEAVY FERMION WEAK DOUBLETS: BETHE-SALPETER EQUATION FORMALISM INCLUDING GOLDSTONE BOSON EXCHANGE

If there are fourth and higher generations of fermion weak isospin doublets, the fractional mass splitting within these ultraheavy doublets [6] is constrained by the standard, three generation model's remarkable survival after two decades of stringent experimental tests [7]. A small mass splitting, or near degeneracy, could lead to a classic isospin-multiplet pattern for the bound state spectrum, and we show in this section that this is indeed the case when the Yukawa couplings of the Higgs and Goldstone bosons [8] are incorporated into the bound state equations-ladder approximation, Bethe-Salpeter equations in our study. We assume that the usual mass generation mechanism operates in the fourth generation sector as well, so the ultraheavy fermion doublets will have strong Yukawa couplings to the standard model Higgs and Goldstone modes. These Yukawa couplings, scalar

for the Higgs boson and pseudoscalar for Goldstone bosons, become the dominant ones when the quark masses reach 400–500 GeV. (The pseudoscalar exchanges actually become competitive only in the tight binding, highly relativistic regime. They decouple in the weak-binding limit.)

We develop below a setup for the two, coupled-channel Bethe-Salpeter equations of a quark doublet-antidoublet that interact through gluon and weak-sector Higgs and Goldstone boson exchange.

Let us consider a doublet of heavy quarks (U, D) interacting in the Feynman gauge by exchange of gluons g , a Higgs boson H , and Goldstone bosons χ^\pm, χ^0 . The photon and W^\pm and Z exchanges are always weak in the gauges chosen, have been checked and found to give insignificant contributions to the binding, and are not included in our development. It is convenient to adopt Feynman gauge in the ladder approximation (perturbative vertices, no crossed graphs), which we will use throughout this work. There are four quark-antiquark channels: $\bar{U}U, \bar{U}D, \bar{D}U,$ and $\bar{D}D$. These channels become coupled when the charged Goldstone boson exchanges are included, as illustrated in Fig. 1, where the Bethe-Salpeter equations are shown graphically. As discussed below, these equations admit a subtraction at fixed q that

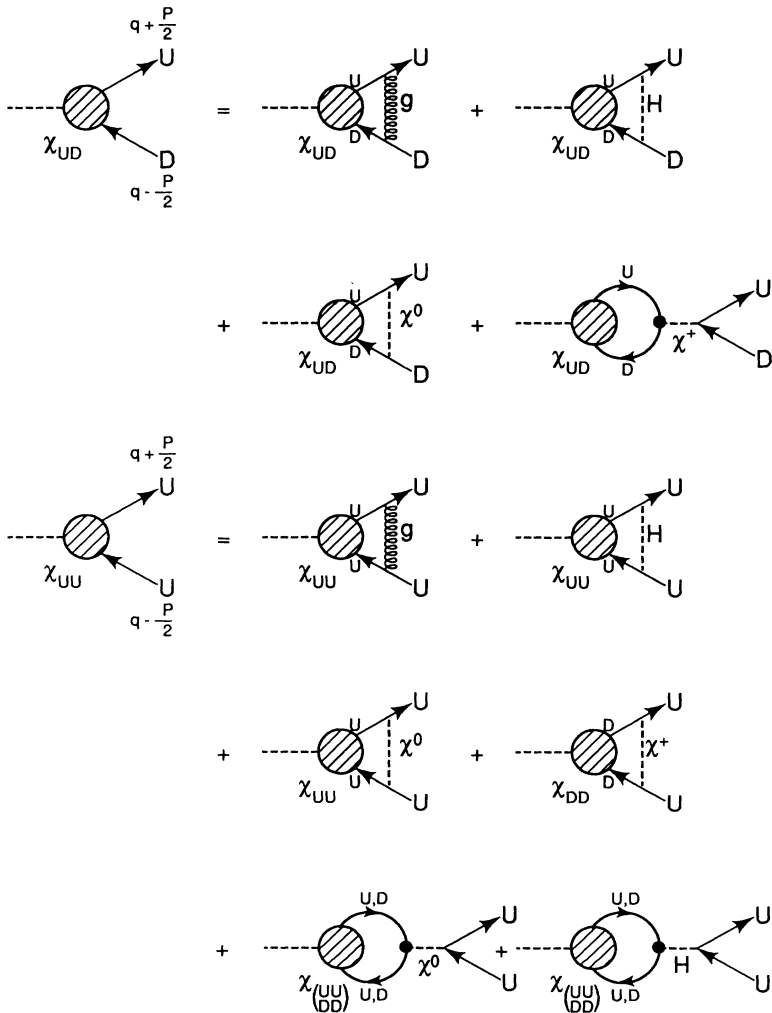


FIG. 1. Bethe-Salpeter equations for the $U\bar{D}$ and $U\bar{U}$ bound states.

eliminates the divergent, q -independent, annihilation graphs which enter the scalar and pseudoscalar bound state channels [9]. The corresponding gluon annihilation graph does not require consideration because we study only color singlet bound states.

The coupled-channel, doublet quark-antiquark bound state equations shown pictorially in Fig. 1 are displayed

$$\begin{aligned}
& S^{-1}(q_+) \chi_{UU}(q, P) S^{-1}(q_-) \\
&= \int \frac{d^4 k}{(2\pi)^4} \left[-\gamma_\mu \chi_{UU}(k, P) \gamma_\nu G^{\mu\nu}(k-q) - \frac{g_2^2 m^2}{4 M_W^2} \chi_{UU}(k, P) \frac{i}{(k-q)^2 - M_H^2} \right. \\
&\quad + \frac{g_2^2 m^2}{4 M_W^2} \gamma_5 \chi_{UU}(k, P) \gamma_5 \frac{i}{(k-q)^2 - M_Z^2} + \frac{g_2^2 m^2}{2 M_W^2} \gamma_5 \chi_{DD}(k, P) \gamma_5 \frac{i}{(k-q)^2 - M_Z^2} \\
&\quad \left. + \frac{g_2^2 m^2}{4 M_W^2} \frac{i}{P^2 - M_Z^2} \gamma_5 \text{Tr}[(\chi_{UU} - \chi_{DD}) \gamma_5] + \frac{g_2^2 m^2}{4 M_W^2} \frac{i}{P^2 - M_H^2} \text{Tr}(\chi_{UU} + \chi_{DD}) \right]. \tag{2.1}
\end{aligned}$$

\bar{D} - D Channel: interchange $U \leftrightarrow D$ in (2.1).

\bar{U} - D Channel:

$$\begin{aligned}
& S^{-1}(q_+) \chi_{UD}(q, P) S^{-1}(q_-) = \int \frac{d^4 k}{(2\pi)^4} \left[-\gamma_\mu \chi_{UD}(k, P) \gamma_\nu G^{\mu\nu}(k-q) - \frac{g_2^2 m^2}{4 M_W^2} \chi_{UD}(k, P) \frac{i}{(k-q)^2 - M_H^2} \right. \\
&\quad - \frac{g_2^2 m^2}{4 M_W^2} \gamma_5 \chi_{UD}(k, P) \gamma_5 \frac{i}{(k-q)^2 - M_Z^2} \\
&\quad \left. + \frac{g_2^2 m^2}{2 M_W^2} \frac{i}{P^2 - M_W^2} \gamma_5 \text{Tr}[\chi_{UD}(k, P) \gamma_5] \right]. \tag{2.2}
\end{aligned}$$

\bar{D} - U Channel: interchange $U \leftrightarrow D$ in Eq. (2.2).

In Eqs. (2.1) and (2.2), $\chi_{UU}(q, P)$ etc., represent the Bethe-Salpeter amplitudes for the $\bar{U}U$ etc., systems with relative four-momentum q and total four-momentum P . The quark masses, m , are assumed to be degenerate, $q_\pm \equiv q \pm P/2$ and $S^{-1}(q_\pm)$ are the inverse quark propagators with momenta q_\pm . Here $G^{\mu\nu}(k-q)$ represents the gluon propagator, assumed to behave like $(k-q)^{-2}$ as

in Eqs. (2.1) and (2.2) below. The subtracted, decoupled versions of these equations are presented later in this section, Eqs. (2.3a) and (2.3b). Let us display in shorthand notation the structure of the ladder approximation, momentum-space Bethe-Salpeter equations for the bound state amplitudes $\chi(q, P)$ in Feynman gauge.

\bar{U} - U channel:

$|k-q| \rightarrow \infty$. The SU(2) coupling constant is denoted by g_2 .

One may decouple the $\bar{U}U$ and $\bar{D}D$ equations by taking the combinations $(\bar{U}U \pm \bar{D}D)/\sqrt{2}$, and the resulting Bethe-Salpeter equations, subtracted as described in Appendix A, for the bound state wave-functions $\chi_{UD}(q, P)$, $\chi_-(q, P)$, and $\chi_+(q, P)$ read as follows [$\chi_{DU}(q, P)$ obeys the same equations as χ_{UD}]:

$$\begin{aligned}
& S^{-1}(q_+) \chi_\pm(q, P) S^{-1}(q_-) = \int \frac{d^4 k}{(2\pi)^4} \left\{ -\gamma_\mu \chi_\pm(k, P) \gamma_\nu G^{\mu\nu}(k-q) - i \frac{m^2 g_2^2}{4 M_W^2} \frac{\chi_\pm(k, P)}{(k-q)^2 - M_H^2} \right. \\
&\quad \left. + \frac{i m^2 g_2^2}{4 M_W^2} \frac{\gamma_5 \chi_\pm(k, P) \gamma_5}{(k-q)^2 - M_Z^2} \pm \frac{i m^2 g_2^2}{2 M_W^2} \frac{\gamma_5 \chi_\pm(k, P) \gamma_5}{(k-q)^2 - M_Z^2} \right\} \tag{2.3a}
\end{aligned}$$

and

$$\begin{aligned}
& S^{-1}(q_+) \chi_{UD}(q, P) S^{-1}(q_-) = \int \frac{d^4 k}{(2\pi)^4} \left\{ -\gamma_\mu \chi_{UD}(k, P) \gamma_\nu G^{\mu\nu}(k-q) \right. \\
&\quad \left. - i \frac{m^2 g_2^2}{4 M_W^2} \frac{\chi_{UD}(k, P)}{(k-q)^2 - M_H^2} - i \frac{m^2 g_2^2}{4 M_W^2} \frac{\gamma_5 \chi_{UD} \gamma_5}{(k-q)^2 - M_Z^2} \right\}. \tag{2.3b}
\end{aligned}$$

The quark mass $m = m_U = m_D$ is taken to be the same for U and D . Experimental support for this assumption is reviewed in Sec. IV.

The important features of Eq. (2.3) are (i) χ_- , χ_{UD} , and

χ_{DU} obey the same equation when $M_Z = M_W$, and they form an isospin triplet of states, (ii) the Goldstone boson contributions to this $\chi_-, \chi_{UD}, \chi_{DU}$ triplet are, like the gluon and Higgs boson contributions *all attractive* in the

ground state, 0^- system (see Sec. III below), and (iii) the χ_+ state obeys a different equation whose Goldstone-boson contributions are all *repulsive* in the ground state, 0^- system.

Equation (2.3) and the fact that pseudoscalar exchange becomes comparable to scalar for relativistic systems combine to produce a clear splitting between the singlet and triplet of ground-state bosons in the strong binding limit. The triplet is below the singlet in mass for the 0^- system. This is very reminiscent of a “ $\pi-\eta$ ” situation, except that this appears to be purely dynamical with no spontaneous symmetry breaking, bound state Goldstone phenomenon at work.

The isosinglet, isotriplet situation is reversed in the 0^+ and 1^- spin-parity channels, as we describe in the next section.

III. RESULTS OF RELATIVISTIC BOUND STATE CALCULATIONS OF ISOSINGLET AND ISOTRIplet MASSES

We employ two different approaches to solving the ladder approximation, Bethe-Salpeter equations for the $J^P=0^-$ ground states of the weak isospin singlet and weak isospin triplet $Q\bar{Q}$ system. We also report results in the same approximations for the $J^P=1^-$ and 0^+ states. The principle feature that emerges, as anticipated in the preceding section, is the splitting between the isosinglet and isotriplet states due to the pseudoscalar exchange (in Feynman gauge): repulsion in the isosinglet case and the attraction in the isotriplet case in the ground-state 0^- channel. The isotriplet pseudoscalar, ground-state mass solutions are found to fall to zero for high enough quark masses in all of the calculations. The reverse is true in the 0^+ and 1^- channels, and the isoscalar masses fall to

zero at high enough quark masses in these states [10,11].

We evaluate the bound state energies (masses) using both the covariant kernel,

$$K(q) = g^2 \frac{1}{q^2 - M^2}, \quad (3.1a)$$

which we refer to as the covariant gauge ladder approximation, and the instantaneous approximation to this kernel,

$$K(|\mathbf{q}|^2) = -g^2 \frac{1}{|\mathbf{q}|^2 + M^2}. \quad (3.1b)$$

We have suppressed any reference to gauge-dependent spin structure in (3.1), and designate generic coupling constants and masses as g and M , respectively. The instantaneous approximation, so called because the coordinate space potentials are instantaneous, reduces the problem to three dimensions, yielding what are referred to as Salpeter's equations. These are coupled equations for positive and negative frequency amplitudes. One can solve the coupled system or set the negative frequency amplitudes to zero and solve the positive frequency problem by itself.

A. Covariant gauge formalism

In the covariant gauge ladder (CGL) formalism, we decompose the 0^- Bethe-Salpeter amplitudes into four independent spinor functions:

$$\chi_P(P, q) = \gamma_5 [\chi_0 + \not{P}\chi_1 + \not{q}\chi_2 + [\not{q}, \not{P}]\chi_3]. \quad (3.2)$$

In our calculation, only χ_0 and χ_1 need to be retained, and the weak coupling relation $\chi_1 = -\chi_0/2m$ can be employed to reduce the system of equations to a single equation for χ_0 . In Euclidean variables, we have

$$\begin{aligned} \chi_0(P, q) = & \frac{4\alpha_s}{3\pi^3} \frac{q^2 + m^2}{D} \int \frac{d^4k}{(2\pi)^4} \frac{\chi_0}{(k-q)^2} + \frac{m^2 g_2^2}{4M_W^2} \frac{1}{D} \left[q^2 + \frac{3M_B^2}{4} + m^2 \right] \int \frac{d^4k}{(2\pi)^4} \frac{\chi_0}{(k-q)^2 + M_H^2} \\ & + \frac{m^2 g_2^2}{4M_W^2} \frac{F^I}{D} \left[q^2 - \frac{M_B^2}{4} + m^2 \right] \int \frac{d^4k}{(2\pi)^4} \frac{\chi_0}{(k-q)^2 + M_Z^2}, \end{aligned} \quad (3.3)$$

where $M_Z = M_W = 90$ GeV has been adopted since including weak boson mass difference affects answers only in the third decimal place, $D \equiv D(q^2, M_B^2, \cos\theta) = [-q^2 + (M_B^2/4) - m^2]^2 + M_B^2 q^2 \cos^2\theta$, $M_B^2 = -P^2$ is the mass squared of the bound state, θ is the angle between P and q , and $F^I = +1, -3$ for isospin $I=1, 0$, respectively. As mentioned in the preceding section, the sign difference in the last term between the $I=1$ and $I=0$ cases produces the extra attraction and repulsion compared to the purely attractive gluon and Higgs potentials. The method of solution is discussed in Ref. [3] in some detail. The resulting bound state mass values as a function of quark mass, m , are shown in Fig. 2 in the curves labeled $S1$ and $T1$. In this calculation, the isosinglet ($S1$) state's binding energy steadily increases, but the bound state mass never sinks to zero in the fermion mass range we

have investigated. The more tightly bound triplet state ($T1$) achieves zero mass when the quark mass reaches about 900 GeV. Above 750 GeV quark mass, the splitting between the different bound state masses becomes quite pronounced. The 0^+ and 1^- calculations proceed in a similar fashion with the isosinglet being more tightly bound than the isotriplet. The details are not presented here, since they do not illuminate the discussion.

B. Salpeter equation formalism

Making the approximation (3.1b), integrating over q_0 in the Bethe-Salpeter equation, and projecting out the 0^- channel amplitude, one obtains coupled equations for the positive and negative frequency amplitudes, $\chi_{\pm}(q)$, where $q \equiv |\mathbf{q}|$. The equations read

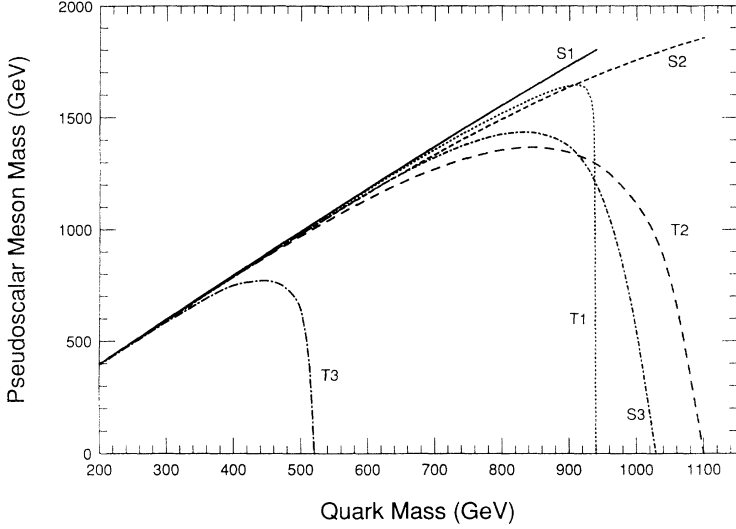


FIG. 2. Isosinglet and isotriplet masses for mesons in the 0^- channel. Curves S1 (isosinglet) and T1 (isotriplet) are the results from covariant gauge formalism. S2 (isosinglet) and T2 (isotriplet) are the results from Salpeter's equation using only positive frequency components. S3 (isosinglet) and T3 (isotriplet) are the results from Salpeter's equation using both positive and negative frequency components.

$$(E - 2\omega)\chi_+(q) = \frac{1}{\pi q} \int dq' q' (V_+ \chi_+ + V_- \chi_-), \quad (3.4)$$

$$(E + 2\omega)\chi_-(q) = \frac{-1}{\pi q} \int dq' q' (V_- \chi_+ + V_+ \chi_-),$$

where $\omega = \sqrt{q^2 + m^2}$, E is the bound state eigenvalue and

$$V_{\pm} = C_V Q_0(Z_V) \frac{2\omega\omega' \mp m^2}{\omega\omega'} + C_P \left\{ Q_0(Z_P) \frac{\omega\omega' \mp m^2}{\omega\omega'} + [Z_P Q_0(Z_P) - 1] \frac{qq'}{\omega\omega'} \right\} + C_S \left\{ Q_0(Z_S) \frac{\omega\omega' \pm m^2}{\omega\omega'} + [Z_S Q_0(Z_S) - 1] \frac{qq'}{\omega\omega'} \right\}.$$

The coefficients C_V and variables Z_V etc. are summarized in Table I. Details on the method of solution are given in Ref. [3].

The 0^- ground-state masses as a function of quark mass is shown for the positive frequency only case in Fig. 2, curves S2 and T2. Again the splitting between isosinglet (S2) and isotriplet (T2) bound state masses is striking for the heavy quark mass region, and the isotriplet system becomes ultrarelativistic in the region above $m = 750$ GeV, producing a zero mass bound state at $m \simeq 1100$ GeV.

The solution for the positive plus negative, fully coupled system is much more tightly bound as is seen in Fig. 2, curves S3 and T3, where a dramatic departure between isotriplet (T3) and isosinglet (S3) masses sets in already at $m = 400$ GeV, and the isotriplet mass plunges to zero at $m = 520$ GeV. Figure 2 also shows the feature of the instantaneous approximation that differs from the CGL approximation, namely the turn-over (and eventual fall to zero) of the isosinglet bound state mass as a function of the quark mass. This effect does not yet appear in Fig. 2 for the isosinglet positive-frequency-only case, but it does occur at a quark mass above 1.20 TeV for this case as well.

Focusing on the isotriplet ground-state mass values versus quark mass curves in Fig. 2 for the three solutions which were discussed above, we see that the general features agree though the different relativistic bound state approximations produce different bound state mass values for a given quark mass. One can conclude that the Goldstone-boson-Higgs-boson exchange plays an important role in the calculations, that deep binding at or above 500 GeV quark mass occurs, and that a dramatic weak isospin mass splitting is produced in the $Q\bar{Q}$ spectrum of a degenerate, or nearly so, U, D doublet system.

With the role of isosinglet and isotriplet reversed, the features just outlined are present also in the $J^P = 1^-$ and 0^+ bound states, and we display results of calculations for this system in Figs. 3 and 4. The principal distinction between 0^- on one hand and 1^- and 0^+ on the other is the isosinglet-triplet splitting reversal and that the latter two are typically less tightly bound.

In the next section we take up the question of ultraheavy quark decay lifetime and intradoublet mass splitting constraints.

IV. CONSTRAINTS ON SUPERHEAVY U, D QUARK LIFETIMES AND ON $U-D$ SPLITTING

Two related questions arise when one considers the phenomenology of superheavy fermions. A crucial consideration for bound state physics is the comparison of

TABLE I. Coefficients for the 0^- channel Salpeter equation kernels, Eq. (3.4) and below. Positive signs indicate attractive interactions and the negative sign a repulsive interaction.

	V	P	S
$C, I=0$	$+\frac{4}{3}\alpha_s$	$\frac{-3}{4\pi} \frac{m^2}{v_0^2}$	$\frac{1}{4\pi} \frac{m^2}{v_0^2}$
$C, I=1$	$+\frac{4}{3}\alpha_s$	$+\frac{1}{4\pi} \frac{m^2}{v_0^2}$	$+\frac{1}{4\pi} \frac{m^2}{v_0^2}$
Z	$\frac{q^2 + q'^2}{2qq'}$	$\frac{q^2 + q'^2 + M_Z^2}{2qq'}$	$\frac{q^2 + q'^2 + M_H^2}{2qq'}$

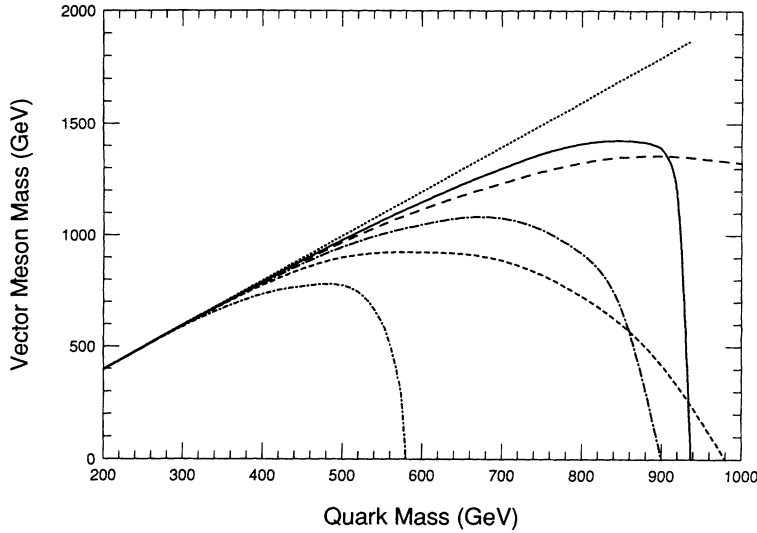


FIG. 3. Isosinglet and isotriplet masses for mesons in the 1^- channel. Solid curve (isosinglet) and dotted curve (isotriplet) are the results from covariant gauge formalism. Short dash curve (isosinglet) and long dash curve (isotriplet) are the results from Salpeter's equation using only positive frequency components. Short dash-dot curve (isosinglet) and long dash-dot (isotriplet) are the results from Salpeter's equation using both positive and negative frequency components.

quark lifetime to the period of bound state motion. The situation for the heavy top quark case has been considered by a number of authors [12], and we apply the argument of Strassler and Peskin to the superheavy, fourth generation in this section. Related to the lifetime question is the U - D mass-splitting question. If the splitting is larger than the W mass, then the higher mass quark can decay to the lower mass quark by direct W emission, presumably with no Cabibbo-Kobayashi-Maskawa (CKM) suppression. The width would then be too broad to permit formation of narrow bound states containing the heavier quark. We review constraints imposed by measured W - and Z -boson properties on the U - D mass splitting, which we would like to take as degenerate to a first approximation.

A. Lifetime for decay to light quarks

The lighter of the U, D doublet partners must decay to lighter generation quarks, q , and we assume that one transition is dominant. Calling the corresponding CKM

factor V_{Qq} , we have

$$\Gamma_{Q \rightarrow q} \sim (180) \text{ MeV} \left[\frac{M_Q}{M_W} \right]^3 |V_{Qq}|^2, \quad (4.1)$$

where $M_Q \gg M_q$, M_W is assumed. To make a conservative estimate, we will include only the gluon binding in estimating the time needed for bound state formation in a nonrelativistic, weak binding approximation. Including the strong binding, Yukawa interactions will only improve the prospects for bound state formation. For pure QCD coupling, the characteristic radius is $a_0 = (\frac{4}{3}\alpha_s M_Q/2)^{-1}$ where $\frac{4}{3}\alpha_s$ is the effective QCD coupling strength for the problem (with a characteristic velocity $v_n \sim \frac{4}{3}\alpha_s/n$ for the n th radial excitation). Taking the ratio of twice the diameter to the velocity as an s -state formation time [12], one has $t_{\text{form}} \sim 9n^3/(2\alpha_s^2 M_Q)$, $n=1,2,\dots$ and in the ground state $t_{\text{form}} \sim 9/2\alpha_s^2 M_Q$. The inverse Bohr radius is the appropriate scale at which to evaluate α_s for the problem, and, using the one-loop parametrization for α_s ,

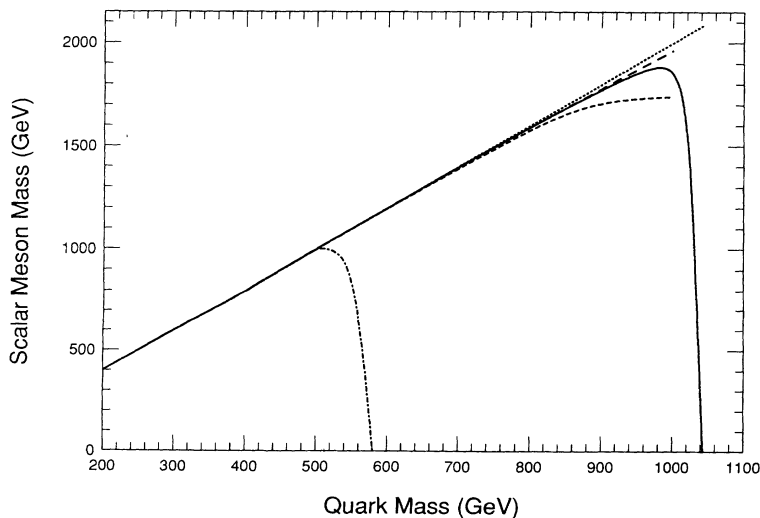


FIG. 4. Isosinglet and isotriplet masses for mesons in the 0^+ channel. Solid curve (isosinglet) and dotted curve (isotriplet) are the results from covariant gauge formalism. Short dash curve (isosinglet) and long dash curve (isotriplet) are the results from Salpeter's equation using only positive frequency components. Dash-dot curve is the isosinglet result from Salpeter's equation with positive and negative frequency components. The isotriplet result for this case is almost identical to the corresponding result obtained by including only the positive frequency components in the Salpeter's equation.

$$\alpha_s(\mu^2) = \frac{\pi d}{\ln(\mu^2/\Lambda_{\text{QCD}}^2)}, \quad (4.2)$$

where $d = 12/(33 - 2n_f)$, along with $a_0^{-1} = [\frac{2}{3}\alpha_s(\mu^2)M_Q]$, leads to a condition on $\alpha_s(\mu^2)$:

$$\frac{2}{\pi d} \ln \left[\frac{2\alpha_s(\mu^2)M_Q}{3\Lambda_{\text{QCD}}} \right] = \alpha_s^{-1}(\mu^2). \quad (4.3)$$

We show a plot of $\alpha_s(1/a_0)$ as a function of M_Q/Λ_{QCD} in Fig. 5. As expected, since the binding scale is much less than the value of M_Q , $\alpha(1/a_0)$ is significantly larger than $\alpha(M_Q)$. The latter is also displayed for comparison purposes in Fig. 5.

Equating the time of formation and the decay lifetime, one obtains an upper bound on the value of $|V_{Qq}|$ for given values of Λ_{QCD} and M_Q . Choosing $\Lambda_{\text{QCD}} = 0.20$ GeV we show $|V_{Qq}|_{\text{max}}$ vs M_Q in Fig. 6.

Figure 6 clearly shows the point, a surprise to us, that the constraints on V_{Qq} are rather weak, permitting for example $V_{Qq} \simeq 0.3 \sim \sin\theta_c \simeq V_{us}$ for $M_Q \simeq 500$ GeV with $\Lambda_{\text{QCD}} = 0.20$ GeV. As mentioned above, including the Higgs boson, Yukawa coupling effects will only improve chances of bound state formation and further weaken the constraints on $(V_{Qq})_{\text{max}}$. This observation is encouraging for prospects of detecting narrow, deeply bound states at higher energies.

B. Mass splitting between U and D

Turning to the question of mass splitting between the U and D quarks, we use the definition [13]

$$\sin^2\theta_w \equiv 1 - \frac{M_W^2}{M_Z^2}$$

and the parametrization of radiative corrections in terms of the factor Δr to write [14]

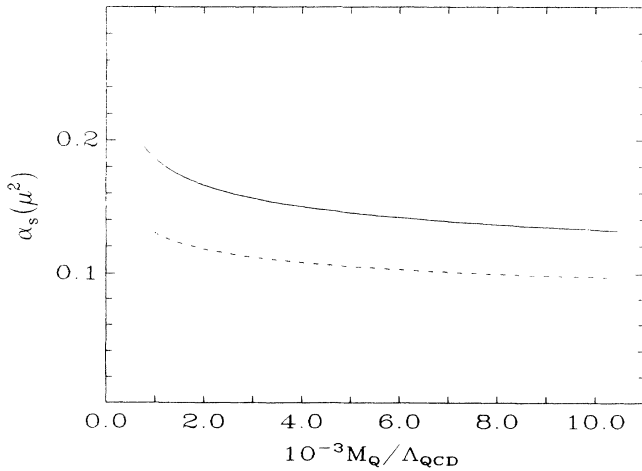


FIG. 5. The value of α_s (solid curve) relevant for the bound state Bethe-Salpeter equation, ignoring Higgs interaction, as a function of the quark mass. The dashed curve shows $\alpha_s(M_Q^2)$ which is significantly smaller than the value of α_s that should be used for bound state calculations.

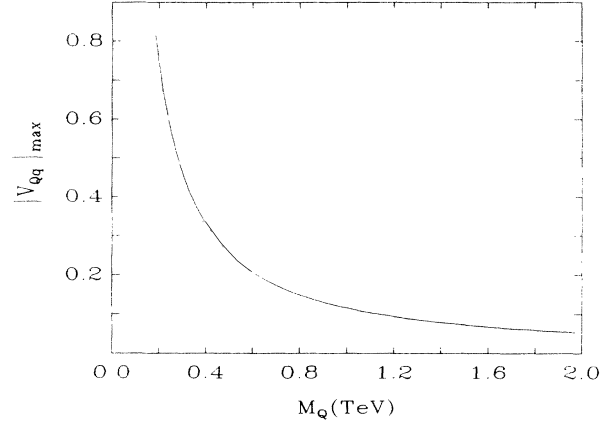


FIG. 6. The maximum value of the CKM matrix element $|V_{Qq}|$ allowed in order for the fourth generation quarks Q to form a meson bound state, q being a lower generation quark. This constraint is derived by including only QCD effects. Inclusion of the Yukawa coupling will allow even higher values of $|V_{Qq}|$.

$$\frac{M_W^2}{M_Z^2} \left[1 - \frac{M_W^2}{M_Z^2} \right] = \frac{\pi\alpha}{\sqrt{2}G_F} \frac{1}{M_Z^2} \frac{1}{1-\Delta r}. \quad (4.4)$$

The quantity Δr can be expressed as

$$\Delta r = 0.071 - \frac{\cos^2\theta_w}{\sin^2\theta_w} \delta_{\rho H}, \quad (4.5)$$

where the heavy fermion factor $\delta_{\rho H}$ is given by

$$\delta_{\rho H} \simeq \frac{3G_F}{8\pi^2\sqrt{2}} \left[M_i^2 + \sum_i \frac{C_i}{3} \Delta M_i^2 \right], \quad (4.6)$$

and

$$\Delta M^2 = M_1^2 + M_2^2 - \frac{4M_1^2 M_2^2}{M_1^2 - M_2^2} \ln \frac{M_1}{M_2}. \quad (4.7)$$

The factor $C_i = 3$ for superheavy quarks and $C_i = 1$ for superheavy leptons [15]. The masses are designated as M_i for the top quark and M_1 and M_2 for superheavy fermion generation members. Using the 1992 Particle Data Group book [15] values for M_W and M_Z and the value $\sin^2\theta_w = 0.230$, we find from Eqs. (4.4)–(4.7) that the one standard deviation constraint is

$$\Delta M_F^2 \equiv M_i^2 + \sum_i \frac{C_i}{3} \Delta M_i^2 \leq (206 \text{ GeV})^2. \quad (4.8)$$

For the case of one superheavy quark generation and a degenerate superheavy lepton generation, the plot of the bound on $|M_U - M_D|$ vs M_i is shown in Fig. 7. There is evidently room for considerable mass splitting in the bound (4.8), where $|M_U - M_D| < 140$ GeV for $M_i = 150$ GeV, for example. The dependence of the bound on the average value of the heavy quark mass is very weak and can be ignored for our purposes.

An independent bound can be obtained from Z decay data. The leptonic and hadronic decay widths, for exam-

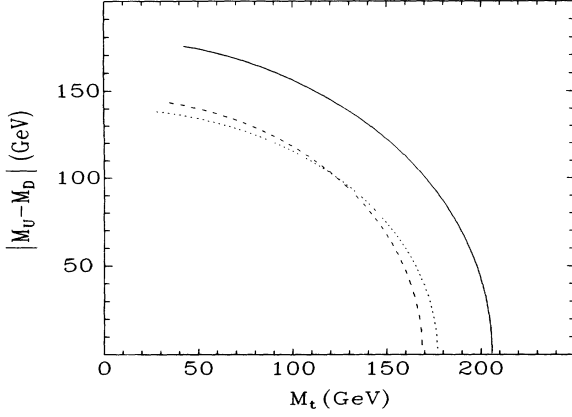


FIG. 7. The maximum allowed value for the fourth generation quark mass difference $|M_U - M_D|$ as a function of the top quark mass M_t . The three curves represent the constraints using ρ parameter (solid curve), $\Gamma(Z \rightarrow \text{hadrons})$ (short dashed curve) and $\Gamma(Z \rightarrow l^+ l^-)$ (long dashed curve).

ple, provide useful constraints. Defining the quantity $\sin^2 \bar{\theta}_w \equiv \kappa(M_Z) \sin^2 \theta_w$, one has, approximately [16],

$$\sin^2 \bar{\theta}_w = \left[\frac{1}{2} - \frac{1}{2} \left[1 - \frac{4\pi\alpha(M_Z)}{\sqrt{2}G_F M_Z^2} \right]^{1/2} \right] \times \left[1 - \frac{3\alpha}{16\pi \sin^2 \theta_w (1 - 2\sin^2 \theta_w)} \frac{\Delta M_F^2}{M_Z^2} \right], \quad (4.9)$$

with $\kappa(M_Z) = 1 + (3\alpha/16\pi \sin^4 \theta_w) (\Delta M_F^2/M_Z^2)$. The value of $\alpha(M_Z)$ is obtained from electroweak corrections to α that do not include the heavy quark effects. Using the value $\Delta r = 0.071$ to define $\alpha(M_Z)$ and adopting the first factor in square brackets in Eq. (4.9) as the value of $\sin^2 \theta_w$ to be used in the expression in the second set of brackets, we find

$$\sin^2 \bar{\theta}_w = 0.235 \left[1 - 3.50 \times 10^{-3} \frac{\Delta M_F^2}{M_Z^2} \right]. \quad (4.10)$$

The expressions for the decay widths are

$$\Gamma(Z \rightarrow l^+ l^-) = \frac{\sqrt{2}G_F M_Z^3}{48\pi} \rho [1 + (1 - 4\sin^2 \bar{\theta}_w)^2] \quad (4.11a)$$

and

$$\Gamma(Z \rightarrow \text{hadrons}) = \frac{\sqrt{2}G_F M_Z^3}{8\pi} \times \rho \left[5 - \frac{28}{3} \sin^2 \bar{\theta}_w + \frac{88}{9} \sin^4 \bar{\theta}_w - \delta_b \right] \times \left[1 + \frac{\alpha_s(M_Z)}{\pi} \right]. \quad (4.11b)$$

The factors ρ and δ_b in Eqs. (4.11) are given by

$$\rho = 1 + \frac{3\alpha}{16\pi \sin^2 \theta_w \cos^2 \theta_w} \frac{\Delta M_F^2}{M_Z^2},$$

and

$$\delta_b = [\alpha(1 + 4\sin^2 \theta_w/3 - 16\sin^4 \theta_w/9)/(8\pi \sin^2 \theta_w \times \cos^2 \theta_w)] (M_t^2/M_Z^2),$$

where $\sin^2 \theta_w = 0.235$ is used from Eq. (4.9) onward to evaluate all of the above expressions. The constraints on $|M_U - M_D|$ provided by the 1992 Particle Data Group values of $\Gamma(Z \rightarrow l^+ l^-)$ and $\Gamma(Z \rightarrow \text{hadrons})$ are illustrated in Fig. 7. We see again that significant mass splitting in a hypothetical fourth generation of fermions is allowed. The constraints on splittings are essentially independent of average mass.

To conclude this section, we note that our assumption of degeneracy between superheavy generation fermions is consistent with the present bounds, but that there could be shifts in our isotriplet vs isosinglet $Q\bar{Q}$ bound state masses due to ultraheavy quark nondegeneracy effects. The pattern of a mass splitting between isosinglet and isotriplet states of a given spin and parity would remain, however.

V. DISCUSSION AND CONCLUSION

In the highly relativistic, strong binding regime, we have shown that the contributions of the pseudoscalar, Goldstone boson degrees of freedom to the binding of ultraheavy $Q\bar{Q}$ systems play a crucial role in determining the bound state spectrum. Even though these pseudoscalar interactions become negligible compared to the gluon and Higgs boson exchanges in the weak binding limit, they are solely responsible for the large isosinglet vs isotriplet $Q\bar{Q}$ bound state splitting which we found for ultraheavy, fourth generation, $Q = (U, D)$ quarks. Depending upon the specific approximation scheme used, we saw (Figs. 2–4) that the splitting between $I=0$ and $I=1$ $Q\bar{Q}$ states become large for M_Q in the range $0.4 \text{ TeV} < M_Q < 0.8 \text{ TeV}$.

Whether a splitting between bound states is large depends of course on the decay width of the (quasi) bound states, and our estimates of $|V_{Qq}|$ in Sec. IV and the analyses of decay widths of similar states into $q\bar{q}$, H , Z , etc. in Refs. [17–21] indicate that the splittings between states will be large compared to the decay widths for the M_Q values in the range given above. The states should, therefore, be clearly separated.

For the production and decay of neutral $Q\bar{Q}$ (and lepton-antilepton) ultraheavy bound states, Refs. [2] and [17–21] provide an encouraging picture of the production and decay signals for such states. The binding indicated by our relativistic, bound state calculations is much stronger than that considered in Refs. [17,18] (gluon nonrelativistic potential model only) or [3,19–21] (Higgs nonrelativistic model or Higgs plus gluon nonrelativistic potential model), so the prospects for detecting new heavy bound states, should a fourth generation exist, are made even brighter by our results.

Avenues for further work include the detailed phenomenology of the isomultiplet system, in particular the possible production and decay of the $I=1$ state, and the application of these techniques to TeV scale supersymmetry

bound state physics. We plan to address these issues in the future.

Note added in proof. After submission of this article, we became aware of a paper relevant to our discussion in Sec. IV on U - D mass splitting by P. Q. Hung [Phys. Rev. Lett. **69**, 3143 (1992)], who argues that vector meson bound states of $Q\bar{Q}$ mix with W and Z and may weaken ρ -parameter restrictions of U - D mass splitting.

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APPENDIX A

We describe here the subtraction procedure used in regulating the Bethe-Salpeter (BS) equation. Referring to Eqs. (2.1) and (2.2) in the text, we see that the direct, P^2 pole terms in each equation are independent of q . All of the other terms in the integrands depend on $k-q$ and they decrease like $(k-q)^{-2} \sim q^{-2}$ as $|q| \rightarrow \infty$ for fixed k . Therefore, if the integrals exist, one finds that, for fixed P , taking (2.1) in particular,

$$\lim_{|q| \rightarrow \infty} S^{-1}(q_+) \chi_{UV}(q, P) S^{-1}(q_-) \rightarrow \frac{g_2^2 m^2}{4 M_W^2} \frac{i}{P^2 - M_Z^2} \gamma_5 \text{Tr} \left[\int \frac{d^4 k}{(2\pi)^4} \chi_{UV}(k, P) \gamma_5 \right] + \frac{g_2^2 m^2}{4 M_W^2} \frac{i}{P^2 - M_H^2} \text{Tr} \left[\int \frac{d^4 k}{(2\pi)^4} \chi_{UV}(k, P) \right]. \quad (\text{A1})$$

If $S^{-1}(q_+)_{|q| \rightarrow \infty} \rightarrow \not{q}$ as is the case for the perturbative propagators which we use in this paper, then the fixed P , large q limit gives

$$\chi(q, P) \rightarrow \frac{1}{q^2}.$$

In this case, the integral $\int d^4 k \chi(k, P)$ does not exist and the integral equation has no solutions, in accordance with direct numerical studies which we have carried out.

If the q -independent direct (p)-channel pole terms are absent, however, then the q^{-2} dependence of the in-

tegrands of all of the other terms in the large q^2 limit means that $\chi(q, P)_{|q| \rightarrow \infty} \rightarrow q^{-4}$ behavior is consistent with the existence of solutions, and we find that this is indeed the case.

To render finite the equations (2.1), (2.2), and their counterparts with U and D interchanged, it is sufficient to subtract each equation at a fixed q , say $q=0$, because this subtraction removes the q -independent, divergent term. To make this subtraction systematic, one regulates (i.e., cutoff) the integrals, makes the subtraction, and then removes the regularization. The subtracted equation is

$$S^{-1}(q_+) \chi_{UV}(q, P) S^{-1}(q_-) - S^{-1} \left[\frac{P}{2} \right] \chi_{UV}(0, P) S^{-1} \left[\frac{-P}{2} \right] = \int \frac{d^4 k}{(2\pi)^4} \left[-\gamma_\mu \chi_{UV}(k, P) \gamma_\nu G^{\mu\nu}(k-q) + \frac{g_2^2 m^2}{4 M_W^2} \left(\chi_{UV}(k, P) \frac{i}{(k-q)^2 - M_H^2} + \gamma_5 \chi_{UV}(k, P) \gamma_5 \frac{i}{(k-q)^2 - M_Z^2} + 2\gamma_5 \chi_{DD}(k, P) \gamma_5 \frac{i}{(k-q)^2 - M_W^2} \right) \right] - \int \frac{d^4 k}{(2\pi)^4} \left[-\gamma_\mu \chi_{UV}(k, P) \gamma_\nu G^{\mu\nu}(k) - \frac{g_2^2 m^2}{4 M_W^2} \times \left(\chi_{UV}(k, P) \frac{i}{k^2 - M_H^2} + \gamma_5 \chi_{UV}(k, P) \gamma_5 \frac{i}{k^2 - M_Z^2} + 2\gamma_5 \chi_{DD}(k, P) \gamma_5 \frac{i}{k^2 - M_W^2} \right) \right]. \quad (\text{A2})$$

If the first term on the left-hand side of (A2) is equal to the first integral on the right-hand side for each q (i.e., a solution for each q is found), then the companion terms, where $q=0$, on each side are guaranteed to be equal and the result is an eigenvalue equation for P and q . This

equation is, of course, just the original equation without the offending, divergent term and we use this form as our regulated, finite Bethe-Salpeter equation which is to be projected into states of definite quantum numbers.

APPENDIX B

1. Running of the Yukawa coupling—the Landau pole

In this appendix we briefly discuss the effects that arise because of the momentum dependence, or running, of the Yukawa couplings. We work at the one-loop level to see where the Landau pole occurs when the quark masses are in the deep binding region that we probe, $0.5 \text{ TeV} \lesssim M_Q \lesssim 1.0 \text{ TeV}$.

The one-loop renormalization group equations for the gluon and Yukawa couplings, g_s and g_Y , without the weak gauge interaction effects, read [22]

$$\frac{d}{dt} g_s = -\frac{1}{4\pi^2 d} g_s^2, \quad (\text{B1a})$$

and

$$\frac{d}{dt} g_Y = \frac{1}{4\pi^2} \left(\frac{9}{8} g_Y^3 - 2g_s^2 g_Y \right), \quad (\text{B1b})$$

where $d = 12/(33 - 4n_g)$, n_g is the number of generations, and $t = \frac{1}{2} \ln(q^2/\mu^2)$ with μ an arbitrary renormalization scale.

Parametrizing the solution to (B1a) in the standard fashion [23],

$$g_s^2(q^2) = \frac{4\pi^2 d}{\ln(q^2/\Lambda_{\text{QCD}}^2)} \equiv \frac{2\pi^2 d}{t}$$

where $\Lambda_{\text{QCD}} \simeq 0.2 \text{ GeV}$ and $\mu = \Lambda_{\text{QCD}}$ is adopted, we can rewrite (B1b) as

$$\frac{dg_Y}{dt} = \frac{9}{32\pi^2} g_Y^3 - \frac{d}{t} g_Y. \quad (\text{B2})$$

The solution to (B2) can be written in terms of an integration constant C , to be fixed by the boundary condition:

$$g_Y^2(t) = \frac{1}{C t^{2d} - (9/16\pi^2) [t/(1-2d)]}. \quad (\text{B3})$$

Checking several limiting cases of (B3), we see that

$$g_Y^2(t) = \frac{g_Y^2(0)}{1 - (9/16\pi^2) g_Y^2(0) t}$$

when $\alpha_{\text{QCD}} = 0$, $d = 0$, as it should. The characteristic Landau pole appears at $t = (16\pi^2/9) [1/g_Y^2(0)]$. In the weak Yukawa coupling limit, which is the light quark case, where only the g_s^2 effect is kept,

$$g_Y^2(t) = \frac{C^{-1}}{t^{2d}},$$

so the quark mass, proportional to g_Y and the weak scale v , behaves as

$$M_Q \simeq g_Y(t) \frac{v}{\sqrt{2}} \rightarrow \frac{1}{t^d}, \quad t \rightarrow \infty,$$

which is the well-known leading-log, asymptotic behavior of the light quark masses in QCD [24].

The constant C in Eq. (B3) can be fixed in terms of the input quark mass, M_Q , by requiring the condition [25]

$$M_Q^2 = g_Y^2(2M_Q) \frac{v^2}{2}, \quad (\text{B4})$$

for example. The requirement (B4) can be solved for C to give

$$C = t_Q^{-2d} \left[\frac{v^2}{2M_Q^2} - \frac{9}{16\pi^2} \frac{1}{2d-1} t_Q \right], \quad (\text{B5})$$

with

$$t_Q \equiv \ln \frac{2M_Q}{\Lambda_{\text{QCD}}}.$$

The expression (B3) with C evaluated by (B5) can now be used to assess the large $|q| > M_Q$ behavior of $g_Y(t)$. We can use the results (B5) and (B3) to determine C_Q and q_{pole} , the location of the singularity for various M_Q values. There is no longer a pole at large t , but the coupling still blows up at large t . Interesting values are: $M_Q = 0.5 \text{ TeV}$, where $C_Q = -0.282$ and $q_{\text{pole}} = 13 \text{ TeV}$; and $M_Q = 1.0 \text{ TeV}$, where $C_Q = -0.288$ and $q_{\text{pole}} = 3.5 \text{ TeV}$. The $t=0$ divergence in (B3) is an artifact of the standard parametrization of g_s , which is only applicable when $|q| \gg \Lambda_{\text{QCD}}$. g_Y should be sensibly constant for small momentum transfers.

The momentum values that are important in the wave function in the Bethe-Salpeter integration are of the order of the inverse Bohr radius, as a rough rule of thumb. Only in those cases when $M_Q \gtrsim 1 \text{ TeV}$ and in the limit as the bound state mass plunges to zero does the g_Y singularity come near to the relevant range of integration in the Bethe-Salpeter equation. Consequently, the binding energies as a function of M_Q should not be substantially affected by the g_Y singularity below the point where the bound state masses fall to zero, even for the $M_Q \gtrsim 1 \text{ TeV}$ cases.

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