Corrections to chiral dynamics of heavy hadrons: $1/M$ correction

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In earlier publications we have analyzed the strong and radiative decays of heavy hadrons in a formalism which incorporates both heavy-quark and chiral symmetries. In particular, we have derived a heavy-hadron chiral Lagrangian whose coupling constants are related by the heavy-quark flavor-spin symmetry arising from the QCD Lagrangian with infinitely massive quarks. In this paper, we reexamine the structure of the above chiral Lagrangian by including the effects of $1/m_Q$ corrections in the heavy-quark effective theory. The relations among the coupling constants, originally derived in the heavy-quark limit, are modified by heavy-quark symmetry-breaking interactions in QCD. Some of the implications are discussed.

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I. INTRODUCTION

In this and a subsequent paper, we would like to examine various symmetry-breaking corrections to the strong and electromagnetic decays of heavy hadrons. There are two different kinds of symmetry-breaking effects on the chiral dynamics of heavy hadrons: the $1/m_Q$ corrections from the heavy quarks and the finite-mass effects from the light quarks. We will focus on the $1/m_Q$ corrections in this work and leave the discussion on SU(3)-breaking effects to the forthcoming paper [1].

As is well known, the QCD dynamics in the limit of infinite quark mass exhibits a new spin-flavor symmetry which is known as heavy-quark symmetry (HQS) [2,3]. Corrections to this symmetry limit can be systematically incorporated into the heavy quark effective theory (HQET) of QCD where symmetry-breaking effects are summarized by higher-dimensional operators suppressed by powers of $1/m_Q$ [4–9]. Such an effective theory has been a powerful tool to analyze weak-transition form factors of heavy hadrons containing one single heavy quark [10]. We have recently, among others, initiated a study of strong and radiative decays of heavy hadrons by deriving a heavy-hadron chiral Lagrangian which obeys constraints from the heavy quark symmetry [11–17]. As the idea of synthesizing the heavy-quark and chiral symmetries receives growing attention, there remain important issues to be explored. Especially, implications of the aforementioned $1/m_Q$ corrections to the structure of the heavy-hadron chiral Lagrangian have not been systematically studied [18]. Since the charmed quark is not particularly heavy compared to the QCD scale, such corrections can be important in the chiral Lagrangian for charmed hadrons.

As an example to illustrate the issues involved, consider the heavy-meson chiral Lagrangian given by Eq. (2.16) of Ref. [17]:

$$\begin{aligned}\mathcal{L}^{(1)}_{PP^*} = & D_\mu P D^\mu P^\dagger - M_P^2 PP^\dagger \\ & + f\sqrt{M_P M_{P^*}}(P\mathcal{A}^\mu P^{*\dagger}_\mu + P^*_\mu \mathcal{A}^\mu P^\dagger) \\ & -\tfrac{1}{2}P^{*\mu\nu}P^{*\dagger}_{\mu\nu} + M^2_{P^*}P^{*\mu}P^{*\dagger}_\mu \\ & +\tfrac{1}{2}g\,\epsilon_{\mu\nu\lambda\kappa}(P^{*\mu\nu}\mathcal{A}^\lambda P^{*\kappa\dagger} + P^{*\kappa}\mathcal{A}^\lambda P^{*\mu\nu\dagger})\ ,\end{aligned} \tag{1.1}$$

where P and P^* are the ground-state heavy mesons with quantum numbers $J^P = 0^-$ and 1^-, respectively, and

$$P^{*\dagger}_{\mu\nu} = D_\mu P^{*\dagger}_\nu - D_\nu P^{*\dagger}_\mu\ , \tag{1.2a}$$

$$D_\mu P^{*\dagger}_\nu = \partial_\mu P^{*\dagger}_\nu + \mathcal{V}_\mu P^\dagger_\nu - ieA_\mu(P^{*\dagger}_\nu Q' - QP^{*\dagger}_\nu)\ , \tag{1.2b}$$

and a similar definition for the covariant derivative $D_\mu P^\dagger$. In Eq. (1.1) A_μ is the electromagnetic field whereas $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$ are, respectively, the chiral vector and chiral axial fields (see Ref. [17] for more detail). The prediction from heavy-quark symmetry consists of two parts. The flavor symmetry implies that the coupling constants f and g are the same for any heavy flavor. The spin symmetry relates the two parameters by

$$g = \tfrac{1}{2}f\ . \tag{1.3}$$

Similar predictions have also been obtained for the heavy baryon chiral Lagrangian. These predictions help reduce the number of unknowns in the heavy-hadron chiral Lagrangian. For instance, the $D^*D\pi$ and $D^*D\gamma$ coupling constants are related to those of $D^*D^*\pi$ and $D^*D^*\gamma$, respectively. This is crucial since the latter two couplings are very difficult to measure in practice. With the knowledge of the above coupling strengths, the predictive power of the heavy meson chiral Lagrangian is greatly enhanced. However, success of such a scheme demands an assessment of how large the $1/m_Q$ corrections are. The purpose of this paper is to study such type of $1/m_Q$

corrections which modify the various HQS relations among the coupling constants. At present, we do not attempt to give a quantitative predictions on the sizes of various $1/m_Q$ effects. A quantitative analysis will be presented in a future publication.

As is well known, there are two energy scales in the chiral perturbation theory involving a heavy hadron: the mass of the heavy hadron M_H and the chiral symmetry-breaking scale Λ_χ. In principle, one may expand the theory in inverse powers of these two scales. However, because the heavy hadrons have large masses, the derivatives acting on the heavy-hadron fields will produce large momentum factors. This complicates the power counting procedure. This difficulty is overcome by a simple observation. Strong and electromagnetic interactions at low energies of a heavy hadron with other light hadrons are governed by the energy scale $\Lambda_{\rm QCD}$ which is much smaller than M_H. Consequently, the four-momentum of a heavy hadron has only fluctuations of the order of $\Lambda_{\rm QCD}$ throughout its history. Its momentum can, therefore, be parametrized as

$$P=M_H v+k, \quad v^2=1 , \tag{1.4}$$

where k is of order $\Lambda_{\rm QCD}$. In accordance with the parametrization, one introduces a velocity-dependent field $H_v(x)$ by [12,19]

$$H(x)=e^{-iM_H v\cdot x}H_v(x) , \tag{1.5}$$

where $H(x)$ is the standard field operator for a heavy hadron. The velocity-dependent field $H_v(x)$ carries only the residual momentum k. It follows from (1.5) that

$$\partial_\mu H(x)=e^{-iM_H v\cdot x}[-iM_H v_\mu H_v(x)+\partial_\mu H_v(x)] . \tag{1.6}$$

The dependence on the large mass M_H is now made explicit: the second term in (1.6) is of order k/M_H relative to the first one. In terms of $H_v(x)$, derivatives acting on the heavy hadron and Goldstone boson fields are treated on equal footing, and a consistent $1/M_H$ and $1/\Lambda_\chi$ expansion can be developed for the heavy-hadron chiral Lagrangian.

The velocity-dependent fields for the 0^- and 1^- heavy hadrons of (1.1) are

$$P(x)=e^{-iM_{P^*}v\cdot x}P_v(x) , \tag{1.7a}$$

$$P^*_\mu(x)=e^{-iM_{P^*}v\cdot x}P^*_{v,\mu}(x) , \tag{1.7b}$$

$$v\cdot P^*(v)=0 . \tag{1.7c}$$

To simplify our notation, we write $P(v)\equiv P_v(x)$ and $P^*_\mu(v)\equiv P^*_{v,\mu}(x)$. Retaining only the leading terms, we obtain

$$\mathcal{L}^{(1)}_{v,PP^*}=-2iM_{P^*}P(v)v\cdot DP^\dagger(v)+2iM_{P^*}P^{*\mu}(v)v\cdot DP^{*\dagger}_\mu(v)+\Delta M^2P(v)P^\dagger(v)$$
$$+f\sqrt{M_PM_{P^*}}[P(v)\mathcal{A}^\mu P^{*\dagger}_\mu(v)+P^*_\mu(v)\mathcal{A}^\mu P^\dagger(v)]+2iM_{P^*}g\epsilon_{\mu\nu\lambda\kappa}P^{*\mu}(v)v^\nu\mathcal{A}^\lambda P^{*\kappa\dagger}(v) , \tag{1.8}$$

with

$$\Delta M^2=M^2_{P^*}-M^2_P . \tag{1.9}$$

Note that we have neglected terms which are suppressed by $1/M_{P^*}$ comparing with the leading contributions. Therefore, $\mathcal{L}^{(1)}_{v,PP^*}$ is the leading-order heavy-meson chiral Lagrangian in the double expansions of $1/M_{P^*}$ and Goldstone-boson momenta. Before proceeding further, we should like to make two remarks on the Lagrangian $\mathcal{L}^{(1)}_{v,PP^*}$. First of all, the parameters M_P and M_{P^*} in Eq. (1.8) are taken to be the physical masses of the heavy mesons P and P^*, respectively. This accounts for the appearance of $\Delta M^2P(v)P^\dagger(v)$ in Eq. (1.8). Theoretically, we expect

$$M_{P^*}-M_P=O\left[\frac{\Lambda^2_{\rm QCD}}{m_Q}\right] , \tag{1.10}$$

so ΔM^2 is of order $\Lambda^2_{\rm QCD}$ and it is a simplest $1/M_H$ correction to the leading terms of $\mathcal{L}^{(1)}_{v,PP^*}$ which we keep. Second, the coupling constants f and g are no longer assumed to satisfy the spin symmetry relation (1.3).

It has been noted by Luke and Manohar [20] that the structure of the $1/M_H$ expansion must satisfy the "reparametrization invariance" which is a consequence of the nonuniqueness of the parametrization (1.4). The four-velocity v and the residual momentum k can be arbitrarily chosen so long as $v^2=1$ and $k\sim\Lambda_{\rm QCD}\ll M_H$. For consistency, the heavy-meson chiral theory must be invariant under the transformation

$$v\to v+r/M_{P^*}, \quad k\to k-r , \tag{1.11a}$$

$$(v+r/M_{P^*})^2=1 . \tag{1.11b}$$

This leads to the conclusion that a reparametrization-invariant heavy-meson chiral Lagrangian, which we denote as $\tilde{\mathcal{L}}_{PP^*}$, must have the structure [20]

$$\tilde{\mathcal{L}}_{PP^*}=\sum_v\tilde{\mathcal{L}}_{v,PP^*}(P(v),\tilde{P}^{*\mu}(v),\mathcal{W}^\mu) , \tag{1.12}$$

where

$$\mathcal{W}_\mu=v_\mu+iD_\mu/M_{P^*} , \tag{1.13a}$$

$$\tilde{P}^{*\mu}(v)=P^{*\mu}(v)-v^\mu\frac{iD\cdot P^*(v)}{M_{P^*}} . \tag{1.13b}$$

Therefore, to maintain the reparametrization invariance to order $O(1/M_{P^*})$, the Lagrangian is augmented to be

$$\tilde{\mathcal{L}}^{(1)}_{v,PP^*} = -M_{P^*}^2 P(v)(\mathcal{W}^2-1)P^\dagger(v) + M_{P^*}^2 \tilde{P}^{*\mu}(v)(\mathcal{W}^2-1)\tilde{P}^{*\dagger}_{\mu}(v) + \Delta M^2 P(v)P^\dagger(v)$$
$$+ f\sqrt{M_P M_{P^*}}[P(v)\mathcal{A}^\mu \tilde{P}^{*\dagger}_\mu(v) + \tilde{P}^*_\mu(v)\mathcal{A}^\mu P^\dagger(v)] + iM_{P^*} g\epsilon_{\mu\nu\lambda\kappa}[\tilde{P}^{*\mu}(v)\overleftarrow{\mathcal{W}}^\nu \mathcal{A}^\lambda \tilde{P}^{*\kappa\dagger}(v) + \tilde{P}^{*\mu}(v)\mathcal{A}^\lambda \mathcal{W}^\nu \tilde{P}^{*\kappa\dagger}(v)] . \tag{1.14}$$

The prescription (1.13) uniquely determines the terms of order $1/M_{P^*}$ necessary to ensure the reparametrization invariance of $\tilde{\mathcal{L}}^{(1)}_{v,PP^*}$. These $1/M_{P^*}$ corrections are essentially kinematic in nature. They are important, but they can be retrieved by following the prescription (1.13). However, there are other $1/M_{P^*}$ contributions which are reparametrization invariant by themselves, but at least contain two derivatives. It should be pointed out that the original Lagrangian (1.1) is reparametrization invariant. Equation (1.14) follows simply from Eq. (1.1) by keeping the first two leading orders in the $1/M_H$ expansion using Eq. (1.7) [in particular, Eq. (1.7c) should hold to order $1/M_H$]. The requirement of reparametrization invariance will become more useful as the $1/M_H$ expansion is carried out to higher orders, or when we deal with new situations [21].

There is another type of $1/M_{P^*}$ corrections which will be the focus of the present work. In contrast with the previous corrections, these are dynamical in nature and they arise from taking into account the $1/m_Q$ terms in HQET. It is well known that the following two operators in HQET break the heavy-quark spin-flavor symmetry at the order of $1/m_Q$ [7,8]:

$$\mathcal{L} = \bar{h}_v iv\cdot D h_v + \mathcal{L}_I , \tag{1.15a}$$

$$\mathcal{L}_I = O_1 + O_2 , \tag{1.15b}$$

$$O_1 = \frac{1}{2m_Q}\bar{h}_v (iD)^2 h_v , \tag{1.15c}$$

$$O_2 = \frac{1}{2m_Q}\bar{h}_v \left[-\frac{1}{2} g_s \sigma_{\mu\nu} G^{\mu\nu} \right] h_v . \tag{1.15d}$$

Specifically, the operator O_1 breaks the flavor symmetry, and the operator O_2 breaks both the flavor and spin symmetries. To the first order in the Goldstone-boson momentum, the only effects of O_1 and O_2 are to make $1/m_Q$ corrections to the coupling constants f and g which appear in Eq. (1.14). To order $1/m_Q$, we may write

$$f = f_0 + f_c \frac{\Lambda}{2m_Q} , \tag{1.16a}$$

$$g = g_0 + g_c \frac{\Lambda}{2m_Q} , \tag{1.16b}$$

where $g_0 = \frac{1}{2} f_0$, and Λ is an arbitrary mass scale. Presumably, the value for Λ should be chosen in such a way that $f_c \approx f_0$ and $g_c \approx g_0$. Under this requirement, it has recently been argued that the parameter Λ is of order Λ_χ rather than $\Lambda_{\rm QCD}$ [22]. However, we will take no position on this point as it is still not widely accepted.

We observe that the two types of $1/M_H$ $(1/m_Q)$ corrections discussed above have distinct characteristics. The $1/M_H$ correction demanded by reparametrization invariance introduces new structures which modify the leading-order Lagrangian. The other, dynamical corrections of order $1/m_Q$ produce heavy-quark symmetry-breaking contributions to the coupling constants in the leading-order Lagrangian but they do not alter the structure of the Lagrangian. The two effects together provide the complete $1/M_H$ $(1/m_Q)$ corrections to heavy-quark symmetry. Since

$$1/M_H = 1/m_Q + O(1/m_Q^2) ,$$

there is no need to keep the difference between $1/M_H$ and $1/m_Q$ at this order.

So far, we have used the heavy-meson dynamics as an example to discuss the various issues in the $1/M_H(1/m_Q)$ corrections to the heavy-quark symmetry. Clearly, we can carry out a similar discussion for heavy baryons on reparametrization invariance and dynamical corrections to the coupling constants.

With the issues in the $1/M_H$ $(1/m_Q)$ corrections clearly defined, we will concentrate our attention in what follows on interactions between the heavy hadrons and the Goldstone bosons with a single derivative. Section II is devoted to a study of the $O(1/m_Q)$ correction to the coupling constants for both strong and electromagnetic interactions in the heavy meson sector. A similar study for heavy baryons is carried out in Sec. III. We will employ the method of interpolating fields extensively utilized in Ref. [17]. We find that all the heavy-quark spin symmetry relations among the coupling constants (both strong and electromagnetic) are completely broken by $1/m_Q$ corrections.

Finally, in Sec. IV we make some concluding remarks and we shall comment on the work done by Randall and Sather [23] concerning the SU(3)-violating corrections to the heavy-meson hyperfine splitting, which is a typical $O(1/m_Q)$ phenomenon. As we shall point out, the calculation performed in Ref. [23] is incomplete; namely, it does not include all the corrections of order $1/m_Q$.

II. $1/m_Q$ CORRECTIONS TO THE DYNAMICS OF HEAVY MESONS

In this section we shall study the $1/m_Q$ corrections to the coupling constants of the heavy-meson chiral Lagrangian given by Eqs. (2.16) and (2.19) of Ref. [17]. First of all, we shall rewrite the chiral Lagrangians $\mathcal{L}^{(1)}_{PP^*}$ and $\mathcal{L}^{(2)}_{PP^*}$ in terms of velocity-dependent fields and retain only the leading terms in the $1/M_H$ expansion. The velocity-dependent version of $\mathcal{L}^{(1)}_{PP^*}$ is given by Eq. (1.8), which we recall here for convenience:

$$\mathcal{L}^{(1)}_{v,PP^*} = -2iM_{P^*}P(v)v\cdot DP^\dagger(v) + 2iM_{P^*}P^{*\mu}(v)v\cdot DP^{*\dagger}_\mu(v) + \Delta M^2 P(v)P^\dagger(v)$$
$$+ f\sqrt{M_P M_{P^*}}[P(v)\mathcal{A}^\mu P^{*\dagger}_\mu(v) + P^*_\mu(v)\mathcal{A}^\mu P^\dagger(v)] + 2iM_{P^*} g\epsilon_{\mu\nu\lambda\kappa}P^{*\mu}(v)v^\nu \mathcal{A}^\lambda P^{*\kappa\dagger}(v) , \tag{2.1}$$

with

$$\Delta M^2 = M^2_{P^*} - M^2_P . \tag{2.2}$$

Substituting Eqs. (1.7a) and (1.7b) into $\mathcal{L}^{(2)}_{PP^*}$ which describes the radiative transitions, we obtain (see Ref. [17] for notation)

$$\mathcal{L}^{(2)}_{v,PP^*} = \sqrt{M_P M_{P^*}}\epsilon_{\mu\nu\alpha\beta}v^\alpha P^{*\beta}(v)[\tfrac{1}{2}d(\xi^\dagger Q\xi + \xi Q\xi^\dagger) + d'Q']F^{\mu\nu}P^\dagger(v) + \text{H.c.}$$
$$+ id'' M_{P^*}F_{\mu\nu}P^{*\nu}(v)[\gamma Q' - \tfrac{1}{2}(\xi^\dagger Q\xi + \xi Q\xi^\dagger)]P^{*\mu\dagger}(v) , \tag{2.3}$$

where Q' is the heavy-quark charge and Q denotes the charge matrix of the light quarks:

$$Q = \begin{bmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{bmatrix} . \tag{2.4}$$

Note that, contrary to Eq. (2.19) of Ref. [17], we do not need to subtract from $\mathcal{L}^{(2)}_{v,PP^*}$ the normal magnetic moment term of P^*_μ induced by the minimum substitution. This is because such contributions are not among the leading terms kept in (2.3).

As indicated in Eqs. (1.16a) and (1.16b), every coupling constant in $\mathcal{L}^{(1)}_{v,PP^*}$ and $\mathcal{L}^{(2)}_{v,PP^*}$ can be expanded in powers of $1/m_Q$. In particular, we have written there the expansion for coupling constants f and g:

$$f = f_0 + f_c\frac{\Lambda}{2m_Q} \tag{2.5}$$

and

$$g = g_0 + g_c\frac{\Lambda}{2m_Q} . \tag{2.6}$$

The zeroth-order contributions f_0 and g_0 are related by HQS [11]: namely,

$$g_0 = \tfrac{1}{2}f_0 . \tag{2.7}$$

To compute the$1/m_Q$ corrections to f_0 and g_0, we insert operators O_1 and O_2, defined in Eqs. (1.15c) and (1.15d), into the relevant decay amplitudes:

$$\Delta M \equiv \Delta M[P^*(v,\varepsilon)\to P(v)+\pi^a(q)]$$
$$= \frac{1}{f_\pi}q^\mu\left\langle P(v)\left| iT\int d^4x\,[O_1(x)+O_2(x)]\mathcal{A}^a_\mu(0)\right| P^*(v,\varepsilon)\right\rangle , \tag{2.8}$$
$$\Delta M' \equiv \Delta M[P^*(v,\varepsilon)\to P^*(v,\varepsilon')+\pi^a(q)]$$
$$= \frac{1}{f_\pi}q^\mu\left\langle P^*(v,\varepsilon')\left| iT\int d^4x\,[O_1(x)+O_2(x)]\mathcal{A}^a_\mu(0)\right| P^*(v,\varepsilon)\right\rangle . \tag{2.9}$$

To determine the general Lorentz structure of Eqs. (2.8) and (2.9), we recall that the interpolating fields for pseudoscalar and vector mesons are given by [24]

$$P_i(v) = \bar{q}\gamma_5 h^i_v\sqrt{M_P} , \tag{2.10a}$$

$$P^*_i(v,\varepsilon) = \bar{q}\not{\varepsilon} h^i_v\sqrt{M_{P^*}} . \tag{2.10b}$$

Since we will keep only leading terms in the $1/M_H$ expansion, we can simply neglect the $1/M_H$ corrections needed for reparametrization invariance. For the same reason, we can also neglect residual momenta k and k' in Eqs. (2.8)–(2.10). Furthermore, we shall treat contributions from O_1 and O_2 separately. Since O_1 preserves heavy-quark spin symmetry, its contributions to both amplitudes must be of the form

$$\Delta M_1 = \frac{ag_s}{f_\pi m_Q}\sqrt{M_P M_{P^*}}(\varepsilon\cdot q)u(P^*)^*\frac{\tau^a}{2}u(P) , \tag{2.11a}$$

$$\Delta M_1' = -\frac{ag_s}{f_\pi m_Q}M_{P^*}u(P^*)^*\frac{\tau^a}{2}u(P'^*)i\epsilon_{\mu\nu\lambda\kappa}q^\mu\varepsilon'^\nu v^\lambda\varepsilon^\kappa , \tag{2.11b}$$

where $u(P^*)$, $u(P)$, and $u(P'^*)$ are isospin wave functions of the heavy mesons and a is a constant independent of heavy-quark masses.

The contributions from O_2 are given by

$$\Delta M_2 = -\frac{g_s q^\mu}{4m_Q f_\pi}\left\langle P(v)\left| iT\int d^4x\, \bar{h}_v \sigma^{\alpha\beta} G_{\alpha\beta} h_v(x) \mathcal{A}^a_\mu(0)\right| P^*(v,\varepsilon)\right\rangle , \tag{2.12a}$$

$$\Delta M'_2 = -\frac{g_s q^\mu}{4m_Q f_\pi}\left\langle P^*(v,\varepsilon')\left| iT\int d^4x\, \bar{h}_v \sigma^{\alpha\beta} G_{\alpha\beta} h_v(x) \mathcal{A}^a_\mu(0)\right| P^*(v,\varepsilon)\right\rangle . \tag{2.12b}$$

To evaluate ΔM_2 and $\Delta M'_2$, we make use of Eqs. (2.10a) and (2.10b) to obtain

$$\Delta M'_2 = -\frac{g_s q^\mu \sqrt{M_{P^*} M_{P^*}}}{4m_Q f_\pi}\left\langle 0\left| iT\int d^4x\, \bar{q}_v \not{\varepsilon}' h_v \bar{h}_v \sigma^{\alpha\beta} G_{\alpha\beta} h_v(x) \mathcal{A}^a_\mu(0) \bar{h}_v \not{\varepsilon} q_v \right| 0\right\rangle$$

$$= \frac{g_s q^\mu M_{P^*}}{4m_Q f_\pi} \mathrm{tr}\left[\not{\varepsilon}' \frac{1+\not{v}}{2} \sigma^{\alpha\beta} \frac{1+\not{v}}{2} \not{\varepsilon} \left\langle 0\left| iT\int d^4x\, q_v G_{\alpha\beta} \mathcal{A}^a_\mu \bar{q}_v \right| 0\right\rangle \right] . \tag{2.13b}$$

$$\Delta M'_2 = -\frac{g_s q^\mu \sqrt{M_{P^*} M_{P^*}}}{4m_Q f_\pi}\left\langle 0\left| iT\int d^4x\, \bar{q}_v \not{\varepsilon}' h_v \bar{h}_v \sigma^{\alpha\beta} G_{\alpha\beta} h_v(x) \mathcal{A}^a_\mu(0) \bar{h}_v \not{\varepsilon} q_v \right| 0\right\rangle$$

$$= \frac{g_s q^\mu M_{P^*}}{4m_Q f_\pi} \mathrm{tr}\left[\not{\varepsilon}' \frac{1+\not{v}}{2} \sigma^{\alpha\beta} \frac{1+\not{v}}{2} \not{\varepsilon} \left\langle 0\left| iT\int d^4x\, q_v G_{\alpha\beta} \mathcal{A}^a_\mu \bar{q}_v \right| 0\right\rangle \right] . \tag{2.13b}$$

Both Eqs. (2.13a) and (2.13b) contain a matrix element which describes the dynamics of light constituents:

$$M_{\alpha\beta\mu} = \left\langle 0\left| iT\int d^4x\, q_v G_{\alpha\beta} \mathcal{A}^a_\mu \bar{q}_v \right| 0\right\rangle$$

$$= u(P^*)^* \frac{\tau^a}{2} u(P)[\epsilon_{\alpha\beta\mu\nu}(bv^\nu + c\gamma^\nu + d\gamma^\nu \not{v}) + e_1\gamma_5\sigma_{\alpha\beta}v_\mu + ie_2\gamma_5(g_{\alpha\mu}\gamma_\beta - g_{\beta\mu}\gamma_\alpha)] , \tag{2.14}$$

where b, c, d, e_1, and e_2 are constants independent of heavy quark masses. Note that we have suppressed the q-dependent terms since they correspond to higher-dimensional terms in chiral expansion. The right-hand side of Eq. (2.14) is the most general expression consistent with the symmetry properties of its left-hand side. Substituting Eq. (2.14) into Eq. (2.13) yields

$$\Delta M_2 = -\frac{g_s\sqrt{M_P M_{P^*}}}{4m_Q f_\pi}(\varepsilon\cdot q)4u(P^*)^*\frac{\tau^a}{2} \times u(P)(b - c + d + 2e_2) , \tag{2.15a}$$

$$\Delta M'_2 = -\frac{g_s M_{P^*}}{4m_Q f_\pi}4iu(P^*)^*\frac{\tau^a}{2}u(P'^*) \times (b - c + d)\epsilon_{\mu\nu\alpha\beta}q^\mu\varepsilon'^\nu v^\alpha \varepsilon^\beta . \tag{2.15b}$$

Since ΔM and $\Delta M'$ can also be computed through the chiral Lagrangian by expanding

$$\mathcal{A}_\mu = -\frac{1}{f_\pi}\partial_\mu\left[\frac{1}{2}\tau^a\pi^a\right] + \cdots ; \tag{2.16}$$

hence, the ΔM and $\Delta M'$ given by (2.11) and (2.15) imply

$$f_c = \frac{2g_s}{\Lambda}[a - (b - c + d + 2e_2)] \tag{2.17a}$$

and

$$g_c = \frac{g_s}{\Lambda}[a + (b - c + d)] . \tag{2.17b}$$

It is clear that $g_c \neq \frac{1}{2}f_c$, in general.

We note that the same combination $b - c + d$ appears in both f_c and g_c. It means that the corrections f_c and g_c are characterized by *three* parameters a, $b - c + d$, and e_2. We will now show that e_2 is zero, so actually there are only two unknowns to describe the two coupling constants f_c and g_c. The two processes $P^* \to P + \pi$ and $P \to P^* + \pi$ are related by charge conjugation, and the appropriate coupling constants are f and f^*, respectively. In Eqs. (1.1) and (1.8), it is implicitly assumed that f is real; this can always be accomplished with a judicious choice of phases for the field operators of the heavy mesons. We will assume that this is done. Let us denote

$$\Delta M'' \equiv \Delta M[P(v) \to P^*(v,\varepsilon) + \pi^a(q)] , \tag{2.18}$$

which can be computed by the same procedure for computing ΔM. We find that $\Delta M''$ also depends on $M_{\alpha\beta\mu}$ given by (2.14). Indeed, we obtain

$$\Delta M'' = -\frac{g_s\sqrt{M_P M_{P^*}}}{4m_Q f_\pi}(\varepsilon\cdot q)4u(P^*)^*\frac{\tau^a}{2} \times u(P)(b - c + d - 2e_2) , \tag{2.19}$$

which gives

$$f_c^* = \frac{2g_s}{\Lambda}[a - (b - c + d - 2e_2)] . \tag{2.20}$$

We now demand that $f_c = f_c^*$. A comparison of (2.17) and (2.20) yields $e_2 = 0$. Finally,

$$f_c=\frac{2g_s}{\Lambda}(a-b') \tag{2.21a}$$

and

$$g_c=\frac{g_s}{\Lambda}(a+b') , \tag{2.21b}$$

where $b'=b-c+d$.

To discuss $1/m_Q$ corrections to the coupling constants in $\mathcal{L}^{(2)}_{v,PP^*}$, we shall treat the heavy-quark and light-quark electromagnetic currents separately. In the case of the heavy-quark electromagnetic current, the relevant coefficients d' and $d''\gamma$ are both of order $1/m_Q$ because they arise from the magnetic moment of the heavy quark. As pointed out by us [17] and by others [15,16], these couplings are rigorously determined by the heavy-quark effective theory. For completeness, we shall reproduce the results here. First of all, the heavy-quark electromagnetic current in the effective theory to order $1/m_Q$ can be written as [9]

$$\begin{aligned} J_\mu^{\rm em}&=\bar h_{v'}\gamma_\mu h_v-\frac{i}{2m_Q}\bar h_{v'}(\overleftarrow{\not D}\gamma_\mu-\gamma_\mu\not D)h_v\\ &=\bar h_{v'}\gamma_\mu h_v-\frac{i}{2m_Q}\bar h_{v'}(\overleftarrow{\not D}\gamma_\mu+\gamma_\mu\overleftarrow{\not D})h_v\\ &\quad+\frac{i}{2m_Q}\partial^\nu(\bar h_{v'}\gamma_\mu\gamma_\nu h_v) . \end{aligned} \tag{2.22}$$

Using the identity

$$\gamma_\mu\gamma_\nu=g_{\mu\nu}-i\sigma_{\mu\nu} , \tag{2.23}$$

the Gordon decomposition

$$\bar h_{v'}\gamma_\mu h_v=\tfrac{1}{2}\bar h_{v'}(v'+v)_\mu h_v+\tfrac{1}{2}i\bar h_{v'}\sigma_{\mu\nu}(v'-v)^\nu h_v , \tag{2.24}$$

and the identity

$$\begin{aligned}&\langle H_f(v')|\partial_\mu(\bar h_{v'}\Gamma h_v)|H_i(v)\rangle\\ &\qquad=i\bar\Lambda(v'-v)_\mu\langle H_f(v')|\bar h_{v'}\Gamma h_v|H_i(v)\rangle , \end{aligned} \tag{2.25}$$

with H_f and H_i being generic hadronic states and $\bar\Lambda=M_{H_f}-m_Q=M_{H_i}-m_Q$, we finally arrive at

$$J_\mu^{\rm em}\doteq\tfrac{1}{2}\bar h_{v'}(v'+v)_\mu h_v-\frac{i}{2m_Q}k^\nu\bar h_{v'}\sigma_{\mu\nu}h_v+O_\mu^{vv'} , \tag{2.26}$$

where $k^\nu=-M_H(v'-v)^\nu$, and

$$O_\mu^{vv'}=-\frac{i}{m_Q}\bar h_{v'}\overleftarrow{D}_\mu h_v-\frac{\bar\Lambda}{2m_Q}(v'-v)^\nu\bar h_{v'}\gamma_\mu\gamma_\nu h_v . \tag{2.27}$$

In Eq. (2.26), we have used the notation $\doteq$ to remind the reader that such a relation holds only after taking the matrix element. The contribution due to $O_\mu^{vv'}$ is negligible since it cannot change the normalization of $J_\mu^{\rm em}$ at $v=v'$, and its contribution to the anomalous magnetic coupling is necessarily of order $1/m_Q^2$. The first term in Eq. (2.26) corresponds to the convection current of the heavy quark, which has already been taken into account in $\mathcal{L}^{(1)}_{v,PP^*}$. The second term, which is of order $1/m_Q$, will contribute to the coefficients d' and $d''\gamma$ in $\mathcal{L}^{(2)}_{v,PP^*}$. There is another source of $1/m_Q$ corrections which arises when one evaluates the time-ordered products of $\frac{1}{2}\bar h_{v'}(v'+v)_\mu h_v$ with the symmetry-breaking operators O_1 and O_2. However, these contributions vanish at $v=v'$ since the normalization of the vector current is already fixed at the leading order. Consequently, to order $1/m_Q$, the parameters d' and $d''\gamma$ are solely induced by the second term on the right-hand side (RHS) of Eq. (2.26):

$$F_\mu^Q=\left\langle P^*(v',\varepsilon')\left|\frac{ieQ'}{2m_Q}k^\nu\bar h_{v'}\sigma_{\mu\nu}h_v\right|P^*(v,\varepsilon)\right\rangle\Bigg|_{v=v'} , \tag{2.28a}$$

$$\tilde F_\mu^Q=\left\langle P(v')\left|\frac{ieQ'}{2m_Q}k^\nu\bar h_{v'}\sigma_{\mu\nu}h_v\right|P^*(v,\varepsilon)\right\rangle\Bigg|_{v=v'} . \tag{2.28b}$$

The evaluation of F_μ^Q and $\tilde F_\mu^Q$ is straightforward with the aid of Eqs. (2.10a) and (2.10b). Taking F_μ^Q as an example, we convert the matrix element in Eq. (2.28a) into

$$\begin{aligned} F_\mu^Q&=\frac{ieQ'}{2m_Q}k^\nu M_{P^*}\langle 0|\bar q_{v'}\not\varepsilon' h_{v'}\bar h_{v'}\sigma_{\mu\nu}h_v\bar h_v\not\varepsilon q_v|0\rangle|_{v=v'}\\ &=-\frac{ieQ'}{2m_Q}k^\nu M_{P^*}\\ &\quad\times\mathrm{tr}\left[\not\varepsilon'\frac{1+\not v}{2}\sigma_{\mu\nu}\frac{1+\not v}{2}\not\varepsilon\langle 0|q_v\bar q_{v'}|0\rangle|_{v=v'}\right] , \end{aligned} \tag{2.29}$$

where [2]

$$\langle 0|q_v\bar q_{v'}|0\rangle|_{v=v'}=\xi(v\cdot v'=1)=1 . \tag{2.30}$$

Working out the trace, we obtain

$$F_\mu^Q=-\frac{eQ'M_{P^*}}{m_Q}(\varepsilon_\mu\varepsilon'\cdot k-\varepsilon'_\mu\varepsilon\cdot k) . \tag{2.31a}$$

Similarly, we have

$$\tilde F_\mu^Q=-\frac{ieQ'\sqrt{M_PM_{P^*}}}{m_Q}\epsilon_{\mu\nu\alpha\beta}k^\nu\varepsilon^\alpha v^\beta . \tag{2.31b}$$

Comparing Eq. (2.31) with Eq. (2.3), we obtain

$$d'=-\frac{e}{2m_Q},\quad d''\gamma=\frac{e}{m_Q} . \tag{2.32}$$

To determine d and d'', we need to consider the form factors induced by the light-quark electromagnetic current. To order $1/m_Q$, we have

$$d=d_0+d_c\frac{\Lambda}{2m_Q} , \tag{2.33a}$$

$$d''=d''_0+d''_c\frac{\Lambda}{2m_Q} . \tag{2.33b}$$

As discussed in Ref. [17], one can apply the heavy-quark spin symmetry to obtain

$$d_0 = -\frac{1}{2} d_0'' \ . \tag{2.34}$$

To see whether d_c and d_c'' obey the same relation, we evaluate the following magnetic form factors induced by the light quark electromagnetic current $j_\mu^{\rm em} \equiv e\bar{q}Q\gamma_\mu q$:

$$F_\mu = \left\langle P^*(v,\varepsilon') \left| iT \int d^4x \, [O_1(x)+O_2(x)] j_\mu^{\rm em}(0) \right| P^*(v,\varepsilon) \right\rangle_m , \tag{2.35a}$$

$$\tilde{F}_\mu = \left\langle P(v) \left| iT \int d^4x \, [O_1(x)+O_2(x)] j_\mu^{\rm em}(0) \right| P^*(v,\varepsilon) \right\rangle_m , \tag{2.35b}$$

where the subscript m indicates the fact that we keep only the magnetic interactions. To evaluate F_μ and $\tilde{F}_\mu$, we again employ the technique of interpolating fields [24] to obtain

$$F_\mu = \left\langle 0 \left| iT \int d^4x \, \bar{q}_v \not{\varepsilon}' h_v [O_1(x)+O_2(x)] j_\mu^{\rm em}(0) \bar{h}_v \not{\varepsilon} q_v \right| 0 \right\rangle_m \tag{2.36a}$$

and

$$\tilde{F}_\mu = \left\langle 0 \left| iT \int d^4x \, \bar{q}_v \gamma_5 h_v [O_1(x)+O_2(x)] j_\mu^{\rm em}(0) \bar{h}_v \not{\varepsilon} q_v \right| 0 \right\rangle_m . \tag{2.36b}$$

For convenience, we shall treat contributions by $O_1(x)$ and $O_2(x)$ separately. Their contributions are denoted by F_μ^1 ($\tilde{F}_\mu^1$) and F_μ^2 ($\tilde{F}_\mu^2$), respectively. As $O_1(x)$ preserves the heavy-quark spin symmetry, F_μ^1 is related to $\tilde{F}_\mu^1$ in such a way that

$$F_\mu^1 = \frac{2a_1 g_s}{m_Q} M_{P^*} (\varepsilon_\mu \varepsilon' \cdot k - \varepsilon'_\mu \varepsilon \cdot k) \tag{2.37a}$$

and

$$\tilde{F}_\mu^1 = \frac{2ia_1 g_s}{m_Q} \sqrt{M_P M_{P^*}} \epsilon_{\mu\nu\alpha\beta} k^\nu v^\alpha \varepsilon^\beta , \tag{2.37b}$$

with a_1 being a constant independent of the heavy-quark mass. For simplicity, we have set the charge matrix $Q = \mathbb{1}$ and suppressed the flavor quantum numbers while obtaining Eqs. (2.35) and (2.37). To compute F_μ^2 and $\tilde{F}_\mu^2$, we apply Eqs. (2.10a) and (2.10b) to obtain

$$\begin{aligned} F_\mu^2 &= -\frac{g_s M_{P^*}}{4m_Q} \left\langle 0 \left| iT \int d^4x \, \bar{q}_v \not{\varepsilon}' h_v \bar{h}_v \sigma^{\alpha\beta} G_{\alpha\beta} h_v(x) j_\mu^{\rm em}(0) \bar{h}_v \not{\varepsilon} q_v \right| 0 \right\rangle_m \\ &= \frac{g_s M_{P^*}}{4m_Q} {\rm tr} \left[\not{\varepsilon}' \frac{1+\not{v}}{2} \sigma^{\alpha\beta} \frac{1+\not{v}}{2} \not{\varepsilon} \left\langle 0 \left| iT \int d^4x \, q_v G_{\alpha\beta} j_\mu^{\rm em} \bar{q}_v \right| 0 \right\rangle \right]_m , \end{aligned} \tag{2.38a}$$

$$\begin{aligned} \tilde{F}_\mu^2 &= -\frac{g_s \sqrt{M_P M_{P^*}}}{4m_Q} \left\langle 0 \left| iT \int d^4x \, \bar{q}_v \gamma_5 h_v \bar{h}_v \sigma^{\alpha\beta} G_{\alpha\beta} h_v(x) j_\mu^{\rm em}(0) \bar{h}_v \not{\varepsilon} q_v \right| 0 \right\rangle_m \\ &= \frac{g_s \sqrt{M_P M_{P^*}}}{4m_Q} {\rm tr} \left[\gamma_5 \frac{1+\not{v}}{2} \sigma^{\alpha\beta} \frac{1+\not{v}}{2} \not{\varepsilon} \left\langle 0 \left| iT \int d^4x \, q_v G_{\alpha\beta} j_\mu^{\rm em} \bar{q}_v \right| 0 \right\rangle \right]_m . \end{aligned} \tag{2.38b}$$

The nonperturbative dynamics of the light constituents can be parametrized as

$$\begin{aligned} M'_{\alpha\beta\mu} &= \left\langle 0 \left| iT \int d^4x \, q_v G_{\alpha\beta} j_\mu^{\rm em} \bar{q}_v \right| 0 \right\rangle \\ &= -ib_1 (g_{\alpha\mu} k_\beta - g_{\beta\mu} k_\alpha) , \end{aligned} \tag{2.39}$$

where k is the outgoing photon momentum and b_1 is a constant independent of the heavy-quark mass. In Eq. (2.39), we kept only structures linear in k relevant to magnetic interactions. With this to be understood, Eq. (2.39) then represents the most general Lorentz structure for $M'_{\alpha\beta\mu}$ which is consistent with gauge invariance, parity conservation, and the constraint $M'_{\alpha\beta\mu} = -M'_{\beta\alpha\mu}$. Substituting Eq. (2.39) into Eqs. (2.38a) and (2.38b), we obtain

$$F_\mu^2 = \frac{-g_s M_{P^*}}{m_Q} b_1 (\varepsilon_\mu \varepsilon' \cdot k - \varepsilon'_\mu \varepsilon \cdot k) , \tag{2.40a}$$

$$\tilde{F}_\mu^2 = \frac{ig_s \sqrt{M_P M_{P^*}}}{m_Q} b_1 \epsilon_{\mu\nu\alpha\beta} k^\nu v^\alpha \varepsilon^\beta . \tag{2.40b}$$

Since the results in Eqs. (2.40a) and (2.40b) can also be obtained from the Lagrangian $\mathcal{L}^{(2)}_{v,PP^*}$, we can hence make the identifications

$$d_c=\frac{g_s}{\Lambda}(2a_1+b_1)\ , \tag{2.41a}$$

$$d_c''=\frac{2g_s}{\Lambda}(-2a_1+b_1)\ . \tag{2.41b}$$

It is clear that $d_c\neq -\frac{1}{2}d_c''$, in general.

Equations (2.21) and (2.41) are the main results in this section.

III. $1/m_Q$ CORRECTIONS TO THE DYNAMICS OF HEAVY BARYONS

In this section we study the $1/m_Q$ corrections to the coupling constants appearing in the heavy-baryon chiral Lagrangian $\mathcal{L}_B^{(1)}$ and $\mathcal{L}_B^{(2)}$ given by Eqs. (3.8) and (3.9), respectively, in Ref. [17]. In terms of velocity-dependent fields,

$$\begin{aligned}\mathcal{L}^{(1)}_{v,B}=&\ \tfrac{1}{2}\mathrm{tr}[\bar B_{\bar 3}(v)(iv\cdot D)B_{\bar 3}(v)]+\mathrm{tr}[\bar B_6(v)(iv\cdot D)B_6(v)]-\mathrm{tr}[\bar B^{*\mu}_6(v)(iv\cdot D)B^*_{6\mu}(v)]\\&+g_1\mathrm{tr}[\bar B_6(v)\gamma_\mu\gamma_5\mathcal{A}^\mu B_6(v)]+g_2\mathrm{tr}[\bar B_6(v)\gamma_\mu\gamma_5\mathcal{A}^\mu B_{\bar 3}(v)]+\mathrm{H.c.}+g_3\mathrm{tr}[\bar B^*_{6\mu}(v)\mathcal{A}^\mu B_6(v)]+\mathrm{H.c.}\\&+g_4\mathrm{tr}[\bar B^{*\mu}_6(v)\mathcal{A}_\mu B_{\bar 3}(v)]+\mathrm{H.c.}+g_5\mathrm{tr}[\bar B^{*\nu}_6(v)\gamma_\mu\gamma_5\mathcal{A}^\mu B^*_{6\nu}(v)]+g_6\mathrm{tr}[\bar B_{\bar 3}(v)\gamma_\mu\gamma_5\mathcal{A}^\mu B_{\bar 3}(v)]\end{aligned} \tag{3.1}$$

with

$$\begin{aligned}D_\mu B(v)=&\ \partial_\mu B(v)+\mathcal{V}_\mu B(v)+B(v)\mathcal{V}_\mu^T\\&+ieQ'A_\mu B(v)+ieA_\mu\{Q,B(v)\}\ ,\end{aligned} \tag{3.2}$$

where, as before,

$$Q=\mathrm{diag}(\tfrac{2}{3},-\tfrac{1}{3},-\tfrac{1}{3})$$

is the charge matrix of light quarks and Q' is the charge of the heavy quark Q. Note that we have omitted those terms which are induced by mass differences between various baryons.

It is well known that baryons do not behave much like Dirac point particles. As a result, they can have large anomalous magnetic moments. The most general gauge-invariant Lagrangian for magnetic transitions of heavy baryons is given by

$$\begin{aligned}\mathcal{L}^{(2)}_{v,B}=&\ a_1\mathrm{tr}[\bar B_6(v)Q\sigma\cdot FB_6(v)]+a_1'\mathrm{tr}[\bar B_6(v)Q'\sigma\cdot FB_6(v)]+a_2\mathrm{tr}[\bar B_6(v)Q\sigma\cdot FB_{\bar 3}(v)]+\mathrm{H.c.}\\&+a_2'\mathrm{tr}[\bar B_6(v)Q'\sigma\cdot FB_{\bar 3}(v)]+\mathrm{H.c.}+a_3\mathrm{tr}[\epsilon_{\mu\nu\lambda\kappa}\bar B^{*\mu}_6(v)Q\gamma^\nu F^{\lambda\kappa}B_6(v)]+\mathrm{H.c.}\\&+a_3'\mathrm{tr}[\epsilon_{\mu\nu\lambda\kappa}\bar B^{*\mu}_6(v)Q'\gamma^\nu F^{\lambda\kappa}B_6(v)]+\mathrm{H.c.}+a_4\mathrm{tr}[\epsilon_{\mu\nu\lambda\kappa}\bar B^{*\mu}_6(v)Q\gamma^\nu F^{\lambda\kappa}B_{\bar 3}(v)]+\mathrm{H.c.}\\&+a_4'\mathrm{tr}[\epsilon_{\mu\nu\lambda\kappa}\bar B^{*\mu}_6(v)Q'\gamma^\nu F^{\lambda\kappa}B_{\bar 3}(v)]+\mathrm{H.c.}+a_5\mathrm{tr}[\bar B^{*\mu}_6(v)Q\sigma\cdot FB^*_{6\mu}(v)]\\&+a_5'\mathrm{tr}[\bar B^{*\mu}_6(v)Q'\sigma\cdot FB^*_{6\mu}(v)]+a_6\mathrm{tr}[\bar B_{\bar 3}(v)Q\sigma\cdot FB_{\bar 3}(v)]+a_6'\mathrm{tr}[\bar B_{\bar 3}(v)Q'\sigma\cdot FB_{\bar 3}(v)]\ .\end{aligned} \tag{3.3}$$

The Lagrangian $\mathcal{L}^{(2)}_{v,B}$ is also the most general chiral-invariant one provided that one makes the replacement

$$Q\to\tfrac{1}{2}(\xi^\dagger Q\xi+\xi Q\xi^\dagger),\quad Q'\to Q'\ . \tag{3.4}$$

Note that, contrary to Eq. (3.9) of Ref. [17], we do not need to subtract from Eq. (3.3) the Dirac magnetic moments of the heavy baryons, because $\mathcal{L}^{(1)}_{v,B}$ is now expressed in terms of velocity-dependent fields and consequently contains no Dirac magnetic moments to the lowest order. The magnetic couplings a_i are induced by light-quark electromagnetic currents whereas a_i' are induced by heavy-quark ones and they are of order $1/m_Q$.

To incorporate $1/m_Q$ corrections, we expand the coupling constants in $\mathcal{L}^{(1)}_{v,B}$ and $\mathcal{L}^{(2)}_{v,B}$ as follows:

$$g_i=g_i^0+g_i^c\frac{\Lambda}{2m_Q}\ , \tag{3.5a}$$

$$a_i=a_i^0+a_i^c\frac{\Lambda}{2m_Q}\ . \tag{3.5b}$$

Let us first focus on the coupling constants g_i's in $\mathcal{L}^{(1)}_{v,B}$. The relations among the leading terms g_i^0 are governed by HQS [11]. They have been derived by evaluating the decay amplitudes $B_{\bar 3}\to B_{\bar 3}+\pi$, $B_6(B_6^*)\to B_{\bar 3}+\pi$, and $B_6(B_6^*)\to B_6(B_6^*)+\pi$. The results can be summarized as

$$g_3^0=\frac{\sqrt{3}}{2}g_1^0,\quad g_5^0=-\frac{3}{2}g_1^0\ , \tag{3.6a}$$

$$g_4^0=-\sqrt{3}g_2^0\ , \tag{3.6b}$$

$$g_6^0=0 \; . \tag{3.6c}$$

The result $g_6^0=0$ follows from the fact that, in the heavy-quark spin symmetry limit, the strong transition between antitriplet baryons is forbidden by parity conservation. To relate the subleading coefficients g_i^c's, we insert the operators O_1 and O_2 of Eqs. (1.15c) and (1.15d) into the relevant matrix elements. First of all, the subleading amplitude for $B_{\bar 3}\to B_{\bar 3}+\pi$ is given by

$$\Delta M_{\bar 3}[B_{\bar 3}(v,s)\to B_{\bar 3}(v,s')+\pi^a(q)]=\frac{1}{f_\pi}q^\mu\left\langle B_{\bar 3}(v,s')\left| iT\int d^4x\,[O_1(x)+O_2(x)]\mathcal{A}_\mu^a(0)\right| B_{\bar 3}(v,s)\right\rangle \; . \tag{3.7}$$

To determine the general Lorentz structure of $M_{\bar 3}$, we employ the following interpolating field for antitriplet baryons [25]:

$$B_{\bar 3}(v,s)=\bar u(v,s)\phi_v h_v \; , \tag{3.8}$$

where ϕ_v is a Lorentz scalar. One can easily show that O_1 does not contribute to $M_{\bar 3}$. Denoting the O_1's contribution as $\Delta M_{\bar 3}^1$, we then have

$$\Delta M_{\bar 3}^1[B_{\bar 3}(v,s)\to B_{\bar 3}(v,s')+\pi^a(q)]=\frac{1}{2f_\pi m_Q}q^\mu\left\langle B_{\bar 3}(v,s')\left| iT\int d^4x\,\bar h_v(iD)^2h_v(x)\mathcal{A}_\mu^a(0)\right| B_{\bar 3}(v,s)\right\rangle \; . \tag{3.9}$$

By Eq. (3.8) we may rewrite (3.9) as

$$\Delta M_{\bar 3}^1=\frac{1}{2f_\pi m_Q}q^\mu\bar u(v,s')u(v,s)\times\left\langle 0\left| iT\int d^4x\,\phi_v(iD)^2\mathcal{A}_\mu^a\phi_v^\dagger\right|0\right\rangle \; . \tag{3.10}$$

Since we cannot construct an axial vector out of v and q, we conclude that

$$\left\langle 0\left| iT\int d^4x\,\phi_v(iD)^2\mathcal{A}_\mu^a\phi_v^\dagger\right|0\right\rangle=0 \; , \tag{3.11}$$

and hence $\Delta M_{\bar 3}^1=0$. The situation is different in the case of O_2 insertion. The amplitude $\Delta M_{\bar 3}^2$ is given by

$$\Delta M_{\bar 3}^2=-\frac{g_s q^\mu}{4m_Q f\pi}\left\langle B_{\bar 3}(v,s')\left| iT\int d^4x\,\bar h_v\sigma^{\alpha\beta}G_{\alpha\beta}h_v(x)\mathcal{A}_\mu^a(0)\right| B_{\bar 3}(v,s)\right\rangle \; . \tag{3.12}$$

Applying Eq. (3.8) gives

$$\Delta M_{\bar 3}^2=-\frac{g_s q^\mu}{4m_Q f_\pi}\bar u(v,s')\frac{1+\not v}{2}\sigma^{\alpha\beta}\frac{1+\not v}{2}u(v,s)\left\langle 0\left| iT\int d^4x\,\phi_v G_{\alpha\beta}\phi_v^\dagger\mathcal{A}_\mu^a\right|0\right\rangle \; . \tag{3.13}$$

Since the diquark field ϕ_v is a Lorentz scalar, we may parametrize the matrix element of light constituents as

$$\left\langle 0\left| iT\int d^4x\,\phi_v G_{\alpha\beta}\phi_v^\dagger\mathcal{A}_\mu^a\right|0\right\rangle=-r\epsilon_{\alpha\beta\mu\nu}v^\nu \; , \tag{3.14}$$

where r is a constant independent of the heavy-quark mass. We have also neglected the flavor wave functions of incoming and outgoing baryons as they are irrelevant to our discussion. Substituting Eq. (3.14) into Eq. (3.13) yields

$$\Delta M_{\bar 3}^2=\frac{rg_s}{2m_Q f_\pi}\bar u(v,s')\not q\gamma_5 u(v,s) \; . \tag{3.15}$$

Comparing this result with Eqs. (3.1) and (3.5), we find

$$g_6^c=rg_s/\Lambda \; , \tag{3.16}$$

which is nonvanishing, in general. This shows that the decay $B_{\bar 3}\to B_{\bar 3}+\pi$, while forbidden in the infinitely heavy-quark limit, is allowed in the subleading order. As a similar conclusion has also been arrived at by Cho [18], we would like to compare our result with his in some details.

Cho constructed the following operator to describe the decay $B_{\bar 3}\to B_{\bar 3}+\pi$ to the order of $1/m_Q$:

$$O_{TTA}=\frac{i}{m_Q}\epsilon_{\mu\nu\sigma\lambda}\bar T^j(v)\sigma^{\mu\nu}D^\sigma T_i(v)(\mathcal{A}^\lambda)_j^i \; , \tag{3.17}$$

where

$$T(v)_i=\epsilon_{ijk}[B_{\bar 3}(v)]_{jk} \; , \tag{3.18}$$

apart from an overall normalization. In Eq. (3.18), $(B_{\bar 3})_{jk}$

is the jk matrix element in the baryon matrix $B_{\bar{3}}$ [11]. The operator O_{TTA} is not reparametrization invariant itself. Therefore, it should be part of one which is. We will now show that O_{TTA} is a reparametrization-invariant partner to the interaction term in (3.1) with the coupling constant g_6:

$$\mathcal{L}_6=g_6\mathrm{tr}(\bar{B}_{\bar{3}}\gamma_\mu\gamma_5\mathcal{A}^\mu B_{\bar{3}}) , \tag{3.19}$$

where it is understood that $B_{\bar{3}}$ and $\bar{B}_{\bar{3}}$ have velocity v. The reparametrization-invariant generalization of $\mathcal{L}_6$ is obtained through the substitution [20]

$$B_{\bar{3}}\to\left[1+\frac{i\not{D}}{2M_{\bar{3}}}\right]B_{\bar{3}} . \tag{3.20}$$

The result is

$$\tilde{\mathcal{L}}_6=g_6\mathrm{tr}\left[\bar{B}_{\bar{3}}\left[1-\frac{i\overleftarrow{\not{D}}}{2M_{\bar{3}}}\right]\gamma_\mu\gamma_5\mathcal{A}^\mu\left[1+\frac{i\not{D}}{2M_{\bar{3}}}\right]B_{\bar{3}}\right]$$
$$=\mathcal{L}_6+\mathcal{L}_6' , \tag{3.21}$$

where $\mathcal{L}_6$ is given by (3.19) and

$$\mathcal{L}_6'=-\frac{ig_6}{2M_{\bar{3}}}\mathrm{tr}[\bar{B}_{\bar{3}}(\overleftarrow{\not{D}}\gamma_\mu\mathcal{A}^\mu+\gamma_\mu\mathcal{A}^\mu\not{D})\gamma_5B_{\bar{3}}] . \tag{3.22}$$

The identity

$$\gamma_\mu\gamma_\nu=g_{\mu\nu}-i\sigma_{\mu\nu} \tag{3.23}$$

reduces $\mathcal{L}_6'$ to

$$\mathcal{L}_6'=-\frac{ig_6}{2M_{\bar{3}}}\mathrm{tr}[\bar{B}_{\bar{3}}(\overleftarrow{D}_\mu\mathcal{A}^\mu+\mathcal{A}^\mu D_\mu)\gamma_5B_{\bar{3}}$$
$$+i\bar{B}_{\bar{3}}\sigma_{\mu\nu}(\overleftarrow{D}^\nu\mathcal{A}^\mu-\mathcal{A}^\mu D^\nu)\gamma_5B_{\bar{3}}] . \tag{3.24}$$

The first term in (3.24) vanishes as a result of the two identities

$$(\bar{B}_{\bar{3}})_{ij}(\overleftarrow{\partial}_\mu+\partial_\mu)\gamma_5(B_{\bar{3}})_{kl}=\partial_\mu[(\bar{B}_{\bar{3}})_{ij}\gamma_5(B_{\bar{3}})_{kl}] , \tag{3.25a}$$

$$(\bar{B}_{\bar{3}})_{ij}\gamma_5(B_{\bar{3}})_{kl}=0 . \tag{3.25b}$$

The second term in (3.24) can be further transformed with the aid of the identity

$$\epsilon_{\mu\nu\sigma\lambda}\sigma^{\mu\nu}=2i\sigma_{\sigma\lambda}\gamma_5 . \tag{3.26}$$

Finally, we obtain

$$\mathcal{L}_6'=\frac{ig_6}{4M_{\bar{3}}}\epsilon_{\mu\nu\sigma\lambda}\mathrm{tr}[\bar{B}_{\bar{3}}\sigma^{\mu\nu}(\overleftarrow{D}^\sigma\mathcal{A}^\lambda-\mathcal{A}^\lambda D^\sigma)B_{\bar{3}}] , \tag{3.27}$$

which, aside from the factor g_6, is O_{TTA} in a somewhat different notation. We see that as a reparametrization-invariant partner to $\mathcal{L}_6$, the coupling constant for O_{TTA} is given by g_6 which is of order $O(1/m_Q)$. Since the operator O_{TTA} already contains a factor of $1/m_Q$, its contribution is smaller by one power of $1/m_Q$ relative to $\mathcal{L}_6$.

Our next task is to relate g_2^c to g_4^c. To do this, we evaluate the amplitudes of $B_6\to B_{\bar{3}}+\pi$ and $B_6^*\to B_{\bar{3}}+\pi$. With our previous notations, the subleading contribution is

$$\Delta M_{6\bar{3}}[B_6(v,s,\kappa)\to B_{\bar{3}}(v,s')+\pi^a(q)]=\frac{1}{f_\pi}q^\mu\left\langle B_{\bar{3}}(v,s')\left|iT\int d^4x[O_1(x)+O_2(x)]\mathcal{A}_\mu^a(0)\right|B_6(v,s,\kappa)\right\rangle , \tag{3.28}$$

where κ is used to specify the spin of sextet baryons: $\kappa=1$ corresponds to spin-$\frac{1}{2}$ baryons whereas $\kappa=2$ denotes spin-$\frac{3}{2}$ ones. To evaluate $\Delta M_{6\bar{3}}$, we employ the interpolating fields [17]

$$B_6(v,s,\kappa)=\bar{B}_\mu(v,s,\kappa)\phi_v^\mu h_v , \tag{3.29}$$

where ϕ_v^μ is an axial-vector field. The wave function $\bar{B}_\mu$ is given by

$$\bar{B}_\mu(v,s,\kappa=1)=\frac{1}{\sqrt{3}}\bar{u}(v,s)\gamma_5(\gamma_\mu+v_\mu) , \tag{3.30a}$$

$$\bar{B}_\mu(v,s,\kappa=2)=\bar{u}_\mu(v,s) , \tag{3.30b}$$

with $u_\mu(v,s)$ and $u(v,s)$ being the Rarita-Schwinger vector spinor and usual Dirac spinor, respectively. The contribution from the operator O_2 gives

$$\Delta M_{6\bar{3}}^2=-\frac{g_sq^\mu}{4m_Qf_\pi}\left\langle B_{\bar{3}}(v,s')\left|iT\int d^4x\;\bar{h}_v\sigma^{\alpha\beta}G_{\alpha\beta}h_v(x)\mathcal{A}_\mu^a(0)\right|B_6(v,s,\kappa)\right\rangle$$

$$=-\frac{g_sq^\mu}{4m_Qf_\pi}\bar{u}(v,s')\frac{1+\not{v}}{2}\sigma^{\alpha\beta}\frac{1+\not{v}}{2}B^\nu(v,s,\kappa)\left\langle 0\left|iT\int d^4x\;\phi_vG_{\alpha\beta}\mathcal{A}_\mu^a\phi_{v,\nu}^\dagger\right|0\right\rangle . \tag{3.31}$$

The matrix element for light constituents may be parametrized as

$$M_{\mu\nu\alpha\beta}=\langle 0|\phi_v G_{\alpha\beta} A^a_\mu \phi^\dagger_{v,\nu}|0\rangle$$
$$= r_1(g_{\mu\alpha}v_\beta - g_{\mu\beta}v_\alpha)v_\nu + r_2(g_{\nu\alpha}v_\beta - g_{\nu\beta}v_\alpha)v_\mu + ir_3(g_{\alpha\mu}g_{\beta\nu} - g_{\beta\mu}g_{\alpha\nu}) , \quad (3.32)$$

where r_1, r_2, and r_3 are constants independent of the heavy-quark mass, and the flavor wave functions are neglected for simplicity. This is the most general Lorentz structure for $M_{\mu\nu\alpha\beta}$ which is antisymmetric in α and β. With Eq. (3.32) we can immediately conclude that contributions from r_1 and r_2 are zero because of the identity

$$\frac{1+\not{v}}{2}\sigma^{\alpha\beta}\frac{1+\not{v}}{2}v_\alpha=0 . \quad (3.33)$$

The contribution due to r_3 is

$$\Delta M^2_{6\bar{3}} = -i\frac{r_3 g_s q^\mu}{2m_Q f_\pi}\bar{u}\sigma_{\mu\nu}B^\nu(v,s,\kappa) . \quad (3.34)$$

Using (3.30) for the wave function B^ν, we find

$$\Delta M^2_{6\bar{3}}(6^*\to\bar{3}) = -\frac{r_3 g_s}{2m_Q f_\pi}\bar{u}q^\mu u_\mu , \quad (3.35a)$$

$$\Delta M^2_{6\bar{3}}(6\to\bar{3}) = -\frac{r_3 g_s}{\sqrt{3}m_Q f_\pi}\bar{u}_{\bar{3}}\not{q}\gamma_5 u_6 , \quad (3.35b)$$

where we have used the notations 6^* and 6 to denote a spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ baryon in the sextet, respectively. Equation (3.35) implies the following corrections to the coupling constants g_2 and g_4:

$$g_2^c = \frac{g_s}{\Lambda}\left[r' - \frac{2}{\sqrt{3}}r_3\right] , \quad (3.36a)$$

$$g_4^c = \frac{g_s}{\Lambda}(-\sqrt{3}r' - r_3) , \quad (3.36b)$$

where we have added the contributions proportional to r' coming from the operator O_1 which preserves the spin symmetry.

Finally, we discuss strong transitions among sextet baryons. The contribution due to O_2 gives

$$\Delta M^2_{66} = -\frac{g_s q^\mu}{4m_Q f_\pi}\left\langle B_6(v,s',\kappa')\left| iT\int d^4x\, \bar{h}_v\sigma^{\alpha\beta}G_{\alpha\beta}h_v(x)\mathcal{A}^a_\mu(0)\right| B_6(v,s,\kappa)\right\rangle$$
$$= -\frac{g_s q^\mu}{4m_Q f_\pi}\bar{B}^\rho(v,s',\kappa')\frac{1+\not{v}}{2}\sigma^{\alpha\beta}\frac{1+\not{v}}{2}B^\nu(v,s,\kappa)\left\langle 0\left| iT\int d^4x\, \phi_{v,\rho}G_{\alpha\beta}\mathcal{A}^a_\mu\phi^\dagger_{v,\nu}\right|0\right\rangle . \quad (3.37)$$

The matrix elements of the light constituents can be parametrized as

$$M_{\alpha\beta\mu\lambda\kappa} = \left\langle 0\left| iT\int d^4x\, \phi_{v,\lambda}G_{\alpha\beta}\mathcal{A}^a_\mu\phi^\dagger_{v,\kappa}\right|0\right\rangle$$
$$= v_\mu\epsilon_{\lambda\alpha\beta\kappa}s + \epsilon_{\alpha\beta\mu\nu}v^\nu g_{\lambda\kappa}s_1 + \epsilon_{\alpha\beta\kappa\nu}v^\nu g_{\lambda\mu}s_2 + \epsilon_{\alpha\beta\lambda\nu}v^\nu g_{\mu\kappa}s_3 + (g_{\alpha\mu}\epsilon_{\beta\nu\lambda\kappa} - g_{\beta\mu}\epsilon_{\alpha\nu\lambda\kappa})v^\nu s_4$$
$$+ (g_{\alpha\lambda}\epsilon_{\beta\nu\mu\kappa} - g_{\beta\lambda}\epsilon_{\alpha\nu\mu\kappa})v^\nu s_5 + (g_{\alpha\kappa}\epsilon_{\beta\nu\mu\lambda} - g_{\beta\kappa}\epsilon_{\alpha\nu\mu\lambda})v^\nu s_6 , \quad (3.38)$$

where s and s_i are constants independent of the heavy-quark mass, and the flavor wave functions are again neglected. This is the most general Lorentz structure for $M_{\alpha\beta\mu\lambda\kappa}$ which conserves parity, and is antisymmetric with respect to the indices α and β. Let us write

$$\Delta M^2_{66} = -\frac{g_s q_\mu}{4m_Q f_\pi}\Delta \quad (3.39)$$

with

$$\Delta = \bar{B}_\lambda(v,s',\kappa')\sigma_{\alpha\beta}B_\rho(v,s,\kappa)M^{\alpha\beta\mu\lambda\rho} . \quad (3.40)$$

We further denote

$$\Delta = \Delta_s + \Delta_1 + \Delta_2 + \cdots + \Delta_6 \quad (3.41)$$

for contributions due to $s, s_1, s_2, \ldots, s_6$, respectively.

Using the properties of the wave functions

$$v^\mu B_\mu(v,s,\kappa)=0 , \quad (3.42a)$$

$$(\not{v}-1)B_\mu(v,s,\kappa)=0 , \quad (3.42b)$$

$$\gamma^\mu u_\mu(v,s)=0 , \quad (3.42c)$$

the identity (3.26), and

$$i\epsilon^{\mu\nu\lambda\kappa}\gamma_\kappa = (-\gamma^\mu\gamma^\nu\gamma^\lambda + g^{\mu\nu}\gamma^\lambda - g^{\mu\lambda}\gamma^\nu + g^{\nu\lambda}\gamma^\mu)\gamma_5 , \quad (3.43)$$

we obtain

$$\Delta_s = 0 , \quad (3.44)$$

$$\Delta_1(6^*\to 6^*) = 2s_1\bar{u}^\lambda\gamma^\mu\gamma_5 u_\lambda , \quad (3.45a)$$

$$\Delta_1(6^*\to 6) = \frac{4}{\sqrt{3}}s_1\bar{u}u^\mu , \quad (3.45b)$$

$$\Delta_1(6\to 6^*) = \frac{4}{\sqrt{3}}s_1\bar{u}^\mu u , \quad (3.45c)$$

$$\Delta_1(6\to 6)=\frac{2}{3}s_1\bar{u}\gamma^\mu\gamma_5 u\ , \tag{3.45d}$$

$$\Delta_2(6^*\to 6^*)=\Delta_2(6^*\to 6)=0\ , \tag{3.46a}$$

$$\Delta_2(6\to 6^*)=2\sqrt{3}s_2\bar{u}^\mu u\ , \tag{3.46b}$$

$$\Delta_2(6\to 6)=-2s_2\bar{u}\gamma^\mu\gamma_5 u\ , \tag{3.46c}$$

$$\Delta_3(6^*\to 6^*)=\Delta_3(6\to 6^*)=0\ , \tag{3.47a}$$

$$\Delta_3(6^*\to 6)=2\sqrt{3}s_3\bar{u}u^\mu\ , \tag{3.47b}$$

$$\Delta_3(6\to 6)=-2s_3\bar{u}\gamma^\mu\gamma_5 u\ , \tag{3.47c}$$

$$\Delta_4(6^*\to 6^*)=\Delta_4(6\to 6)=0\ , \tag{3.48a}$$

$$\Delta_4(6^*\to 6)=2\sqrt{3}s_4\bar{u}u^\mu\ , \tag{3.48b}$$

$$\Delta_4(6\to 6^*)=-2\sqrt{3}s_4\bar{u}^\mu u\ , \tag{3.48c}$$

$$\Delta_5(6^*\to 6^*)=2s_5\bar{u}^\lambda\gamma^\mu\gamma_5 u_\lambda\ , \tag{3.49a}$$

$$\Delta_5(6^*\to 6)=\frac{4}{\sqrt{3}}s_5\bar{u}u^\mu\ , \tag{3.49b}$$

$$\Delta_5(6\to 6^*)=-\frac{2}{\sqrt{3}}s_5\bar{u}^\mu u\ , \tag{3.49c}$$

$$\Delta_5(6\to 6)=\tfrac{8}{3}s_5\bar{u}\gamma^\mu\gamma_5 u\ , \tag{3.49d}$$

$$\Delta_6(6^*\to 6^*)=2s_6\bar{u}^\lambda\gamma^\mu\gamma_5 u_\lambda\ , \tag{3.50a}$$

$$\Delta_6(6^*\to 6)=-\frac{2}{\sqrt{3}}s_6\bar{u}u^\mu\ , \tag{3.50b}$$

$$\Delta_6(6\to 6^*)=\frac{4}{\sqrt{3}}s_6\bar{u}^\mu u\ , \tag{3.50c}$$

$$\Delta_6(6\to 6)=\tfrac{8}{3}s_6\bar{u}\gamma^\mu\gamma_5 u\ . \tag{3.50d}$$

Collecting all the terms, we find

$$\Delta M^2(6^*\to 6^*)=-\frac{g_s}{2m_Q f_\pi}(s_1+s_5+s_6)\bar{u}^\lambda \not{q}\gamma_5 u_\lambda\ , \tag{3.51a}$$

$$\Delta M^2(6^*\to 6)=-\frac{g_s}{2m_Q f_\pi}\left[\frac{2}{\sqrt{3}}s_1+\sqrt{3}s_2+\sqrt{3}s_4+\frac{2}{\sqrt{3}}s_5-\frac{1}{\sqrt{3}}s_6\right]\bar{u}q^\mu u_\mu\ , \tag{3.51b}$$

$$\Delta M^2(6\to 6^*)=-\frac{g_s}{2m_Q f_\pi}\left[\frac{2}{\sqrt{3}}s_1+\sqrt{3}s_2-\sqrt{3}s_4-\frac{1}{\sqrt{3}}s_5+\frac{2}{\sqrt{3}}s_6\right]\bar{u}_\mu q^\mu u\ , \tag{3.51c}$$

$$\Delta M^2(6\to 6)=-\frac{g_s}{2m_Q f_\pi}\left[\frac{1}{3}s_1-s_2-s_3+\frac{4}{3}s_5+\frac{4}{3}s_6\right]\times\bar{u}\not{q}\gamma_5 u\ , \tag{3.51d}$$

where we have dropped the subscripts 66 in the corrections to the matrix elements ΔM^2. When Eq. (3.51) is compared with (3.1), we find

$$g_1^c=-\frac{g_s}{\Lambda}\left[s'+\frac{1}{3}s_1-s_2-s_3+\frac{4}{3}s_5+\frac{4}{3}s_6\right]\ , \tag{3.52a}$$

$$g_3^c=-\frac{g_s}{\Lambda}\left[\frac{\sqrt{3}}{2}s'+\frac{2}{\sqrt{3}}s_1+\sqrt{3}s_2-\sqrt{3}s_4-\frac{1}{\sqrt{3}}s_5+\frac{2}{\sqrt{3}}s_6\right]\ , \tag{3.52b}$$

$$g_3^{c*}=-\frac{g_s}{\Lambda}\left[\frac{\sqrt{3}}{2}s'+\frac{2}{\sqrt{3}}s_1+\sqrt{3}s_3+\sqrt{3}s_4+\frac{2}{\sqrt{3}}s_5-\frac{1}{\sqrt{3}}s_6\right]\ , \tag{3.52c}$$

$$g_5^c=-\frac{g_s}{\Lambda}\left[-\frac{3}{2}s'+s_1+s_5+s_6\right]\ , \tag{3.52d}$$

where we have added a term proportional to s' due to the operator O_1 which preserves the spin symmetry. As in the heavy-meson case, we will assume that the phases for the field operators of the heavy baryons have been so chosen that all the coupling constants are real. Then, $g_3^c=g_3^{c*}$ gives

$$s_2-s_4+s_6=s_3+s_4+s_5\ . \tag{3.53}$$

We can rewrite (3.52) in terms of the combinations

$$s_2'=s_2+s_3\ , \tag{3.54a}$$

$$s_3'=s_5+s_6\ . \tag{3.54b}$$

Finally, we obtain

$$g_1^c=-\frac{g_s}{\Lambda}\left[s'+\frac{1}{3}s_1-s_2'+\frac{4}{3}s_3'\right]\ , \tag{3.55a}$$

$$g_3^c=g_3^{c*}=-\frac{g_s}{\Lambda}\left[\frac{\sqrt{3}}{2}s'+\frac{2}{\sqrt{3}}s_1+\frac{\sqrt{3}}{2}s_2'+\frac{1}{2\sqrt{3}}s_3'\right]\ , \tag{3.55b}$$

$$g_5^c=-\frac{g_s}{\Lambda}\left[-\frac{3}{2}s'+s_1+s_3'\right]\ . \tag{3.55c}$$

Equation (3.55) shows that the spin symmetry relations (3.6a)–(3.6c) are completely broken at order $O(1/m_Q)$ due to the presence of the parameters s_1, s_2', and s_3'.

We now turn to the $1/m_Q$ corrections to the radiative interactions $\mathcal{L}_{v,B}^{(2)}$, we shall treat the heavy- and light-quark electromagnetic currents separately. It is known that the magnetic couplings a_1'-a_6', induced by heavy-quark electromagnetic currents, can be rigorously determined by the heavy-quark effective theory [15,17]. As in the meson case, coefficients a_1'-a_6' arise entirely from the magnetic moment part of $J_\mu^{\rm em}$ shown in Eq. (2.26). For antitriplet baryons, we evaluate the magnetic form factor

$$G_{\bar{3}\mu}^Q=\left\langle B_{\bar{3}}(v,s')\left|\frac{ieQ'}{2m_Q}k^\nu\bar{h}_v\sigma_{\mu\nu}h_v\right|B_{\bar{3}}(v,s)\right\rangle\ . \tag{3.56}$$

Notice that we have taken $v=v'$ while maintaining a finite photon momentum k^{ν}. Applying Eq. (3.8) yields

$$G^{Q}_{\bar{3}\mu}=\frac{ieQ'}{2m_Q}k^{\nu}\bar{u}(v,s')\frac{1+\slashed{v}}{2}\sigma_{\mu\nu}\frac{1+\slashed{v}}{2}u(v,s)\langle 0|\phi_v\phi^{\dagger}_{v}|0\rangle \tag{3.57}$$

with [25]

$$\langle 0|\phi_v\phi^{\dagger}_{v'}|0\rangle=\zeta(v\cdot v') \tag{3.58}$$

and $\zeta(1)=1$. After simplifying the gamma matrices and comparing the result with that given by $\mathcal{L}^{(2)}_{v,B}$, we obtain

$$a'_6=-\frac{1}{4}\left[\frac{e}{2m_Q}\right] . \tag{3.59}$$

For magnetic transitions between sextet and antitriplet baryons, we evaluate the matrix element

$$G^{Q}_{6\bar{3}\mu}=\left\langle B_{\bar{3}}(v,s')\left|\frac{ieQ'}{2m_Q}k^{\nu}\bar{h}_v\sigma_{\mu\nu}h_v\right|B_6(v,s,\kappa)\right\rangle . \tag{3.60}$$

Application of Eqs. (3.8) and (3.29) yields

$$G^{Q}_{6\bar{3}\mu}=\frac{ieQ'}{2m_Q}k^{\nu}\bar{u}(v,s')\frac{1+\slashed{v}}{2}\sigma_{\mu\nu}\frac{1+\slashed{v}}{2}\times B^{\alpha}(v,s,\kappa)\langle 0|\phi_v\phi^{\dagger}_{v,\alpha}|0\rangle . \tag{3.61}$$

Since the diquark fields ϕ and ϕ_μ are scalar and axial-vector fields, respectively, the matrix elements $\langle 0|\phi_v\phi^{\dagger}_{v,\alpha}|0\rangle$ must vanish due to conservation of parity. This renders

$$a'_2=0,\quad a'_4=0 . \tag{3.62}$$

Finally, we evaluate the following matrix elements to determine the couplings a'_1, a'_3, and a'_5:

$$G^{Q}_{66\mu}=\left\langle B_6(v,s',\kappa')\left|\frac{ieQ'}{2m_Q}k^{\nu}\bar{h}_v\sigma_{\mu\nu}h_v\right|B_6(v,s,\kappa)\right\rangle . \tag{3.63}$$

Applying Eq. (3.29), we obtain

$$G^{Q}_{66\mu}=\frac{ieQ'}{2m_Q}k^{\nu}\bar{B}^{\alpha}(v,s',\kappa')\frac{1+\slashed{v}}{2}\sigma_{\mu\nu}\frac{1+\slashed{v}}{2}\times B^{\beta}(v,s,\kappa)\langle 0|\phi_{v,\alpha}\phi^{\dagger}_{v,\beta}|0\rangle , \tag{3.64}$$

with [25]

$$\langle 0|\phi_{v,\alpha}\phi^{\dagger}_{v',\beta}|0\rangle=-g_{\alpha\beta}\xi_1(v\cdot v')+v'_\alpha v_\beta\xi_2(v\cdot v')+\cdots , \tag{3.65}$$

where terms proportional to v_α or/and v'_β are not shown, as they do not contribute to (3.64). The normalization of ξ_1 is given by

$$\xi_1(v\cdot v'=1)=1 . \tag{3.66}$$

In the $v=v'$ limit, the function ξ_2 does not contribute to $G^{Q}_{66\mu}$ because

$$v^{\mu}B_{\mu}=0 . \tag{3.67}$$

One can explicitly work out $G^{Q}_{66\mu}$ by substituting Eq. (3.30) into Eq. (3.64). Comparing results obtained in this manner with those given by the relevant couplings in $\mathcal{L}^{(2)}_{v,B}$, we arrive at

$$a'_1=\frac{1}{6}\left[\frac{e}{2m_Q}\right],\quad a'_3=\frac{-1}{\sqrt{3}}\left[\frac{e}{2m_Q}\right],\quad a'_5=\frac{1}{2}\left[\frac{e}{2m_Q}\right] . \tag{3.68}$$

The results (3.59), (3.62), and (3.68) agree with the quark model calculations [17].

Next we tackle the light-quark electromagnetic currents, which give rise to the magnetic couplings a_1-a_6. In the heavy-quark mass expansion, we again write

$$a_i=a_i^0+a_i^c\frac{\Lambda}{2m_Q} , \tag{3.69}$$

where $i=1,2,\ldots,6$. The relations among a_i^0's were derived in Ref. [17] by evaluating the matrix elements for $B_{\bar{3}}\to B_{\bar{3}}+\gamma$, $B_6(B_g^*)\to B_{\bar{3}}+\gamma$, and $B_6(B_6^*)\to B_6(B_6^*)+\gamma$. We find

$$a_3^0=-\frac{\sqrt{3}}{2}a_1^0,\quad a_5^0=-\frac{3}{2}a_1^0,\quad a_4^0=\sqrt{3}a_2^0,\quad a_6^0=0 . \tag{3.70}$$

We shall follow the previous procedure to obtain the subleading contributions. For $B_{\bar{3}}\to B_{\bar{3}}+\gamma$, we have

$$G_{\bar{3}\mu}=\left\langle B_{\bar{3}}(v,s')\left|iT\int d^4x\,[O_1(x)+O_2(x)]j^{\rm em}_{\mu}(0)\right|B_{\bar{3}}(v,s)\right\rangle_m , \tag{3.71}$$

where the subscript m indicates magnetic contributions only. As in the previous section, we shall set the charge matrix $Q=\mathbb{1}$ and suppress all the flavor quantum numbers in the subsequent discussions.

Since the operator O_1 does not alter the Lorentz structure of light constituents' matrix element, the magnetic form factors in Eq. (3.71) receive no contributions from O_1 [17]. The contribution from O_2 is given by

$$\begin{aligned}G^{2}_{\bar{3}\mu}&=-\frac{g_s}{4m_Q}\left\langle B_{\bar{3}}(v,s')\left|iT\int d^4x\,\bar{h}_v\sigma_{\alpha\beta}G^{\alpha\beta}h_vj^{\rm em}_{\mu}(0)\right|B_{\bar{3}}(v,s)\right\rangle_m\\&=-\frac{g_s}{4m_Q}\bar{u}(v,s')\sigma^{\alpha\beta}u(v,s)(M^{\bar{3}}_{\alpha\beta\mu})_m ,\end{aligned} \tag{3.72}$$

where

$$M^{\bar{3}}_{\alpha\beta\mu}=\left\langle 0\left|iT\int d^4x\,\phi_v G_{\alpha\beta}j^{\rm em}_\mu\phi^\dagger_v\right|0\right\rangle . \quad (3.73)$$

Since $M^{\bar{3}}_{\alpha\beta\mu}$ must be antisymmetric with respect to the indices α,β and obeys constraints from both parity and electromagnetic current conservation, one concludes

$$M^{\bar{3}}_{\alpha\beta\mu}=-i\delta(g_{\mu\alpha}k_\beta-g_{\mu\beta}k_\alpha) , \quad (3.74)$$

where δ is a constant independent of the heavy-quark mass and k is the photon's momentum. In parametrizing $M^{\bar{3}}_{\alpha\beta\mu}$, we restrict ourselves to the structures linear in k since we are only interested in magnetic interactions. The same simplification will be assumed in the subsequent discussions. Substituting Eq. (3.74) into Eq. (3.72), we arrive at

$$G^2_{\bar{3}\mu}=\frac{i\delta g_s}{2m_Q}\bar{u}(v,s')\sigma_{\mu\nu}k^\nu u(v,s) . \quad (3.75)$$

Equation (3.75) corresponds to a change in the transition amplitude for $B_{\bar{3}}\to B_{\bar{3}}+\gamma$:

$$\Delta\Gamma(B_{\bar{3}}\to B_{\bar{3}}+\gamma)=\left\langle B_{\bar{3}}\gamma(k,\varepsilon)\left|iT\int d^4x\,O_2(x)[-j^{\rm em}_\mu(0)A^\mu(0)]\right|B_{\bar{3}}\right\rangle$$
$$=\frac{g_s\delta}{4m_Q}\bar{u}\sigma_{\mu\nu}F^{\mu\nu}u , \quad (3.76)$$

where

$$F_{\mu\nu}\equiv i(k_\mu\varepsilon_\nu-k_\nu\varepsilon_\mu) . \quad (3.77)$$

Comparing Eq. (3.76) with $\mathcal{L}^{(2)}_{v,B}$, we conclude that

$$a^c_6=g_s\delta/2\Lambda . \quad (3.78)$$

Since $a^0_6=0$, the amplitude for the magnetic transition $B_{\bar{3}}\to B_{\bar{3}}+\gamma$ is suppressed by $\Lambda/2m_Q$. For $B_6(B^*_6)\to B_{\bar{3}}+\gamma$, the operator O_2 gives a contribution to the electromagnetic form factor:

$$G^2_{6\bar{3}\mu}=-\frac{g_s}{4m_Q}\left\langle B_{\bar{3}}(v,s')\left|iT\int d^4x\,\bar{h}_v\sigma_{\alpha\beta}G^{\alpha\beta}h_v j^{\rm em}_\mu(0)\right|B_6(v,s,\kappa)\right\rangle$$
$$=-\frac{g_s}{4m_Q}\bar{u}(v,s')\sigma^{\alpha\beta}\bar{B}^\nu(v,s,\kappa)(M_{\alpha\beta\mu\nu})_m , \quad (3.79)$$

where

$$M_{\alpha\beta\mu\nu}=\left\langle 0\left|iT\int d^4x\,\phi_v G_{\alpha\beta}j^{\rm em}_\mu\phi^\dagger_{v,\nu}\right|0\right\rangle . \quad (3.80)$$

The most general structure of $M_{\alpha\beta\mu\nu}$ relevant to the magnetic transition is given by

$$M_{\alpha\beta\mu\nu}=(g_{\alpha\nu}\epsilon_{\mu\beta\lambda\kappa}-g_{\beta\nu}\epsilon_{\mu\alpha\lambda\kappa})k^\lambda v^\kappa t , \quad (3.81)$$

where t is a constant independent of the heavy-quark mass. Equations (3.79) and (3.81) give

$$\Delta\Gamma^2(6^*\to\bar{3})=\left\langle B_{\bar{3}}\gamma(k,\varepsilon)\left|iT\int d^4x\,O_2(x)[-j^{\rm em}_\mu(0)A^\mu(0)]\right|B^*_6\right\rangle$$
$$=\frac{g_s t}{4m_Q}\epsilon_{\mu\nu\lambda\kappa}\bar{u}\gamma^\nu F^{\lambda\kappa}u^\mu , \quad (3.82a)$$

$$\Delta\Gamma^2(6\to\bar{3})=-\frac{g_s t}{2m_Q}\frac{1}{\sqrt{3}}\bar{u}_{\bar{3}}\sigma_{\mu\nu}F^{\mu\nu}u_6 . \quad (3.82b)$$

Comparing Eq. (3.82) with $\mathcal{L}^{(2)}_{v,B}$, we find

$$a^c_2=\frac{g_s}{\Lambda}\left[t'-\frac{t}{\sqrt{3}}\right] , \quad (3.83a)$$

$$a^c_4=\frac{g_s}{\Lambda}\left[\sqrt{3}t'+\frac{t}{2}\right] , \quad (3.83b)$$

where the spin-symmetry-preserving contributions proportional to t' come from the operator O_1.

Finally we consider the couplings a^c_1, a^c_3, and a^c_5, which are relevant to magnetic transitions among sextet baryons.

The relevant matrix element is

$$G_{66\mu}=\left\langle B_6(v',s',\kappa')\left|iT\int d^4x\,[O_1(x)+O_2(x)]j^{\rm em}_\mu(0)\right|B_6(v,s,\kappa)\right\rangle_m \ . \tag{3.84}$$

Particularly, we shall focus on the contribution from the operator O_2, which is given by

$$\begin{aligned}G^2_{66\mu}&=-\frac{g_s}{4m_Q}\left\langle B_6(v,s',\kappa')\left|iT\int d^4x\,\bar h_v\sigma_{\alpha\beta}G^{\alpha\beta}h_v j^{\rm em}_\mu(0)\right|B_6(v,s,\kappa)\right\rangle_m\\&=-\frac{g_s}{4m_Q}\bar B^\nu(v,s',\kappa')\sigma^{\alpha\beta}B^\lambda(v,s,\kappa)(M_{\alpha\beta\nu\lambda\mu})_m \ ,\end{aligned}\tag{3.85}$$

where

$$M_{\alpha\beta\nu\lambda\mu}=\langle 0|\phi_{v,\nu}G_{\alpha\beta}j^{\rm em}_\mu\phi^\dagger_{v,\lambda}|0\rangle \ . \tag{3.86}$$

One can parametrize $M_{\alpha\beta\nu\lambda\mu}$ as follows:

$$\begin{aligned}M_{\alpha\beta\nu\lambda\mu}=&\ iw_1g_{\nu\lambda}(g_{\mu\alpha}k_\beta-g_{\mu\beta}k_\alpha)+iw_2[g_{\alpha\nu}(g_{\beta\mu}k_\lambda-g_{\lambda\mu}k_\beta)-g_{\beta\nu}(g_{\alpha\mu}k_\lambda-g_{\lambda\mu}k_\alpha)]\\&+iw_3[g_{\alpha\lambda}(g_{\beta\mu}k_\nu-g_{\mu\nu}k_\beta)-g_{\beta\lambda}(g_{\alpha\mu}k_\nu-g_{\mu\nu}k_\alpha)] \ ,\end{aligned}\tag{3.87}$$

where the constants w_1, w_2, and w_3 are independent of the heavy-quark mass. Equations (3.85) and (3.87) give a contribution to the photon transition amplitude:

$$\begin{aligned}\Delta\Gamma^2[B_6(v,s,\kappa)\to B_6(v',s',\kappa')+\gamma(k,\varepsilon)]&=-\left\langle B_6(v',s',\kappa')\gamma(k,\varepsilon)\left|iT\int d^4x\,O_2(x)j^{\rm em}_\mu(0)A^\mu(0)\right|B_6(v,s,\kappa)\right\rangle\\&=-\frac{g_s}{4m_Q}\bar B_\nu(v',s',\kappa')(w_1\sigma^{\alpha\beta}F_{\alpha\beta}g^{\nu\lambda}+2w_2\sigma^{\nu\beta}F_\beta{}^\lambda+2w_3\sigma^{\lambda\beta}F_\beta{}^\nu)B_\lambda(v,s,\kappa) \ .\end{aligned}\tag{3.88}$$

Let us denote

$$\Delta\Gamma^2=-\frac{g_s}{4m_Q}(\delta_1+\delta_2+\delta_3) \ , \tag{3.89}$$

where

$$\delta_1=w_1\bar B^\nu(v',s',\kappa')\sigma^{\alpha\beta}F_{\alpha\beta}B_\nu(v,s,\kappa) \ , \tag{3.90a}$$

$$\delta_2=2w_2\bar B_\nu(v',s',\kappa')\sigma^{\nu\beta}F_{\beta\lambda}B^\lambda(v,s,\kappa) \ , \tag{3.90b}$$

$$\delta_3=2w_3\bar B^\nu(v',s',\kappa')\sigma^{\lambda\beta}F_{\beta\nu}B_\lambda(v,s,\kappa) \ . \tag{3.90c}$$

Making uses of the identities for the Dirac matrices stated earlier and

$$\{\sigma_{\mu\nu},\gamma_\lambda\}=2\epsilon_{\mu\nu\lambda\kappa}\gamma^\kappa\gamma_5 \ , \tag{3.91a}$$

$$[\sigma_{\mu\nu},\gamma_\lambda]=-2i(g_{\mu\lambda}\gamma_\nu-g_{\nu\lambda}\gamma_\mu) \ , \tag{3.91b}$$

$$\gamma^\lambda\sigma_{\mu\nu}\gamma_\lambda=0 \ , \tag{3.91c}$$

$$\bar u^\lambda(v)\sigma_{\mu\nu}F^{\mu\nu}u_\lambda(v)=2i\bar u^\mu(v)F_{\mu\nu}u^\nu(v) \ , \tag{3.91d}$$

we obtain

$$\delta_1(6^*\to6^*)=w_1\bar u^\nu\sigma^{\alpha\beta}F_{\alpha\beta}u_\nu \ , \tag{3.92a}$$

$$\delta_1(6^*\to6)=-\frac{2}{\sqrt3}w_1\epsilon_{\mu\nu\lambda\kappa}\bar u\gamma^\nu F^{\lambda\kappa}u^\mu \ , \tag{3.92b}$$

$$\delta_1(6\to6^*)=-\frac{2}{\sqrt3}w_1\epsilon^{\mu\nu\lambda\kappa}\bar u_\mu\gamma_\nu F_{\lambda\kappa}u \ , \tag{3.92c}$$

$$\delta_1(6\to6)=\tfrac13 w_1\bar u\sigma^{\alpha\beta}F_{\alpha\beta}u \ , \tag{3.92d}$$

$$\delta_2(6^*\to6^*)=-w_2\bar u^\nu\sigma^{\alpha\beta}F_{\alpha\beta}u_\nu \ , \tag{3.93a}$$

$$\delta_2(6^*\to6)=\frac{2}{\sqrt3}w_2\epsilon_{\mu\nu\lambda\kappa}\bar u\gamma^\nu F^{\lambda\kappa}u^\mu \ , \tag{3.93b}$$

$$\delta_2(6\to6^*)=-\frac{1}{\sqrt3}w_2\epsilon^{\mu\nu\lambda\kappa}\bar u_\mu\gamma_\nu F_{\lambda\kappa}u \ , \tag{3.93c}$$

$$\delta_2(6\to6)=-\tfrac43 w_2\bar u\sigma^{\alpha\beta}F_{\alpha\beta}u \ , \tag{3.93d}$$

$$\delta_3(6^*\to6^*)=-w_3\bar u^\nu\sigma^{\alpha\beta}F_{\alpha\beta}u_\nu \ , \tag{3.94a}$$

$$\delta_3(6^*\to6)=-\frac{1}{\sqrt3}w_3\epsilon_{\mu\nu\lambda\kappa}\bar u\gamma^\nu F^{\lambda\kappa}u^\mu \ , \tag{3.94b}$$

$$\delta_3(6\to6^*)=\frac{2}{\sqrt3}w_3\epsilon^{\mu\nu\lambda\kappa}\bar u_\mu\gamma_\nu F_{\lambda\kappa}u \ , \tag{3.94c}$$

$$\delta_3(6\to6)=-\tfrac43 w_3\bar u\sigma^{\alpha\beta}F_{\alpha\beta}u \ . \tag{3.94d}$$

Collecting all the terms yields

$$\Delta\Gamma^2(6^*\to6^*)=-\frac{g_s}{4m_Q}(w_1-w_2-w_3)\bar u^\nu\sigma^{\alpha\beta}F_{\alpha\beta}u_\nu \ , \tag{3.95a}$$

$$\Delta\Gamma^2(6^*\to6)=-\frac{g_s}{4m_Q}\left[-\frac{2}{\sqrt3}w_1+\frac{2}{\sqrt3}w_2-\frac{1}{\sqrt3}w_3\right]\times\epsilon_{\mu\nu\lambda\kappa}\bar u\gamma^\nu F^{\lambda\kappa}u^\mu \ , \tag{3.95b}$$

$$\Delta\Gamma^2(6\to 6^*) = -\frac{g_s}{4m_Q}\left[-\frac{2}{\sqrt{3}}w_1 - \frac{1}{\sqrt{3}}w_2 + \frac{2}{\sqrt{3}}w_3\right] \times \epsilon_{\mu\nu\lambda\kappa}\bar{u}^\mu\gamma^\nu F^{\lambda\kappa}u \ , \tag{3.95c}$$

$$\Delta\Gamma^2(6\to 6) = -\frac{g_s}{4m_Q}\left[\frac{1}{3}w_1 - \frac{4}{3}w_2 - \frac{4}{3}w_3\right]\bar{u}\sigma^{\alpha\beta}F_{\alpha\beta}u \ . \tag{3.95d}$$

When Eq. (3.95) is compared with $\mathcal{L}^{(2)}_{v,B}$, we have

$$a_1^c = -\frac{g_s}{2\Lambda}\left[w + \frac{1}{3}(w_1 - 4w_2 - 4w_3)\right] , \tag{3.96a}$$

$$a_3^c = -\frac{g_s}{2\Lambda}\left[-\frac{\sqrt{3}}{2}w + \frac{1}{\sqrt{3}}(-2w_1 - w_2 + 2w_3)\right] , \tag{3.96b}$$

$$a_3^{c*} = -\frac{g_s}{2\Lambda}\left[-\frac{\sqrt{3}}{2}w + \frac{1}{\sqrt{3}}(-2w_1 + 2w_2 - w_3)\right] , \tag{3.96c}$$

$$a_5^c = -\frac{g_s}{2\Lambda}\left[-\frac{3}{2}w + (w_1 - w_2 - w_3)\right] , \tag{3.96d}$$

where we have included the contributions proportional to w from the spin-symmetry-preserving operator O_1. Again, we will assume that the phases of the heavy baryon fields have been so chosen that the coupling constants are real. Then $a_3^c = a_3^{c*}$ gives

$$w_2 = w_3 \ , \tag{3.97}$$

and

$$a_1^c = -\frac{g_s}{2\Lambda}\left[w + \frac{1}{3}w_1 - \frac{8}{3}w_2\right] , \tag{3.98a}$$

$$a_3^c = a_3^{c*} = -\frac{g_s}{2\Lambda}\left[-\frac{\sqrt{3}}{2}w - \frac{2}{\sqrt{3}}w_1 + \frac{1}{\sqrt{3}}w_2\right] , \tag{3.98b}$$

$$a_5^c = -\frac{g_s}{2\Lambda}\left[-\frac{3}{2}w + w_1 - 2w_2\right] . \tag{3.98c}$$

It is clear from Eqs. (3.78), (3.83), and (3.98) that to order $O(1/m_Q)$ all the spin symmetry relations among the coupling constants a_i for radiative transitions are broken.

Equations (3.16), (3.36), and (3.55) are the main results in this section for the strong-coupling constants $g_1,\dots,g_6$, and (3.78), (3.83), and (3.98) for the electromagnetic transition couplings $a_1,\dots,a_6$.

IV. DISCUSSION

In this work we have carried out a systematic theoretical study of the order $1/m_Q$ effects to the heavy-meson and heavy-baryon chiral Lagrangian for strong and electromagnetic interactions. There are two distinct corrections at this order. The first is a kinematic correction required by reparametrization invariance. In practice, this effect for simple processes such as decays can be largely taken into account by using the full momentum P of a heavy particle and the corresponding polarization vector or Dirac spinor instead of parametrizing it by $P = M_H v + k$ and dropping the residual momentum k. The second effect is a dynamic correction induced by the order $1/m_Q$ terms in the QCD Lagrangian which break the flavor-spin symmetry of the heavy quarks. As in our earlier publications [11,17], we focus our attention on the interactions involving only the first order in the momentum of a Goldstone boson or a photon. To this order, not surprisingly, the heavy-quark symmetry-breaking interactions of QCD do not produce any new types of interactions for the heavy hadrons with the Goldstone bosons or photons. Instead, their effects make order $1/m_Q$ corrections to the coupling constants in the heavy-hadron chiral Lagrangian.

To order $1/m_Q$, QCD contains one operator O_1 [see Eq. (1.15c)] which breaks only the heavy flavor symmetry, and a second operator O_2 [see Eq. (1.15d)] which breaks the flavor-spin symmetry of heavy quarks. For a given heavy flavor, the effects due to O_1 can be absorbed by the coupling constants which satisfy the heavy-quark spin symmetry. Effectively, the operator O_1 does not introduce any new unknowns. On the other hand, the operator O_2 introduces new unknowns of order $1/m_Q$ which break the spin symmetry relations among the coupling constants. In the heavy meson sector, there is one new unknown each in the strong interactions and electromagnetic interactions, respectively. In the heavy-baryon sector, there are five new unknowns of order $1/m_Q$ to describe the six strong interaction coupling constants $g_1,\dots,g_6$. There are four new unknowns to describe the six radiative transition coupling constants $a_1,\dots,a_6$. In particular, the reactions $B_{\bar{3}}\to B_{\bar{3}}+\pi$ and $B_{\bar{3}}\to B_{\bar{3}}+\gamma$ which are forbidden in the infinitely heavy-quark limit have coupling strength of order $1/m_Q$. In reducing the number of unknowns, we have appealed to the charge conjugation symmetry for certain processes and the reality of the coupling constants associated with them. This can always be accomplished by a proper choice of the phases for the field operators of the heavy hadrons. For example, in our quark model calculations in Refs. [11,17], all the coupling constants are indeed real.

These new unknowns depend on the QCD's long-distance dynamics of light quarks and gluons. In principle, they are calculable numerically in lattice QCD. At a more phenomenological level, the quark model has no simple predictions for them either, unlike the coupling constants in the infinitely heavy-quark limit. Nevertheless, it is important to consider the sizes of these corrections as they affect the strong and electromagnetic interaction physics of the heavy hadrons, especially the charmed mesons and baryons. For this purpose, it is perhaps useful to calculate those corrections in the quark model and the MIT bag model with some specific phenomenological wave functions for the heavy hadrons.

We will illustrate the last point with a problem of practical interest. For the heavy-meson chiral Lagrangian $\mathcal{L}^{(1)}_{v,PP^*}$, the HQS relation $f=2g$ is modified by $1/m_Q$ corrections. The splitting of f and $2g$, $\delta\equiv 2g-f$, will contribute to SU(3)-violating corrections to the heavy-meson hyperfine splitting. Such corrections are characterized by the parameter [26]

$$\Delta_P \equiv (M_{P_s^*}-M_{P_s})-(M_{P_d^*}-M_{P_d}) .$$

In the charmed meson case, experimental data give [27,28]

$$\Delta_D = 0.9\pm 1.9 \text{ MeV} . \tag{4.1}$$

On the theoretical side, a one-loop calculation based on the heavy-meson chiral Lagrangian given by Eq. (2.1) has recently been performed [23]. In this work Δ_D is obtained by evaluating the self-energy diagrams of D and D^*, where each diagram contains one insertion of the residual mass term $\Delta M^2 P(v)P^\dagger(v)$ appearing in $\mathcal{L}^{(1)}_{v,PP^*}$. By retaining $m_s \ln m_s$ and $m_s^{3/2}$ corrections, it was found that, to the order of $1/m_c$ [23],

$$\Delta_D = +95 \text{ MeV} . \tag{4.2}$$

In comparison with the experimental value given by Eq. (4.1), this result is larger by almost two orders of magnitude. In addition to obtaining Eq. (4.2), the authors of Ref. [23] also estimated other contributions to Δ_D, which are of the same order or one order higher. As they pointed out, one had to include an additional contribution which is quadratic in Goldstone-boson masses. Such corrections arise from the chiral loops mentioned above plus the counterterms listed below (see also Ref. [1]):

$$\begin{aligned} \mathcal{O}^P &= \alpha_1 M_P P(\xi\mathcal{M}^\dagger\xi+\xi^\dagger\mathcal{M}\xi^\dagger)P^\dagger , \\ \mathcal{O}^{P^*} &= \alpha_2 M_{P^*} P^*_\mu(\xi\mathcal{M}^\dagger\xi+\xi^\dagger\mathcal{M}\xi^\dagger)P^{*\mu\dagger} , \end{aligned} \tag{4.3}$$

where

$$\mathcal{M} = \begin{bmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{bmatrix} . \tag{4.4}$$

The counterterms $\mathcal{O}^P$ and $\mathcal{O}^{P^*}$ contribute to Δ_D because there will be a deviation from the spin-symmetry relation $\alpha_1=-\alpha_2$ at the order of $1/m_c$. By a naive dimensional argument, Randall and Sather estimated this contribution to be about 20 MeV in magnitude. Furthermore, they noted that the $O(1/m_c^2)$ contributions to Δ_D could also be as large as 10 MeV in magnitude. These large individual corrections raise interesting questions on the reliability of chiral perturbation theory and/or $1/m_Q$ expansion. Given all these large contributions, the authors suggested that there may be accidental cancellations among various terms so that the resultant Δ_D is small. Although this might indeed be the case, we would like to point out that the calculation done in Ref. [23] is not complete at order $1/m_c$. Specifically, it missed those effects induced by the splitting of f and $2g$ in the self-energy diagram depicted in Fig. 1. At the order of $1/m_c$, δ is finite, and D and D^* will acquire different mass shifts which contributes to the hyperfine splittings of heavy mesons. Since the difference between D and D^* mass shifts is SU(3) flavor dependent, it therefore contributes to the parameter Δ_D. Denoting this extra contribution as Δ'_D, we find

$$\begin{aligned} \Delta'_D = {} & \left[\frac{1}{32}\frac{m_\pi^3}{\pi f_\pi^2}-\frac{1}{48}\frac{m_K^3}{\pi f_K^2}-\frac{1}{96}\frac{m_\eta^3}{\pi f_\eta^2}\right]x \\ & -\frac{3}{32}\frac{m_K^2}{\pi^2 f_K^2}\ln\left[\frac{m_K^2}{\Lambda_\chi^2}\right](M_{D_s}-M_{D_d})x , \end{aligned} \tag{4.5}$$

where $x=f\delta$, and we have suppressed contributions proportional to $(M_{D_s}-M_{D_d})^2$ or higher since they are found to be negligible. The first term in Eq. (4.5) can be easily obtained from Eq. (4.3) of our forthcoming paper [1] (see the footnote there for details). The second term emerges as one takes into account the splitting of the strange and nonstrange heavy-meson masses. Now for numerical analyses, we shall take $\Lambda_\chi \approx 1$ GeV [29], and the fitted value of [27]

$$(M_{D_s}-M_{D_d}) = 99.5\pm 0.6 \text{ MeV} .$$

In view of large positive value in Eq. (4.2), one would favor a negative Δ'_D to counteract it. This requires x to be positive or, in other words, $4g^2>f^2$. At this point, we do not plan to perform any model calculation of x. Nevertheless, a crude estimation of x can be obtained by dimensional arguments. If we assume that the heavy-quark expansion at the hadronic level is governed by the parameter $\Lambda_\chi/2m_Q$ [22], we would roughly expect that

$$\left|\frac{\delta}{f}\right| = O\left[\frac{\Lambda_\chi}{2m_c}\right] . \tag{4.6}$$

Taking $m_c=1.8$ GeV and $f^2=2$ [30], we obtain

$$\frac{|x|}{f^2} = \left|\frac{\delta}{f}\right| \approx 0.3 . \tag{4.7}$$

If one simply assumes $x=0.3f^2=0.6$, then $\Delta'_D=-62$ MeV. If x is indeed positive, this would provide a substantial cancellation to the result of Eq. (4.2). The cancellation would be further enhanced if the contribution from

FIG. 1. An additional chiral-loop diagram which contributes to the hyperfine splitting of charmed mesons. The dashed line denotes a light meson which can be strange or nonstrange. The solid line represents a charmed meson which can be strange or nonstrange, spin 0 or spin 1. In this case, the propagator of the heavy meson contains no insertion of the mass-difference term, $\Delta M^2 P(v)P^\dagger(v)$.

Eq. (4.3) is also negative. Unfortunately, there still exists no data to support this claim. In fact, there is also no experimental evidence for a positive x. Certainly, a negative x would make the situation even more troublesome. At any rate, we want to emphasize that one should include the effect of Fig. 1 when computing the parameter Δ_D. Whether or not this effect is adequate to resolve the puzzle posed by Eq. (4.2) is not yet clear until one has more experimental data, and a better theoretical understanding [31].

From the above example, we have seen that the splitting of f and $2g$ at the order of $1/m_Q$ could give important effects. Similar situations may also occur in other parts of the heavy-hadron chiral Lagrangian. Since there are insufficient data to fix the parameters of the theory, it would be helpful to combine the results of this paper with a certain model estimation of parameters, so that quantitative predictions of the $1/m_Q$ correction can be made. The results in this paper provide constraints that individual contributions must satisfy in any model calculation. To cite just one example, consider the corrections to the heavy-meson strong interaction coupling constants f and g. From Eq. (2.21) we see that the corrections due to the operator O_1 satisfy the spin symmetry relation, and those due to the operator O_2 also satisfy a relation, albeit a different one. We shall leave such model studies to a future investigation.

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