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Decay of a heavy chargino into a neutralino and a quark-antiquark pair, in the minimal supersymmetric standard model

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The differential and total decay rates for the process $\tilde{\chi}_j^+ \to \tilde{\chi}_i^0 q\bar{q}$ are calculated numerically for a range of supersymmetric parameters in the minimal supersymmetric extension of the standard model. Explicit analytical results are also given for the total decay rates in certain parameter limits.

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I. INTRODUCTION

Recent developments [1] in supersymmetric grand unification make it plausible that the minimal supersymmetric extension of the standard model (MSSM) can provide a consistent picture of what lies beyond the standard model and also not contradict any current experimental results. For a compelling exposition of this point of view see Kane [2]. With our confidence in minimal supersymmetry thus reinforced, we present in this paper some results of a calculation of the decays chargino $\to$ neutralino quark antiquark in the MSSM.

Our original starting point in this investigation was a desire to "supersymmetrize" the early work of Mikaelian and Grose [3] on the three-body decays in the standard model: $W \to q\bar{q}\gamma$, $W \to q\bar{q}g$, $Z \to q\bar{q}\gamma$, and $Z \to q\bar{q}g$. In view of the present experimental limits on the gluino and the squarks, the "tilded" versions of processes 2 and 4 probably do not occur, but "tilded" versions of 1 and 3 are of possible experimental interest.

In this paper we shall calculate the "tilded" version of 1, namely, $\tilde{\chi}^+ \to \tilde{\chi}^0 q\bar{q}$ for the heavier chargino and the lightest and second lightest neutralino, and for a wide range of parameters in the MSSM. Of course in general the two-body decays of the chargino such as $\tilde{\chi}^+ \to \tilde{\chi}^0 W^+, \tilde{\chi}^+ \to \tilde{\chi}^0 H^+, \tilde{\chi}^- \to \tilde{q}\bar{q}$, etc., are dominant. These have been studied some time ago by Gunion and Haber [4]. But, as they noted in [4], for certain ranges of parameters these two-body decays are forbidden, and the three-body decays then dominate.

A remarkable feature of the decay $W \to q\bar{q}\gamma$, noted in [3], is the existence of a radiation amplitude zero in the differential decay rate. In the supersymmetric limit in which $\tilde{\chi}^0$ becomes exactly a massless photino, the decay $\tilde{\chi}^+ \to \tilde{\chi}^0 q\bar{q}$ also has a radiation zero. It is amusing to study at what rate this feature goes away as supersymmetry breaking is switched on.

In Sec. II we establish notation, review the chargino and neutralino mass eigenstates, and introduce some experimental and theoretical discussion of constraints on the supersymmetry- (SUSY-) breaking parameters. In Sec. III we give explicit formulas for the matrix elements for our decay process. In Sec. IV we look at the radiation amplitude zeros (RAZ's). In Sec. V we review the structure of the Dalitz plots of the differential decay rates and also give some analytical results for the total decay rates in certain limits. In Sec. VI we state our conclusions.

II. THE NEUTRALINO AND CHARGINO MASS EIGENSTATES

In this section we survey the mass eigenstates for the neutralino and chargino sectors as a function of the supersymmetry-breaking parameters in the minimal supersymmetric standard model (MSSM) [5]. This is a necessary preliminary to our detailed discussion of the partial differential and total decay rates in the subsequent sections. The material in this section has been investigated previously by many authors [5–8]. Our discussion here is tailored to our particular needs in exploring the decay processes $\tilde{\chi}_j^+ \to \tilde{\chi}_i^0 q\bar{q}$ (chargino $\to$ neutralino quark antiquark) and is included to make the paper self-contained and also to establish continuity of notation with previous authors to whom we later shall refer.

First we discuss the neutralino mass matrix. There are four neutral gauge and Higgs fermions which mix in the MSSM [5]: namely, $\tilde{\gamma}$, $\tilde{Z}$, $\tilde{H}_1^0$, and $\tilde{H}_2^0$ or the photino, the Z-ino, and the two neutral Higgsinos. These differ in general from the four neutralino mass eigenstates which are denoted by $\tilde{\chi}_i^0, i = 1, ..., 4$, in order of increasing mass. Their masses and decomposition in terms of the neutral gaugino-Higgsino states can be determined by diagonalizing the 4 × 4 neutral gaugino-Higgsino mixing matrix [6,7]. This matrix depends on four parameters M, M', μ, and $\tan\beta = v_2/v_1$. Here M and M' are the Majorana mass terms for the W-ino and b-ino, while μ is a supersymmetric Higgs boson mass term from the superpotential. The vacuum expectation values of the two Higgs doublets are v_1 and v_2 corresponding to the U(1) hypercharges $-\frac{1}{2}$ and $+\frac{1}{2}$, respectively. These four parameters are crucial in determining the masses and couplings of the neutralino mass eigenstates. Three of the parameters (M, μ, $\tan\beta$) also uniquely specify the mass matrix for the charginos as we shall discuss shortly. We shall assume, as has become the custom, the grand unification constraint [9]

$$M' = \tfrac{5}{3}\tan^2\theta_W M, \tag{1}$$

where θ_W is the Weinberg angle and

$$\sin^2\theta_W = 0.233 \tag{2}$$

and conventionally take M and M' to be positive. Then μ may have either sign, which possibility we shall allow for.

Since the SUSY breaking parameters are at present unknown we have performed the numerical calculations of decay rates for a wide range of parameters. Specifically we have taken $1 < \tan\beta < 10$, $0 < |\mu|, M < 450$ GeV and 150 GeV $< \tilde{m} < 500$ GeV.

Under specific assumptions as to the model, etc., the experimental failure to detect any SUSY decays can be used to constrain the SUSY parameters. For example, M and M' cannot be too small since they are related (per the grand unification hypothesis) to the gluino mass via the equation

$$M_{\text{gluino}} = 16\sin^2\theta_W M. \tag{3}$$

Experimental limits set $M_{\text{gluino}} > 106$ GeV [10] or > 179 GeV [11]. The ALEPH searches [12] for the decay $Z^0 \to \tilde{\chi}^0\tilde{\chi}^{0\prime}$ have also been used to constrain certain regions of $M-\mu$ parameter space. It seems likely that the lightest supersymmetric particle (LSP) mass exceeds 20 GeV [13], thus also constraining the parameter space. It must be emphasized that all such "experimental" constraints are somewhat or highly model and assumption dependent.

An intriguing but somewhat loose phenomenological contraint arises from consideration of dark matter in astrophysics [2,14]. It seems that the LSP is a very promising (but not unique) candidate for the dark matter or at least a considerable portion of it. The LSP would then have to be a b-ino corresponding to large $\mu \gg M$ [14].

There also exist theoretical arguments constraining SUSY parameters. String-inspired arguments based on $N = 1$ supergravity [15] strongly favor $\tan\beta > 1$; in fact $\tan\beta = O(M_{\text{top}}/M_{\text{bottom}})$ is suggested. In the context of the MSSM and with careful use of renormalization group and grand unification Roberts and Ross [1] have come up with specific values for the SUSY parameters for a given top quark mass. They present two solutions, the first (case Z) based on $M_{\text{top}} = 160$ GeV which gives $\tan\beta = 21$, $\mu = 190$ GeV, $M = 95$ GeV, $m_{\text{gluino}} = 354$ GeV, and $\tilde{m} = 364$ GeV while their second solution (case X) with $M_{\text{top}} = 100$ GeV gives $\tan\beta = 5$, $\mu = -120$ GeV, $M = 150$ GeV, $M_{\text{gluino}} = 559$ GeV, and $\tilde{m} = 504$ GeV. We have later calculated the decay rates for these two specific solutions since they arise from a fairly convincing and well-established theoretical basis.

For the neutralino mass matrix we shall work in the basis [6]

$$\psi_j^0 = (-i\lambda_\gamma, -i\lambda_Z, \psi_H^a, \psi_H^b), \qquad j = 1,2,3,4, \tag{4}$$

where the rotated Higgsino states are

$$\psi_H^a = \psi_{H_1}^1\cos\beta - \psi_{H_2}^2\sin\beta \tag{5}$$

and

$$\psi_H^b = \psi_{H_1}^1\sin\beta + \psi_{H_2}^2\cos\beta. \tag{6}$$

Here λ_γ, λ_Z , $\psi_{H_1}^1$, and $\psi_{H_2}^2$ are the two-component spinors of the photino, Z-ino, and the neutral Higgsinos $\tilde{H}_1^0$ and $\tilde{H}_2^0$. We shall follow the notation of Bartl *et al.* [6]. The mass term has the form

$$\mathcal{L}^0 = -\frac{M_Z}{2}\psi_i^0 Y_{ij}\psi_j^0 + \text{H.c.}, \qquad i,j = 1,2,3,4 \tag{7}$$

and the mass matrix is

$$Y = \begin{bmatrix} \lambda(\sin^2\theta_W + \alpha\cos^2\theta_W) & (1-\alpha)\lambda\sin\theta_W\cos\theta_W & 0 & 0 \\ (1-\alpha)\lambda\sin\theta_W\cos\theta_W & \lambda(\cos^2\theta_W + \alpha\sin^2\theta_W) & 1 & 0 \\ 0 & 1 & \nu\sin 2\beta & -\nu\cos 2\beta \\ 0 & 0 & -\nu\cos 2\beta & -\nu\sin 2\beta \end{bmatrix}. \tag{8}$$

Here M_Z is the Z^0 boson mass, θ_W the Weinberg angle, and we introduce the dimensionless quantities

$$\lambda = \frac{M}{M_Z},\ \nu = \frac{\mu}{M_Z},\ \alpha = \frac{M'}{M}. \tag{9}$$

Y is a real, symmetric matrix which can be diagonalized by a 4×4 matrix N. The mass eigenstates are given by

$$\tilde{\chi}_i^0 = N_{ij}\psi_j^0, \tag{10}$$

where

$$N_{ij}Y_{jk}N_{lk} = \omega_l\delta_{il} \tag{11}$$

with $\omega_j = \frac{m_j}{M_Z}$ and m_j is the mass of the state $\tilde{\chi}_j^0$. Until our study was completed, no analytical formula was available for diagonalizing Y. Therefore we used a program to find N and ω_j numerically. Recently El Kheishen *et al.* [16] have found a general analytical result with which our numbers agree when checked. An extensive and very interesting discussion of the various options as one ranges through the SUSY parameter space is given in [6]. A number of special cases are discussed there which have simple analytical solutions.

In the chargino sector the mass term for the mixing of the W-ino and charged Higgsino has the form

$$\mathcal{L}_M = -\frac{M_Z}{2}[\psi^+\psi^-]\begin{bmatrix} 0 & X^T \\ X & 0 \end{bmatrix}\begin{bmatrix} \psi^+ \\ \psi_- \end{bmatrix} + \text{H.c.}, \quad (12)$$

where

$$X = \begin{bmatrix} \lambda & \sqrt{2}\cos\theta_W\cos\beta \\ \sqrt{2}\cos\theta_W\sin\beta & \nu \end{bmatrix} \quad (13)$$

and

$$\psi_j^+ = \left[-i\lambda^+, \psi_{H_2}^1\right], \quad \psi_j^- = \left[-i\lambda^-, \psi_{H_1}^2\right], \quad j = 1,2. \quad (14)$$

Here $\psi^{+,-}$, $\psi_{H_2}^1$, and $\psi_{H_1}^2$ denote the two-component spinor fields of the W-ino and the charged Higgsinos. The mass eigenstates are defined by

$$\tilde{\chi}_j^+ = V_{ij}\psi_j^+, \quad \tilde{\chi}_j^- = U_{ij}\psi_j^-, \quad i = 1,2, \quad (15)$$

where U and V are unitary matrices which satisfy

$$U^* X V^{-1} = M_D, \quad (16)$$

where M_D is the diagonal chargino mass matrix. Explicit formulas for these matrices and the mass eigenvalues are given in [5].

Although in general the neutralino mass eigenstates are a complicated mixture of the original states (the photino, Z-ino, and Higgsinos), for extensive areas of parameter space the lowest mass eigenstate is often largely one or other pure state. For example, if λ is very small and ν not too large, the lowest mass state is almost pure photino. We have prepared plots of the value of the mass of the lowest eigenstate for a range of ν and λ and a given $\tan\beta$. For completeness, we have also given mass plots for the second lightest neutralino. These are all displayed in Figs. 1–4. Similar plots are to be found in Griest and Haber [6], Bartl *et al.* [6], and Olive and Srednicki [17].

As discussed in detail in [6,17], one has a number of distinct regions of almost pure neutralino states which we indicate below.

(1) $\nu = 0$ *or very small.* This is the light Higgsino scenario. The lowest mass eigenstate is almost pure ψ_H^b.

(2) $\lambda = 0$ *or very small.* This is the light photino scenario. The lowest mass eigenstate is almost pure photino. This scenario is virtually ruled out by experiment [13] but is interesting theoretically as we shall demonstrate later.

(3) λ, ν *satisfy* $\alpha\lambda\nu = \sin 2\beta(\sin^2\theta_W + \alpha\cos^2\theta_W)$. In this case, a remarkable zero appears [6], while neither λ nor ν need be particularly small for this to happen. If $\lambda < 1$, the state is predominantly Z-ino; if $\lambda > 1$ the state is predominantly a Higgsino mixture. This is the light Higgsino-Z-ino scenario. This may also be ruled out by experiment [13].

(4) *Large* λ. This is the symmetric Higgsino scenario. The lowest mass eigenstate is $\tilde{H}^0_{(1,2)} = (\tilde{H}_{H_1} + \tilde{H}_{H_2})/\sqrt{2}$.

(5) *Large* ν. This is the b-ino scenario. The lowest mass eigenstate is a b-ino, $\tilde{B} = \tilde{\gamma}\cos\theta_W - \tilde{Z}\sin\theta_W$.

For moderate values of λ and ν, the lowest mass eigenstate is a mixture. This is the mixed scenario, which we shall also consider subsequently.

III. THE DECAYS $\tilde{\chi}_i^+ \to \tilde{\chi}_k^0 q\bar{q}$

In this paper we treat the decays[1] of the chargino into a neutralino and a quark-antiquark pair. Such decays have been considered before [18] but not in numerical detail. We shall examine the partial differential decay rates as a function of the quark energies for a range of SUSY breaking parameters. We find very different Dalitz plot structures for the different SUSY parameter scenarios. In particular, the light photino scenario (for small λ) provides an amusing generalization of the radiation amplitude zero (RAZ) discovered by Grose and Mikaelian [3] in the Dalitz plot for the decay $W^- \to \gamma q\bar{q}'$. Such a SUSY zero is expected because of a theorem of Brown

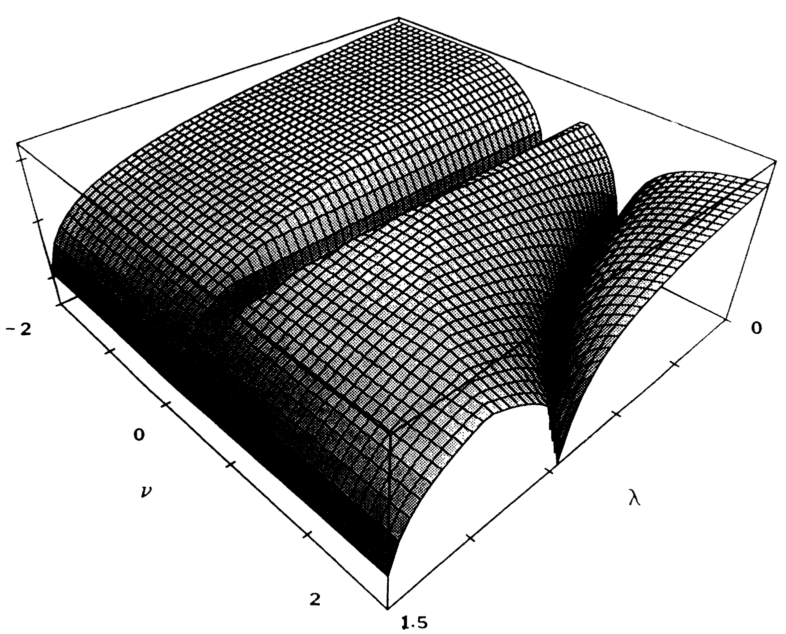

FIG. 1. Surface plot of the logarithm of the mass of the lightest neutralino versus the parameters ν and λ for $\tan\beta = 1$. Limits on ν and λ are $-2 \le \nu \le 2$, $0 \le \lambda \le 1.5$.

[1] All decay rates have been calculated for one quark family and one color. Decay rates for two families (top excluded) and three colors are obtained by multiplying by 6.

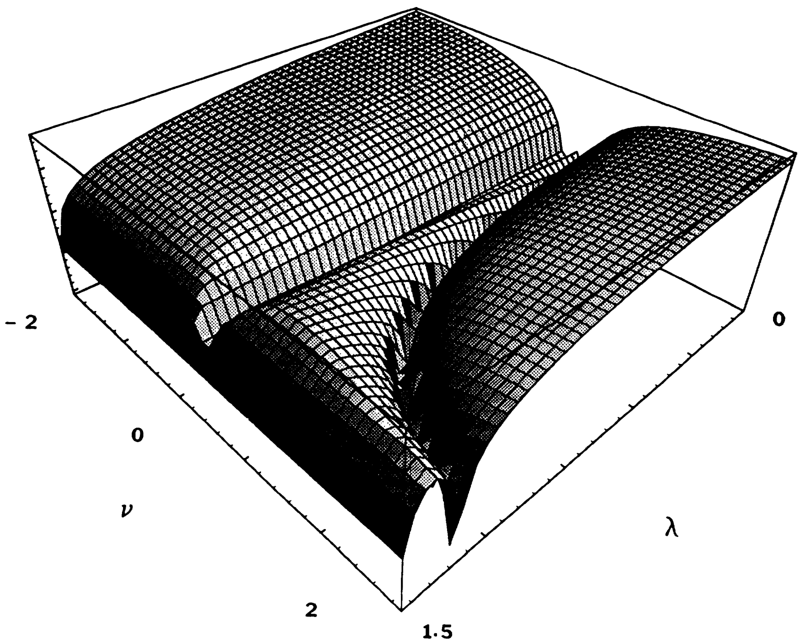

FIG. 2. Surface plot of the logarithm of the mass of the lightest neutralino versus the parameters ν and λ for $\tan\beta = 5$. Limits on ν and λ are $-2 \leq \nu \leq 2$, $0 \leq \lambda \leq 1.5$.

and Kowalski [19] which generalizes the Brodsky-Brown-Kowalski theorem [20] to supersymmetry. We shall discuss this and other SUSY scenarios in some detail later.

We consider

$$\tilde{\chi}_i^+(p) \to \tilde{\chi}_k^0(p_1) + q(p_2) + \bar{q}'(p_3), \tag{17}$$

where $i = 1, 2$ and $k = 1, 2, 3, 4$ are the indices of the charginos and neutralinos, respectively, in order of increasing mass, and the p's are the respective four-momenta. We shall take the quark masses to be light compared to the chargino and neutralino masses, so the top quark is excluded as a final state in our analysis here. We introduce the variables

$$s = (p_2 + p_3)^2 = (p - p_1)^2, \tag{18}$$

$$t = (p_1 + p_3)^2 = (p - p_2)^2, \tag{19}$$

$$u = (p_1 + p_2)^2 = (p - p_3)^2, \tag{20}$$

where, in the massless quark approximation,

$$s + t + u = m_i^2 + m_k^2 \tag{21}$$

and m_i and m_k denote the masses of the chargino and neutralino, respectively. We shall evaluate the partial differential decay rates for this process in terms of the variables u and t. The kinematic constraints defining the phase space then take the attractively simple form

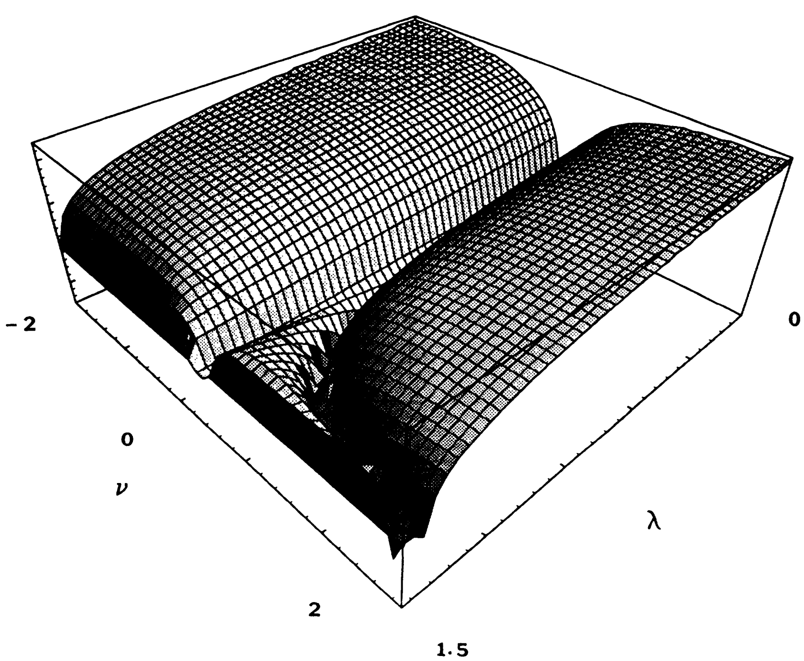

FIG. 3. Surface plot of the logarithm of the mass of the lightest neutralino versus the parameters ν and λ for $\tan\beta = 20$. Limits on ν and λ are $-2 \leq \nu \leq 2$, $0 \leq \lambda \leq 1.5$.

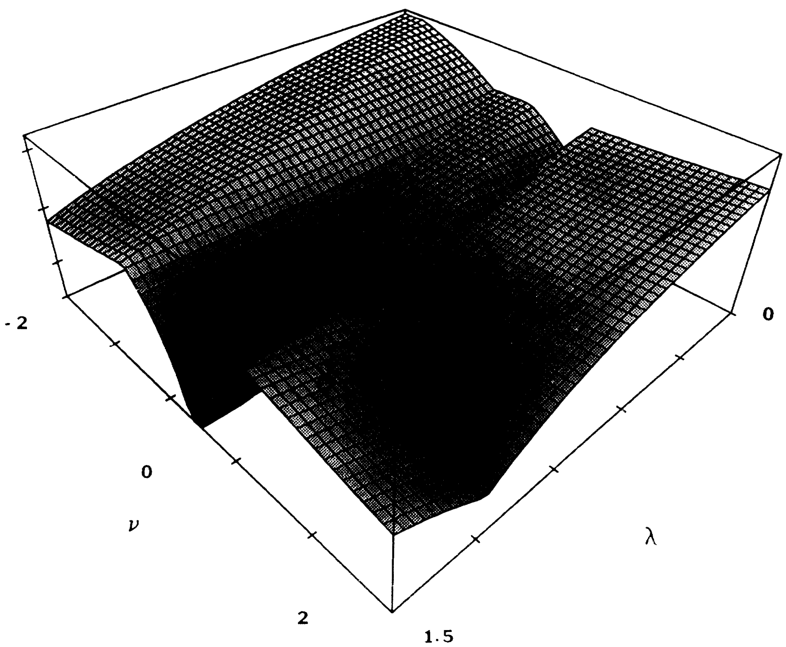

FIG. 4. Surface plot of the logarithm of the mass of the second lightest neutralino versus the parameters ν and λ for $\tan\beta = 1$. Limits on ν and λ are $-2 \le \nu \le 2$, $0 \le \lambda \le 1.5$.

$$ut \ge m_i^2 m_k^2 \tag{22}$$

and

$$u + t \le m_i^2 + m_k^2. \tag{23}$$

The upper and lower limits for the variables s, t, and u are

$$0 \le s \le (m_i - m_k)^2, \tag{24}$$

and

$$\tfrac{1}{2}\left[m_i^2 + m_k^2 - s - \lambda^{1/2}(m_i^2, m_k^2, s)\right]$$
$$\le u, t \le \tfrac{1}{2}\left[m_i^2 + m_k^2 - s + \lambda^{1/2}(m_i^2, m_k^2, s)\right], \tag{25}$$

where

$$\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc. \tag{26}$$

The Feynman diagrams contributing to this decay process to lowest order are depicted in Fig. 5. In the case of the light quarks considered in this paper, the Feynman diagrams with an intermediate charged Higgs bosons are negligible because the Higgs-quark coupling is proportional to the quark masses. If we had considered the top quark in the final state, then the diagram with an intermediate charged Higgs boson would be important. The expression below for the partial differential decay rate does not include the contribution of the charged Higgs diagram. The differential decay rate in the case of the intermediate W and squarks with zero width has already been worked out by Bartl *et al.* [18]. Using the standard Feynmann rules for the MSSM [5,6], we have extended their result to include a finite width for the W and squarks as is necessary to avoid an unphysical divergence in the decay rate. Our result is

$$\frac{d^2\Gamma}{du\,dt} = \frac{\alpha_{\rm em}^2}{32\pi \sin^4\theta_W m_i^3}\left[W_s + W_t + W_u + W_{tu} + W_{st} + W_{su}\right], \tag{27}$$

where

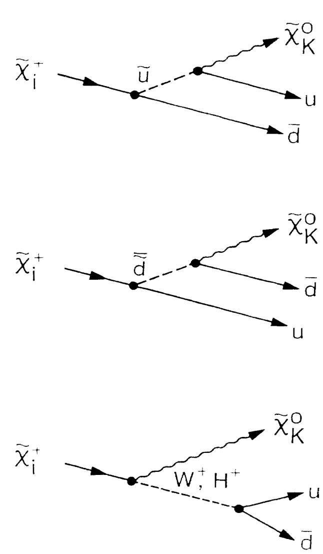

FIG. 5. Feynman diagrams for the decay process.

$$W_s = \frac{A_s(m_i^2 - t)(t - m_k^2) + B_s(m_i^2 - u)(u - m_k^2) + C_s 2\eta_i\eta_k m_i m_k s}{(s - M_W^2)^2 + \Gamma_W^2 M_W^2}, \tag{28}$$

$$W_t = \frac{A_t(m_i^2 - t)(t - m_k^2)}{(t - \tilde{m}^2)^2 + \tilde{\Gamma}^2\tilde{m}^2}, \tag{29}$$

$$W_u = \frac{A_u(m_i^2 - u)(u - m_k^2)}{(u - \tilde{m}^2)^2 + \tilde{\Gamma}^2\tilde{m}^2}, \tag{30}$$

$$W_{tu} = \frac{2A_{tu}\eta_i\eta_k m_i m_k s\left[(t - \tilde{m}^2)(u - \tilde{m}^2) + \tilde{\Gamma}^2\tilde{m}^2\right]}{\left[(t - \tilde{m}^2)^2 + \tilde{\Gamma}^2\tilde{m}^2\right]\left[(u - \tilde{m}^2)^2 + \tilde{\Gamma}^2\tilde{m}^2\right]}, \tag{31}$$

$$W_{st} = \frac{2\left[A_{st}(m_i^2 - t)(t - m_k^2) + B_{st}\eta_i\eta_k m_i m_k s\right]\left[(s - M_W^2)(t - \tilde{m}^2) + \Gamma_W\tilde{\Gamma}M_W\tilde{m}\right]}{\left[(s - M_W^2)^2 + \Gamma_W^2 M_W^2)\right]\left[(t - \tilde{m}^2)^2 + \tilde{\Gamma}^2\tilde{m}^2)\right]}, \tag{32}$$

and

$$W_{su} = \frac{2\left[A_{su}(m_i^2 - u)(u - m_k^2) + B_{su}\eta_i\eta_k m_i m_k s\right]\left[(s - M_W^2)(u - \tilde{m}^2) + \Gamma_W\tilde{\Gamma}M_W\tilde{m}\right]}{\left[(s - M_W^2)^2 + \Gamma_W^2 M_W^2)\right]\left[(u - \tilde{m}^2)^2 + \tilde{\Gamma}^2\tilde{m}^2)\right]}. \tag{33}$$

In the above M_W and Γ_W are the W mass and width, $\tilde{m}$ and $\tilde{\Gamma}$ the squark mass and width (left and right masses assumed degenerate here), and η_i and η_k are the sign factors for the mass eigenvalues m_i and m_k for the chargino and neutralino, respectively.

The coefficients A, B, and C are related to the matrices N, U, and V which diagonalize the neutralino and chargino mass matrices [see Eqs. (10) and (15)]. Explicitly these are

$$A_s = 6(O_{ki}^L)^2, \tag{34}$$
$$B_s = 6(O_{ki}^R)^2, \tag{35}$$
$$C_s = -6O_{ki}^L O_{ki}^R, \tag{36}$$
$$A_t = 3(f_{uk}^L V_{il})^2, \tag{37}$$
$$A_u = 3(f_{dk}^L U_{il})^2, \tag{38}$$
$$A_{tu} = 3f_{uk}^L f_{dk}^L V_{il}U_{il}, \tag{39}$$
$$A_{st} = -3\sqrt{2}f_{uk}^L V_{il}O_{ki}^L, \tag{40}$$
$$A_{su} = 3\sqrt{2}f_{dk}^L U_{il}O_{ki}^R, \tag{41}$$
$$B_{st} = 3\sqrt{2}f_{uk}^L V_{il}O_{ki}^R \tag{42}$$

and

$$B_{su} = -3\sqrt{2}f_{dk}^L U_{il}O_{ki}^L, \tag{43}$$

where

$$f_{uk}^L = -\sqrt{2}\left[\frac{1}{\cos\theta_W}\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right)N_{k2} + \frac{2}{3}\sin\theta_W N_{k1}\right], \tag{44}$$

$$f_{dk}^L = -\sqrt{2}\left[\frac{1}{\cos\theta_W}\left(-\frac{1}{2} - \frac{1}{3}\sin^2\theta_W\right)N_{k2} - \frac{1}{3}\sin\theta_W N_{k1}\right], \tag{45}$$

$$O_{ki}^L = -\frac{1}{\sqrt{2}}\left[\cos\beta N_{k4} - \sin\beta N_{k3}\right]V_{i2} + \left[\sin\theta_W N_{k1} + \cos\theta_W N_{k2}\right]V_{i1}, \tag{46}$$

and

$$O_{ki}^R = \frac{1}{\sqrt{2}}\left[\sin\beta N_{k4} + \cos\beta N_{k3}\right]U_{i2} + \left[\sin\theta_W N_{k1} + \cos\theta_W N_{k2}\right]U_{i1}. \tag{47}$$

In the above, N_{kj}, V_{im}, and U_{in} are elements of the mass diagonalizing matrices introduced and defined in Sec. II.

This completes the formulas we shall need for our subsequent discussions. In Sec. IV we shall give a detailed discussion of the radiation amplitude zero which occurs in the supersymmetric limit.

IV. RADIATION AMPLITUDE ZEROS

A radiation amplitude zero (RAZ) may occur if a massless gauge or gaugino particle is radiated [19,20]. On setting the squark and W widths to zero, the decay rate (27) involving a massless neutralino may be expressed as

$$\frac{d^2\Gamma}{dudt} = \frac{\alpha_{\text{em}}^2}{32\pi\sin^4\theta_W m_i^3}\left\{t(m_i^2 - t)\left[\frac{\sqrt{A_s}}{s - M_W^2} - \frac{\sqrt{A_t}}{t - \tilde{m}^2}\right]^2 + u(m_i^2 - u)\left[\frac{\sqrt{B_s}}{s - M_W^2} + \frac{\sqrt{A_u}}{u - \tilde{m}^2}\right]^2\right\}. \tag{48}$$

A massless neutralino is obtained in the supersymmetric limit, and we now consider this case. In general the masses of the charginos m_i are given by

$$m_i^2 = \frac{M_Z^2}{2}\left\{\lambda^2+\nu^2+\frac{2M_W^2}{M_Z^2}\pm\left[(\lambda^2-\nu^2)^2+\frac{4M_W^4}{M_Z^4}\cos^2 2\beta+\frac{4M_W^2}{M_Z^2}(\lambda^2+\nu^2+2\lambda\nu\sin 2\beta)\right]^{\frac{1}{2}}\right\}. \tag{49}$$

The supersymmetric limit requires that $\lambda,\nu \to 0$ and $\tan\beta = 1$. If we set $\lambda=\nu=0$, Eq. (49) yields

$$m_i^2 = 2M_W^2\sin^2\beta \quad\text{or}\quad m_i^2 = 2M_W^2\cos^2\beta. \tag{50}$$

Then if $\tan\beta=1$, $m_i = \pm M_W$. We shall show explicitly that in this supersymmetric limit, the partial differential decay rate for $\tilde{\chi}^+ \to \tilde{\chi}^0 q\bar{q}'$ exhibits an RAZ. Since we have assumed massless quarks, we must also set the squark masses to zero, $\tilde{m}=0$. The RAZ may also exist for nonzero quark masses, provided that the squark and corresponding quark masses are equal.

In the supersymmetric limit the lightest neutralino is a pure photino (see Sec. II). In this case the matrix N which diagonalizes the mass matrix becomes [see Eqs. (10) and (11)]

$$N_{kl} = \delta_{l1}. \tag{51}$$

The expressions in Eqs. (44)–(47) then become

$$O_{ki}^L = \sin\theta_W V_{i1}, \tag{52}$$

$$O_{ki}^R = \sin\theta_W U_{i1}, \tag{53}$$

$$f_{uk}^L = -\sqrt{2}\sin\theta_W Q_u \tag{54}$$

and

$$f_{dk}^L = -\sqrt{2}\sin\theta_W Q_d. \tag{55}$$

Substituting these expressions into those for A and B, the differential decay rate, Eq. (48) becomes

$$\frac{d^2\Gamma}{dudt} = \frac{3\alpha_{\rm em}^2}{16\pi\sin^2\theta_W m_i^2}\left\{V_{il}^{\,2}t(m_i^2-t)\left[\frac{1}{s-M_W^2}+\frac{Q_u}{t}\right]^2 + U_{il}^2u(m_i^2-u)\times\left[\frac{1}{s-M_W^2}-\frac{Q_d}{u}\right]^2\right\}, \tag{56}$$

where the widths of the squark and W have been set to zero. The RAZ factors are contained in the square brackets of the above equation. Both square brackets must vanish simultaneously for a zero. In the supersymmetric limit our decay becomes $\tilde{W} \to \tilde{\gamma}q\bar{q}$ with the charges of the W-ino and the quark given by $Q_{\tilde{W}}$ and Q_q, respectively. We must also set $\tan\beta=1$ to get the RAZ. Our result is that one obtains an RAZ for

$$u = \left[-1+\frac{Q_{\tilde{W}}}{Q_q}\right]t. \tag{57}$$

We note that Eqs. (21) and (50) have been used in the derivation. It follows that for $\tilde{W}^+ \to \tilde{\gamma}u\bar{d}$ the RAZ occurs at $t=2u$ and for $\tilde{W}^- \to \tilde{\gamma}d\bar{u}$ it occurs for $u=2t$.

In passing we note that in the limit $\nu\to 0$, a massless Higgsino boson is also obtained in addition to the photino. There is however no RAZ in the decay to the Higgsino boson.

For completeness we present the decay rate for $\tilde{W}\to\tilde{\gamma}q\bar{q}$. Following Grose and Mikaelian, to facilitate comparison with the decay rate of the W boson to a quark, antiquark, and photon we require that the quark and antiquark each carry a minimum energy $E_{\rm min} = \epsilon m_i$, where ϵ is a small positive number. The decay rate with this energy cutoff is

$$\begin{aligned}\Gamma(\epsilon) &= \int_{\epsilon M_W^2}^{M_W^2(1-2\epsilon)} dt \int_{\epsilon M_W^2}^{M_W^2(1-\epsilon)-t} du \frac{d^2\Gamma}{dudt} \\ &= \frac{3\alpha_{\rm em}^2 M_W}{16\pi\sin^2\theta_W} \\ &\quad\times\left\{Q_{\tilde{W}}^2\left[(1-3\epsilon)\left(\frac{1-3\epsilon+3\epsilon^2}{3(1-\epsilon)}+\frac{3}{2}\epsilon\right)-\epsilon(\epsilon+1)\ln\frac{1-\epsilon}{2\epsilon}\right]\right. \\ &\quad + Q_{\tilde{W}}(Q_u-Q_d)\left(2\epsilon\ln\frac{1-\epsilon}{2\epsilon}-\frac{3}{4}(1-3\epsilon)(1+\epsilon)\right) \\ &\quad \left. +\frac{Q_u^2+Q_d^2}{2}\left(-\frac{3}{2}(1-3\epsilon)(1-\epsilon)+(1-2\epsilon)\ln\frac{1-2\epsilon}{\epsilon}\right)\right\}.\end{aligned} \tag{58}$$

As a check note that $\Gamma(\epsilon)$ in Eq. (58) vanishes exactly in the limit $\epsilon\to\frac{1}{3}$ when the phase space vanishes. In the limit $\epsilon\to 0$ the total decay rate diverges. This is an indication of the presence of infrared and/or collinear singularities arising from the masslessness of the neutralino in the final state. Similar behavior in the $\epsilon\to 0$ limit is seen in the expression in Ref. [3] for the total decay rate for $W\to\gamma q\bar{q}$.

V. DALITZ PLOTS AND DECAY RATES

The partial differential decay rate for $\tilde{\chi}^+ \to \tilde{\chi}^0 q\bar{q}$ exhibits interesting structure when plotted as a function of u and t (Dalitz plot). The variables u and t are simply related to the antiquark and quark energies in the rest frame of the chargino as can be seen from Eqs. (19) and (20). In fact,

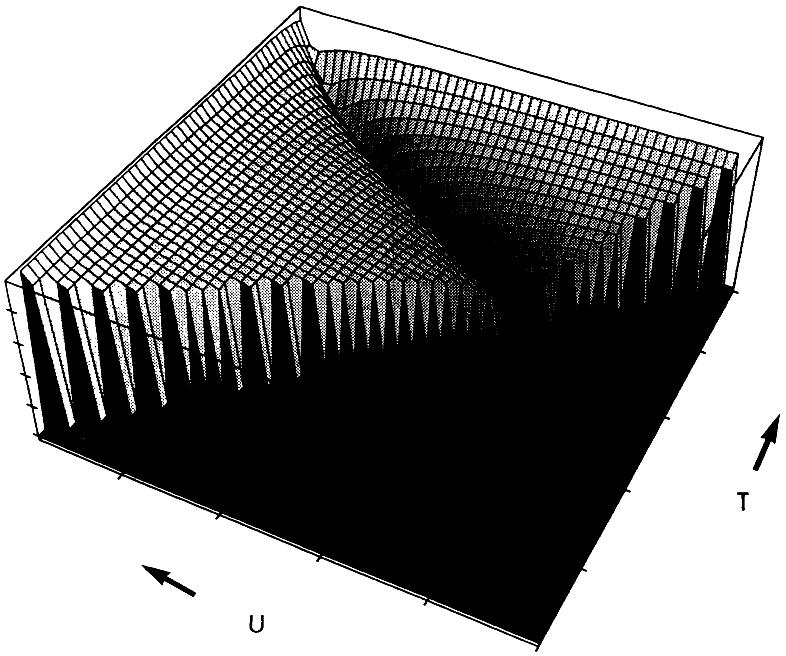

FIG. 6. The logarithm of the partial differential decay rate for $\tilde{\chi}^+ \to \tilde{\chi}^0 q\bar{q}$ as a function of t and u. The heavier chargino (mass 81.7 GeV) goes to the lightest neutralino (mass 0.545 GeV). $\tan\beta = 1$, $\nu = \lambda = 0.01$, average squark mass = 1 GeV, $\Gamma_W = 1$ GeV, and $\tilde{\Gamma} = 1$ GeV. The total decay rate is 0.246 $\times 10^{-2}$ GeV. The amplitude zero is very pronounced in this plot.

$$E_q = \frac{m_i^2 - t}{2m_i}, \; E_{\bar{q}} = \frac{m_i^2 - u}{2m_i}, \tag{59}$$

so u and t are natural variables to use.

In Sec. IV we pointed out the existence of an RAZ along the line $t = 2u$ in the Dalitz plot for $\tilde{\chi}^+ \to \tilde{\chi}^0 q\bar{q}$ when exact supersymmetry obtains. This is the supersymmetric analog of the RAZ found for $W^+ \to \gamma u\bar{d}$ in [3]. When supersymmetry is broken this zero will disappear, but how rapidly does this occur as the supersymmetry-breaking parameters are increased? For mild supersymmetry breaking a clearly detectable depletion zone remains along the RAZ line in the Dalitz plot. We have illustrated this in Figs. 6, 7, and 8 which give Dalitz plots for various supersymmetry-breaking parameters. That such a feature remains is remarkable for two reasons: firstly, in the presence of supersymmetry breaking the lightest neutralino is no longer a pure photino but a linear combination of the Z-ino, photino, and Higgsino, and secondly, the lightest neutralino has a mass very different from zero. For sufficiently strong supersymmetry breaking (see Fig. 9) this RAZ "valley" disappears as expected.

Some other interesting features in the Dalitz plot are

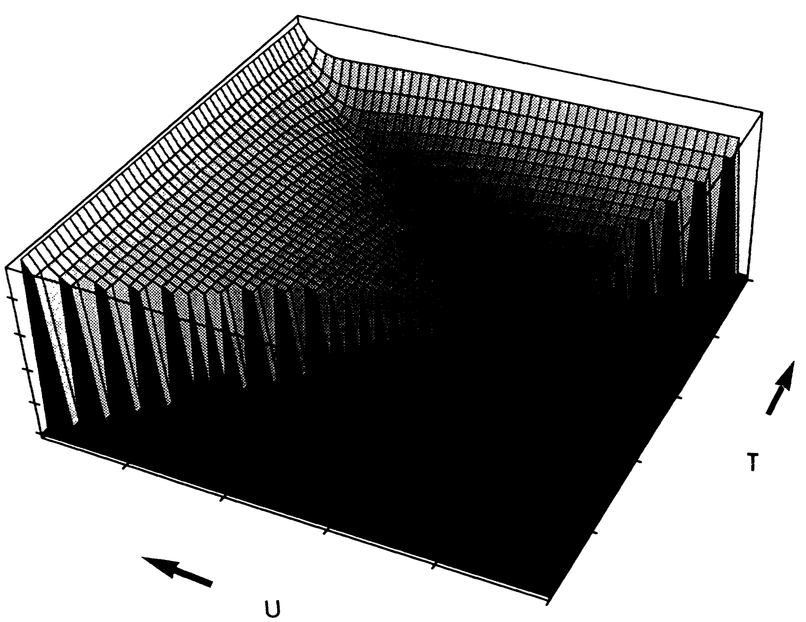

FIG. 7. The logarithm of the partial differential decay rate for $\tilde{\chi}^+ \to \tilde{\chi}^0 q\bar{q}$ as a function of t and u. The heavier chargino goes to the lightest neutralino. $\tan\beta = 1$, $\nu = \lambda = 0.01$, average squark mass = 50 GeV, $\Gamma_W = 1$ GeV, and $\tilde{\Gamma} = 5$ GeV. The total decay rate is 0.459 $\times 10^{-2}$ GeV. The amplitude zero valley is still present.

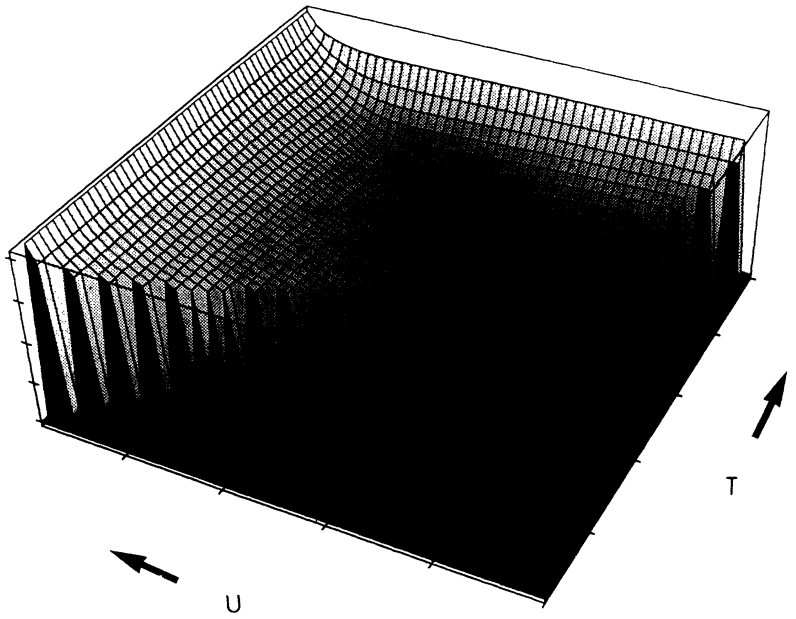

FIG. 8. The logarithm of the partial differential decay rate for $\tilde{\chi}^+ \to \tilde{\chi}^0 q\bar{q}$ as a function of t and u. The heavier chargino goes to the lightest neutralino. $\tan\beta = 1$, $\nu = \lambda = 0.01$, average squark mass = 100 GeV, $\Gamma_W = 1$ GeV, and $\tilde{\Gamma} = 5$ GeV. The total decay rate is 0.644×10^{-3} GeV. The amplitude zero valley is still present but less sharp.

the enhancements which occur because of real (not virtual) intermediate states. This can be seen from the Feynman diagrams of Fig. 5 and the equations expressing the differential decay rate, Eqs. (27)–(33). There are pole terms in s for an intermediate W and in t and u for intermediate squarks. Whether these real intermediate states will occur depends on the choice of supersymmetry-breaking parameters which control the masses of the relevant particles.

If $M_{\tilde{\chi}^+} > M_W + M_{\tilde{\chi}^0}$, then we have an enhancement along the lines $s = M_W^2$ or equivalently $u+t = m_i^2 + m_k^2 - M_W^2$ in the Dalitz plot. If both $M_{\tilde{\chi}^+} > \tilde{m}$ and $\tilde{m} > M_{\tilde{\chi}^0}$ then we can see enhancement along the lines $u = \tilde{m}^2$ and $t = \tilde{m}^2$.

We have also calculated the total decay rate for the process using numerical integration. The total decay rate may be determined analytically if the particle widths vanish and the radiated neutralino is massless. Since this is a useful limiting case, we quote the result below. From (48) we obtain, in this special case,

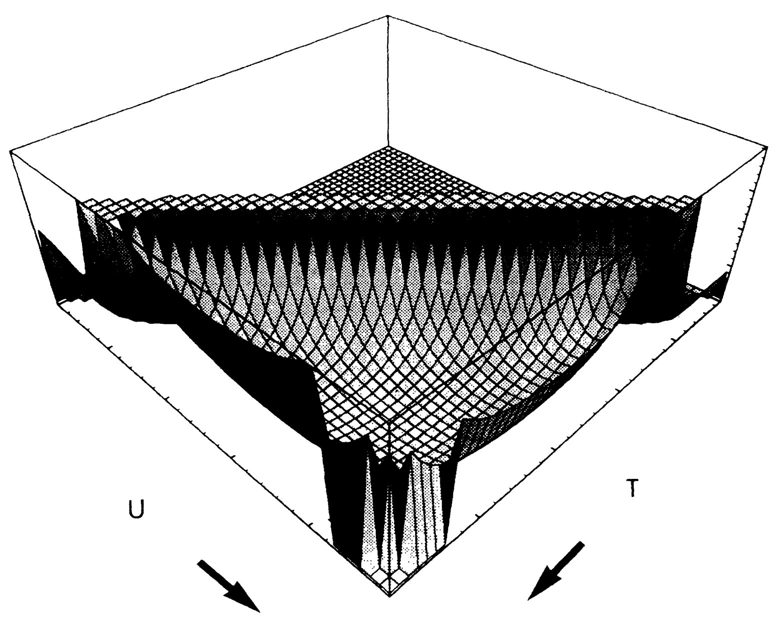

FIG. 9. The logarithm of the partial differential decay rate for $\tilde{\chi}^+ \to \tilde{\chi}^0 q\bar{q}$ as a function of t and u. The heavier chargino (mass 173 GeV) goes to the lightest neutralino (mass 9.34 GeV). $\tan\beta = 1$, $\nu = \lambda = 1$, average squark mass = 100 GeV, $\Gamma_W = 1$ GeV, and $\tilde{\Gamma} = 5$ GeV. The total decay rate is 0.103 GeV. The amplitude zero valley has now vanished.

TABLE I. Decay width Γ for $\tilde{\chi}_i^+ \to \tilde{\chi}_k^0 q\bar{q}$ for the heavier chargino to the lightest neutralino with $\tan\beta = 1$. The widths of the W and squark are 2.7 and 5 GeV, respectively.

λ	ν	m_i (GeV)	m_k (GeV)	Γ (GeV) $\tilde{m}$ = 150 GeV	$\tilde{m}$ =300 GeV	$\tilde{m}$ = 500 GeV
0	0	80.7	0.0 ($\tilde{\gamma}$)	4.36×10^{-4}	3.90×10^{-4}	3.81×10^{-4}
	0	80.7	0.0 (ψ_H^b)	4.09×10^{-4}	4.09×10^{-4}	4.09×10^{-4}
	± 1	139	0.0	0.110	0.109	0.109
($m_{\tilde{g}}$ = 0)	± 2	214	0.0	0.658	0.608	0.608
	± 3	298	0.0	1.96	1.81	1.81
	± 4	385	0.0	4.32	4.18	4.07
	± 5	474	0.0	8.07	8.04	7.72
1	-5	472	46.2	0.463	0.463	0.414
	-4	382	46.7	0.321	0.302	0.283
($m_{\tilde{g}}$ = 339 GeV)	-3	292	47.4	0.188	0.162	0.162
	-2	206	48.3	6.85×10^{-2}	5.92×10^{-2}	5.92×10^{-2}
	-1	122	49.8	8.51×10^{-5}	6.02×10^{-5}	5.74×10^{-5}
	0	139	0.0	3.99×10^{-2}	3.99×10^{-2}	3.99×10^{-2}
	1	173	9.34	3.49×10^{-2}	2.89×10^{-2}	2.89×10^{-2}
	2	231	18.2	1.02	0.961	0.961
	3	306	29.5	1.35	1.27	1.27
	4	390	34.4	1.52	1.48	1.44
	5	477	36.9	1.68	1.68	1.59
2	-5	470	89.6	0.111	0.124	9.23×10^{-2}
	-4	380	90.2	7.14×10^{-2}	6.86×10^{-2}	5.77×10^{-2}
($m_{\tilde{g}}$ = 677 GeV)	-3	290	90.9	3.60×10^{-2}	2.80×10^{-2}	2.80×10^{-2}
	-2	201	92.1	1.56×10^{-3}	4.89×10^{-4}	4.85×10^{-4}
	-1	206	92.0	3.09×10^{-2}	3.09×10^{-2}	3.09×10^{-2}
	0	214	0.0	9.37×10^{-2}	9.37×10^{-2}	9.37×10^{-2}
	1	231	21.8	9.41×10^{-2}	6.50×10^{-2}	6.50×10^{-2}
	2	265	59.5	2.91×10^{-2}	1.17×10^{-2}	1.17×10^{-2}
	3	323	73.0	0.557	0.537	0.534
	4	398	78.0	0.508	0.504	0.483
	5	482	80.3	0.508	0.522	0.478
5	-5	467	219	7.59×10^{-4}	4.50×10^{-3}	7.42×10^{-4}
	-4	468	219	5.80×10^{-3}	1.33×10^{-2}	5.76×10^{-3}
($m_{\tilde{g}}$ = 1692 GeV)	-3	469	220	7.72×10^{-3}	1.52×10^{-2}	7.68×10^{-3}
	-2	470	184	8.45×10^{-2}	8.45×10^{-2}	8.45×10^{-2}
	-1	472	92.0	0.161	0.161	0.161
	0	474	0.0	0.244	0.244	0.244
	1	477	62.9	0.303	0.309	0.269
	2	482	139	0.242	0.277	0.239
	3	490	187	6.71×10^{-2}	8.70×10^{-2}	6.67×10^{-2}
	4	507	202	9.48×10^{-4}	1.27×10^{-2}	1.04×10^{-3}
	5	541	208	1.78×10^{-2}	2.57×10^{-2}	1.93×10^{-2}

TABLE II. Decay width Γ for $\tilde{\chi}_i^+ \to \tilde{\chi}_k^0 q\bar{q}$ for the heavier chargino to the lightest neutralino with $\tan\beta = 5$. The widths of the W and squark are 2.7 and 5 GeV, respectively.

λ	ν	m_i (GeV)	m_k (GeV)	Γ (GeV) $\tilde{m}$ = 150 GeV	$\tilde{m}$ =300 GeV	$\tilde{m}$ = 500 GeV
0	0	112	0 ($\tilde{\gamma}$)	4.92×10^{-2}	4.86×10^{-2}	4.85×10^{-2}
	0	112	0 (ψ_H^b)	0	0	0
	± 1	146	0.0	0.141	0.139	0.139
($m_{\tilde{g}}$ = 0)	± 2	216	0.0	0.663	0.616	0.616
	± 3	299	0.0	1.96	1.82	1.82
	± 4	385	0.0	4.32	4.18	4.07
	± 5	474	0.0	8.07	8.04	7.72
1	-5	473	44.3	9.29×10^{-2}	9.22×10^{-2}	5.78×10^{-2}
	-4	385	44.5	6.37×10^{-2}	5.11×10^{-2}	3.91×10^{-2}
($m_{\tilde{g}}$ = 339 GeV)	-3	298	44.7	3.63×10^{-2}	2.26×10^{-2}	2.25×10^{-2}
	-2	218	44.7	1.36×10^{-2}	9.63×10^{-3}	9.61×10^{-3}
	-1	159	41.3	1.22×10^{-2}	1.22×10^{-2}	1.22×10^{-2}

TABLE II. (*Continued*).

λ	ν	m_i (GeV)	m_k (GeV)	Γ (GeV) $\tilde{m}$ = 150 GeV	$\tilde{m}$ =300 GeV	$\tilde{m}$ = 500 GeV
1	0	146	0.0	7.66×10^{-2}	7.66×10^{-2}	7.66×10^{-2}
	1	169	16.3	0.242	0.235	0.235
($m_{\tilde{g}}$ = 339 GeV)	2	227	33.6	0.297	0.275	0.275
	3	303	38.3	0.282	0.253	0.253
	4	388	40.1	0.287	0.269	0.251
	5	475	40.9	0.300	0.298	0.256
2	-5	474	87.3	2.63×10^{-2}	3.79×10^{-2}	8.90×10^{-3}
	-4	386	87.3	1.81×10^{-2}	1.59×10^{-2}	5.96×10^{-3}
($m_{\tilde{g}}$ = 677 GeV)	-3	304	87.0	1.14×10^{-2}	5.12×10^{-3}	4.95×10^{-3}
	-2	241	85.4	2.11×10^{-2}	1.92×10^{-2}	1.92×10^{-2}
	-1	218	64.8	0.226	0.220	0.220
	0	216	0.0	0.199	0.199	0.199
	1	227	41.1	8.18×10^{-2}	6.82×10^{-2}	6.82×10^{-2}
	2	257	72.6	0.216	0.205	0.205
	3	316	80.2	0.161	0.149	0.148
	4	394	82.7	0.132	0.128	0.115
	5	479	83.8	0.127	0.139	0.106
5	-5	510	216	3.30×10^{-2}	3.86×10^{-2}	3.35×10^{-2}
	-4	484	215	3.84×10^{-2}	4.63×10^{-2}	3.82×10^{-2}
($m_{\tilde{g}}$ = 1692 GeV)	-3	476	209	0.143	0.152	0.142
	-2	474	167	0.884	0.897	0.883
	-1	473	83.3	0.933	0.938	0.924
	0	474	0.0	0.464	0.464	0.464
	1	475	72.1	0.238	0.245	0.219
	2	479	150	0.180	0.203	0.180
	3	485	197	3.54×10^{-2}	4.78×10^{-2}	3.56×10^{-2}
	4	498	208	8.93×10^{-3}	1.76×10^{-2}	9.20×10^{-3}
	5	529	211	0.126	0.134	0.126

TABLE III. Decay width Γ for $\tilde{\chi}_i^+ \rightarrow \tilde{\chi}_k^0 q\bar{q}$ for the heavier chargino to the lightest neutralino with $\tan\beta = 10$. The widths of the W and squark are 2.7 and 5 GeV, respectively.

λ	ν	m_i (GeV)	m_k (GeV)	Γ (GeV) $\tilde{m}$ = 150 GeV	$\tilde{m}$ =300 GeV	$\tilde{m}$ = 500 GeV
0	0	114	0 ($\tilde{\gamma}$)	5.38×10^{-2}	5.32×10^{-2}	5.31×10^{-2}
	0	114	0 (ψ_H^b)	0.0	0.0	0.0
	$\pm$ 1	146	0.0	0.146	0.144	0.144
($m_{\tilde{g}}$ = 0)	$\pm$ 2	216	0.0	0.664	0.617	0.617
	$\pm$ 3	299	0.0	1.96	1.82	1.82
	$\pm$ 4	385	0.0	4.32	4.18	4.07
	$\pm$ 5	474	0.0	8.08	8.04	7.73
1	-5	474	43.7	4.30×10^{-2}	4.20×10^{-2}	9.23×10^{-3}
	-4	386	43.7	2.87×10^{-2}	1.66×10^{-2}	5.22×10^{-3}
($m_{\tilde{g}}$ = 339 GeV)	-3	300	43.5	1.59×10^{-2}	2.33×10^{-3}	2.19×10^{-3}
	-2	220	42.8	4.91×10^{-3}	6.64×10^{-4}	6.55×10^{-4}
	-1	162	36.3	2.58×10^{-2}	2.54×10^{-2}	2.54×10^{-2}
	0	146	0.0	7.74×10^{-2}	7.74×10^{-2}	7.74×10^{-2}
	1	168	23.2	0.178	0.174	0.173
	2	225	37.2	0.148	0.135	0.135
	3	302	40.3	0.124	0.104	0.103
	4	387	41.4	0.124	0.109	9.47×10^{-2}
	5	475	41.9	0.129	0.127	9.11×10^{-2}
2	-5	475	86.5	1.85×10^{-2}	2.96×10^{-2}	8.46×10^{-4}
	-4	388	86.3	1.31×10^{-2}	1.08×10^{-2}	6.25×10^{-4}
($m_{\tilde{g}}$ = 677 GeV)	-3	307	85.6	8.21×10^{-3}	1.53×10^{-3}	1.29×10^{-3}
	-2	245	82.7	3.15×10^{-2}	2.91×10^{-2}	2.90×10^{-2}
	-1	220	58.9	0.315	0.306	0.306
	0	216	0.0	0.212	0.212	0.212
	1	225	46.8	7.71×10^{-2}	6.67×10^{-2}	6.67×10^{-2}

TABLE III. (*Continued*).

λ	ν	m_i (GeV)	m_k (GeV)	Γ (GeV) $\tilde{m}=150$ GeV	$\tilde{m}=300$ GeV	$\tilde{m}=500$ GeV
2	2	253	76.2	0.151	0.142	0.142
	3	313	82.1	9.32×10^{-2}	8.37×10^{-2}	8.31×10^{-2}
($m_{\tilde{g}}=677$ GeV)	4	392	83.9	7.37×10^{-2}	7.10×10^{-2}	5.89×10^{-2}
	5	478	84.7	6.86×10^{-2}	7.94×10^{-2}	4.89×10^{-2}
5	-5	515	215	4.02×10^{-2}	4.60×10^{-2}	4.09×10^{-2}
	-4	487	213	7.25×10^{-2}	8.06×10^{-2}	7.22×10^{-2}
($m_{\tilde{g}}=1692$ GeV)	-3	479	206	0.245	0.255	0.245
	-2	475	163	1.09	1.11	1.09
	-1	474	80.6	1.18	1.18	1.17
	0	474	0.0	0.492	0.492	0.492
	1	475	74.9	0.226	0.232	0.210
	2	478	154	0.163	0.183	0.163
	3	483	200	2.84×10^{-2}	3.92×10^{-2}	2.86×10^{-2}
	4	495	210	1.12×10^{-2}	1.94×10^{-2}	1.15×10^{-2}
	5	525	212	9.73×10^{-2}	0.105	9.74×10^{-2}

TABLE IV. Decay width Γ for $\tilde{\chi}_i^+ \to \tilde{\chi}_k^0 q'\bar{q}$ for the heavier chargino to the second lightest neutralino with $\tan\beta = 1$. The widths of the W and squark are 2.7 and 5 GeV, respectively.

λ	ν	m_i (GeV)	m_k (GeV)	Γ (GeV) $\tilde{m}=150$ GeV	$\tilde{m}=300$ GeV	$\tilde{m}=500$ GeV
0	-5	473	17.7	26.5	26.4	25.5
	-4	385	21.7	14.1	13.7	13.4
($m_{\tilde{g}}=0$)	-3	298	27.9	6.07	5.68	5.68
	-2	214	38.1	1.70	1.58	1.58
	-1	139	56.9	8.95×10^{-3}	7.77×10^{-3}	7.66×10^{-3}
	1	139	57.0	0.110	0.107	0.106
	2	214	38.1	2.25	2.13	2.13
	3	298	27.9	6.55	6.15	6.15
	4	385	21.7	14.5	14.1	13.8
	5	474	17.7	26.9	26.7	25.8
1	-5	472	104	27.4	27.9	26.9
	-4	382	107	12.6	12.7	12.4
($m_{\tilde{g}}=339$ GeV)	-3	293	110	3.79	3.67	3.67
	-2	206	116	9.72×10^{-2}	8.26×10^{-2}	8.26×10^{-2}
	-1	122	92.0	1.27×10^{-5}	1.27×10^{-5}	1.27×10^{-5}
	0	139	52.0	3.35×10^{-3}	3.05×10^{-3}	3.02×10^{-3}
	1	173	56.4	5.24×10^{-2}	4.65×10^{-2}	4.64×10^{-2}
	2	231	62.6	1.28	1.19	1.19
	3	306	69.0	5.49	5.22	5.21
	4	390	73.8	14.5	14.3	14.0
	5	477	77.2	29.9	30.2	29.2
2	-5	470	194	17.0	17.5	17.0
	-4	380	196	5.08	5.24	5.08
($m_{\tilde{g}}=677$ GeV)	-3	290	198	0.129	0.134	0.132
	-2	201	184	8.15×10^{-7}	8.15×10^{-7}	8.15×10^{-7}
	-1	206	94.0	1.05×10^{-2}	7.65×10^{-3}	7.62×10^{-3}
	0	214	43.6	7.58×10^{-2}	6.00×10^{-2}	6.01×10^{-2}
	1	231	92.0	8.74×10^{-2}	8.74×10^{-2}	8.74×10^{-2}
	2	265	125	0.153	0.145	0.145
	3	323	145	2.91	2.94	2.91
	4	398	157	9.19	9.43	9.18
	5	482	164	21.8	22.3	21.7
5	-5	467	460	1.08×10^{-8}	1.08×10^{-8}	1.08×10^{-8}
	-4	468	368	5.52×10^{-3}	5.52×10^{-3}	5.52×10^{-3}
($m_{\tilde{g}}=1692$ GeV)	-3	469	276	3.28×10^{-2}	3.28×10^{-2}	3.28×10^{-2}
	-2	470	199	7.32×10^{-2}	7.86×10^{-2}	7.26×10^{-2}

TABLE IV. (*Continued.*)

λ	ν	m_i (GeV)	m_k (GeV)	Γ (GeV) $\tilde{m}$ = 150 GeV	$\tilde{m}$ =300 GeV	$\tilde{m}$ = 500 GeV
5	-1	472	109	0.150	0.160	0.142
	0	474	21.7	0.249	0.245	0.214
($m_{\tilde{g}}$ = 1692 GeV)	1	477	92.0	0.318	0.318	0.318
	2	482	184	0.353	0.353	0.353
	3	490	274	0.304	0.308	0.304
	4	507	333	0.314	0.315	0.316
	5	541	385	0.288	0.288	0.296

TABLE V. Decay width Γ for $\tilde{\chi}_i^+ \to \tilde{\chi}_k^{0\prime} q\bar{q}$ for the heavier chargino to the second lightest neutralino with $\tan\beta = 5$. The widths of the W and squark are 2.7 and 5 GeV, respectively.

λ	ν	m_i (GeV)	m_k (GeV)	Γ (GeV) $\tilde{m}$ = 150 GeV	$\tilde{m}$ =300 GeV	$\tilde{m}$ = 500 GeV
0	-5	474	6.81	26.2	26.1	25.2
	-4	385	8.33	13.8	13.4	13.0
($m_{\tilde{g}}$ = 0)	-3	299	10.6	5.99	5.58	5.58
	-2	216	14.2	1.79	1.66	1.66
	-1	146	18.0	0.199	0.196	0.196
	1	146	18.0	0.296	0.293	0.293
	2	216	14.2	1.96	1.84	1.84
	3	299	10.6	6.16	5.75	5.75
	4	385	8.33	13.9	13.5	13.2
	5	474	6.81	26.4	26.2	25.3
1	-5	473	94.7	28.6	29.1	28.1
	-4	385	94.5	13.1	13.1	12.7
($m_{\tilde{g}}$ = 339 GeV)	-3	298	93.3	4.47	4.28	4.27
	-2	218	87.9	0.660	0.595	0.594
	-1	159	64.6	5.21×10^{-2}	4.81×10^{-2}	4.80×10^{-2}
	0	146	52.2	1.29×10^{-2}	1.22×10^{-2}	1.21×10^{-2}
	1	169	57.7	0.123	0.115	0.115
	2	226	69.0	1.37	1.28	1.28
	3	303	77.2	5.64	5.37	5.36
	4	388	81.6	14.7	14.6	14.2
	5	475	84.2	29.3	29.7	28.6
2	-5	474	184	17.5	17.9	17.4
	-4	386	181	5.93	6.11	5.93
($m_{\tilde{g}}$ = 677 GeV)	-3	304	175	0.806	0.816	0.809
	-2	241	148	8.22×10^{-2}	8.38×10^{-2}	8.35×10^{-2}
	-1	218	104	8.82×10^{-2}	8.16×10^{-2}	8.14×10^{-2}
	0	216	43.6	0.193	0.176	0.176
	1	226	103	0.131	0.128	0.128
	2	257	132	0.414	0.413	0.412
	3	316	155	2.25	2.28	2.26
	4	394	167	8.10	8.32	8.09
	5	479	172	20.5	21.0	20.4
5	-5	510	419	0.127	0.126	0.128
	-4	484	354	0.471	0.471	0.472
($m_{\tilde{g}}$ = 1692 GeV)	-3	476	275	0.807	0.809	0.807
	-2	474	194	0.166	0.170	0.166
	-1	473	105	0.220	0.227	0.214
	0	474	21.7	0.610	0.606	0.574
	1	475	97.9	0.667	0.671	0.662
	2	479	189	0.463	0.466	0.463
	3	485	275	0.227	0.231	0.227
	4	498	341	0.191	0.191	0.191
	5	529	397	0.867	0.867	0.874

$$
\begin{aligned}
\Gamma = & \int_{\epsilon M_W^2}^{m_i^2-2\epsilon M_W^2} dt \int_{\epsilon M_W^2}^{m_i^2-t-\epsilon M_W^2} du \frac{d^2\Gamma}{dudt} \\
= & \frac{\alpha_{\rm em}^2 m_i}{32\pi \sin^4\theta_W}\Bigg[(A_s+B_s)\Bigg[1-3\epsilon \\
& +\left(1-\epsilon-\frac{M_W^2}{m_i^2}\right)\ln\left\{\frac{\frac{M_W^2}{m_i^2}-\epsilon}{\frac{M_W^2}{m_i^2}-(1-2\epsilon)}\right\} - \left(1-\epsilon-\frac{M_W^2}{m_i^2}\right)^2\ln\left\{\frac{\frac{M_W^2}{m_i^2}-\epsilon}{\frac{M_W^2}{m_i^2}-(1-2\epsilon)}\right\} \\
& -2(1-3\epsilon)\left(1-\epsilon-\frac{M_W^2}{m_i^2}\right)-\frac{1}{2}\left\{\left(\frac{M_W^2}{m_i^2}-\epsilon\right)^2-\left(\frac{M_W^2}{m_i^2}-1+2\epsilon\right)^2\right\}+\frac{1-3\epsilon}{\epsilon-\frac{M_W^2}{m_i^2}}\left\{\frac{1-\epsilon}{2}-\frac{1-3\epsilon+3\epsilon^2}{3}\right\}\Bigg] \\
& +(A_t+A_u)\left((1-2\epsilon)\ln\frac{1-2\epsilon}{\epsilon}-\frac{3}{2}(1-\epsilon)(1-3\epsilon)\right) \\
& +2(\sqrt{A_sA_t}-\sqrt{B_sA_u})\Bigg[\frac{(1-3\epsilon)(1+\epsilon)}{2}\ln\left(\frac{M_W^2}{m_i^2}-\epsilon\right) \\
& -\left(\epsilon+\frac{M_W^2}{m_i^2}\right)\Bigg\{\left(\frac{M_W^2}{m_i^2}-\epsilon\right)\left[\ln\left(\frac{M_W^2}{m_i^2}-\epsilon\right)-1\right]-\left(\frac{M_W^2}{m_i^2}-1+2\epsilon\right)\left[\ln\left(\frac{M_W^2}{m_i^2}-1+2\epsilon\right)-1\right] \\
& +\frac{1}{2}\left(\frac{M_W^2}{m_i^2}-\epsilon\right)^2\left(\ln\left(\frac{M_W^2}{m_i^2}-\epsilon\right)-\frac{1}{2}\right)-\frac{1}{2}\left(\frac{M_W^2}{m_i^2}-1+2\epsilon\right)^2\left(\ln\left(\frac{M_W^2}{m_i^2}-1+2\epsilon\right)-\frac{1}{2}\right)\Bigg\}\Bigg]\Bigg]. \qquad (60)
\end{aligned}
$$

Our numerical results (see footnote 1) are displayed in Tables I–VI. We have obtained the decay rate by numerical integration for a variety of choices of squark masses and parameters controlling the masses of the gauginos. These cover the mass ranges favored for these particles by minimal supersymmetric unification considerations [1]. The widths of the W and squark have been fixed at 2.7 and 5 GeV, respectively, although strictly one should allow for a significantly smaller squark width should the mass of the gluino exceed that of the squark. While the influence of the parameters ν, λ, and $\tan\beta$ via phase-space restrictions is evident from the tables, the behavior of the decay width as a function of these parameters is clearly more complex. In examining the two-body decay $\tilde{\chi}^+ \to \tilde{\chi}^0 W^+$ [4], Gunion and Haber found that for large areas of parameter space the decay to the lightest neutralino does not dominate. This is also the case here where the decay width of the chargino to the second lightest neutralino is larger in some areas of parameter space than that of the decay to the lightest neutralino. The mass of the intermediate squark does not play a large role in determining this three-body decay rate.

Apart from these tabulated decay widths, we have also calculated the widths corresponding to the solutions of Ross and Roberts [1]. Case Z leads to a heavier chargino of mass 229 GeV and the two lightest neutralinos have masses of 40.5 and 77.6 GeV. The width of the decay to the lightest neutralino is 0.0762 GeV, while the decay width to the second lightest neutralino is 1.28 GeV. In case X the heavier chargino has a mass of 198 GeV, while the masses of the two lightest neutralinos are 66.0 and 102 GeV in this case. The decay widths of the chargino to the lightest and second lightest neutralinos are then 0.0416 and 0.0888 GeV, respectively.

VI. CONCLUSION

We have calculated the absolute decay rates in GeV for the processes $\tilde{\chi}^+ \to \tilde{\chi}^0 q\bar{q}$ and $\tilde{\chi}^+ \to \tilde{\chi}^{0\prime} q\bar{q}$ for a wide range of the parameters $\tan\beta$, ν, λ, and $\tilde{m}$. We have also calculated the decay rates for the renormalization group solutions X and Z of Ross and Roberts [1] as noted in Sec. V.

Inspection of these tables shows the results to depend only mildly on the average squark mass $\tilde{m}$ and on $\tan\beta$. But the decay rates increase very rapidly with increasing ν. As ν and λ are varied one sees roughly a 4 to 5 order of magnitude variation in width, reaching a maximum of tens of GeV. For comparison, the total decay rate for the analogous non-SUSY decay $W \to \gamma q\bar{q}$ can be obtained as a function of a cutoff ϵ from Fig. 7 of Ref. [3]. For $\epsilon = 0.1$ the decay rate is of order 10^{-3} GeV comparable to the decay rates for $\tilde{\chi}^+ \to \tilde{\chi}^0 q\bar{q}$ when one has $\nu, \lambda \approx 0$ and $\tan\beta = 1$, in the SUSY limit.

For the two solutions of Ross and Roberts [1] the decay rate to the LSP is of order 0.1 GeV, while to the next lightest neutralino it is of order 1 GeV, several orders of

TABLE VI. Decay width Γ for $\tilde{\chi}_i^+ \rightarrow \tilde{\chi}_k^{0\prime} q\bar{q}$ for the heavier chargino to the second lightest neutralino with $\tan\beta = 10$. The widths of the W and squark are 2.7 and 5 GeV, respectively.

λ	ν	m_i (GeV)	m_k (GeV)	Γ (GeV) $\tilde{m} = 150$ GeV	$\tilde{m} = 300$ GeV	$\tilde{m} = 500$ GeV
0	-5	474	3.50	26.1	26.0	25.0
	-4	385	4.29	13.7	13.3	13.0
($m_{\tilde{g}} = 0$)	-3	299	5.47	5.95	5.53	5.53
	-2	216	7.30	1.79	1.66	1.66
	-1	146	9.15	0.229	0.226	0.226
	1	146	9.15	0.278	0.275	0.275
	2	216	7.30	1.88	1.75	1.75
	3	299	5.47	6.03	5.62	5.62
	4	385	4.29	13.7	13.4	13.0
	5	474	3.50	26.2	26.0	25.1
1	-5	474	92.0	28.5	28.6	28.0
	-4	386	91.1	13.5	13.4	13.1
($m_{\tilde{g}} = 339$ GeV)	-3	300	88.8	4.41	4.42	4.41
	-2	220	81.9	0.777	0.702	0.701
	-1	162	61.6	7.96×10^{-2}	7.38×10^{-2}	7.36×10^{-2}
	0	146	52.2	1.43×10^{-2}	1.36×10^{-2}	1.35×10^{-2}
	1	168	58.4	0.121	0.113	0.113
	2	225	72.3	1.33	1.23	1.23
	3	302	80.5	5.53	5.27	5.27
	4	387	84.4	14.3	14.2	13.9
	5	475	86.5	29.0	29.4	28.3
2	-5	475	181	18.1	18.6	18.0
	-4	388	178	6.16	6.36	6.16
($m_{\tilde{g}} = 677$ GeV)	-3	307	169	0.968	0.980	0.971
	-2	245	143	0.159	0.160	0.160
	-1	220	104	0.105	9.75×10^{-2}	9.73×10^{-2}
	0	216	43.6	0.240	0.222	0.222
	1	225	105	5.30×10^{-2}	4.46×10^{-2}	4.44×10^{-2}
	2	253	135	0.352	0.353	0.352
	3	313	159	2.00	2.02	2.00
	4	392	170	7.77	7.98	7.76
	5	478	175	19.6	20.1	19.6
5	-5	515	413	0.274	0.274	0.276
	-4	487	351	0.682	0.682	0.683
($m_{\tilde{g}} = 1692$ GeV)	-3	479	275	0.995	0.998	0.995
	-2	475	193	0.195	0.199	0.194
	-1	474	103	0.239	0.245	0.233
	0	474	21.7	0.753	0.750	0.717
	1	475	99.6	0.755	0.760	0.749
	2	478	190	0.500	0.504	0.499
	3	482	275	0.203	0.206	0.203
	4	495	344	0.156	0.157	0.156
	5	525	402	0.674	0.674	0.680

magnitude larger than the analogous $W \rightarrow \gamma q\bar{q}$ decay. This illustrates the point that the decay to the LSP will not necessarily dominate. We noted in Sec. V how the RAZ, present in the Dalitz plot in the SUSY limit slowly disappears as $\tilde{m}$ is varied but disappears more rapidly as ν and λ are increased from zero. This is consistent with our earlier observations on the relative sensitivity of the decay rate to the various parameters.

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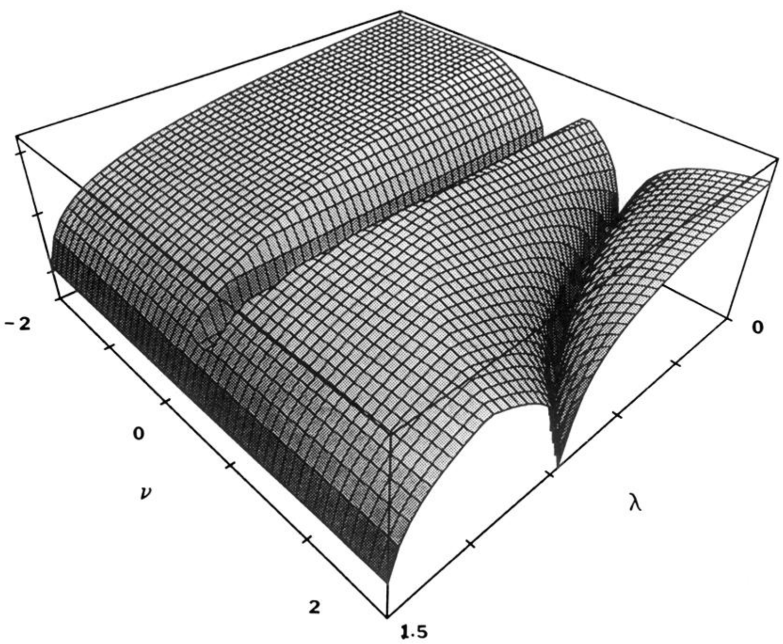

FIG. 1. Surface plot of the logarithm of the mass of the lightest neutralino versus the parameters ν and λ for $\tan\beta = 1$. Limits on ν and λ are $-2 \le \nu \le 2$, $0 \le \lambda \le 1.5$.

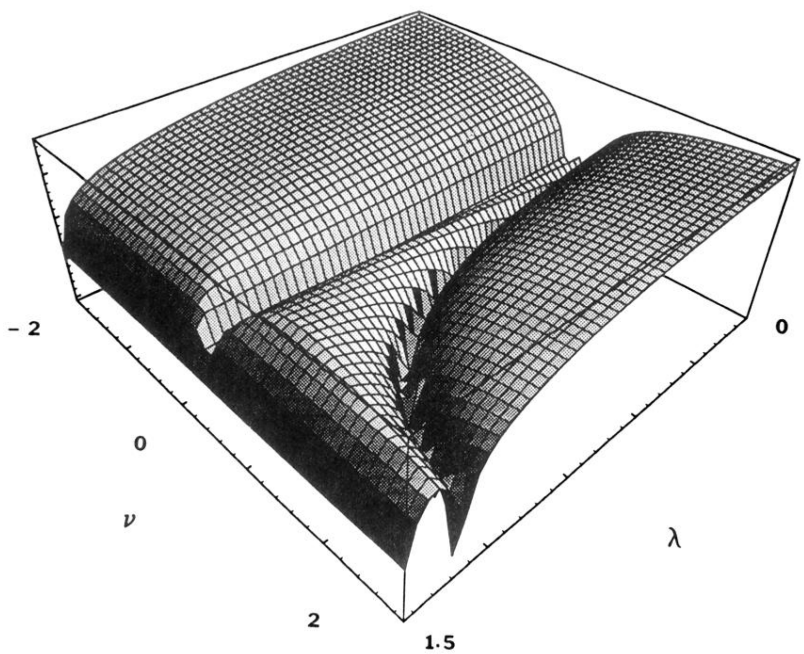

FIG. 2. Surface plot of the logarithm of the mass of the lightest neutralino versus the parameters ν and λ for $\tan\beta = 5$. Limits on ν and λ are $-2 \leq \nu \leq 2$, $0 \leq \lambda \leq 1.5$.

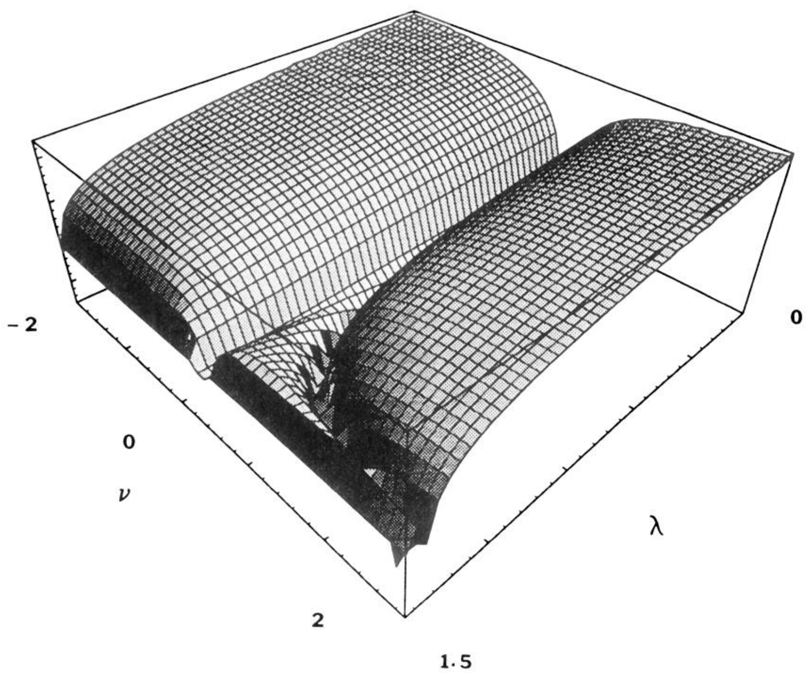

FIG. 3. Surface plot of the logarithm of the mass of the lightest neutralino versus the parameters ν and λ for $\tan\beta = 20$. Limits on ν and λ are $-2 \le \nu \le 2$, $0 \le \lambda \le 1.5$.

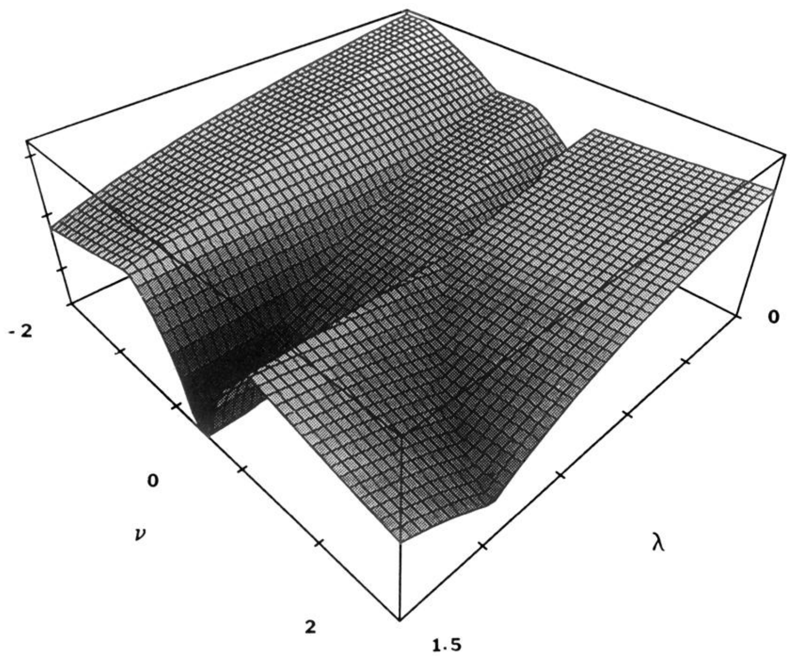

FIG. 4. Surface plot of the logarithm of the mass of the second lightest neutralino versus the parameters ν and λ for $\tan\beta = 1$. Limits on ν and λ are $-2 \le \nu \le 2$, $0 \le \lambda \le 1.5$.

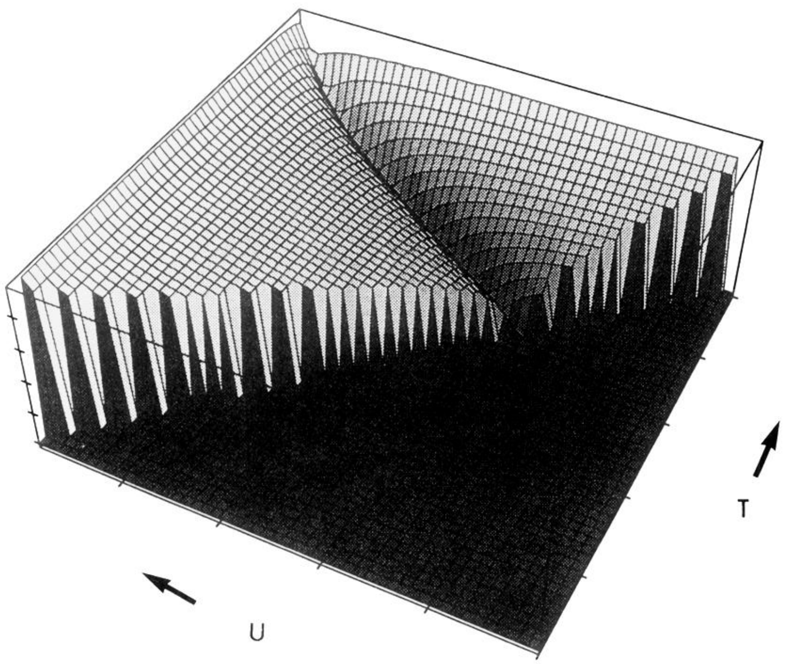

FIG. 6. The logarithm of the partial differential decay rate for $\tilde{\chi}^+ \to \tilde{\chi}^0 q\bar{q}$ as a function of t and u. The heavier chargino (mass 81.7 GeV) goes to the lightest neutralino (mass 0.545 GeV). $\tan\beta = 1$, $\nu = \lambda = 0.01$, average squark mass = 1 GeV, $\Gamma_W = 1$ GeV, and $\tilde{\Gamma} = 1$ GeV. The total decay rate is 0.246 $\times 10^{-2}$ GeV. The amplitude zero is very pronounced in this plot.

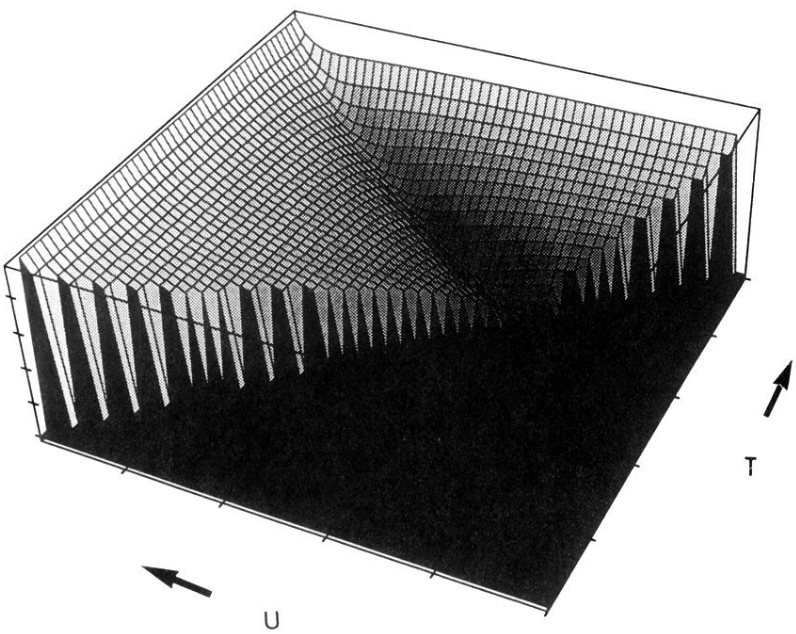

FIG. 7. The logarithm of the partial differential decay rate for $\tilde{\chi}^+ \to \tilde{\chi}^0 q\bar{q}$ as a function of t and u. The heavier chargino goes to the lightest neutralino. $\tan\beta = 1$, $\nu = \lambda = 0.01$, average squark mass = 50 GeV, $\Gamma_W = 1$ GeV, and $\tilde{\Gamma} = 5$ GeV. The total decay rate is 0.459 $\times 10^{-2}$ GeV. The amplitude zero valley is still present.

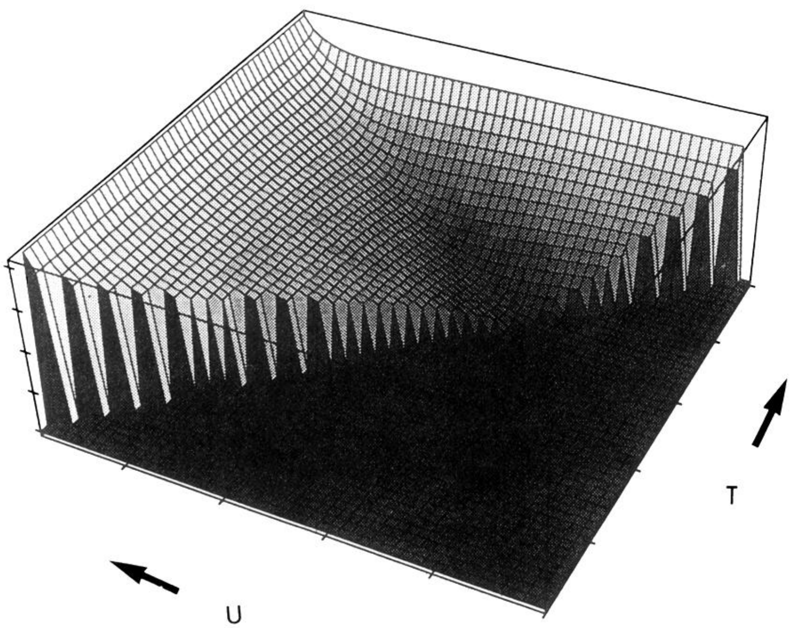

FIG. 8. The logarithm of the partial differential decay rate for $\tilde{\chi}^+ \to \tilde{\chi}^0 q\bar{q}$ as a function of t and u. The heavier chargino goes to the lightest neutralino. $\tan\beta = 1$, $\nu = \lambda = 0.01$, average squark mass = 100 GeV, $\Gamma_W = 1$ GeV, and $\tilde{\Gamma} = 5$ GeV. The total decay rate is 0.644×10^{-3} GeV. The amplitude zero valley is still present but less sharp.

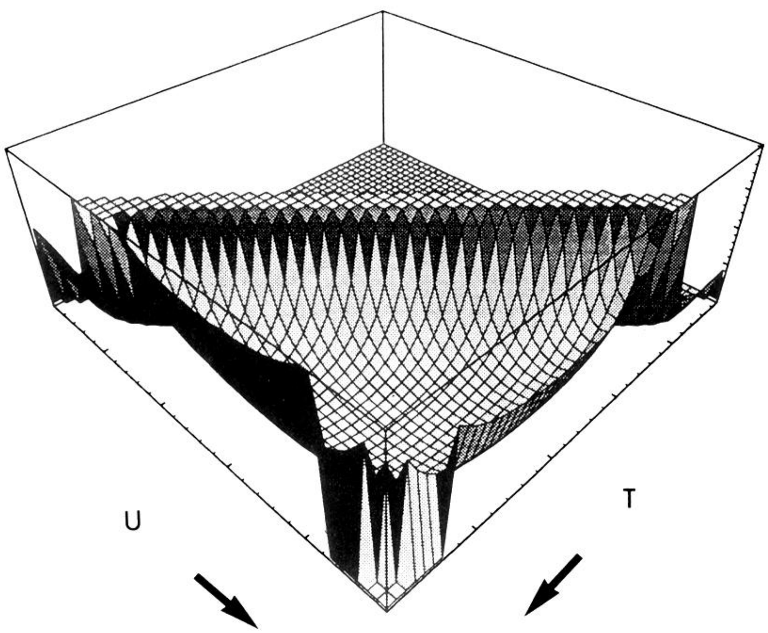

FIG. 9. The logarithm of the partial differential decay rate for $\tilde{\chi}^+ \to \tilde{\chi}^0 q\bar{q}$ as a function of t and u. The heavier chargino (mass 173 GeV) goes to the lightest neutralino (mass 9.34 GeV). $\tan\beta = 1$, $\nu = \lambda = 1$, average squark mass = 100 GeV, $\Gamma_W = 1$ GeV, and $\tilde{\Gamma} = 5$ GeV. The total decay rate is 0.103 GeV. The amplitude zero valley has now vanished.