PHYSICAL REVIEW D VOLUME 49, NUMBER 1 1 JANUARY 1994

Phenomenology of B_s decays

P. Blasi and P. Colangelo*
Istituto Nazionale di Fisica Nucleare, Sezione di Bari, Italy

G. Nardulli
Istituto Nazionale di Fisica Nucleare, Sezione di Bari, Italy
and Dipartimento di Fisica, Universitá di Bari, Italy

N. Paver
Dipartimento di Fisica Teorica, Universitá di Trieste, Italy
and Istituto Nazionale di Fisica Nucleare, Sezione di Trieste, Italy

(Received 7 June 1993)

Using the QCD sum rules technique we study several aspects of the phenomenology of the b-flavored strange meson $\overline{B}_s^0$. In particular, we evaluate the mass of the particle, the leptonic constant, and the form factors of the decays $\overline{B}_s^0 \to D_s^+ l^- \bar{\nu}$, $\overline{B}_s^0 \to D_s^{*+} l^- \bar{\nu}$, $\overline{B}_s^0 \to K^{*+} l^- \bar{\nu}$. We also calculate, in the factorization approximation, a number of two-body nonleptonic $\overline{B}_s^0$ decays. Finally, we compare our evaluation of the $SU(3)_F$-breaking effects in the $\overline{B}_s^0$ channel to other estimates.

PACS number(s): 13.20.Jf, 11.50.Li, 13.25.+m

I. INTRODUCTION

The interest in the b-flavored strange meson $\overline{B}_s^0(b\bar{s})$ has been recently prompted by the reported evidence for the production of this particle in the hadronic Z^0 decays at the CERN e^+e^- collider LEP [1–3]. A signal of correlated $D_s^+ l^-$ pairs ($l = \mu, e$) has been observed [4], with the lepton having a large momentum and a large momentum component with respect to the b quark direction; this signal can be attributed to the semileptonic process

$$\overline{B}_s^0 \to D_s^+ l^- \bar{\nu}_l X \ , \tag{1}$$

which, by analogy with the $B_{u,d}$ case, is expected to occur at the 10% level. The indication of B_s mesons is confirmed by the observation of an excess of inclusive D_s^+ production, whose measured value is larger than the expected production from $B_{u,d}$.

Evidence for the B_s production at $\Upsilon(5S)$ was already reported by the CUSB Collaboration at the Cornell Electron Storage Ring (CESR) [5]. Moreover, indications for B_s have been deduced from the measurement of the rate of same sign dileptons at the hadron $p\bar{p}$ colliders [6] and at LEP [7]: since this rate is larger than the corresponding quantity measured at $\Upsilon(4S)$ [8], the difference can be attributed to the presence of $\overline{B}_s^0$ and B_s^0 mesons with a (nearly) maximal mixing.

Ongoing measurements will soon provide us with a value for the mass difference $m_{B_s} - m_{B_d}$ by reconstructing nonleptonic decay channels; as for the lifetime τ_{B_s}, the measured value [9]

$$\tau_{B_s} = 1.1 \pm 0.5 \text{ ps} \tag{2}$$

is still dominated by the statistical error, so that no information on the possible role of nonspectator effects in this channel is available yet.

From the theoretical standpoint, the interest in the $\overline{B}_s^0$ meson stems from the possibility of clarifying the size of the light flavor $SU(3)_F$-breaking effects in the b quark sector. In the charm sector some hints on such effects can be obtained by comparing D^+ and D_s; the difference [10]

$$m_{D_s} - m_{D^+} = (99.5 \pm 0.6) \text{ MeV} \tag{3}$$

shows that these effects are of the order of 5% for the mass of the particles. In the b system the $SU(3)_F$ breaking terms, which account for the deviations from unity of the ratios m_{B_s}/m_{B_d}, f_{B_s}/f_{B_d}, etc., play a significant role in the possibility of constraining the Cabibbo-Kobayashi-Maskawa matrix and, consequently, the quark sector of the standard model. As a matter of fact, within the standard model the mixing between $\overline{B}_s^0$ and B_s^0 occurs with the parameter $x_s = (\Delta M/\Gamma)_{B_s}$ given by [11]

$$x_s = \frac{G_F^2}{6\pi^2} \tau_{B_s} m_W^2 m_{B_s} (f_{B_s}^2 B_{B_s}) \eta_{B_s} |V_{ts}^* V_{tb}|^2 y_t f_2(y_t) \ . \tag{4}$$

Equation (4) shows that the ratio x_s/x_d is independent of the (still unknown) top quark mass m_t; the experimental determination of this ratio implies a measurement of $|V_{ts}/V_{td}|$ once $(f_{B_s}^2 B_{B_s})/(f_{B_d}^2 B_{B_d})$, m_{B_s}/m_{B_d}, and τ_{B_s}/τ_{B_d} have been calculated and/or measured [12].

The ratios m_{B_s}/m_{B_d} and f_{B_s}/f_{B_d} are available presently from potential models for the quark-antiquark systems [13]. The quantity $(f_{B_s}^2 B_{B_s})/(f_{B_d}^2 B_{B_d})$ has been estimated also by using heavy-quark effective chiral perturbation theory [14].

In this paper we calculate m_{B_s}, f_{B_s}, and the ratios

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*Electronic address: COLANGELO@BARI.INFN.IT

m_{B_s}/m_{B_d}, and f_{B_s}/f_{B_d} by QCD sum rules [15]. This method is deeply rooted in the QCD framework of the strong interactions, and has been successfully applied to different aspects of the light [16,17] and heavy hadrons [18]. It avoids the notion of wave function for a system of constituent quarks, and directly relates hadronic properties (masses, leptonic constants, etc.) to fundamental QCD quantities like current quark masses, α_s, and a set of parameters, the "condensates," which describe the deviations from the asymptotically free behavior at short distances by allowing the inclusion of a series of power corrections.

In the QCD sum rules approach the $SU(3)_F$-breaking effects in the static parameters of the heavy mesons can be systematically taken into account. Moreover, this technique permits the calculation of a number of dynamical heavy system properties, e.g., the form factors that describe the semileptonic decays $\bar{B}_s^0 \to D_s^+(D_s^{*+})l^-\nu$ and their deviations from the analogous quantities related to $\bar{B}_d^0 \to D^+(D^{*+})l^-\nu$.

The plan of the paper is as follows. In Sec. II we evaluate the mass and the leptonic constant of the $\bar{B}_s^0$ meson by two-point function QCD sum rules. An analysis of the ratios m_{B_s}/m_{B_d} and f_{B_s}/f_{B_d} allows us to estimate the size of $SU(3)_F$ breaking in these quantities. Since the calculation can be extended in a straightforward way to the D_s meson, we calculate f_{D_s} and compare our findings with a number of recent experimental and theoretical determinations. By using three point function QCD sum rules we calculate in Sec. III the hadronic matrix elements that describe the semileptonic decays $\bar{B}_s^0 \to D_s^+(D_s^{*+})l^-\nu$ and $\bar{B}_s^0 \to K^{*+}l^-\nu$. Also in this case we evaluate the light flavor symmetry breaking effects. In Sec. IV we estimate, in the factorization hypothesis, the width of several two-body nonleptonic B_s decays.

II. B_s MASS AND LEPTONIC CONSTANT

A number of estimates of the leptonic constants for the heavy-quark–light-quark mesonic systems can be found in the literature. In particular, QCD sum rules have been used to evaluate the B meson leptonic constant f_B both for a finite [19,20] and an infinite heavy-quark mass m_b [21,22]. Here we apply this method to the calculation of f_{B_s} defined by the matrix element

$$\langle 0|\bar{b}i\gamma_5 s|\bar{B}_s^0\rangle = \frac{f_{B_s} m_{B_s}^2}{m_b + m_s} \tag{5}$$

(m_b and m_s are the b and s quark masses). As usual in the QCD sum rules approach, the starting point is the correlator of quark currents:

$$\Pi(q^2) = i\int dx\, e^{iqx}\langle 0|T[J_5(x)J_5^\dagger(0)]|0\rangle \tag{6}$$

with $J_5 = \bar{b}i\gamma_5 s$. This correlator can be evaluated in two different ways.

First, it is evaluated by a short-distance operator product expansion in QCD ($q^2 \to -\infty$), which gives the perturbative (P) contribution, written through a dispersion relation

$$\Pi^P(q^2) = \frac{1}{\pi}\int ds \frac{\rho_P(s)}{s - q^2}\ , \tag{7}$$

and nonperturbative (NP) power corrections parametrized by vacuum matrix elements of quark and gluon field operators. These terms are ordered according to the dimension; they represent the breaking of asymptotic freedom. Therefore, the QCD form of the correlator reads

$$\begin{aligned}\Pi_{\mathrm{QCD}}(q^2) &= \Pi^P(q^2) + \Pi^{\mathrm{NP}}(q^2)\\ &= \Pi^P(q^2) + C_3(q^2)\langle \bar{s}s\rangle + C_4(q^2)\left\langle \frac{\alpha_s}{\pi}G^2\right\rangle\\ &\quad + C_5(q^2)\langle \bar{s}g\sigma G s\rangle + \cdots\ .\end{aligned} \tag{8}$$

The perturbative spectral function $\rho_P(s)$ is given to the lowest order in α_s by

$$\begin{aligned}\rho_P(s) &= \frac{3}{8\pi}\frac{\sqrt{\lambda(s, m_b^2, m_s^2)}}{s}[s - (m_b - m_s)^2]\\ &\quad \times \Theta[s - (m_b + m_s)^2]\ ,\end{aligned} \tag{9}$$

where λ is the triangular function; the $O(\alpha_s)$ corrections can be found in Ref. [17]. The coefficients C_3, C_4, and C_5 in Eq. (8) can be calculated using the fixed-point technique [23] with the result

$$C_3 = \frac{m_b}{q^2 - m_b^2} - \frac{m_s}{2}\frac{q^2 - 2m_b^2}{(q^2 - m_b^2)^2} + \frac{m_s^2 m_b^3}{(q^2 - m_b^2)^3}\ , \tag{10}$$

$$\begin{aligned}C_4 &= \frac{1}{12}\frac{1}{(q^2 - m_b^2)}\\ &\quad \times \left[-1 + 6\frac{m_s m_b^3}{(q^2 - m_b^2)^2}\ln\frac{|q^2 - m_b^2|}{m_s m_b}\right.\\ &\quad \left. + \frac{m_s}{m_b}\left[1 - \frac{2m_b^2}{(q^2 - m_b^2)} - \frac{6m_b^4}{(q^2 - m_b^2)^2}\right]\right]\ ,\end{aligned} \tag{11}$$

$$C_5 = -\frac{1}{2}\left[\frac{m_b}{(q^2 - m_b^2)^2} + \frac{m_b^3}{(q^2 - m_b^2)^3}\right]\ . \tag{12}$$

Actually, the main contribution comes from the $D = 3$ and 5 terms.

The second evaluation of the correlator is obtained by writing the spectral function $\rho(s)$ in terms of hadronic (H) resonances and of a continuum of states; assuming the dominance of the lowest-lying resonance, one writes

$$\rho_H(s) = \pi\left[\frac{f_{B_s} m_{B_s}^2}{m_b + m_s}\right]^2 \delta(s - m_{B_s}^2) + \rho_{\mathrm{cont}}(s)\Theta(s - s_0)\ , \tag{13}$$

where s_0 is an effective threshold which separates the contribution of the resonance from the continuum. According to duality, the continuum spectral function can be modeled as in perturbative QCD: therefore, in (13), $\rho_{\mathrm{cont}}(s) = \rho_P(s)$.

In the QCD sum rules approach, a region in q^2 (duality window) has to be found where the hadronic and the QCD expressions for the correlator match with each other. The matching can be improved by a Borel transformation defined by the operator

$$\mathcal{B}=\frac{(-Q^2)^n}{(n-1)!}\left[\frac{d}{dQ^2}\right]^n \tag{14}$$

in the limit $Q^2 \to \infty$ ($Q^2=-q^2$), $n \to \infty$, and $Q^2/n=M^2$ fixed, applied to both the hadronic and QCD sides of the rule. One obtains

$$\frac{1}{\pi}\int ds\,\rho(s)\frac{e^{-s/M^2}}{M^2}=\Pi^{P}(M^2)+\Pi^{\mathrm{NP}}(M^2) \tag{15}$$

and a daughter sum rule for the mass of the meson by differentiating Eq. (15) with respect to $1/M^2$.

Let us discuss the values of the parameters appearing in the sum rule Eq. (15). The strange quark mass m_s and the strange quark condensate $\langle \bar{s}s \rangle$ are responsible for the deviation of Eq. (15) from the analogous expression for the B meson. Both these parameters are fixed by the analysis of the baryonic states given in Ref. [24]: $m_s=0.14-0.15$ GeV and $\langle \bar{s}s \rangle=0.8\langle \bar{d}d \rangle$ with $\langle \bar{d}d \rangle=(-0.23\ \mathrm{GeV})^3$; the mixed $D=5$ condensate can be expressed in terms of $\langle \bar{s}s \rangle$: $\langle \bar{s}g\sigma Gs \rangle=m_0^2\langle \bar{s}s \rangle$ with $m_0^2=0.8\ \mathrm{GeV}^2$.

The (pole) mass of the b quark plays a crucial role in the sum rule. We use the value fixed in Ref. [19] by analyzing the Υ system (see also Ref. [25]): $m_b=4.6-4.7$ GeV [26].

The last QCD input parameter is α_s; we use the value obtained at the scale m_b with $\Lambda_{\mathrm{QCD}}=150-200$ MeV.

There are now two quantities that must be fixed: the effective threshold s_0 and the duality window in the Borel parameter M^2. The range of acceptable M^2 values can be fixed by requiring a hierarchical structure in the contributions of the operator-product expansion (OPE) and in the resonance-continuum hadronic side. On the other hand, the value of s_0 can be changed in a small interval: we use $s_0=33-36\ \mathrm{GeV}^2$. The typical curves are depicted in Fig. 1; our result is

$$\begin{aligned} m_{B_s}&=(5.4\pm0.1)\ \mathrm{GeV}\ ,\\ f_{B_s}&=(190\pm20)\ \mathrm{MeV}\ , \end{aligned} \tag{16}$$

where the uncertainties are due to the variation of the parameters in their allowed intervals.

Before discussing these results let us observe that the same calculation can be performed for the $D_s^+(c\bar{s})$ meson. Using $m_c=1.35$ GeV, $s_0=6-7\ \mathrm{GeV}^2$, and α_s at the scale m_c we get

$$\begin{aligned} m_{D_s}&=(2.0\pm0.1)\ \mathrm{GeV}\ ,\\ f_{D_s}&=(195\pm20)\ \mathrm{MeV}\ . \end{aligned} \tag{17}$$

Within the uncertainties the result for the leptonic constant is compatible with the value obtained in Ref. [27] by a numerical calculation on the lattice: $f_{D_s}=(230\pm50)$ MeV. Moreover, it is in agreement with the measurement of the WA75 Collaboration [28],

$$f_{D_s}=(232\pm45\pm20\pm48)\ \mathrm{MeV}\ , \tag{18}$$

obtained by the observation of leptonic decays $D_s^+ \to \mu^+\nu$ in emulsion. Another estimate of f_{D_s} has been given in Refs. [29,30] using the nonleptonic decay channel $B \to D(D^*)D_s^+$ and the factorization hypothesis, with a similar result.

As stated above, the uncertainties in Eqs. (16) and (17) are due to the variation of s_0 and M^2 in the stability window. Trying to reduce this error (mainly in the prediction of m_{B_s}) we have studied the ratios m_{B_s}/m_{B_d} and f_{B_s}/f_{B_d} by writing the ratios of the corresponding rules with two different continuum thresholds s_0 (33–36 GeV^2 for B_s and 32–35 GeV^2 for B). These quantities display a softer dependence on the parameters and are remarkably stable in M^2 as shown in Fig. 2. This allows us to predict

$$m_{B_s}/m_{B_d}=1.005\pm0.002\ , \tag{19}$$

$$f_{B_s}/f_{B_d}=1.09\pm0.03 \tag{20}$$

with the uncertainty reduced by a factor of 2 with respect to Eqs. (16). The conclusion is that the size of $\mathrm{SU}(3)_F$ breaking effects are of 0.5% for the B_s mass and less than 10% for the leptonic constant; these effects mainly come from the value of the $\langle \bar{s}s \rangle$ condensate.

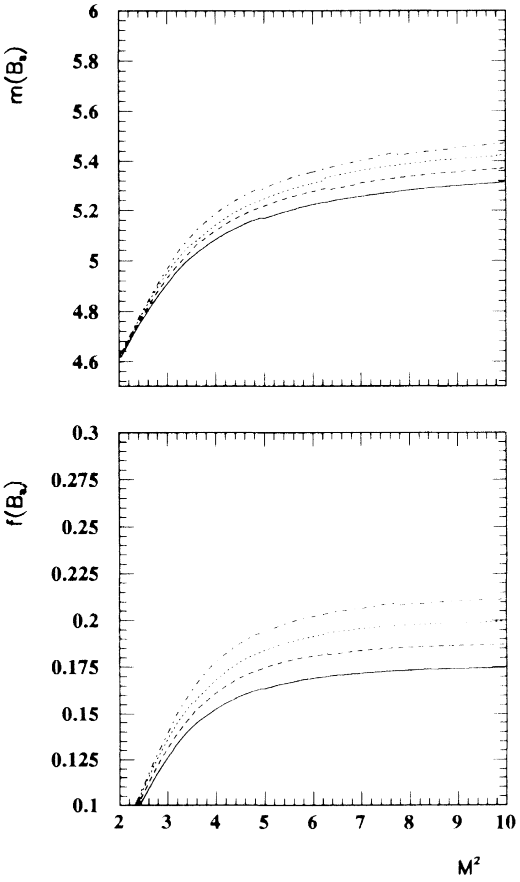

FIG. 1. Stability analysis for the mass and the leptonic constant of the B_s meson. The solid line corresponds to $s_0=33\ \mathrm{GeV}^2$, the dashed line to $s_0=34\ \mathrm{GeV}^2$, the dotted line to $s_0=35\ \mathrm{GeV}^2$ and the dashed-dotted line to $s_0=36\ \mathrm{GeV}^2$. M, f_{B_s}, and m_{B_s} are in GeV.

III. SEMILEPTONIC FORM FACTORS

The hadronic matrix elements of the transitions $\overline{B}_s^0 \to P^+ e^- \overline{\nu}_e$ and $\overline{B}_s^0 \to V^+ e^- \overline{\nu}_e$ (P and V are strange pseudoscalar and vector mesons, respectively) can be written in terms of form factors using the decomposition in Ref. [31]:

$$\langle P^+(p_P)|V_\mu|\overline{B}_s^0(p_{B_s})\rangle = F_1(q^2)(p_{B_s}+p_P)+\frac{m_{B_s}^2-m_P^2}{q^2}q_\mu[F_0(q^2)-F_1(q^2)] \ , \tag{21}$$

$$\langle V^+(p_V)|J_\mu|\overline{B}_s^0(p_{B_s})\rangle = \frac{2V(q^2)}{m_{B_s}+m_V}\epsilon_{\mu\alpha\rho\sigma}\epsilon^{*\alpha}p_{B_s}^\rho p_V^\sigma - i\left[(m_{B_s}+m_V)A_1(q^2)\epsilon_\mu^* - \frac{A_2(q^2)}{m_{B_s}+m_V}(\epsilon^*\cdot p_{B_s})(p_{B_s}+p_V)_\mu \right.$$
$$\left. -(\epsilon^*\cdot p_{B_s})\frac{2m_V}{q^2}q_\mu[A_3(q^2)-A_0(q^2)]\right] \ , \tag{22}$$

where $q^2=(p_{B_s}-p_{P,V})^2$ and $J_\mu=\overline{q}\gamma_\mu(1-\gamma_5)b$ ($q=c,u$); ϵ is the V^+ meson polarization vector. The conditions

$$F_1(0)=F_0(0) \ ,$$
$$A_3(0)=A_0(0) \tag{23}$$

must be implemented in Eqs. (21) and (22) in order to avoid unphysical poles at $q^2=0$; A_3 can be expressed in terms of A_1 and A_2:

$$A_3(q^2)=\frac{m_{B_s}+m_V}{2m_V}A_1(q^2)-\frac{m_{B_s}-m_V}{2m_V}A_2(q^2) \ . \tag{24}$$

In the limit of massless charged leptons the relevant form factors are F_1, V, A_1, and A_2. Their calculation by QCD sum rules [32] can be done by considering the three-point correlators

$$\Pi_\mu(p_{B_s},p_P,q)=(i)^2\int dx\, dy\, e^{i(p_P x - p_{B_s} y)} \times\langle 0|T[J^P(x)V_\mu(0)J^{B_s\dagger}(y)]|0\rangle \tag{25}$$

and

$$\Pi_{\mu\nu}^{V,A}=(i)^2\int dx\, dy\, e^{i(p_V x - p_{B_s} y)} \times\langle 0|T[J_\nu^V(x)J_\mu^{V,A}(0)J^{B_s\dagger}(y)]|0\rangle \ , \tag{26}$$

where

$$J^{B_s}(y)=\overline{s}(y)i\gamma_5 b(y) \ , \quad J^P(x)=\overline{s}(x)i\gamma_5 q(x) \ ,$$

$$J_\nu^V(x)=\overline{s}(x)\gamma_\nu q(x) \ .$$

For $q=c$ the last two currents interpolate the D_s^+ and D_s^{*+} meson, respectively, whereas, for $q=u$, J_ν^V interpolates K^{*+}.

The correlators in (25) and (26) can be decomposed in Lorentz-invariant structures:

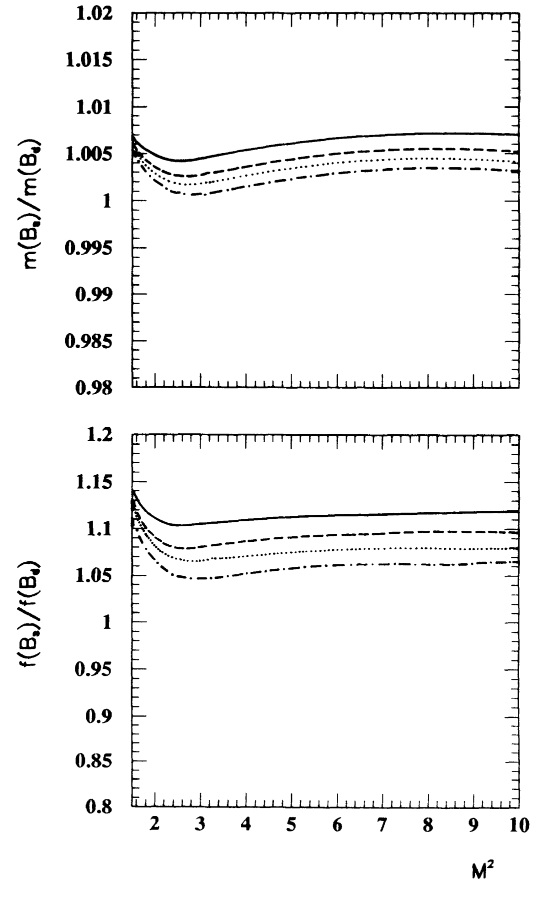

FIG. 2. Stability analysis for the ratios m_{B_s}/m_{B_d} and f_{B_s}/f_{B_d}. The symbols are the same as in Fig. 1.

$$\Pi_\mu(p_{B_s},p_P,q)=(p_{B_s}+p_P)_\mu\Pi+(p_{B_s}-p_P)_\mu\Pi' , \tag{27}$$

$$\Pi^V_{\mu\nu}(p_{B_s},p_V,q)=\epsilon_{\mu\nu\rho\sigma}p^\rho_V p^\sigma_{B_s}\Pi_V , \tag{28}$$

$$\Pi^A_{\mu\nu}(p_{B_s},p_V,q)=i[g_{\mu\nu}\Pi_1-(p_{B_s}+p_V)_\mu p_{B_{s\nu}}\Pi_2-(p_{B_s}-p_V)_\mu p_{B_{s\nu}}\Pi_3-p_{V\mu}(p_{B_s}+p_V)_\nu\Pi_4-p_{V\mu}(p_{B_s}-p_V)_\nu\Pi_5] . \tag{29}$$

The saturation of the p_{B_s} and $p_{P,V}$ channels by hadronic states provides the hadronic side of the sum rules. For the invariant structures Π, Π_V, Π_1, and Π_2 the following expressions can be written, keeping the contribution of the lowest lying resonances only:

$$\Pi^H=\left[\frac{f_{B_s}m^2_{B_s}}{m_b+m_s}\right]\left[\frac{f_P m^2_P}{m_q+m_s}\right]F_1(q^2)\frac{1}{p^2_{B_s}-m^2_{B_s}+i\epsilon}\,\frac{1}{p^2_P-m^2_P+i\epsilon} , \tag{30}$$

$$\Pi^H_V=\left[\frac{f_{B_s}m^2_{B_s}}{m_b+m_s}\right]\frac{m^2_V}{g_V}\,\frac{2V(q^2)}{m_{B_s}+m_V}\,\frac{1}{p^2_{B_s}-m^2_{B_s}+i\epsilon}\,\frac{1}{p^2_V--m^2_V+i\epsilon} , \tag{31}$$

$$\Pi^H_1=\left[\frac{f_{B_s}m^2_{B_s}}{m_b+m_s}\right]\frac{m^2_V}{g_V}(m_{B_s}+m_V)A_1(q^2)\frac{1}{p^2_{B_s}-m^2_{B_s}+i\epsilon}\,\frac{1}{p^2_V-m^2_V+i\epsilon} , \tag{32}$$

$$\Pi^H_1=\left[\frac{f_{B_s}m^2_{B_s}}{m_b+m_s}\right]\frac{m^2_V}{g_V}\,\frac{A_2(q^2)}{m_{B_s}+m_V}\,\frac{1}{p^2_{B_s}-m^2_{B_s}+i\epsilon}\,\frac{1}{p^2_V-m^2_V+i\epsilon} , \tag{33}$$

where

$$\langle 0|J^V_\mu|V(p_V,\epsilon)\rangle=(m^2_V/g_V)\epsilon_\mu .$$

On the other hand, the correlators can be computed, for $p^2_{B_s}, p^2_{P,V}\to-\infty$, by an operator product expansion in QCD in terms of a perturbative contribution and nonperturbative power corrections. For example, the perturbative contribution to Π in Eq. (27) reads

$$\Pi^P(p^2_{B_s},p^2_P,q^2)=\frac{1}{\pi^2}\int ds\,ds'\frac{\rho_P(s',s,q^2)}{(s'-p^2_{B_s})(s-p^2_P)} , \tag{34}$$

where

$$\rho_P(s,s',q^2)=\frac{3}{2\chi^{3/2}}\left[\frac{\chi}{2}(\Delta+\Delta')-\chi m_s(2m_s-m_b-m_\mu)\right. \tag{35}$$

$$-[2(s\Delta'+s'\Delta)-u(\Delta+\Delta')] \tag{36}$$

$$\left.\times(m^2_s-u/2+m_b m_q-m_q m_s-m_b m_s)\right] \tag{37}$$

with $\Delta=s-m^2_q+m^2_s$, $\Delta'=s'-m^2_b+m^2_s$, $\chi=(s+s'+q^2)-4ss'$, and $u=s+s'+q^2$; the integral in s,s' is within the domain bordered by the curves:

$$s(s')_\pm=\frac{2s'(m^2_q-m^2_s)-s'(m^2_b-m^2_s-s')\pm s'\sqrt{(m^2_b-m^2_s-s')^2-4s'm^2_s}}{2s'+(m^2_b-m^2_s-s')\pm\sqrt{(m^2_b-m^2_s-s')^2-4s'm^2_s}} . \tag{38}$$

The power corrections to Π [33], given in terms of quark and gluon condensates, read

$$\begin{aligned}\Pi^{\text{NP}}=&-\frac{\langle\bar{s}s\rangle}{2rr'}(m_b+m_q)\\&+\left[-m^2_s\langle\bar{s}s\rangle+\frac{1}{2}\langle\bar{s}\sigma gGs\rangle\right]\left[\frac{m^2_b(m_b+m_q)}{2rr'^3}-\frac{m_q}{4rr'^2}+\frac{m^2_q(m_b+m_q)}{2r^3r'}\right.\\&\left.\qquad-\frac{m_b}{4r^2r'}+\frac{(m_b+m_q)(m^2_b+m^2_q+Q^2)}{4r^2r'^2}+\frac{m_b}{4r^2r'}+\frac{m_q}{4rr'^2}\right]\\&+\frac{\langle\bar{s}\sigma gGs\rangle}{24}\left[\frac{4(m_b+2m_q)}{r^2r'}+\frac{4(m_q+2m_b)}{rr'^2}+\frac{(m_b+m_q)[(m_b-m_q)^2+Q^2]}{r^2r'^2}\right] ,\end{aligned} \tag{39}$$

where $r = p_P^2 - m_q^2$ and $r' = p_{B_s}^2 - m_b^2$. The perturbative spectral densities ρ_V, ρ_1, and ρ_2 and the power corrections to Π_V, Π_1, and Π_2 can be found in the Appendix.

We improve the matching between the hadronic side and the QCD side of the sum rule,

$$\Pi^H = \Pi^P + \Pi^{\rm NP} , \tag{40}$$

by performing a double Borel transform to the variables M'^2 and M^2 (conjugated to $-p_{P,V}^2$ and $-p_{B_s}^2$). This suppresses the higher-order power corrections in the QCD side of the sum rule by factorials, and enhances the contribution of the lowest-lying resonances in the hadronic side. By requiring stability in the variables M^2 and M'^2 and hierarchy in the power corrections and in the resonance-continuum contributions, a prediction for the form factors at $Q^2=0$ can be obtained. The quark masses, the condensates and the effective thresholds are the same as in the previous section (for $B_s \to K^*$ we use $s_0 = 1.2-1.3$ GeV^2); as for the leptonic constants of the vector mesons, we use $g_{D_s^*} = 8.3$ [from the relation $f_{D_s}/f_D = (m_{D_s^*}/g_{D_s^*})/(m_{D^*}/g_{D^*})$, with $g_{D^*} = 7.8$] and $g_{K^*} = 4.3$.

The results for the form factors of the transitions $B_s \to D_s, D_s^*$ at $Q^2=0$ are collected in Table I [$F_0(0)$, $A_0(0)$ and $A_3(0)$ are obtained from Eqs. (23) and (24)]. One can see that these values qualitatively agree with the predictions of the Bauer-Stech-Wirbel (BSW) model [31]. As for the Q^2 dependence, it can be obtained in principle by QCD sum rules. However, to avoid the relevant numerical uncertainties we prefer to assume a polar dependence dominated by the nearest resonance. These resonances are $\bar{b}c$ mesons whose mass, for the lowest-lying states, has been estimated in Ref. [34]; the 0^+ and 1^+ states are 500 MeV above 0^- and 1^-, as suggested by the splitting between S and P states in the D channel. In any case, the results for the semileptonic widths, as well as for the nonleptonic widths calculated in the following section, are quite insensitive to the exact position of the poles. As for the Cabibbo-suppressed transition $B_s \to K^*$, the results for the form factors at $Q^2=0$ are $V(0) = 0.12 \pm 0.02$, $A_1(0) = 0.3 \pm 0.1$, and $A_2(0) \simeq 0$; however, a test of the predictions based on these form factors is difficult.

Using the form factors in Table I we predict, for $V_{cb} = 0.045$,

$$\Gamma(\overline{B}_s^0 \to D_s^+ l^- \bar{\nu}) = (1.35 \pm 0.21) \times 10^{-14}\ \text{GeV} , \tag{41}$$

$$\Gamma(\overline{B}_s^0 \to D_s^{*+} l^- \bar{\nu}) = (2.5 \pm 0.1) \times 10^{-14}\ \text{GeV} . \tag{42}$$

An estimate of the SU(3)$_F$-breaking effects can be obtained by studying the ratios $F_1(B_s \to D_s)/F_1(B \to D)$, etc., with the result

$$\frac{F_1(B_s \to D_s)}{F_1(B \to D)} = 1.12 \pm 0.04 , \tag{43}$$

$$\frac{V(B_s \to D_s^*)}{V(B \to D^*)} = 1.3 \pm 0.1 , \tag{44}$$

$$\frac{A_1(B_s \to D_s^*)}{A_1(B \to D^*)} = 0.9 \pm 0.1 , \tag{45}$$

$$\frac{A_2(B_s \to D_s^*)}{A_2(B \to D^*)} = 1.3 \pm 0.1 . \tag{46}$$

IV. TWO-BODY NONLEPTONIC B_s DECAYS

We consider the two-body nonleptonic $\overline{B}_s^0$ decays induced by the effective weak Hamiltonian

$$H_W = \frac{G_F}{\sqrt{2}} V_{cb} V_{q_2 q_1}^* [c_1 (\bar{c}b)_L (\bar{q}_1 q_2)_L + c_2 (\bar{c} q_2)_L (\bar{q}_1 b)_L] . \tag{47}$$

The Wilson coefficients c_1 and c_2, evaluated at the b-quark mass scale $m_b \simeq 5$ GeV, are given by [35]

$$c_1(m_b) = 1.1, \quad c_2(m_b) = -0.24 . \tag{48}$$

The usual way to evaluate the matrix element of the operator (47) between the $\overline{B}_s^0$ state and, e.g., the $D_s^+ \pi^-$ state is to assume a factorization in the product of the $\langle D_s^+ | (\bar{c}b)_L | \overline{B}_s^0 \rangle$ matrix element and the $\langle \pi^- | (\bar{d}u)_L | 0 \rangle$ matrix element. One obtains

$$\langle D_s^+ \pi^- | H_W | \overline{B}_s^0 \rangle = \frac{G_F}{\sqrt{2}} V_{cb} V_{q_2 q_1}^* a_1 \langle D_s^+ | (\bar{c}b)_L | \overline{B}_s^0 \rangle \times \langle \pi^- | (\bar{d}u)_L | 0 \rangle , \tag{49}$$

with $a_1 = c_1 + c_2/N_c$ (N_c is the number of colors). In this way the nonleptonic amplitude is given in terms of the semileptonic matrix element parametrized in Eq. (21) and of the pion leptonic constant $f_\pi = 132$ MeV. As for the

TABLE I. Values at $q^2=0$ of the form factors appearing in the matrix elements of the decays $\overline{B}_s^0 \to D_s^+ (D_s^{*+}) l^- \bar{\nu}_l$. The quantum numbers and the mass of the poles which determine the q^2 dependence of the form factors is also shown.

$B_s \to D_s, D_s^*$	Value at $q^2=0$	J^P of the pole	Pole mass (GeV)
F_1	0.7 ± 0.1	1^-	6.3
F_0	0.7 ± 0.1	0^+	6.8
V	0.63 ± 0.05	1^-	6.3
A_0	0.52 ± 0.06	0^-	6.3
A_1	0.62 ± 0.01	1^+	6.8
A_2	0.75 ± 0.07	1^+	6.8
A_3	0.52 ± 0.06	1^+	6.8

TABLE II. Two-body nonleptonic $\bar{B}_s^0$ decay widths. The branching ratios are obtained for $\tau_{B_s}=1.2$ ps.

Decay mode	Width $\times(V_{cb}/0.045)^2$ (GeV)	Branching ratio
$\bar{B}_s^0 \to D_s^{*+}D_s^{*-}$	9×10^{-15}	1.6×10^{-2}
$\bar{B}_s^0 \to D_s^{*+}\rho^-$	7×10^{-15}	1.3×10^{-2}
$\bar{B}_s^0 \to D_s^{*+}D^{*-}$	5×10^{-16}	8×10^{-4}
$\bar{B}_s^0 \to D_s^{*+}K^{*-}$	4×10^{-16}	6×10^{-4}
$\bar{B}_s^0 \to D_s^{+}\rho^-$	7×10^{-15}	1.3×10^{-2}
$\bar{B}_s^0 \to D_s^{+}a_1$	6×10^{-15}	1.1×10^{-2}
$\bar{B}_s^0 \to D_s^{+}D_s^-$	6×10^{-15}	1×10^{-2}
$\bar{B}_s^0 \to D_s^{+}D_s^{*-}$	4×10^{-15}	8×10^{-3}
$\bar{B}_s^0 \to D_s^{+}\pi^-$	3×10^{-15}	5×10^{-3}
$\bar{B}_s^0 \to D_s^{*+}\pi^-$	1×10^{-15}	2×10^{-3}
$\bar{B}_s^0 \to D_s^{*+}D_s^-$	2×10^{-15}	4×10^{-3}
$\bar{B}_s^0 \to D_s^{*+}K^-$	1×10^{-16}	2×10^{-4}
$\bar{B}_s^0 \to D_s^{*+}D^-$	1×10^{-16}	2×10^{-4}
$\bar{B}_s^0 \to D_s^{+}D^-$	3×10^{-16}	5×10^{-4}
$\bar{B}_s^0 \to D_s^{+}K^-$	2×10^{-16}	4×10^{-4}
$\bar{B}_s^0 \to D_s^{+}D^{*-}$	2×10^{-16}	4×10^{-4}
$\bar{B}_s^0 \to D_s^{+}K^{*-}$	4×10^{-16}	6×10^{-4}

coefficient a_1, the analysis of nonleptonic $B_{u,d}$ decays shows that the rule of discarding $1/N_c$ corrections should be adopted [36]; we follow this rule and use $a_1=c_1=1.1$. The relevant leptonic constants are the same as in Sec. II, or they are fixed from the experimental data. The resulting nonleptonic widths for several two body $\bar{B}_s^0$ decays are collected in Table II; the branching ratios in the same table are obtained using $\tau_{B_s}=1.2$ ps [37].

It is worth observing that the channels with largest branching ratio, e.g., $\bar{B}_s^0\to D_s^+\pi^-$ or $\bar{B}_s^0\to D_s^+D_s^-$, could be revealed in the LEP experiments [38].

V. CONCLUSIONS

We have studied several aspects of the B_s meson phenomenology by QCD sum rules. Our main result concerns the possibility of obtaining the size of the SU(3)$_F$-breaking effects in this channel; we have shown that the method is sensitive to such effects and can predict them carefully.

The deviation of f_{B_s} from the leptonic constant of the B_d meson is around 10%; such deviation is of the same order as predicted by the heavy-quark effective chiral theory [14] but its origin is different since in the QCD sum rules approach it must be ascribed to the finite strange quark mass and to the value of the strange quark condensate, whereas in Ref. [14] the deviation is connected to chiral loops. SU(3)$_F$-breaking effects are at 10–20% level in the semileptonic form factors, i.e., at the same level as predicted by the heavy-quark effective chiral theory [39].

Finally, we have calculated the width of several nonleptonic B_s decays; some of them are in the LEP discovery potential.

APPENDIX

The perturbative spectral densities ρ in Eqs. (34) can be computed by applying the Cutkosky rule

$$\frac{i}{k^2-m^2}\to 2\pi\delta_+(k^2-m^2) \qquad (A1)$$

to the triangle diagrams corresponding to the three-point functions in Eqs. (25) and (26). For the vector and axial-vector current correlators these spectral densities are

$$\rho_V(s,s',q^2)=\frac{3}{\chi^{3/2}}[(2s'\Delta-u\Delta')(m_s-m_q)+(2s\Delta'-u\Delta)(m_s-m_b)+m_s\chi] , \qquad (A2)$$

$$\rho_1(s,s',q^2)=\frac{3}{\chi^{1/2}}\left[(m_b-m_s)\left[m_s^2+\frac{1}{\chi}(s'\Delta^2+s\Delta'^2-u\Delta\Delta')\right]-m_q\left[m_s^2-\frac{\Delta'}{2}\right]\right.$$
$$\left.-m_b\left[m_s^2-\frac{\Delta}{2}\right]+m_s\left[m_s^2-\frac{1}{2}(\Delta+\Delta'-u)+m_bm_q\right]\right] , \qquad (A3)$$

$$\begin{aligned}\rho_2(s,s',q^2)=\frac{3}{2\chi^{3/2}}\{&m_b(2s\Delta'-u\Delta+4\Delta\Delta'+2\Delta^2)+m_b m_s^2(4s-2u)+m_q(2s'\Delta-u\Delta')\\&-m_s[2(3s\Delta'+s'\Delta)-u(3\Delta+\Delta')+\chi+4\Delta\Delta'+2\Delta^2+m_s^2(4s-2u)]\\&+\frac{6}{\chi}(m_b-m_s)[4ss'\Delta\Delta'-u(2s\Delta\Delta'+s'\Delta^2+s\Delta'^2)+2s(s'\Delta^2+s\Delta'^2)]\}\ .\end{aligned}\tag{A4}$$

The nonperturbative power corrections can be computed by applying the fixed-point technique [23]. The result is

$$\Pi_V^{\langle\bar{s}s\rangle}=-\langle\bar{s}s\rangle\left[\frac{1}{rr'}-\frac{2m_b^2m_s^2}{rr'^3}-\frac{2m_q^2m_s^2}{r^3r'}+\frac{m_s^2(m_b^2+m_q^2+Q^2)}{r^2r'^2}\right],\tag{A5}$$

$$\Pi_V^{\langle\bar{s}\sigma Gs\rangle}=\frac{1}{6}\langle\bar{s}\sigma gGs\rangle\left[\frac{3m_q^2}{r^3r'}+\frac{3m_b^2}{rr'^3}-\frac{2}{rr'^2}+\frac{1}{r^2r'^2}(2m_q^2+2m_b^2-m_qm_b+2Q^2)\right],\tag{A6}$$

$$\begin{aligned}\Pi_1^{\langle\bar{s}s\rangle}=-\langle\bar{s}s\rangle\Bigg[&\frac{1}{rr'}\left[\frac{1}{2}[(m_b+m_q)^2+Q^2]-\frac{m_s^2}{2}\right]+\frac{1}{2r}+\frac{1}{2r'}+\frac{m_s^2}{4}\frac{1}{r^2r'}[(m_b+m_q)^2+Q^2]\\&+\frac{m_s^2}{4}\frac{1}{rr'^2}[(m_b+m_q)^2+Q^2]+\frac{m_q^2m_s^2}{r^3r'}(m_bm_q+m_b^2+m_q^2+Q^2)+\frac{m_b^2m_s^2}{rr'^3}(m_bm_q+m_b^2+m_q^2+Q^2)\\&+\frac{m_s^2}{4}\frac{m_b^2+m_q^2+Q^2}{r^2r'^2}[(m_b+m_q)^2+Q^2]-\frac{m_s^2m_q^2}{2r^3}-\frac{m_s^2m_b^2}{2r'^3}\Bigg],\end{aligned}\tag{A7}$$

$$\begin{aligned}\Pi_1^{\langle\bar{s}\sigma Gs\rangle}=\frac{1}{12}\langle\bar{s}\sigma gGs\rangle\Bigg[&\frac{3m_q^2}{r^3r'}(m_q^2+m_b^2+2m_bm_q+Q^2)+\frac{3m_b^2}{rr'^3}(m_b^2+m_q^2+2m_bm_q+Q^2)\\&+\frac{1}{r^2r'^2}\{3m_bm_q(m_b^2+m_q^2+Q^2)+2[(m_b^2+m_q^2+Q^2)^2-m_bm_q]\}\\&+\frac{1}{r^2r'}[3m_q(m_b+m_q)+2(m_b^2+Q^2)]+\frac{1}{rr'^2}[3m_b(3m_q+m_b)+4(m_q^2+Q^2)]-\frac{2}{rr'}\\&+\frac{3m_q^2}{r^3}+\frac{3m_b^2}{r'^3}+\frac{2}{r'^2}\Bigg],\end{aligned}\tag{A8}$$

$$\Pi_2^{\langle\bar{s}s\rangle}=-\frac{1}{2}\langle\bar{s}s\rangle\left[\frac{1}{rr'}+\frac{m_s^2m_q^2}{r^3r'}+\frac{m_s^2m_b^2}{rr'^3}+\frac{(m_b^2+m_q^2+Q^2)m_s^2}{2r^2r'^2}-\frac{m_s^2}{rr'^2}\right],\tag{A9}$$

$$\Pi_2^{\langle\bar{s}\sigma Gs\rangle}=\frac{1}{12}\langle\bar{s}\sigma gGs\rangle\left[\frac{3m_q^2}{r^3r'}+\frac{3m_b^2}{rr'^3}-\frac{2}{rr'^2}+\frac{1}{r^2r'^2}(2m_q^2+2m_b^2+2Q^2-m_bm_q)\right].\tag{A10}$$

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