Simply modeling $\bar{B} \to \bar{K}^* \gamma$

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A simple relativistic model of heavy-light mesons is applied to the rare decay $\bar{B} \to \bar{K}^* \gamma$ in the standard model. We find $\Gamma(\bar{B} \to \bar{K}^* \gamma)/\Gamma(b \to s\gamma) = (17 \pm 4)\%$ and $B(\bar{B} \to \bar{K}^* \gamma) = (4.8 \pm 1.9) \times 10^{-5}(|V_{cb}|/0.04)^2$. These numbers are reduced by only 20% when the relevant form factor is replaced by its value in the heavy-quark limit $m_b, m_s \to \infty$.

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We have recently developed a relativistic model of mesons containing a heavy quark Q and a light antiquark $\bar{q}$ [1, 2]. Matrix elements for $Q_1\bar{q} \to Q_2\bar{q}$ meson transitions are represented by quark loop graphs with $\bar{Q}_2 Q_1$-type operator insertions on the heavy-quark line. The external mesons are joined to the loop with vertices of the form $Z^2/(-k^2+\Lambda^2)$, where k is the light-quark momentum. These vertices suppress large momentum flow into the light quark, which is the essential physical effect of the light-quark wave function. The Z's and Λ's are different for each meson flavor and spin, but are not arbitrary parameters. They are fixed in terms of the heavy and light masses appearing in the standard quark propagators by requiring the meson self-energies to vanish and have unit slope at the physically measured meson masses.

Central to the application of a free quark model is the assumption that QCD confinement is characterized by somewhat smaller momentum scales than typical light-quark momenta in a heavy-light meson. This situation is realized by our model, and it thus suggests the possibility that confinement may not play an essential role. Indeed, our model results are obtained by simply dropping the imaginary parts arising from the free quark loop diagrams. The success or failure of such a model will shed light on the role played by confinement. Thus far we have found that the model yields a differential decay spectrum for $\bar{B} \to D^* \ell \bar{\nu}$ whose shape compares well with the data when $m_b = 4800$ MeV, $m_c = 1440$ MeV, and $m_q = 250$ MeV [3]. The model may also be expanded in inverse powers of heavy-quark masses and the vector- and axial-vector current form factors have been shown in [1] to be consistent with all heavy-quark symmetry constraints through order $1/m_Q$ [4, 5].

The purpose of this paper is to apply our model to the rare decay $\bar{B} \to \bar{K}^* \gamma$ [6] within the context of the standard model. We will be mainly interested in the results of our full, unexpanded model, but we will also compare these results to those obtained in the heavy-quark limit. Because the strange quark is not particularly heavy, there is no *a priori* reason to expect there to be any resemblance. We will see, however, that certain quantities have surprisingly small net corrections.

The relevant $\bar{s}b$-type operators are [7]

$$\mathcal{O}_\mu = \bar{s}i\sigma_{\mu\nu}q^\nu b \quad \text{and} \quad \mathcal{O}_{5\mu} = \bar{s}i\sigma_{\mu\nu}q^\nu \gamma_5 b. \qquad (1)$$

Form factors for $\bar{B}(M, V) \to \bar{K}^*(m, v, \varepsilon)$ may be defined by

$$\langle \bar{K}^* | \mathcal{O}_\mu | \bar{B} \rangle = \sqrt{Mm}(M + m)h(\omega)\varepsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}V^\rho v^\sigma , \qquad (2)$$

$$\langle \bar{K}^* | \mathcal{O}_{5\mu} | \bar{B} \rangle = -i\sqrt{Mm} \left\{ [(M - m)(\omega + 1)g_{\mu\nu} - M(V + v)_\mu V_\nu] h_5(\omega) \right. $$
$$\left. + [(M + m)(\omega - 1)g_{\mu\nu} + M(V - v)_\mu V_\nu] h_5'(\omega) \right\} \varepsilon^{*\nu} , \qquad (3)$$

where $\omega = V \cdot v$ is the product of the meson four-velocities. The three form factors h, h_5, and h_5' are not independent at the physical recoil point $\omega_0 = (M^2 + m^2)/2Mm = 3.04$, where they satisfy $h(\omega_0) = h_5(\omega_0) + (M - m)(M + m)^{-1}h_5'(\omega_0)$. In the heavy-quark limit $m_b, m_s \to \infty$, we find $h(\omega) = h_5(\omega) = \xi(\omega)$ and $h_5'(\omega) = 0$, where ξ is the same Isgur-Wise function appearing in meson semileptonic decays.

As described below we will fix the m_b and m_s masses

by going to a point of minimal sensitivity in the m_b-m_s plane. It is reassuring to find that this point occurs at the physically reasonable values $m_b = 4830$ MeV and $m_s = 400$ MeV. As in our earlier work, we choose $m_q = 250$ MeV as a reasonable light-quark constituent mass [2]. This is somewhat smaller than the usual 330 MeV and it models the fact that the actual momentum-dependent light-quark mass has fallen somewhat at typical light-quark momenta in the loop. We will find little sensitivity to the choice of m_q. The vertices are given explicitly by

$$\gamma_5 \frac{Z_B^2}{-k^2 + \Lambda_B^2} \quad \text{and} \quad -i\gamma_\nu \frac{Z_{K^*}^2}{-k^2 + \Lambda_{K^*}^2}, \qquad (4)$$

and the various constants are determined by the physical

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masses $M = 5279$ MeV and $m = 892$ MeV to be $\Lambda_B = 577$ MeV, $Z_B = 1013$ MeV, $\Lambda_{K^*} = 617$ MeV, and $Z_{K^*} = 871$ MeV. These values of Λ characterize the typical light-quark momenta.

The form factors computed using the full vertices of Eq. (4) are shown along with ξ in Fig. 1. Numerical results are shown in (5) at the zero recoil point $\omega = 1$ and at the physical recoil point ω_0:

ω	h	h_5	h_5'	ξ
1	1.27	0.964	0.207	1
ω_0	0.262	0.205	0.080	0.235

$$(5)$$

We see that the net deviations from the heavy-quark limit are in general not as large as one might have expected. At the physical recoil point h and h_5 differ from ξ by less than 15%. But this does not mean that the $1/m_s$ expansion makes any sense; indeed if we write

$$h(\omega_0) = \xi(\omega_0) \{1 + A/m_s + B/m_b\} , \qquad (6)$$

we find numerically that A and B are of order -150 MeV. This gives a correction going in the opposite direction from the full result, and thus the higher-order terms must be significant.

We also find that the leading-order corrections to h and h_5 vanish at zero recoil in the model; this is analogous to Luke's theorem [4]. In particular, $h(1) - 1 = 0.27$ is entirely due to effects at order $1/m_Q^2$ and beyond.

The $b \to s\gamma$ vertex in the effective theory obtained by integrating out the W boson and the top quark in the standard model is [7]

$$\Gamma_\mu = \kappa[(1 + r)\mathcal{O}_\mu + (1 - r)\mathcal{O}_{5\mu}], \qquad (7)$$

where $\kappa = eG_F V_{ts}^* V_{tb} \bar{F}_2 m_b / 8\sqrt{2}\pi^2$ and $r = m_s/m_b$. The coefficient $\bar{F}_2$ depends on the mass m_t of the top quark and contains the effects of QCD scaling from $\mu = M_W$ down to $\mu = m_b$. In the leading logarithmic approximation it is given by [8]

$$\bar{F}_2 \approx \eta^{-16/23} \left\{ F_2(m_t^2/M_W^2) + 116(\eta^{10/23} - 1)/135 + 58(\eta^{28/23} - 1)/189 \right\}, \qquad (8)$$

where $\eta = \alpha_s(m_b)/\alpha_s(M_W)$ and [7]

$$F_2(x) = \frac{8x^3 + 5x^2 - 7x}{12(x-1)^3} - \frac{3x^3 - 2x^2}{2(x-1)^4} \ln x. \qquad (9)$$

(We note that results in the next-to-leading-logarithmic approximation have been computed and are not drastically different [9].)

Using $\eta = \ln(M_W/\Lambda_{\rm QCD})/\ln(m_b/\Lambda_{\rm QCD}) = 1.96$ with $\Lambda_{\rm QCD} = 250$ MeV, we find $\bar{F}_2/F_2 = 1.92$ for $m_t = 135$ GeV. This is a well-known large enhancement factor from short-distance QCD [8, 10, 11]. We note in passing that attempts have been made to estimate the contribution of internal $\psi - \gamma$ conversion to $\bar{B} \to \bar{K}^* \gamma$ via vector meson dominance [12]. However, the Wilson coefficient of the relevant four-quark operator at $\mu = m_b$ is suppressed by roughly a factor of 3 compared with its value at $\mu = M_W$ [11]. We therefore neglect this contribution compared with the QCD-enhanced short-distance one.
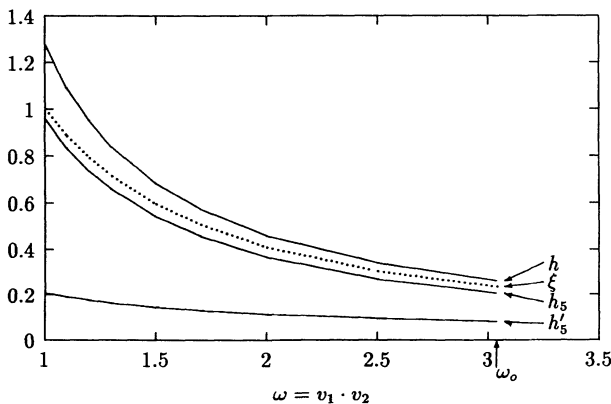
The width for $\bar{B} \to \bar{K}^* \gamma$ is given by

$$\Gamma(\bar{B} \to \bar{K}^* \gamma) = M^3 |\kappa|^2 (16\pi R)^{-1} (1 - R^2)^3 \times (1 + R)^2 (1 + r^2) h(\omega_0)^2, \qquad (10)$$

where[1] $R = m/M$. A fundamental quantity is the ratio of the exclusive $\bar{B} \to \bar{K}^* \gamma$ decay width to that of the inclusive $\bar{B} \to X_s \gamma$ decay. This may be taken to be equal to the quark-level $b \to s\gamma$ width, given by

$$\Gamma(b \to s\gamma) = m_b^3 |\kappa|^2 (4\pi)^{-1} (1 - r^2)^3 (1 + r^2), \qquad (11)$$

where $r = m_s/m_b$. (The corrections to this relation were shown in [13] to be of order $1/m_b^2$ and may be neglected here.) The Kobayashi-Maskawa elements, top quark mass, and QCD scaling effects cancel in the ratio, and we find

$$\Gamma(\bar{B} \to \bar{K}^* \gamma)/\Gamma(b \to s\gamma) = 17\%. \qquad (12)$$

Because $V_{ts}^* V_{tb}$ is not directly measured, it is convenient to use unitarity and the smallness of V_{ub} to write $V_{ts}^* V_{tb} \approx -V_{cs}^* V_{cb}$ and express the branching ratio in terms of V_{cb}. With $|V_{cs}| = 0.974$, $\tau_B = 1.5 \times 10^{-12}$ s, and $m_t = 135$ GeV, we find

$$B(\bar{B} \to \bar{K}^* \gamma) = 4.8 \times 10^{-5} (|V_{cb}|/0.04)^2. \qquad (13)$$

The uncertainties due to a ± 100 MeV shift in $\Lambda_{\rm QCD}$ and a ± 25 GeV shift in m_t are $\pm 10\%$ each. The rate for $\bar{B} \to \bar{K}^* \gamma$ is found to decrease by 20% if the form factor $h(\omega_0)$ in (10) is replaced by its value in the heavy-quark limit.

FIG. 1. Model results for form factors h, h_5 and h_5' and Isgur-Wise function ξ. The physical recoil point is $\omega = \omega_0$.

[1] The apparent $1/m$ behavior of Eq. (10) as $m \to 0$ is canceled by the decrease in $h(\omega_0)$ as $\omega_0 \propto 1/m \to \infty$.
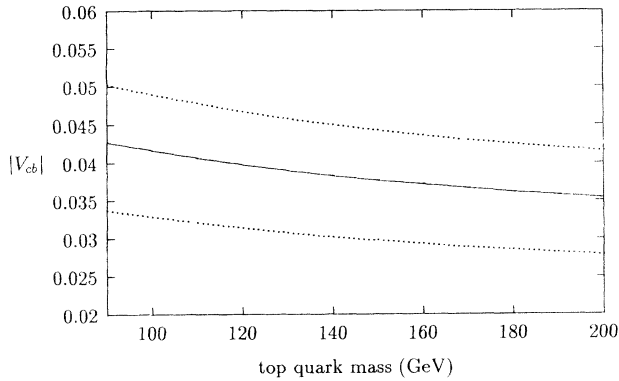
FIG. 2. Region in $|V_{cb}|$-m_t plane (between dotted lines) allowed by data. The solid line corresponds to the central value of the branching ratio. The figure is plotted for $m_b = 4830$ MeV and $m_s = 400$ MeV.

The CLEO Collaboration [14] has recently reported a branching ratio of $(4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$. We show in Fig. 2 how $|V_{cb}|$ is constrained by the data as a function of m_t.

We now discuss the sensitivity of these results to the quark masses. When $h(\omega_0)$ is plotted as a function of m_b and m_s with m_q held fixed at 250 MeV, there is a saddle point at $m_b = 4830$ MeV and $m_s = 400$ MeV. This point represents the point of minimal sensitivity to the choice of quark masses, and we adopt it as our standard reference point. (Parameters appearing in the QCD sum rule approach are handled similarly.) In the circu-

lar region of radius 40 MeV centred at this point, we find that $h(\omega_0)$ varies by less than 7.5% from its value at the saddle point. We thus estimate a 15% uncertainty in the branching ratio due to the b and s quark masses. The heightened sensitivity to the quark masses which occurs farther away from the saddle point has been discussed in [3] in the context of $\bar{B} \to D^{(*)}$ semileptonic decays. There, it was stressed that the sensitivity is the expected result of constraining the meson masses to their physical values, and not a breakdown of the heavy-quark expansion in the model. We also find that varying the light-quark mass m_q by 40 MeV, with the b and s masses fixed, changes the branching ratio by less than 5%. It will be possible in the future to more accurately determine the quark masses appropriate to the model from heavy-meson and -baryon semileptonic decays.

A recent lattice calculation [15] found $h(\omega_0) = 0.20^{+0.17}_{-0.20}$. The result was observed to be approximately independent of the spectator quark mass m_q, in agreement with the present analysis. The authors of [15] extracted $h(\omega_0) = 0.21^{+0.07}_{-0.05}$ when this independence was assumed, consistent with the present value of 0.26.

In conclusion, we find in our model $\Gamma(\bar{B} \to \bar{K}^*\gamma)/\Gamma(b \to s\gamma) = (17 \pm 4)\%$ and $B(\bar{B} \to \bar{K}^*\gamma) = (4.8 \pm 1.9) \times 10^{-5}(|V_{cb}|/0.04)^2$. These compare well with values of $(20 \pm 6)\%$ and $(6.8 \pm 2.4) \times 10^{-5}(|V_{cb}|/.035)^2$ obtained in one recent QCD sum rule approach [16], and $(17 \pm 5)\%$ and $(4 \pm 1) \times 10^{-5}$ in another [17], and with recent lattice results.

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