Decay $B \to \pi \ell \nu$ in heavy quark effective theory

Gustavo Burdman,[1] Zoltan Ligeti,[2] Matthias Neubert,[3] and Yosef Nir[2]
[1] *Department of Physics and Astronomy, University of Massachusetts, Amherst, Massachusetts 01003*
[2] *Weizmann Institute of Science, Physics Department, Rehovot 76100, Israel*
[3] *Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309*
(Received 16 September 1993)

We present a systematic analysis of the $B^{(*)} \to \pi \ell \nu$ weak decay form factors to order $1/m_b$ in the heavy quark effective theory, including a discussion of renormalization-group effects. These processes are described by a set of ten universal functions (two at leading order, and eight at order $1/m_b$), which are defined in terms of matrix elements of operators in the effective theory. In the soft pion limit, the effective theory yields normalization conditions for these functions, which generalize the well-known current algebra relations derived from the combination of heavy quark and chiral symmetries to next-to-leading order in $1/m_b$. In particular, the effects of the nearby B^* pole are correctly contained in the form factors of the effective theory. We discuss the prospects for a model-independent determination of $|V_{ub}|$ and the $BB^*\pi$ coupling constant from these processes.

PACS number(s): 13.20.He, 12.39.Hg, 14.40.Nd

I. INTRODUCTION

Of the three independent mixing angles in the Cabibbo–Kobayashi–Maskawa matrix, $|V_{ub}|$ is the most poorly determined. Chiral symmetry provides an absolute normalization of the hadronic form factor in the decay $K \to \pi \ell \nu$, allowing a precise and model-independent determination of $|V_{us}|$ [1]. Heavy quark symmetry provides absolute normalization and various relations among the form factors in the decays $B \to D^{(*)} \ell \nu$, allowing a precise and model-independent determination of $|V_{cb}|$ [2–6]. Neither of these symmetries is as powerful in heavy-to-light transitions such as $b \to u \ell \bar{\nu}$. Consequently, the present determination of $|V_{ub}|$ from the end point of the lepton spectrum in semileptonic B decays suffers from large theoretical uncertainties and strong model dependence [7,8].

It was suggested that the exclusive semileptonic decay mode $B \to \pi \ell \nu$ could be used for a more reliable determination of $|V_{ub}|$ [9]. The basis for this hope is the fact that, to leading order in the heavy quark expansion and over a limited kinematic range, the corresponding form factors are related to those of $D \to \pi \ell \nu$ by heavy quark flavor symmetry. The applicability of this idea depends, besides experimental considerations, on the importance of symmetry-breaking corrections of order $1/m_Q$. For the related case of leptonic decays of heavy mesons, there are indications from lattice gauge theory [10–13] and QCD sum-rule calculations [14–17] that these power corrections can be significant.

Our purpose in this study is to work out the structure of $1/m_b$ corrections for the $B^{(*)} \to \pi \ell \nu$ decay form factors using the heavy quark effective theory. The main points of our analysis are as follows.

(i) Eight universal functions are needed to describe the $1/m_b$ corrections to these processes. They are defined in terms of matrix elements of dimension-four operators in the effective theory.

(ii) The renormalization-group improvement of these low-energy parameters is discussed in detail.

(iii) The behavior of the universal functions in the soft pion limit is derived using standard current algebra techniques.

(iv) It is shown explicitly that the B^*-pole contribution is correctly contained in the heavy quark effective theory.

At leading order in the heavy quark expansion, the two form factors which parametrize $B \to \pi \ell \nu$ decays have been investigated by several authors [9,18–21]. It is well known that, in this limit, the soft-pion behavior is fully determined by the decay constant of the B meson and the $BB^*\pi$ coupling constant. Here we generalize these results to next-to-leading order in $1/m_b$. In particular, we show that when one uses the physical meson decay constants and the physical $BB^*\pi$ coupling constants (as opposed to their asymptotic values in the $m_b \to \infty$ limit), there are neither $1/m_b$ nor short-distance QCD corrections to the soft pion relations. We also derive the general structure of the decay form factors at larger pion momenta, where a chiral expansion is no longer valid.

The paper is organized as follows: The formalism of heavy quark effective theory relevant to our work is reviewed in Sec. II. In Sec. III we then construct the heavy quark expansion for the $B^{(*)} \to \pi \ell \nu$ decay form factors to next-to-leading order in $1/m_b$, including a detailed analysis of renormalization-group effects. In Sec. IV we derive the normalization conditions for the universal functions of the effective theory, which arise in the soft pion limit. We compare our results to the predictions of the so-called heavy meson chiral perturbation theory [19,20]. Section V contains a summary and some concluding remarks concerning the prospects and possibilities to obtain a model-independent measurement of $|V_{ub}|$ and the $BB^*\pi$ coupling constant. Technical details related to the renormalization-group improvement and the soft pion limit are described in two Appendixes.

II. THE $1/m_Q$ EXPANSION

Our goal in this paper is to analyze the dependence of the hadronic form factors describing $B \to \pi\,\ell\,\nu$ decays on the mass of the b quark, in the limit where $m_b \gg \Lambda_{\rm QCD}$. A convenient tool to make this dependence explicit is provided by the heavy quark effective theory (HQET) [22–30]. It is based on the construction of an effective low-energy Lagrangian of QCD, which is appropriate to describe the soft interactions of a heavy quark with light degrees of freedom. In the effective theory, a heavy quark bound inside a hadron moving at velocity v is described by a velocity-dependent field h_v, which is related to the conventional quark field in QCD by [24]

$$h_v(x) = \exp(im_Q\,v\cdot x)\,\frac{1+\not v}{2}\,Q(x)\,. \qquad (1)$$

By means of the phase redefinition one removes the large part of the heavy quark momentum from the new field. When the total momentum is written as $P = m_Q\,v + k$, the field h_v carries the residual momentum k, which results from soft interactions of the heavy quark with light degrees of freedom and is typically of order $\Lambda_{\rm QCD}$. The matrix $\frac{1}{2}(1+\not v)$ projects out the heavy quark (rather than antiquark) components of the spinor. The antiquark components are integrated out to obtain the effective Lagrangian [22,24,25,28]

$$\mathcal{L}_{\rm eff} = \bar h_v\,iv\cdot D\,h_v + \frac{1}{2m_Q}\Big[\mathcal{O}_{\rm kin} + C_{\rm mag}(\mu)\,\mathcal{O}_{\rm mag}\Big] + O(1/m_Q^2)\,, \qquad (2)$$

where $D^\mu = \partial^\mu - ig_s T_a A_a^\mu$ is the gauge-covariant derivative. The operators appearing at order $1/m_Q$ are

$$\mathcal{O}_{\rm kin} = \bar h_v\,(iD)^2 h_v\,, \qquad \mathcal{O}_{\rm mag} = \frac{g_s}{2}\,\bar h_v\,\sigma_{\mu\nu}G^{\mu\nu}h_v\,. \quad (3)$$

Here $G^{\mu\nu}$ is the gluon field-strength tensor defined by $[iD^\mu, iD^\nu] = ig_s G^{\mu\nu}$. In the rest frame of the hadron, it is readily seen that $\mathcal{O}_{\rm kin}$ describes the kinetic energy resulting from the residual motion of the heavy quark, whereas $\mathcal{O}_{\rm mag}$ describes the chromomagnetic coupling of the heavy quark spin to the gluon field. One can show that, to all orders in perturbation theory, the kinetic operator $\mathcal{O}_{\rm kin}$ is not renormalized [31]. The renormalization factor $C_{\rm mag}(\mu)$ of the chromomagnetic operator has been calculated in leading logarithmic approximation and is given by [28]

$$C_{\rm mag} = x^{-3/\beta}\,, \qquad \beta = 11 - \tfrac{2}{3}\,n_f\,, \qquad (4)$$

where $x = \alpha(\mu)/\alpha(m_Q)$, μ denotes the renormalization scale, and n_f is the number of light quarks with mass below m_Q.

Any operator of the full theory that contains one or more heavy quark fields can be matched onto a short-distance expansion in terms of operators of the effective theory. In particular, the expansion of the heavy-light vector current reads

$$\bar q\,\gamma^\mu\,Q \cong \sum_i C_i(\mu)\,J_i + \frac{1}{2m_Q}\sum_j B_j(\mu)\,\mathcal{O}_j + O(1/m_Q^2)\,, \qquad (5)$$

where the symbol $\cong$ is used to indicate that this is an equation that holds on the level of matrix elements. The operators $\{J_i\}$ form a complete set of local dimension-three current operators with the same quantum numbers as the vector current in the full theory. In HQET there are two such operators:

$$J_1 = \bar q\,\gamma^\mu h_v\,, \qquad J_2 = \bar q\,v^\mu h_v\,. \qquad (6)$$

Similarly, $\{\mathcal{O}_j\}$ denotes a complete set of local dimension-four operators. It is convenient to use the background field method, which ensures that there is no mixing between gauge-invariant and gauge-dependent operators. Moreover, operators that vanish by the equations of motion are irrelevant. It is thus sufficient to consider gauge-invariant operators that do not vanish by the equations of motion. A convenient basis of such operators is [29]

$$\begin{aligned}
&\mathcal{O}_1 = \bar q\,\gamma^\mu\,i\not D\,h_v\,, && \mathcal{O}_4 = \bar q\,(-iv\cdot\overleftarrow{D})\,\gamma^\mu h_v\,,\\
&\mathcal{O}_2 = \bar q\,v^\mu\,i\not D\,h_v\,, && \mathcal{O}_5 = \bar q\,(-iv\cdot\overleftarrow{D})\,v^\mu h_v\,, \qquad (7)\\
&\mathcal{O}_3 = \bar q\,iD^\mu h_v\,, && \mathcal{O}_6 = \bar q\,(-i\overleftarrow{D^\mu})\,h_v\,.
\end{aligned}$$

For simplicity, we consider here the limit where the light quark is massless. Otherwise one would have to include two additional operators $\mathcal{O}_7 = m_q\,J_1$ and $\mathcal{O}_8 = m_q\,J_2$. It is convenient to work with a regularization scheme with anticommuting γ_5. This has the advantage that, to all orders in $1/m_Q$, the operator product expansion of the axial vector current can be simply obtained from (5) by replacing $\bar q \to -\bar q\,\gamma_5$ in the HQET operators. The Wilson coefficients remain unchanged. The reason is that in any diagram the γ_5 from the current can be moved outside next to the light quark spinor. For $m_q = 0$, this operation always leads to a minus sign. Hence it is sufficient to consider the case of the vector current.

A "hidden" symmetry of the effective theory, namely, its invariance under reparametrizations of the heavy quark velocity and residual momentum which leave the total momentum unchanged [31], determines three of the coefficients $B_i(\mu)$. It implies that, to all orders in perturbation theory [32],

$$B_1(\mu) = C_1(\mu)\,, \;\; B_2(\mu) = \tfrac{1}{2}\,B_3(\mu) = C_2(\mu)\,. \qquad (8)$$

The remaining coefficients in (5) can be obtained from the solution of the renormalization-group equation that determines the scale dependence of the renormalized current operators in HQET. For our purposes, it will be sufficient to know these coefficients in leading logarithmic approximation. They are [4,29,33]

$$\begin{aligned}
C_1(\mu) &= x^{2/\beta}\,, \qquad C_2(\mu) = 0\,,\\
B_4(\mu) &= \tfrac{34}{27}\,x^{2/\beta} - \tfrac{4}{27}\,x^{-1/\beta} - \tfrac{10}{9} + \tfrac{16}{3\beta}\,x^{2/\beta}\ln x\,,\\
B_5(\mu) &= -\tfrac{28}{27}\,x^{2/\beta} + \tfrac{88}{27}\,x^{-1/\beta} - \tfrac{20}{9}\,,\\
B_6(\mu) &= -2\,x^{2/\beta} - \tfrac{4}{3}\,x^{-1/\beta} + \tfrac{10}{3}\,, \qquad (9)
\end{aligned}$$

where again $x = \alpha(\mu)/\alpha(m_Q)$.

After the effective Lagrangian and currents have been constructed, one proceeds to parametrize the relevant hadronic matrix elements of the HQET operators in terms of universal, m_Q-independent form factors. In the effective theory, hadrons containing a heavy quark can be represented by covariant tensor wave functions, which are determined completely by their transformation properties under the Lorentz group and heavy quark symmetry. In particular, the ground-state pseudoscalar and vector mesons are described by [26,34]

$$\mathcal{M}(v) = \frac{1+\not v}{2}\begin{cases} -\gamma_5 & \text{pseudoscalar meson,} \\ \not\epsilon & \text{vector meson.}\end{cases} \tag{10}$$

Here ϵ^μ is the polarization vector of the vector meson. Any matrix element of an operator of the effective theory can be written as a trace over such wave functions, whose structure is determined by symmetry and by the Feynman rules of the effective theory.

We will now develop this formalism for $B \to \pi$ transitions. Matrix elements of the leading-order currents J_i in (6) can be written as (see, e.g., Ref. [35])

$$\langle\pi(p)|\,\bar q\,\Gamma\,h_v\,|M(v)\rangle = -\mathrm{Tr}\Big\{\Pi(v,p)\,\Gamma\,\mathcal{M}(v)\Big\}\,, \tag{11}$$

where Γ is an arbitrary Dirac matrix. Note that we use a mass-independent normalization of meson states to $2v^0$ (instead of $2p^0$), as this is more convenient when dealing with heavy quark systems. The Feynman rules of HQET imply that there cannot appear any γ matrices on the right-hand side of Γ. The matrix $\Pi(v,p)$ must transform as a pseudoscalar, but is otherwise a general function of v and p. Using the fact that $\mathcal{M}(v)\,\not v = -\mathcal{M}(v)$, we can write down the most general decomposition

$$\Pi(v,p) = \gamma_5\,\Big[A(v\cdot p,\mu) + \not{\widehat p}\,B(v\cdot p,\mu)\Big]\,. \tag{12}$$

We find it convenient to introduce the dimensionless variable

$$\widehat p^\mu = \frac{p^\mu}{v\cdot p}\,, \qquad v\cdot\widehat p = 1\,, \tag{13}$$

so that the scalar functions $A(v\cdot p,\mu)$ and $B(v\cdot p,\mu)$ have the same dimension. These universal form factors depend on the kinematic variable $v\cdot p$. They also depend on the scale μ at which the HQET operators are renormalized, but not on the heavy quark mass m_Q. These functions are the analogs of the celebrated Isgur-Wise function, which describes heavy-to-heavy meson transitions at leading order in HQET [5].

Let us now turn to the study of the leading power corrections proportional to $1/m_Q$, which arise from the corrections both to the currents and to the effective Lagrangian of HQET. We first consider the dimension-four operators in the expansion of the currents (5). Matrix elements of the operators $\mathcal{O}_1$, $\mathcal{O}_2$, and $\mathcal{O}_3$, which contain a covariant derivative acting on the heavy quark field, have the generic structure

$$\begin{aligned}&\langle\pi(p)|\,\bar q\,\Gamma\,iD^\mu h_v\,|M(v)\rangle \\ &\quad = -\mathrm{Tr}\Big\{\Big[(F_1\,v^\mu + F_2\,\widehat p^\mu + F_3\,\gamma^\mu)\,\gamma_5 \\ &\qquad + (F_4\,v^\mu + F_5\,\widehat p^\mu + F_6\,\gamma^\mu)\,\gamma_5\,\not{\widehat p}\Big]\,\Gamma\,\mathcal{M}(v)\Big\}\,.\end{aligned} \tag{14}$$

The functions $F_i(v\cdot p,\mu)$ are new low-energy parameters. They, again, depend only on the kinematic variable $v\cdot p$ and the renormalization scale (although we do not display this dependence for simplicity), but not on the heavy quark mass. Not all of these functions are independent. The equation of motion, $iv\cdot D\,h_v = 0$, implies

$$\begin{aligned}F_1 + F_2 - F_3 &= 0\,, \\ F_4 + F_5 - F_6 &= 0\,.\end{aligned} \tag{15}$$

We may furthermore use the structure of the field redefinition (1) to derive that

$$\begin{aligned}&\langle\pi(p)|\,i\partial^\mu(\bar q\,\Gamma\,h_v)\,|M(v)\rangle \\ &\quad = (\bar\Lambda\,v^\mu - p^\mu)\,\langle\pi(p)|\,\bar q\,\Gamma\,h_v\,|M(v)\rangle\,,\end{aligned} \tag{16}$$

where $\bar\Lambda = m_M - m_Q$ denotes the finite mass difference between a heavy meson and the heavy quark that it contains, in the infinite quark-mass limit [27,29]. This parameter sets the canonical scale for power corrections in HQET. Substituting $\Gamma = \gamma_\mu\,\Gamma'$ into the above relation, and using the equation of motion for the light quark field, $i\not D\,q = 0$, we find

$$\begin{aligned}F_2 - F_4 + 2\,F_6 &= -v\cdot p\,A - \bar\Lambda\,B\,, \\ F_1 - 4\,F_3 + 2\,F_4 + \widehat p^2\,F_5 &= \bar\Lambda\,A + (2\bar\Lambda - v\cdot p\,\widehat p^2)\,B\,.\end{aligned} \tag{17}$$

We shall use the relations (15) and (17) to eliminate F_1, F_2, F_3, and F_4 in favor of F_5 and F_6. Matrix elements of the operators $\mathcal{O}_4$, $\mathcal{O}_5$, and $\mathcal{O}_6$ in (7) can be evaluated along the same lines, using

$$\bar q\,(-i\overleftarrow{D}^\mu)\,\Gamma\,h_v = \bar q\,\Gamma\,(iD^\mu)\,h_v - i\partial^\mu(\bar q\,\Gamma\,h_v) \tag{18}$$

together with (16).

Next we investigate the effects of $1/m_Q$ corrections to the effective Lagrangian of HQET. The operators $\mathcal{O}_{\rm kin}$ and $\mathcal{O}_{\rm mag}$ in (3) can be inserted into matrix elements of the leading-order currents J_i. The corresponding corrections can be described in terms of six additional functions $G_i(v\cdot p,\mu)$, which parametrize the matrix elements of the time-ordered products:

$$\langle\pi(p)|\, i\int dy\, T\Big\{\bar q\,\Gamma\, h_v(0), \mathcal{O}_{\rm kin}(y)\Big\}\,|M(v)\rangle = -\mathrm{Tr}\Big\{\gamma_5\,(G_1+\hat{p\!\!/}\,G_2)\,\Gamma\,\mathcal{M}(v)\Big\}\,,$$

$$\langle\pi(p)|\, i\int dy\, T\Big\{\bar q\,\Gamma\, h_v(0), \mathcal{O}_{\rm mag}(y)\Big\}\,|M(v)\rangle = -\mathrm{Tr}\Big\{\Big[(iG_3\,\hat p_\alpha\gamma_\beta + G_4\,\sigma_{\alpha\beta})\,\gamma_5 + (iG_5\,\hat p_\alpha\gamma_\beta + G_6\,\sigma_{\alpha\beta})\,\gamma_5\,\hat{p\!\!/}\Big] \times\Gamma\,\frac{1+{v\!\!\!/}}{2}\,\sigma^{\alpha\beta}\,\mathcal{M}(v)\Big\}\,. \qquad (19)$$

Using the above definitions and relations, it is a matter of patience to compute the matrix elements relevant to $B^{(*)}\to\pi\,\ell\,\nu$ decays to order $1/m_b$. We will discuss these matrix elements in the following section.

III. MATRIX ELEMENTS

The matrix element of the flavor-changing vector current responsible for the decay $B\to\pi\,\ell\,\nu$ can be parametrized in terms of two invariant form factors, which are conveniently defined as

$$\begin{aligned}\sqrt{m_B}\,\langle\pi(p)|\,\bar q\,\gamma^\mu\,Q\,|B(v)\rangle &= f_+(q^2)\,\bigg[(m_B\,v+p)^\mu - \frac{m_B^2-m_\pi^2}{q^2}\,q^\mu\bigg]\\ &\quad + f_0(q^2)\,\frac{m_B^2-m_\pi^2}{q^2}\,q^\mu\,,\end{aligned} \qquad (20)$$

where $q=m_B\,v-p$. The prefactor $\sqrt{m_B}$ appears since we use a somewhat unconventional normalization of states. In practice, only $f_+(q^2)$ is measurable in $B\to\pi\,\ell\,\nu$ decays into the light leptons e or μ, since the contribution of $f_0(q^2)$ to the decay rate is suppressed by a factor m_ℓ^2/m_B^2. However, both form factors are important in $B\to\pi\,\tau\,\nu$ decays.

As we have seen above, in the context of HQET it is more natural to work with the velocity of the heavy meson, and to consider the form factors as functions of the kinematic variable

$$v\cdot p = \frac{m_B^2+m_\pi^2-q^2}{2m_B}\,. \qquad (21)$$

Accordingly, we define

$$\langle\pi(p)|\,\bar q\,\gamma^\mu\,Q\,|B(v)\rangle = 2\,\Big[f_1(v\cdot p)\,v^\mu + f_2(v\cdot p)\,\hat p^\mu\Big]\,. \qquad (22)$$

The two sets of form factors are related by

$$\begin{aligned} f_+(q^2) &= \sqrt{m_B}\,\bigg\{\frac{f_2(v\cdot p)}{v\cdot p} + \frac{f_1(v\cdot p)}{m_B}\bigg\}\,,\\ f_0(q^2) &= \frac{2}{\sqrt{m_B}}\,\frac{m_B^2}{m_B^2-m_\pi^2}\,\bigg\{\Big[f_1(v\cdot p)+f_2(v\cdot p)\Big]\\ &\quad -\frac{v\cdot p}{m_B}\,\Big[f_1(v\cdot p)+\hat p^2\,f_2(v\cdot p)\Big]\bigg\}\,.\end{aligned} \qquad (23)$$

The fact that in the $m_b\to\infty$ limit the functions $f_{1,2}(v\cdot p)$ become independent of m_b (modulo logarithms) implies the well-known scaling relations [9]

$$f_+\sim\sqrt{m_B}\,,\qquad f_0\sim 1/\sqrt{m_B}\,, \qquad (24)$$

which are valid as long as $v\cdot p$ does not scale with m_B.

By evaluating the traces and using the definitions of the previous section, we find, to next-to-leading order in $1/m_b$, the expressions

$$\begin{aligned} f_1 &= C_1\,A + C_2\,(A+B)\\ &\quad + \frac{1}{2m_b}\,\bigg\{C_1\,\Big[-(\bar\Lambda-2v\cdot p)\,A + v\cdot p\,\hat p^2\,B + 4F_6 + G_1\Big]\\ &\quad + C_2\,\Big[(\bar\Lambda+v\cdot p)\,A + (3\bar\Lambda - v\cdot p\,\hat p^2)\,B + 4F_6 + G_1 + G_2\Big]\\ &\quad - B_4\,(\bar\Lambda-v\cdot p)\,A - B_5\,(\bar\Lambda-v\cdot p)\,(A+B) - B_6\,\Big[(\bar\Lambda-v\cdot p)\,A - 2F_6\Big]\\ &\quad + C_1\,C_{\rm mag}\,\Big[-2G_3 + 6G_4 + 2\hat p^2\,G_5\Big] + C_2\,C_{\rm mag}\,\Big[6G_4 - 2(1-\hat p^2)\,G_5 + 6G_6\Big]\bigg\}\,,\end{aligned} \qquad (25)$$

$$\begin{aligned} f_2 &= C_1\,B + \frac{1}{2m_b}\,\bigg\{C_1\,\Big[-v\cdot p\,A - \bar\Lambda\,B - 4F_6 + G_2\Big] - 2C_2\,\Big[v\cdot p\,A + \bar\Lambda\,B + 2F_6\Big]\\ &\quad - B_4\,(\bar\Lambda-v\cdot p)\,B - B_6\,\Big[(\bar\Lambda-v\cdot p)\,B + 2F_6\Big] + C_1\,C_{\rm mag}\,\Big[2G_3 - 2G_5 + 6G_6\Big]\bigg\}\,.\end{aligned}$$

For simplicity we have omitted the dependence of the universal functions on $v\cdot p$ and μ, and the dependence of the Wilson coefficients on μ.

From the fact that the physical form factors must be independent of the renormalization scale, one can deduce the μ dependence of the universal functions of HQET, since it has to cancel against that of the Wilson coefficients. For the leading-order functions $A(v\cdot p,\mu)$ and $B(v\cdot p,\mu)$, it follows that

$$A_{\rm ren}(v\cdot p) \equiv \Big[\alpha_s(\mu)\Big]^{2/\beta} A(v\cdot p,\mu)\,,$$
$$B_{\rm ren}(v\cdot p) \equiv \Big[\alpha_s(\mu)\Big]^{2/\beta} B(v\cdot p,\mu)\,, \qquad (26)$$

must be μ independent (in leading logarithmic approximation). It is then convenient to define two related functions

$$\hat A(v\cdot p) \equiv \Big[\alpha_s(m_Q)\Big]^{-2/\beta} A_{\rm ren}(v\cdot p) = x^{2/\beta} A(v\cdot p,\mu)\,,$$
$$\hat B(v\cdot p) \equiv \Big[\alpha_s(m_Q)\Big]^{-2/\beta} B_{\rm ren}(v\cdot p) = x^{2/\beta} B(v\cdot p,\mu)\,, \qquad (27)$$

which are clearly also μ independent. These functions are no longer universal since they contain a logarithmic dependence on the heavy quark mass. At the tree level, however, they agree with the original functions A and B.

In order to find the corresponding relations for the subleading universal functions F_i and G_i, the expressions (25) for f_1 and f_2 are not sufficient. We have thus worked out the heavy quark expansion for $B^*\to\pi\,\ell\,\nu$ decays, although these processes have little (if any) phenomenological relevance. We define hadronic form factors h_i by

$$\langle\pi(p)|\,\bar q\,\gamma^\mu\,Q\,|B^*(v)\rangle = 2i\,\varepsilon^{\mu\alpha\beta\gamma}\,\epsilon_\alpha\,\hat p_\beta\,v_\gamma\,h_1(v\cdot p)\,, \qquad (28)$$

$$\langle\pi(p)|\,\bar q\,\gamma^\mu\gamma_5\,Q\,|B^*(v)\rangle = 2\Big[h_2(v\cdot p)\,\epsilon^\mu - h_3(v\cdot p)\,\epsilon\cdot\hat p\,v^\mu - h_4(v\cdot p)\,\epsilon\cdot\hat p\,\hat p^\mu\Big]\,.$$

By studying these form factors at order $1/m_b$, one can derive enough relations to fully determine the μ dependence of the universal functions. We discuss this somewhat technical issue in Appendix A. There we define a set of renormalization-group invariant functions $\hat F_i(v\cdot p)$ and $\hat G_i(v\cdot p)$, which are μ independent and coincide with F_i and G_i at the tree level. In terms of these functions, the form factor relations take a much simpler form. Instead of (25) we find

$$f_1 = \hat A + \frac{1}{2m_b}\Big[-(\bar\Lambda - 2v\cdot p)\,\hat A + v\cdot p\,\hat p^2\,\hat B + 4\hat F_6 + \hat G_1 - 2\hat G_3 + 6\hat G_4 + 2\hat p^2\,\hat G_5\Big]\,,$$
$$f_2 = \hat B + \frac{1}{2m_b}\Big[-v\cdot p\,\hat A - \bar\Lambda\,\hat B - 4\hat F_6 + \hat G_2 + 2\hat G_3 - 2\hat G_5 + 6\hat G_6\Big]\,, \qquad (29)$$

and the $B^*\to\pi$ decay form factors are given by

$$h_1 = \hat B + \frac{1}{2m_b}\Big[v\cdot p\,\hat A + \bar\Lambda\hat B + 2\hat F_6 + \hat G_2 - 2\hat G_6\Big]\,,$$
$$h_2 = (\hat A + \hat B) + \frac{1}{2m_b}\Big[\tfrac13(\bar\Lambda - v\cdot p)\,\hat A + \tfrac13(\bar\Lambda - v\cdot p\,\hat p^2)\,\hat B - \frac23(1-\hat p^2)\,\hat F_5 + \hat G_1 + \hat G_2 - 2\hat G_4 - 2\hat G_6\Big]\,,$$
$$h_3 = \hat B + \frac{1}{2m_b}\Big[-v\cdot p\,\hat A - \bar\Lambda\,\hat B - 2\hat F_5 + \hat G_2 - 2\hat G_3 - 2\hat G_6\Big]\,, \qquad (30)$$
$$h_4 = \frac{1}{2m_b}\Big[2\hat F_5 + 2\hat G_5\Big]\,.$$

Note that $\hat F_6$ appears only in the vector form factors f_1, f_2, and h_1, whereas $\hat F_5$ appears only in the axial form factors h_2, h_3, and h_4.

Most relevant, of course, are the form factors f_+ and f_0 that are usually used to describe $B\to\pi\,\ell\,\nu$ decays. From (23) we obtain, at next-to-leading order in $1/m_b$,

$$f_+ = \frac{\sqrt{m_B}}{v\cdot p}\bigg\{\hat B + \frac{1}{2m_b}\Big[v\cdot p\,\hat A - \bar\Lambda\,\hat B - 4\hat F_6 + \hat G_2 + 2\hat G_3 - 2\hat G_5 + 6\hat G_6\Big]\bigg\}\,,$$
$$f_0 = \frac{2}{\sqrt{m_B}}\bigg\{(\hat A + \hat B) + \frac{1}{2m_b}\Big[-(\bar\Lambda + v\cdot p)\,\hat A - (\bar\Lambda + v\cdot p\,\hat p^2)\,\hat B + \hat G_1 + \hat G_2 + 6\hat G_4 - 2(1-\hat p^2)\hat G_5 + 6\hat G_6\Big]\bigg\}\,. \qquad (31)$$

These relations show how, in a rather complicated way, the $1/m_b$ corrections to f_+ and f_0 are related to matrix elements of operators in HQET. To gain more insight into the structure of the corrections, it is instructive to consider the soft pion limit $v\cdot p\to 0$ and $p^2 = m_\pi^2\to 0$, in which current algebra can be used to derive normalization conditions on the universal functions. This is the subject of the following section.

IV. SOFT PION RELATIONS

In this section we shall derive the normalization conditions for the universal functions of HQET, which arise in the soft pion limit $p\to 0$. Our goal is to reduce, as much as possible, the number of independent parameters upon which our predictions depend. The soft pion relations are derived by using the PCAC (partial conservation of the axial vector current) relation for the pion field. To be specific, let us consider the decay $M^0 \to \pi^+ \ell^- \nu$ (where $M^0 = \bar{B}^0$ or $\bar{B}^{0*}$). Then

$$\pi^+(x) = \frac{1}{f_\pi m_\pi^2}\, \partial^\mu A_\mu(x)\,, \tag{32}$$

where $A_\mu = \bar{d}\,\gamma_\mu \gamma_5\, u$ is the axial vector current, and $f_\pi \simeq 132$ MeV is the pion decay constant. The Lehmann-Symanzik-Zimmermann (LSZ) reduction formalism can be employed to write

$$\langle \pi(p)|\, \mathcal{O}(0)\, |M(v)\rangle = \lim_{p^2\to m_\pi^2} \frac{1}{f_\pi}\, \frac{m_\pi^2 - p^2}{m_\pi^2}\; i \int dx\, e^{ip\cdot x}\, \langle\, 0\,|\, T\left\{\mathcal{O}(0), \partial^\mu A_\mu(x)\right\} |M(v)\rangle\,, \tag{33}$$

where $\mathcal{O}$ may be any operator that couples M to π. The right-hand side can be rewritten using

$$i \int dx\, e^{ip\cdot x}\; T\left\{\mathcal{O}(0), \partial^\mu A_\mu(x)\right\} = p^\mu \int \mathrm{d}x\, e^{ip\cdot x}\; T\left\{A_\mu(x), \mathcal{O}(0)\right\} - i\left[Q_5, \mathcal{O}(0)\right]. \tag{34}$$

Here Q_5 denotes the axial charge, i.e., the spatial integral of the zero component of A_μ: $Q_5 = \int \mathrm{d}^3x\, d^\dagger \gamma_5\, u$. Therefore,

$$\left[Q_5, \mathcal{O}(0)\right] = \mathcal{O}'(0)\,, \tag{35}$$

where the operator $\mathcal{O}'$ is obtained from $\mathcal{O}$ by replacing $\bar{u}$ by $\bar{d}(-\gamma_5)$, i.e., if $\mathcal{O} = \bar{u}\,\gamma^\mu\, b$ then $\mathcal{O}' = \bar{d}\,\gamma^\mu \gamma_5\, b$, etc. The soft pion relation is obtained by analytically continuing (33) to $p \to 0$. In this limit the first term on the right-hand side of (34) is saturated by intermediate states degenerate with the ground state. They lead to poles proportional to $1/v\cdot p$, which cancel the factor p^μ in front of the integral. In the case of $B \to \pi$ transitions, the relevant intermediate state will be the B^* meson, which to leading order in HQET is in fact degenerate with the B meson. We obtain

$$\lim_{p\to 0} \langle \pi(p)|\, \mathcal{O}(0)\, |M(v)\rangle = \frac{1}{f_\pi}\left\{ -\, i\, \langle\, 0\,|\, \mathcal{O}'(0)\, |M(v)\rangle + \lim_{p\to 0} \int \mathrm{d}x\, e^{ip\cdot x}\, \langle\, 0\,|\, T\left\{\mathcal{O}(0), p\cdot A(x)\right\} |M(v)\rangle \right\}. \tag{36}$$

In what follows we shall refer to the first and second terms on the right-hand side as the commutator and the pole contribution, respectively.

A. Soft pion relations for $\hat{A}(v\cdot p)$ and $\hat{B}(v\cdot p)$

Let us now evaluate this relation for the matrix elements arising at leading order in the $1/m_Q$ expansion, where the effective current operators have the generic form $\mathcal{O} = \bar{q}\,\Gamma\, h_v$. Both the commutator and the pole contribution involve a current-induced transition of a heavy meson into the vacuum. At leading order in HQET, the corresponding matrix elements can be written as [14]

$$\langle\, 0\,|\, \bar{q}\,\Gamma'\, h_v\, |M(v)\rangle = \frac{iF(\mu)}{2}\, \mathrm{Tr}\{\Gamma' \mathcal{M}(v)\}\,, \tag{37}$$

where $\Gamma' = -\gamma_5\, \Gamma$ in the commutator term and $\Gamma' = \Gamma$ in the pole term. The prefactor is chosen such that the universal low-energy parameter $F(\mu)$, which is independent of the heavy quark mass, corresponds to the asymptotic value of the scaled meson decay constant: $F \sim f_M \sqrt{m_M}$ (modulo logarithms).

To compute the pole term, we further need the coupling of two heavy mesons to the axial vector current, as shown in Fig. 1. We define

$$\langle M'(v,p)|\, p\cdot A\, |M(v,0)\rangle = g(v\cdot p)\, \mathrm{Tr}\left\{\gamma_5\, \not{p}\, \overline{\mathcal{M}}'(v) \mathcal{M}(v)\right\}, \tag{38}$$

where M' is off-shell by the pion momentum p. The form factor $g(v\cdot p)$ is real and regular as $v\cdot p \to 0$. We define

$$\lim_{p\to 0} g(v\cdot p) = g(0) \equiv g\,. \tag{39}$$

Note that g is renormalization-group invariant. We can now write the pole contribution as

$$\sum_{M'} \langle\, 0\,|\, \bar{q}\,\Gamma\, h_v\, |M'(v)\rangle\, \frac{i}{2v\cdot(-p)}\, \langle M'(v)|\, p\cdot A\, |M(v)\rangle = \frac{F(\mu)\, g(v\cdot p)}{4v\cdot p} \sum_{M'} \mathrm{Tr}\{\Gamma\, \mathcal{M}'(v)\} \times \mathrm{Tr}\left\{\gamma_5\, \not{p}\, \overline{\mathcal{M}}'(v) \mathcal{M}(v)\right\}, \tag{40}$$

where we have used that in the effective theory the intermediate meson propagator is simply given by $i/v\cdot k$, where k stands for the residual momentum. (Recall that

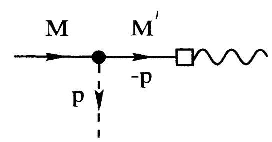

FIG. 1. Pole diagram contributing to the soft pion relations for the universal form factors. The axial vector current is shown as the dashed line, whereas the weak current is drawn as a wiggly line. The black dot represents the strong interaction vertex, the open box the weak interaction matrix element.

we use a mass-independent normalization of states.) To proceed further, we need a relation that allows us to combine the two traces appearing on the right-hand side into a single trace. This is accomplished by the identity

$$\sum_{M'=P,V} \mathrm{Tr}\{X\,\mathcal{M}'(v)\}\,\mathrm{Tr}\Big\{\gamma_5\,\not{p}\,\overline{\mathcal{M}}'(v)\mathcal{M}(v)\Big\} = -2\,\mathrm{Tr}\Big\{\gamma_5\,(\not{p}-v\cdot p)\,X\,\mathcal{M}(v)\Big\}, \quad (41)$$

which is valid for any Dirac matrix X, and for a pseudoscalar or vector meson $M(v)$. The sum extends over the ground-state pseudoscalar and vector mesons $M'(v)$ degenerate with $M(v)$, and summation over polarizations is understood if $M'(v)$ is a vector meson.

Putting together the various pieces and using (11), we obtain the soft pion relation

$$\lim_{p\to 0} \mathrm{Tr}\Big\{\gamma_5\,\Big[A(v\cdot p,\mu)+\hat{\not{p}}B(v\cdot p,\mu)\Big]\,\Gamma\,\mathcal{M}(v)\Big\} = \frac{F(\mu)}{2f_\pi}\left[\mathrm{Tr}\{\gamma_5\,\Gamma\,\mathcal{M}(v)\} + \lim_{p\to 0} g(v\cdot p)\,\mathrm{Tr}\Big\{\gamma_5\,(\hat{\not{p}}-1)\,\Gamma\,\mathcal{M}(v)\Big\}\right], \quad (42)$$

from which we read off the values of the form factors A and B in the soft pion limit:

$$A(0,\mu) = \frac{F(\mu)}{2f_\pi}\,(1-g)\,, \qquad B(0,\mu) = \frac{F(\mu)}{2f_\pi}\,g\,. \quad (43)$$

These relations are preserved by renormalization. In fact, one can define a scale-independent quantity $\hat{F} = x^{2/\beta}\,F(\mu)$, which agrees with F at tree level [14]. As mentioned above, the coupling constant g is not renormalized. From (27), it then follows that

$$\hat{A}(0) = \frac{\hat{F}}{2f_\pi}\,(1-g)\,, \qquad \hat{B}(0) = \frac{\hat{F}}{2f_\pi}\,g\,, \quad (44)$$

which are the desired normalization conditions for the renormalized form factors in the soft pion limit.

B. Soft pion relations for $\hat{F}_i(v\cdot p)$

Let us next consider the soft pion relations for the subleading form factors F_i defined in (14). The only difference from the previous derivation is that now the current contains a covariant derivative. Hence we need the corresponding matrix elements for the case of meson-to-vacuum transitions. They are [14]

$$\langle\,0\,|\,\bar{q}\,\Gamma'\,iD^\mu h_v\,|M(v)\rangle = -\frac{iF(\mu)}{2}\,\frac{\bar{\Lambda}}{3}\,\mathrm{Tr}\Big\{(v^\mu+\gamma^\mu)\,\Gamma'\mathcal{M}(v)\Big\}\,. \quad (45)$$

Using again the trace relation (41), we obtain, for the pole term,

$$\sum_{M'}\langle\,0\,|\,\bar{q}\,\Gamma\,iD^\mu h_v\,|M'(v)\rangle\,\frac{i}{2v\cdot(-p)}\,\langle M'(v)|\,p\cdot A\,|M(v)\rangle = \frac{F(\mu)\,\bar{\Lambda}}{6}\,\frac{g(v\cdot p)}{v\cdot p} \times\mathrm{Tr}\Big\{\gamma_5\,(\not{p}-v\cdot p)\,(v^\mu+\gamma^\mu)\,\Gamma\,\mathcal{M}(v)\Big\}\,. \quad (46)$$

The commutator term is simply given by (45) with $\Gamma' = -\gamma_5\,\Gamma$. Combining the two, we obtain, from a comparison with (14),

$$\begin{aligned} F_1(0,\mu) &= -\frac{F(\mu)}{2f_\pi}\,\frac{\bar{\Lambda}}{3}\,(1-g), & F_4(0,\mu) &= -\frac{F(\mu)}{2f_\pi}\,\frac{\bar{\Lambda}}{3}\,g\,,\\ F_2(0,\mu) &= -\frac{F(\mu)}{2f_\pi}\,\frac{2\bar{\Lambda}}{3}\,g\,, & F_5(0,\mu) &= 0\,,\\ F_3(0,\mu) &= -\frac{F(\mu)}{2f_\pi}\,\frac{\bar{\Lambda}}{3}\,(1+g)\,, & F_6(0,\mu) &= -\frac{F(\mu)}{2f_\pi}\,\frac{\bar{\Lambda}}{3}\,g\,. \end{aligned} \quad (47)$$

Note that the relations (15) and (17), which are consequences of the equations of motion, are satisfied by these expressions. Using the results of Appendix A we find that radiative corrections can again be incorporated in a straightforward manner. The two independent renormalized form factors satisfy

$$\hat{F}_5(0) = 0\,, \qquad \hat{F}_6(0) = -\frac{\hat{F}}{2f_\pi}\,\frac{\bar{\Lambda}}{3}\,g\,. \quad (48)$$

C. Soft pion relations for $\hat{G}_i(v\cdot p)$

Here one encounters the complication that the soft pion relation involves the time-ordered product of three operators: the original heavy-light current, the axial vector current that interpolates the pion field, and one of the operators $\mathcal{O}_{\rm kin}$ and $\mathcal{O}_{\rm mag}$ which appear at order $1/m_Q$ in the effective Lagrangian of HQET. Consequently, there are both single and double pole contributions in addition to the commutator term, and the derivations become more cumbersome. We shall only give the final expressions here and refer the interested reader to Appendix B, where we give details of the calculation.

The $1/m_Q$ insertions from corrections to the effective Lagrangian correct both the meson decay constants and the $MM'\pi$ coupling constant, as shown in Fig. 2(a). The corrections to the decay constant were treated in Ref. [14]. They can be parametrized in terms of two renormalized parameters $\hat{\mathcal{G}}_1$ and $\hat{\mathcal{G}}_2$, which describe the effects of the kinetic and chromomagnetic operator, respectively. To order $1/m_Q$ (and in leading logarithmic approximation), the physical decay constants are

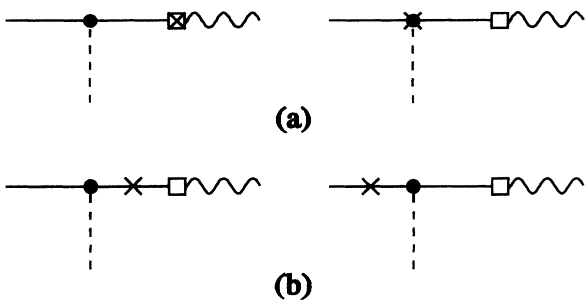

FIG. 2. Single (a) and double (b) pole diagrams contributing to the soft pion relations for the form factors G_i. The notation is the same as in Fig. 1. In addition, a cross represents a $1/m_Q$ insertion of $\mathcal{O}_{\rm kin}$ or $\mathcal{O}_{\rm mag}$.

$$f_P\sqrt{m_P} = \hat F\left\{1+\frac{1}{m_Q}\left(\hat{\mathcal{G}}_1+6\hat{\mathcal{G}}_2-\frac{\bar\Lambda}{2}\right)\right\},$$
$$f_V\sqrt{m_V} = \hat F\left\{1+\frac{1}{m_Q}\left(\hat{\mathcal{G}}_1-2\hat{\mathcal{G}}_2+\frac{\bar\Lambda}{6}\right)\right\}. \qquad (49)$$

Similarly, at next-to-leading order in $1/m_Q$, the coupling of two heavy mesons to the pion receives corrections. Instead of the universal coupling constant g in (39) we write

$$g_{PV\pi} = g_{VP\pi} = g + \frac{1}{2m_Q}\left(g_1+4\hat g_2\right),$$
$$g_{VV\pi} = g + \frac{1}{2m_Q}\left(g_1-4\hat g_2\right), \qquad (50)$$

where P and V stand for a pseudoscalar or vector meson, respectively. The coupling of two pseudoscalar mesons to the pion vanishes by parity invariance of the strong interactions. For the precise definition of the parameters $\mathcal{G}_i$ and g_i and their renormalization the reader is encouraged to consult Appendix B.

In terms of these parameters, we find the following soft pion relations for the renormalized form factors $\hat G_i$:

$$\hat G_1(0) = \frac{\hat F}{2f_\pi}\left[2(1-g)\,\hat{\mathcal{G}}_1 - g_1\right],$$
$$\hat G_2(0) = \frac{\hat F}{2f_\pi}\left[2g\,\hat{\mathcal{G}}_1 + g_1\right],$$
$$\hat G_4(0) = \frac{\hat F}{2f_\pi}\left[2(1-g)\,\hat{\mathcal{G}}_2 - 2\hat g_2\right],$$
$$\hat G_5(0) = 0,$$
$$\hat G_6(0) = \frac{\hat F}{2f_\pi}\left[2g\,\hat{\mathcal{G}}_2 + 2\hat g_2\right], \qquad (51)$$

as well as

$$\lim_{p\to 0}\hat G_3(v\cdot p) = -\frac{\hat F}{2f_\pi}\left[(m_V^2-m_P^2)\,\frac{g}{2v\cdot p} + 8g\,\hat{\mathcal{G}}_2 + 4\hat g_2\right]. \qquad (52)$$

It might seem surprising that $\hat G_3$ develops a pole as $v\cdot p\to 0$, with a residue proportional to the mass splitting between vector and pseudoscalar mesons. However, as we shall see below, this is exactly what is required to recover the correct pole contributions predicted by chiral symmetry. The singular behavior of $\hat G_3$ results from the diagrams depicted in Fig. 2(b). For later purposes we define a regular function $\hat G_3^{\rm reg}(v\cdot p)$ by

$$\hat G_3^{\rm reg}(v\cdot p) \equiv \hat G_3(v\cdot p) + \frac{\hat F\, g}{4f_\pi}\,\frac{(m_V^2-m_P^2)}{v\cdot p},$$
$$\hat G_3^{\rm reg}(0) = -\frac{\hat F}{2f_\pi}\left[8g\,\hat{\mathcal{G}}_2 + 4\hat g_2\right]. \qquad (53)$$

D. Meson form factors in the chiral limit

The soft pion relations derived above will become more transparent when we consider the physical meson form factors f_i and h_i defined in (22) and (28). We start by considering the sum f_1+f_2. In the soft pion limit we obtain

$$f_1(0)+f_2(0) = \frac{\hat F}{2f_\pi}\left\{1+\frac{1}{m_b}\left(\hat{\mathcal{G}}_1+6\hat{\mathcal{G}}_2-\frac{\bar\Lambda}{2}\right)\right\} = \frac{f_B\sqrt{m_B}}{2f_\pi}, \qquad (54)$$

where we have used (49) to write the result in terms of the physical decay constant f_B. Next, consider the form factor f_2. We find

$$\lim_{p\to 0} f_2(v\cdot p) = \frac{\hat F}{2f_\pi}\left\{1+\frac{1}{m_b}\left(\hat{\mathcal{G}}_1-2\hat{\mathcal{G}}_2+\frac{\bar\Lambda}{6}\right)\right\} \times\left\{g+\frac{1}{2m_b}\left(g_1+4\hat g_2\right)\right\}\left(1-\frac{\Delta_B}{v\cdot p}\right), \qquad (55)$$

where

$$\Delta_B = \frac{m_{B^*}^2-m_B^2}{2m_b} \approx m_{B^*}-m_B, \qquad (56)$$

and we have factorized various terms in an educated way, so that it is immediate to identify the decay constant of the B^* meson and the $BB^*\pi$ coupling constant. In fact, using (49) and (50) we can rewrite the result as

$$\lim_{p\to 0} f_2(v\cdot p) = \frac{f_{B^*}\sqrt{m_{B^*}}}{2f_\pi}\, g_{BB^*\pi}\,\frac{v\cdot p}{(v\cdot p+\Delta_B)}. \qquad (57)$$

The resummation of the B^*-pole term, which is allowed to the order we are working, has removed the spurious singularity at $v\cdot p=0$, and we have recovered the physical pole position at $v\cdot p=-\Delta_B$, corresponding to $q^2=m_{B^*}^2$. This becomes apparent when we use (23) to convert to the conventional form factors $f_+(q^2)$ and $f_0(q^2)$. In the soft pion limit, they become

$$\lim_{q^2\to m_B^2} f_+(q^2) = \frac{m_B}{2}\,\frac{f_{B^*}}{f_\pi}\,\frac{g_{BB^*\pi}}{(v\cdot p+\Delta_B)} = \frac{f_{B^*}}{f_\pi}\,\frac{g_{BB^*\pi}}{\left[1-q^2/m_{B^*}^2\right]}\,,$$
$$f_0(m_B^2) = \frac{f_B}{f_\pi}\,, \tag{58}$$

where we have used that $m_{B^*}/m_B = 1+O(1/m_b^2)$. These are the well-known results for the meson form factors in the chiral limit. They have been previously derived in the $m_b\to\infty$ limit by combining HQET with chiral perturbation theory [19,20], or by using current algebra in connection with the fact that the B and B^* mesons are degenerate to leading order in $1/m_b$ [21]. The same relations have also been obtained without a heavy quark expansion, by assuming nearest pole dominance [18,36]. We emphasize, however, that here we have not only recovered these results from a rigorous expansion in QCD, but we have proven them to hold even at next-to-leading order in $1/m_b$, and including short-distance corrections. We find that there are no such corrections to the soft pion relations once one uses the physical values of the meson decay constants and of the $BB^*\pi$ coupling constant, as opposed to their values in the $m_b\to\infty$ limit.

In a similar manner, one can derive the soft pion limit for the $B^*\to\pi$ decay form factors h_i defined in (28). We obtain

$$h_1(0) = \frac{f_{B^*}\sqrt{m_{B^*}}}{2f_\pi}\,g_{B^*B^*\pi}\,,$$
$$h_2(0) = \frac{f_{B^*}\sqrt{m_{B^*}}}{2f_\pi}\,,$$
$$h_3(0) = \frac{f_B\sqrt{m_B}}{2f_\pi}\,g_{B^*B\pi}\,\frac{v\cdot p}{(v\cdot p-\Delta_B)}\,,$$
$$h_4(0) = 0\,. \tag{59}$$

Again, at leading order in $1/m_b$, these relations could also be derived using heavy meson chiral perturbation theory.

V. SUMMARY AND CONCLUSIONS

We have presented a systematic analysis of the $B^{(*)}\to\pi\,\ell\,\nu$ decay form factors to order $1/m_b$ in the heavy quark expansion, including a detailed treatment of short-distance corrections. Similar analyses have been carried out in the past for the semileptonic decays $B\to D^{(*)}\ell\,\nu$ [27] and $\Lambda_b\to\Lambda_c\,\ell\,\nu$ [37], and for heavy meson decay constants [14]. As in these cases, the analysis of the form factors in the context of a heavy quark expansion provides the theoretical framework for a comprehensive investigation of the hadronic physics encoded in the universal functions of HQET, using nonperturbative techniques such as lattice gauge theory or QCD sum rules. For the decays between two heavy mesons, this strategy has been very successful and has led to much insight into the properties of these nonperturbative objects. In particular, analytic (two-loop) predictions have been obtained for the leading and subleading Isgur-Wise functions using QCD sum rules [38,39], and first results for the leading-order Isgur-Wise function are available from lattice gauge theory [40,41]. Previous predictions for the $B\to\pi\,\ell\,\nu$ form factors, on the other hand, were obtained using quark models [42,43], or QCD sum rules in the full theory [44–49]. The next step should be a more detailed analysis in the context of the heavy quark expansion. Recently, calculations incorporating ingredients of heavy quark symmetry were performed in the $m_b\to\infty$ limit [50–52]. One of the purposes of our paper is to allow an extension of this type of calculations to order $1/m_b$.

The main motivation for a study of exclusive heavy-to-light decays is to extract the element $|V_{ub}|$ of the quark mixing matrix in a reliable, model-independent way. The idea is to compare the lepton spectra in the decays $B\to\pi\,\ell\,\nu$ and $D\to\pi\,\ell\,\nu$, which are related to each other by heavy quark flavor symmetry [9]. In the limit of vanishing lepton mass, the differential decay rate is determined by the form factor f_+ defined in (20):

$$\frac{d\Gamma(B\to\pi\,\ell\,\nu)}{d(v\cdot p)} = \frac{G_F^2\,m_B}{12\pi^3}\,|V_{ub}|^2\left[(v\cdot p)^2-m_\pi^2\right]^{3/2}\times|\,f_+\,|^2\,. \tag{60}$$

Hence, the ratio of the two distributions at the same value of $v\cdot p$ is

$$\left.\frac{d\Gamma(B\to\pi\,\ell\,\nu)/d(v\cdot p)}{d\Gamma(D\to\pi\,\ell\,\nu)/d(v\cdot p)}\right|_{\text{same } v\cdot p} = \left|\frac{V_{ub}}{V_{cd}}\right|^2\left(\frac{m_B}{m_D}\right)^2\left|\frac{\sqrt{m_D}\,f_+^{B\to\pi}}{\sqrt{m_B}\,f_+^{D\to\pi}}\right|^2. \tag{61}$$

In the limit of an exact heavy quark flavor symmetry, the last factor on the right-hand side equals unity. It is convenient to rewrite this factor as

$$\frac{\sqrt{m_D}\,f_+^{B\to\pi}}{\sqrt{m_B}\,f_+^{D\to\pi}} \equiv \frac{v\cdot p+\Delta_D}{v\cdot p+\Delta_B}\,R_{BD}(v\cdot p)\,, \tag{62}$$

where $\Delta_B = m_{B^*}-m_B \simeq 0.05$ GeV and $\Delta_D = m_{D^*}-m_D \simeq 0.14$ GeV. This definition of R_{BD} takes into account the dominant momentum dependence for low momenta, which comes from the presence of the nearby vector meson pole. The difference in the pole positions for $B\to\pi\,\ell\,\nu$ and $D\to\pi\,\ell\,\nu$ is formally of order $1/m_Q$, but is significant for $v\cdot p$ close to its minimum value m_π. This effect is explicitly taken into account in (61). The remaining, nontrivial power corrections reside in the quantity R_{BD}. Using (31) we obtain

$$R_{BD}(v\cdot p) = \left(\frac{\alpha_s(m_c)}{\alpha_s(m_b)}\right)^{2/\beta}\left\{1+\frac{\bar\Lambda}{2m_c}\,r_c(v\cdot p) - \frac{\bar\Lambda}{2m_b}\,r_b(v\cdot p)+O(1/m_Q^2)\right\}, \tag{63}$$

where the function

$$r_Q(v\cdot p) = 1+\frac{1}{\bar\Lambda\,\hat B}\left(-v\cdot p\,\hat A+4\hat F_6-\hat G_2-2\hat G_3^{\rm reg} +2\hat G_5-6\hat G_6\right) \tag{64}$$

depends logarithmically on m_Q through the definition of the renormalized form factors in Appendix A. The function $\hat{G}_3^{\rm reg}$ has been defined in (53). The accuracy with which $|V_{ub}|$ can be determined depends crucially upon how well one will be able to estimate the $1/m_Q$ corrections in (64). Thus, a detailed investigation of the leading ($\hat{A}$ and $\hat{B}$) and subleading ($\hat{F}_6$ and $\hat{G}_i$) universal functions is most desirable. Such an analysis is beyond the scope of the present paper. We note, however, that for the related case of $B \to \rho\,\ell\,\nu$ decays, the form factor ratio corresponding to (62) has been calculated in the quark model, using a $1/m_Q$ expansion [53]. The results are encouraging in that the deviations from the flavor symmetry limit turn out to be small, of order 15%. We expect corrections of similar size for the case of $B \to \pi$ transitions. In fact, assuming that r_Q is of order unity, we expect that the scale of power corrections is set by

$$\frac{\bar\Lambda}{2m_c} - \frac{\bar\Lambda}{2m_b} \simeq 11\%\,, \qquad (65)$$

where we have used $\bar\Lambda = 0.5$ GeV, $m_c = 1.5$ GeV, and $m_b = 4.8$ GeV for the sake of argument.

Are there any indications that this estimate might be too optimistic? We think not. The reason is that current algebra puts powerful constraints on the form factors in the soft pion limit. In particular, it fixes the normalization of f_+ at zero recoil. Using (58) one obtains

$$\lim_{p\to 0} R_{BD}(v\cdot p) = \frac{g_{BB^*\pi}}{g_{DD^*\pi}}\,\frac{f_{B^*}\sqrt{m_B}}{f_{D^*}\sqrt{m_D}}\,. \qquad (66)$$

While this relation was derived before in the infinite heavy quark mass limit [19–21,51], we have shown that it is actually valid to next-to-leading order in $1/m_Q$. It is well known that, for pseudoscalar mesons, there are substantial corrections to the asymptotic scaling law $f_B\sqrt{m_B} \approx f_D\sqrt{m_D}$, which enhance the ratio f_B/f_D. Theoretical predictions typically fall in the range $(f_B\sqrt{m_B})/(f_D\sqrt{m_D}) \simeq 1.3 - 1.5$ [10–17]. However, in (66) there appear the decay constants of vector mesons. Both QCD sum rules and lattice gauge theory predict that spin-symmetry-violating corrections decrease the ratio f_{B^*}/f_{D^*} as compared to f_B/f_D. The predictions are $f_{B^*}/f_{D^*} = \kappa(f_B/f_D)$ with $\kappa = 0.79 \pm 0.03$ from QCD sum rules [14], and $\kappa = 0.86 \pm 0.06$ from lattice gauge theory [11]. The total effect is that the scaling violations are much smaller for vector meson decay constants. One expects

$$\frac{f_{B^*}\sqrt{m_B}}{f_{D^*}\sqrt{m_D}} \simeq 1.05 - 1.20\,, \qquad (67)$$

i.e., a rather moderate correction to the flavor symmetry limit. Although we are not able to give a similar estimate for the ratio $g_{BB^*\pi}/g_{DD^*\pi}$ in (66), we see no reason why it should deviate from unity by an anomalously large amount. Hence, we believe that the deviations from the symmetry prediction $R_{BD} = 1$ are of the naively expected order of magnitude. We conclude that from a comparison of the lepton spectra in $B \to \pi\,\ell\,\nu$ and $D \to \pi\,\ell\,\nu$ decays, it should be possible to extract $|V_{ub}|$ in a model-independent way with a theoretical uncertainty of 10–20%. This would already be a major improvement over the current, largely model-dependent determination of $|V_{ub}|$ from inclusive decays. To achieve an even higher precision, it is necessary to study in detail the $1/m_Q$ corrections in (63). It is only at this level that hadronic uncertainties enter the analysis. In this paper we have developed the theoretical framework for such an investigation.

At this point it is necessary to discuss the validity of the various expansions considered in this paper. The heavy quark expansion is valid as long as, in the rest frame of the initial heavy meson, the energy of the light degrees of freedom before and after the weak decay is small compared to (twice) the heavy quark mass.[1] Hence, one must require that

$$\frac{\bar\Lambda}{2m_Q} \ll 1\,, \qquad \frac{v\cdot p}{2m_Q} \ll 1\,, \qquad (68)$$

where $\bar\Lambda$ is the effective mass of the light degrees of freedom in the initial heavy meson [29]. The first ratio is of order 5% for $Q = b$ and 15% for $Q = c$, whereas the second ratio varies roughly between 0 and 1/4 for $m_\pi \le v\cdot p \le \frac{1}{2}(m_B^2 + m_\pi^2)/m_B$. Hence, we expect the heavy quark expansion to hold over essentially the entire kinematic range accessible in semileptonic decays. This assertion is in fact supported by quark model calculations [54,55].

Another important question is over what range in $v\cdot p$ can one trust the leading term in the chiral expansion, i.e., the soft pion relations given in (58) and (66). Since the pion is a pseudo Goldstone boson associated with the spontaneous breaking of chiral symmetry, we expect that the scale for the momentum dependence of the universal form factors of HQET is set by $\Lambda_\chi = 4\pi f_\pi$, which is the characteristic scale of chiral symmetry breaking. Although one should not take this naive dimensional argument too seriously, we may argue that the universal functions are slowly varying in $x = v\cdot p/\Lambda_\chi$, and the leading chiral behavior should be a good approximation until $v\cdot p \sim 1$ GeV. Hence, we expect that (58) and (66) should not only hold near $v\cdot p = 0$, but actually over a rather wide range in $v\cdot p$. Recent QCD sum rule calculations of the q^2 dependence of $f_+^{B\to\pi}(q^2)$ in the full theory support this expectation. The authors of Ref. [47] find that the pole formula (58) gives an excellent fit to their theoretical calculation over the wide range $0 \le q^2 \le 20$ GeV^2. For the residue at $q^2 = 0$, they obtain $f_+^{B\to\pi}(0) = 0.26 \pm 0.03$, which is consistent with other sum rule calculations [44–46,48,49], and with the quark model prediction of Ref. [42].

This result is interesting since, by means of (58), the residue can be translated into a value for the $BB^*\pi$ coupling constant, yielding $g_{BB^*\pi} \simeq 0.17 \times (200\ \text{MeV}/f_{B^*})$. This value is significantly smaller than a naive estimate based on PCAC and the nonrelativistic constituent quark model, which gives $g_{BB^*\pi} \simeq 1$ [3,18,56]. However, it has

[1]For a discussion of the factor 2, see Ref. [30].

been pointed out that this number may indeed be too large. From a generalization of the Nambu–Jona-Lasinio model, the authors of Ref. [57] find $g_{BB^*\pi} \simeq 0.32$. The large spread in the theoretical predictions for the $BB^*\pi$ coupling constant poses the question whether it is possible to obtain experimental information on this parameter. So far, attempts in this direction have focused on the decays of charm mesons, assuming heavy quark symmetry (i.e., neglecting $1/m_c$ corrections). From the width of the D^{*+}, one can derive the rather loose upper bound $g_{DD^*\pi} < 1.7$ [19].[2] The analysis of radiative D^* decays in Refs. [59,60] allows $0 < g_{DD^*\pi} < 1$. Finally, one can combine the measured branching ratio for $D^0 \to \pi^- e^+ \nu$ with the assumption of a monopole behavior of the form factor $f_+^{D\to\pi}(q^2)$ to obtain $g_{DD^*\pi} \simeq (0.40\pm0.15)\times(200\ \text{MeV}/f_{D^*})$ [61,62]. All these determinations have large uncertainties, however. The semileptonic decay $B \to \pi\,\tau\,\nu$, on the other hand, offers a rather clean measurement of $g_{BB^*\pi}$. By measuring the distribution in the decay angle between the pion and the lepton, it is possible to disentangle the contributions of the form factors f_+ and f_0 to the decay rate [36]. By means of (58), such a measurement would determine the ratio $g_{B^*B\pi}(f_{B^*}/f_B) \simeq 1.1\,g_{BB^*\pi}$, where we have used the results of Refs. [11,14] for the ratio of decay constants. This might be one of the best ways to determine this important coupling constant experimentally.

ACKNOWLEDGMENTS

G.B. thanks John Donoghue for helpful discussions and suggestions. M.N. thanks Mark Wise for useful discussions. Z.L. and M.N. wish to thank the organizers of TASI-93 for the stimulating atmosphere and the relaxing ambience, which helped to complete parts of this work. G.B. acknowledges partial support from the U.S. National Science Foundation. M.N. gratefully acknowledges financial support from the BASF Aktiengesellschaft and from the German National Scholarship Foundation. Y.N. was supported in part by the Israel Commission for Basic Research, by the United States–Israel Binational Science Foundation (BSF), and by the Minerva Foundation. This work was also supported by the Department of Energy, Contract No. DE-AC03-76SF00515.

APPENDIX A: RADIATIVE CORRECTIONS

The renormalization-scale dependence of the universal functions of HQET can be derived from the requirement that the physical meson form factors defined in (22) and (28) be μ independent. Using the explicit expressions for the Wilson coefficients given in (4) and (9), we find that, in leading logarithmic approximation, the following combinations of functions are renormalization-group invariant:

$$
\begin{aligned}
z_1(v\cdot p) &= F_5(v\cdot p,\mu) + v\cdot p\, B(v\cdot p,\mu)\,, \quad z_2(v\cdot p) = F_6(v\cdot p,\mu) + \tfrac{1}{3}(\bar\Lambda - v\cdot p)\, B(v\cdot p,\mu)\,,\\
z_3(v\cdot p) &= \frac{G_1(v\cdot p,\mu)}{A(v\cdot p,\mu)} - \frac{16}{3\beta}\,(\bar\Lambda - v\cdot p)\,\ln[\alpha_s(\mu)]\,, \quad z_4(v\cdot p) = \frac{G_2(v\cdot p,\mu)}{B(v\cdot p,\mu)} - \frac{16}{3\beta}\,(\bar\Lambda - v\cdot p)\,\ln[\alpha_s(\mu)]\,,\\
z_5(v\cdot p) &= [\alpha_s(\mu)]^{-1/\beta}\,\Big\{G_3(v\cdot p,\mu) - G_5(v\cdot p,\mu) + G_6(v\cdot p,\mu) + \tfrac{8}{9}(\bar\Lambda - v\cdot p)\, B(v\cdot p,\mu)\Big\}\,,\\
z_6(v\cdot p) &= [\alpha_s(\mu)]^{-1/\beta}\,\Big\{G_4(v\cdot p,\mu) - \tfrac{1}{3}(1-\hat p^2)\, G_5(v\cdot p,\mu) + G_6(v\cdot p,\mu)\\
&\qquad - \tfrac{8}{27}(\bar\Lambda - v\cdot p)\,\Big[A(v\cdot p,\mu) + B(v\cdot p,\mu)\Big]\Big\}\,,\\
z_7(v\cdot p) &= [\alpha_s(\mu)]^{-1/\beta}\,\Big\{G_5(v\cdot p,\mu) - \tfrac{2}{3}\,F_5(v\cdot p,\mu) - \tfrac{2}{3}\,v\cdot p\, B(v\cdot p,\mu)\Big\}\,,\\
z_8(v\cdot p) &= [\alpha_s(\mu)]^{-1/\beta}\,\Big\{G_6(v\cdot p,\mu) + \tfrac{2}{3}\,F_6(v\cdot p,\mu) - \tfrac{2}{27}(\bar\Lambda - v\cdot p)\, B(v\cdot p,\mu)\Big\}\,.
\end{aligned}
\tag{A1}
$$

These renormalized functions are still universal in that they do not depend on the heavy quark mass. In the next step, we define related renormalized functions $\hat F_i(v\cdot p)$ and $\hat G_i(v\cdot p)$ in analogy to (27), by requiring that they be μ independent and agree at tree level with the original functions F_i and G_i. This necessarily introduces logarithmic dependence on the heavy quark mass. We obtain, again in the leading logarithmic approximation,

[2]The tighter bound $g_{DD^*\pi} < 0.7$ is obtained when one uses the value $\Gamma(D^{*+}) < 131$ keV reported in Ref. [58].

$$\hat{F}_5(v\cdot p) = F_5(v\cdot p,\mu) - \left(x^{2/\beta}-1\right) v\cdot p\, B(v\cdot p,\mu)\,, \quad \hat{F}_6(v\cdot p) = F_6(v\cdot p,\mu) - \tfrac{1}{3}\left(x^{2/\beta}-1\right)(\bar\Lambda - v\cdot p)\, B(v\cdot p,\mu)\,,$$

$$\frac{\hat{G}_1(v\cdot p)}{\hat{A}(v\cdot p)} = \frac{G_1(v\cdot p,\mu)}{A(v\cdot p,\mu)} - \frac{16}{3\beta}(\bar\Lambda - v\cdot p)\ln x\,, \quad \frac{\hat{G}_2(v\cdot p)}{\hat{B}(v\cdot p)} = \frac{G_2(v\cdot p,\mu)}{B(v\cdot p,\mu)} - \frac{16}{3\beta}(\bar\Lambda - v\cdot p)\ln x\,,$$

$$\begin{aligned}\hat{G}_3(v\cdot p) &= x^{-1/\beta} G_3(v\cdot p,\mu) - \tfrac{32}{27}\left(x^{2/\beta} - x^{-1/\beta}\right)(\bar\Lambda - v\cdot p)\, B(v\cdot p,\mu)\\ &\quad + \tfrac{2}{3}\left(1-x^{-1/\beta}\right)\left[F_5(v\cdot p,\mu) + F_6(v\cdot p,\mu) + \tfrac{1}{3}(\bar\Lambda + 2v\cdot p)\, B(v\cdot p,\mu)\right],\end{aligned}$$

$$\begin{aligned}\hat{G}_4(v\cdot p) &= x^{-1/\beta} G_4(v\cdot p,\mu) + \tfrac{8}{27}\left(x^{2/\beta} - x^{-1/\beta}\right)(\bar\Lambda - v\cdot p)\, A(v\cdot p,\mu)\\ &\quad + \tfrac{2}{9}\left(1-x^{-1/\beta}\right)\left[(1-\hat{p}^2)\, F_5(v\cdot p,\mu) + 3F_6(v\cdot p,\mu) + (\bar\Lambda - v\cdot p\,\hat{p}^2)\, B(v\cdot p,\mu)\right],\end{aligned}$$

$$\hat{G}_5(v\cdot p) = x^{-1/\beta} G_5(v\cdot p,\mu) + \tfrac{2}{3}\left(1-x^{-1/\beta}\right)\left[F_5(v\cdot p,\mu) + v\cdot p\, B(v\cdot p,\mu)\right],$$

$$\begin{aligned}\hat{G}_6(v\cdot p) &= x^{-1/\beta} G_6(v\cdot p,\mu) + \tfrac{8}{27}\left(x^{2/\beta} - x^{-1/\beta}\right)(\bar\Lambda - v\cdot p)\, B(v\cdot p,\mu)\\ &\quad - \tfrac{2}{3}\left(1-x^{-1/\beta}\right)\left[F_6(v\cdot p,\mu) + \tfrac{1}{3}(\bar\Lambda - v\cdot p)\, B(v\cdot p,\mu)\right], \qquad \text{(A2)}\end{aligned}$$

where $x = \alpha_s(\mu)/\alpha_s(m_Q)$. Using (27) and (A1), it is readily seen that these functions are indeed μ-independent. In terms of them, the $1/m_Q$ expansion of any meson form factor assumes the same form as at tree-level.

APPENDIX B: SOFT PION RELATIONS FOR $\hat{G}_i(v\cdot p)$

In this appendix we derive the soft pion relations for the subleading form factors G_i, which arise from insertions of the $1/m_Q$ corrections in the effective Lagrangian into matrix elements of the leading-order currents. The corresponding corrections to meson decay constants[3] are [14]

$$\langle 0\,|\,i\int dy\, T\{\bar q\,\Gamma\, h_v(0), \mathcal{O}_{\rm kin}(y)\}\,|M(v)\rangle = iF(\mu)\,\mathcal{G}_1(\mu)\,{\rm Tr}\{\Gamma\,\mathcal{M}(v)\}\,, \qquad \text{(B1)}$$

$$\langle 0\,|\,i\int dy\, T\{\bar q\,\Gamma\, h_v(0), \mathcal{O}_{\rm mag}(y)\}\,|M(v)\rangle = 2id_M F(\mu)\,\mathcal{G}_2(\mu)\,{\rm Tr}\{\Gamma\,\mathcal{M}(v)\}\,,$$

where $d_M = 3$ for a pseudoscalar meson, and $d_M = -1$ for a vector meson. The corrections to the coupling of two heavy mesons to the axial vector current can be written as

$$\langle M'(v,p)|\,i\int dy\, T\{\,p\cdot A(0), \mathcal{O}_{\rm kin}(y)\}\,|M(v)\rangle = g_1\,{\rm Tr}\Big\{\gamma_5\,\not p\,\overline{\mathcal{M}}'(v)\mathcal{M}(v)\Big\} + \cdots\,, \qquad \text{(B2)}$$

$$\langle M'(v,p)|\,i\int dy\, T\{\,p\cdot A(0), \mathcal{O}_{\rm mag}(y)\}\,|M(v)\rangle = 2(d_M + d_{M'})\,g_2(\mu)\,{\rm Tr}\Big\{\gamma_5\,\not p\,\overline{\mathcal{M}}'(v)\mathcal{M}(v)\Big\} + \cdots\,,$$

where the ellipses denote terms quadratic and higher order in p.

Let us first work out the pole terms arising from an insertion of $\mathcal{O}_{\rm kin}$. According to Fig. 2(a), there are two single pole contributions:

$$\begin{aligned}&\sum_{M'}\Big[\langle 0\,|\,\bar q\,\Gamma\, h_v\,|M'(v)\rangle\,\frac{i}{2v\cdot(-p)}\,\langle M'(v)|\,i\int dy\, T\{\,p\cdot A(0), \mathcal{O}_{\rm kin}(y)\}\,|M(v)\rangle\\ &\quad + \langle 0\,|\,i\int dy\, T\{\bar q\,\Gamma\, h_v(0), \mathcal{O}_{\rm kin}(y)\}\,|M'(v)\rangle\,\frac{i}{2v\cdot(-p)}\,\langle M'(v)|\,p\cdot A\,|M(v)\rangle\Big]\\ &= -\frac{F(\mu)}{2v\cdot p}\Big[2g\,\mathcal{G}_1(\mu) + g_1\Big]\,{\rm Tr}\Big\{\gamma_5\,(\not p - v\cdot p)\,\Gamma\,\mathcal{M}(v)\Big\} + \cdots\,. \qquad \text{(B3)}\end{aligned}$$

[3]The constants $\mathcal{G}_i(\mu)$ were denoted by $G_i(\mu)$ in the original paper.

As shown in Fig. 2(b), there are also potential double pole contributions. The first diagram gives rise to

$$\sum_{M',M''} \langle 0\,|\,\bar q\,\Gamma\,h_v\,|M''(v)\rangle\,\langle M''(v)|\,\mathcal{O}_{\rm kin}\,|M'(v)\rangle\,\langle M'(v)|\,p\cdot A\,|M(v)\rangle \left(\frac{i}{2v\cdot(-p)}\right)^2$$

$$= -\frac{\lambda_1}{2(v\cdot p)^2}\sum_{M'} \langle 0\,|\,\bar q\,\Gamma\,h_v\,|M'(v)\rangle\,\langle M'(v)|\,p\cdot A\,|M(v)\rangle\,. \quad \text{(B4)}$$

Note that only diagonal terms ($M''=M'$) contribute to the sum. We have introduced the mass parameter λ_1, which parametrizes the matrix element of the kinetic operator. In general, one defines [63]

$$\begin{aligned} \langle M(v)|\,\mathcal{O}_{\rm kin}\,|M(v)\rangle &= 2\,\lambda_1\,,\\ \langle M(v)|\,\mathcal{O}_{\rm mag}\,|M(v)\rangle &= 2d_M\,\lambda_2(\mu)\,. \end{aligned} \quad \text{(B5)}$$

The same matrix elements also determine the $1/m_Q$ corrections to the physical meson masses:

$$m_M = m_Q + \bar\Lambda - \frac{1}{2m_Q}\Big[\lambda_1 + d_M\,C_{\rm mag}(\mu)\,\lambda_2(\mu)\Big] + \cdots\,. \quad \text{(B6)}$$

This induces a mass renormalization which modifies the meson propagator, as shown in the second diagram in Fig. 2(b). The corresponding correction is obtained from the expansion

$$\frac{iM}{[(M+\varepsilon)v+k]^2-M^2} \stackrel{M\to\infty}{=} \frac{i}{2v\cdot k}\left(1-\frac{\varepsilon}{v\cdot k}+\cdots\right), \quad \text{(B7)}$$

where $M = m_Q + \bar\Lambda$. For the kinetic operator, $\varepsilon = -\lambda_1/2m_Q$. (The λ_2 term will be taken into account below.) Combining this with the leading-order pole contribution in (40), we find that the contribution from mass renormalization exactly cancels the double pole contribution (B4). As a result, only the commutator and the single pole terms remain, and we obtain the soft pion relations

$$\begin{aligned} G_1(0,\mu) &= \frac{F(\mu)}{2f_\pi}\Big[2(1-g)\,\mathcal{G}_1(\mu) - g_1\Big]\,,\\ G_2(0,\mu) &= \frac{F(\mu)}{2f_\pi}\Big[2g\,\mathcal{G}_1(\mu) + g_1\Big]\,. \end{aligned} \quad \text{(B8)}$$

Things are slightly more complicated in the case of an insertion of the chromomagnetic operator $\mathcal{O}_{\rm mag}$. The single pole contributions are

$$\frac{F(\mu)}{2v\cdot p}\sum_{M'}\Big[2d_{M'}\,g\,\mathcal{G}_2(\mu) + (d_M+d_{M'})\,g_2(\mu)\Big]\,{\rm Tr}\big\{\Gamma\,\mathcal{M}'(v)\big\}\,{\rm Tr}\Big\{\gamma_5\,\not{p}\,\overline{\mathcal{M}}'(v)\mathcal{M}(v)\Big\}\,. \quad \text{(B9)}$$

To recover the trace structures appearing in (19), we need a second trace identity in addition to (41). It is

$$\sum_{M'}(d_{M'}-d_M)\,{\rm Tr}\big\{\Gamma\,\mathcal{M}'(v)\big\}\,{\rm Tr}\Big\{\gamma_5\,\not{p}\,\overline{\mathcal{M}}'(v)\mathcal{M}(v)\Big\} = -4\,{\rm Tr}\Big\{i\gamma_5\,p_\alpha\gamma_\beta\,\Gamma\,\frac{1+\not{v}}{2}\,\sigma^{\alpha\beta}\mathcal{M}(v)\Big\}\,. \quad \text{(B10)}$$

This allows us to rewrite our result (B9) as

$$-\frac{F(\mu)}{v\cdot p}\Big[g\,\mathcal{G}_2(\mu) + g_2(\mu)\Big]\,{\rm Tr}\Big\{\gamma_5\,\sigma_{\alpha\beta}\,(\not{p}-v\cdot p)\,\Gamma\,\frac{1+\not{v}}{2}\,\sigma^{\alpha\beta}\mathcal{M}(v)\Big\}$$

$$-\frac{2F(\mu)}{v\cdot p}\Big[2g\,\mathcal{G}_2(\mu) + g_2(\mu)\Big]\,{\rm Tr}\Big\{i\gamma_5\,p_\alpha\gamma_\beta\,\Gamma\,\frac{1+\not{v}}{2}\,\sigma^{\alpha\beta}\mathcal{M}(v)\Big\}, \quad \text{(B11)}$$

where we have used that $\frac{1}{2}(1+\not{v})\,\sigma_{\alpha\beta}\mathcal{M}(v)\,\sigma^{\alpha\beta} = 2d_M\,\mathcal{M}(v)$ [14]. The double pole contribution can be calculated in complete analogy to the case of the kinetic operator, except that the contribution from mass renormalization will not cancel the direct double pole term, since the spin of the pole meson can be different from the spin of the external heavy meson. In fact, we find

$$-\frac{\lambda_2}{2(v\cdot p)^2}\sum_{M'}(d_{M'}-d_M)\,\langle 0\,|\,\bar q\,\Gamma\,h_v\,|M'(v)\rangle\,\langle M'(v)|\,p\cdot A\,|M(v)\rangle$$

$$= iF(\mu)\,\lambda_2(\mu)\,\frac{g(v\cdot p)}{(v\cdot p)^2}\,{\rm Tr}\Big\{\gamma_5\,ip_\alpha\gamma_\beta\,\Gamma\,\frac{1+\not{v}}{2}\,\sigma^{\alpha\beta}\,\mathcal{M}(v)\Big\}\,. \quad \text{(B12)}$$

The parameter $\lambda_2(\mu)$ has been defined in (B5). Collecting the commutator, single and double pole contributions and comparing the result with (20), we find the soft pion relations

$$\begin{aligned}\lim_{p\to 0} G_3(v\cdot p) &= \frac{F(\mu)}{2f_\pi}\bigg[-2\lambda_2(\mu)\,\frac{g(v\cdot p)}{v\cdot p} - 8g\,\mathcal{G}_2(\mu) \\ &\quad -4g_2(\mu)\bigg]\,,\\ G_4(0) &= \frac{F(\mu)}{2f_\pi}\Big[2(1-g)\,\mathcal{G}_2(\mu) - 2g_2(\mu)\Big]\,,\\ G_5(0) &= 0\,,\\ G_6(0) &= \frac{F(\mu)}{2f_\pi}\Big[2g\,\mathcal{G}_2 v + 2g_2(\mu)\Big]\,. \end{aligned} \qquad \text{(B13)}$$

To obtain the corresponding relations for the renormalized functions $\hat{G}_i(v\cdot p)$, one first has to renormalize the low-energy parameters appearing on the right-hand side of the soft pion relations. It is easy to see that g, g_1, and λ_1 are not renormalized to all orders in perturbation theory, whereas the μ dependence of $g_2(\mu)$ and $\lambda_2(\mu)$ is compensated by that of the Wilson coefficient of the chromomagnetic operator. The renormalization of $\mathcal{G}_{1,2}$ is slightly more complicated. It is discussed in Ref. [14]. In leading logarithmic approximation, the renormalized low-energy parameters are given by

$$\begin{aligned}&\hat{g}_2 = x^{-3/\beta}\,\hat{g}_2(\mu)\,, \qquad \hat{\lambda}_2 = x^{-3/\beta}\,\lambda_2(\mu)\,,\\ &\hat{\mathcal{G}}_1 = \mathcal{G}_1(\mu) - \frac{8\bar{\Lambda}}{3\beta}\ln x\,, \qquad \hat{\mathcal{G}}_2 = x^{-3/\beta}\left[\mathcal{G}_2(\mu) - \frac{4\bar{\Lambda}}{27}\right],\end{aligned} \qquad \text{(B14)}$$

where $x = \alpha_s(\mu)/\alpha_s(m_Q)$. By means of (B6), $\hat{\lambda}_2$ is related to the mass splitting between vector and pseudoscalar mesons:

$$\hat{\lambda}_2 = \tfrac{1}{4}\,(m_V^2 - m_P^2)\,. \qquad \text{(B15)}$$

The soft pion relations (51) and (52) follow by combining (B8), (B13), (B14), and (A2).

[1] H. Leutwyler and M. Roos, Z. Phys. C **25**, 91 (1984).
[2] E.V. Shuryak, Phys. Lett. **93B**, 134 (1980); Nucl. Phys. **B198**, 83 (1981).
[3] S. Nussinov and W. Wetzel, Phys. Rev. D **36**, 130 (1987).
[4] M.B. Voloshin and M.A. Shifman, Yad. Fiz. **45**, 463 (1987) [Sov. J. Nucl. Phys. **45**, 292 (1987)]; **47**, 801 (1988) [**47**, 511 (1988)].
[5] N. Isgur and M.B. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990).
[6] M. Neubert, Phys. Lett. B **264**, 455 (1991).
[7] J. Bartelt *et al.*, Phys. Rev. Lett. **71**, 4111 (1993).
[8] H. Albrecht *et al.*, Phys. Lett. B **255**, 297 (1991).
[9] N. Isgur and M.B. Wise, Phys. Rev. D **42**, 2388 (1990).
[10] C.R. Allton *et al.*, in *Lattice '90*, Proceedings of the International Symposium, Tallahassee, Florida, 1990, edited by V. M. Heller, A. D. Kennedy, and S. Sanielevici [Nucl. Phys. B (Proc. Suppl.) **20**, 504 (1991)].
[11] A. Abada *et al.*, Nucl. Phys. **B376**, 172 (1992).
[12] C.W. Bernard, C. Heard, J.N. Labrenz, and A. Soni, in *Lattice '91*, Proceedings of the International Symposium, Tsukuba, Japan, 1991, edited by M. Fuhugita *et al.* [Nucl. Phys. B (Proc. Suppl.) **26**, 384 (1992)]; C.W. Bernard, J.N. Labrenz, and A. Soni, this issue, Phys. Rev. D **49**, 2536 (1994).
[13] R.M. Baxter *et al.*, Phys. Rev. D **49**, 1594 (1994).
[14] M. Neubert, Phys. Rev. D **45**, 2451 (1992); **46**, 1076 (1992).
[15] E. Bagan, P. Ball, V.M. Braun, and H.G. Dosch, Phys. Lett. B **278**, 457 (1992).
[16] V. Eletsky and E. Shuryak, Phys. Lett. B **276**, 191 (1992).
[17] C.A. Dominguez and N. Paver, Phys. Lett. B **269**, 169 (1991); **276**, 179 (1992).
[18] N. Isgur and M.B. Wise, Phys. Rev. D **41**, 151 (1990).
[19] M.B. Wise, Phys. Rev. D **45**, 2188 (1992).
[20] G. Burdman and J.F. Donoghue, Phys. Lett. B **280**, 287 (1992).
[21] L. Wolfenstein, Phys. Lett. B **291**, 177 (1992).
[22] E. Eichten and B. Hill, Phys. Lett. B **234**, 511 (1990); **243**, 427 (1990).
[23] B. Grinstein, Nucl. Phys. **B339**, 253 (1990).
[24] H. Georgi, Phys. Lett. B **240**, 447 (1990).
[25] T. Mannel, W. Roberts, and Z. Ryzak, Nucl. Phys. **B368**, 204 (1992).
[26] A.F. Falk, H. Georgi, B. Grinstein, and M.B. Wise, Nucl. Phys. **B343**, 1 (1990).
[27] M.E. Luke, Phys. Lett. B **252**, 447 (1990).
[28] A.F. Falk, B. Grinstein, and M.E. Luke, Nucl. Phys. **B357**, 185 (1991).
[29] A.F. Falk, M. Neubert, and M.E. Luke, Nucl. Phys. **B388**, 363 (1992).
[30] For a comprehensive review, see M. Neubert, Phys. Rep. (to be published).
[31] M. Luke and A.V. Manohar, Phys. Lett. B **286**, 348 (1992).
[32] M. Neubert, Phys. Rev. D **49**, 1542 (1994).
[33] A.F. Falk and B. Grinstein, Phys. Lett. B **247**, 406 (1990).
[34] J.D. Bjorken, in *Gauge Bosons and Heavy Quarks*, Proceedings of the 18th SLAC Summer Institute on Particle Physics, Stanford, California, 1990, edited by J.F. Hawthorne (SLAC Report No. 378, Stanford, 1991), pp. 167.
[35] T. Mannel, in *QCD–20 Years Later*, Proceedings of the Workshop, Aachen, Germany, 1992, edited by P.M. Zerwas and H.A. Kastrup (World Scientific, Singapore, in press).
[36] C.A. Dominguez, J.G. Körner, and K. Schilcher, Phys. Lett. B **248**, 399 (1990).
[37] H. Georgi, B. Grinstein, and M.B. Wise, Phys. Lett. B **252**, 456 (1990).
[38] M. Neubert, Phys. Rev. D **47**, 4063 (1993).
[39] Z. Ligeti, M. Neubert, and Y. Nir, Phys. Lett. B **301**, 101 (1993); M. Neubert, Z. Ligeti, and Y. Nir, Phys. Rev. D **47**, 5060 (1993); Z. Ligeti, Y. Nir, and M. Neubert, *ibid.* **49**, 1302 (1994).
[40] C.W. Bernard, Y. Shen, and A. Soni, Phys. Lett. B **317**, 164 (1993).

[41] S.P. Booth *et al.*, Edinburgh Report No. 93/525, 1993 (unpublished).
[42] M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C **29**, 637 (1985).
[43] N. Isgur, D. Scora, B. Grinstein, and M.B. Wise, Phys. Rev. D **39**, 799 (1989).
[44] C.A. Dominguez and N. Paver, Z. Phys. C **41**, 217 (1988).
[45] A.A. Ovchinnikov, Yad. Fiz. **50**, 831 (1989) [Sov. J. Nucl. Phys. **50**, 519 (1989)]; Phys. Lett. B **229**, 127 (1989).
[46] V.L. Chernyak and I.R. Zhitnitski, Nucl. Phys. **B345**, 137 (1990).
[47] P. Ball, V.M. Braun, and H.G. Dosch, Phys. Lett. B **273**, 316 (1991); P. Ball, Phys. Rev. D **48**, 3190 (1993).
[48] S. Narison, Phys. Lett. B **283**, 384 (1992).
[49] V.M. Belyaev, A. Khodjamirian, and R. Rückl, CERN Report No. CERN-TH.6880/93, 1993 (unpublished).
[50] G. Kramer, T. Mannel, and G.A. Schuler, Z. Phys. C **51**, 649 (1991).
[51] R. Casalbuoni *et al.*, Phys. Lett. B **299**, 139 (1993).
[52] Q.P. Xu, Phys. Lett. B **306**, 363 (1993).
[53] C.O. Dib and F. Vera, Phys. Rev. D **47**, 3938 (1993).
[54] N. Isgur, Phys. Rev. D **43**, 810 (1991).
[55] G. Burdman and J.F. Donoghue, Phys. Lett. B **270**, 55 (1991).
[56] T.-M. Yan *et al.*, Phys. Rev. D **46**, 1148 (1992).
[57] W.A. Bardeen and C.T. Hill, Phys. Rev. D **49**, 409 (1994).
[58] S. Barlag *et al.*, Phys. Lett. B **278**, 480 (1992).
[59] P. Cho and H. Georgi, Phys. Lett. B **296**, 408 (1992).
[60] J.F. Amundson *et al.*, Phys. Lett. B **296**, 415 (1992).
[61] R. Casalbuoni *et al.*, Phys. Lett. B **294**, 106 (1992).
[62] R. Fleischer, Phys. Lett. B **303**, 147 (1993).
[63] A.F. Falk and M. Neubert, Phys. Rev. D **47**, 2965 (1993).