

## Empirical determination of the very high energy heavy quark cross section from nonaccelerator data

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To cosmic rays incident near the horizon the Earth's atmosphere represents a beam dump with a slant depth reaching  $36\,000\text{ g cm}^{-2}$  at  $90^\circ$ . The prompt decay of a heavy quark produced by very high energy cosmic ray showers will leave an unmistakable signature in this dump. We translate the failure of experiments to detect such a signal into an upper limit on the heavy quark hadroproduction cross section in the energy region beyond existing accelerators. Our results disfavor any rapid growth of the cross section or the gluon structure function beyond conservative estimates based on perturbative QCD.

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### I. INTRODUCTION

Understanding the production of heavy quarks, especially the relatively light charm quark, is a subject that is at the forefront of particle physics for several reasons. Leptonic decays of the charm and  $b$  quark are the source of high momentum electrons and muons which represent critical backgrounds in the search for new phenomena at present and future hadron colliders. Perspectives for very interesting neutrino physics at future colliders, including the direct observation of  $\tau$  neutrinos, depend critically on the production of charm [1]. Because of its intermediate mass, the charm quark provides a bridge between light and heavy quark physics. Precisely for this reason the mechanisms for charm production are believed not to be well understood. A perturbative calculation lies beyond the scope of existing QCD technology because it requires resummation of large logarithms of  $1/x$  with  $x \simeq m_c/\sqrt{s}$ [2]. Even the leading order perturbative calculations, which require inclusion of  $O(\alpha_s^2)$  and  $O(\alpha_s^3)$  diagrams, are unreliable because the results are sensitive to the assumed quark mass and the renormalization scale. Because charm is at high energies predominantly produced by gluons, the cross section also critically depends on the low- $x$  behavior of the gluon structure function which is poorly known or undetermined depending on the energy [3]. Charm production is a great laboratory to study these issues on the interphase of perturbative and nonperturbative QCD. The experimental status of open charm production is unfortunately rather unsettled. Large uncertainties are especially associated with the production of charm particles in the Feynman- $x$ ,  $x_F \rightarrow 1$  region which is poorly covered by the highest energy colliders.

Cosmic ray particles have been observed up to energies exceeding  $10^{20}$  eV [4], about 100 times the Superconducting Super Collider (SSC) energy, and thus have a unique potential for studying the cross section for charmed par-

ticle production at energies impossible to achieve with current technology in controlled experiments. More importantly, the leptonic decay of charm particles into high energy muons produces a gold-plated signature leading to the rather unusual circumstance of doing a "clean" particle physics experiment with cosmic particles. Cosmic rays with energy in excess of 100 TeV initiate air showers which can be studied with sea-level particle detectors. The detected flux is a steeply falling function of zenith angle because the depth of atmosphere traversed by a cascade reaching the ground rises rapidly from 1030 to  $36\,000\text{ g cm}^{-2}$  as the zenith angle varies from zero to  $90^\circ$ . Thus near the horizon ( $90^\circ$ ) close to 1000 radiation lengths of matter separate the interaction from the detector and the configuration of the experiment is identical to that of any accelerator-based beam dump experiment. Most secondaries such as pions and kaons are absorbed in the dump and only penetrating particles, such as muons and neutrinos produced in the initial interaction, reach the detector. For very high energy interactions the decay of charm particles is the dominant source of high energy secondary muons. So counting high energy muons at large zenith angles, typically larger than  $60^\circ$ , determines the charm cross section. There is no background from the semileptonic decay of pions and kaons which, as a result of time dilation, interact and lose energy rather than decay into high energy muons. This background has been extensively studied and is well understood [5].

It should also be pointed out that the muon signature can be sharply defined, even when using a conventional air shower array as a detector. The high energy muon will traverse the atmosphere and occasionally lose energy by catastrophic photon bremsstrahlung. If the photon shower is produced close to the detector, it will be recorded and is referred to as a horizontal air shower. The origin of such a signal can be verified as (i) it must be independent of zenith angle unlike any potential background, (ii) it must have a low muon content as the

shower is purely electromagnetic, and (iii) it is initiated by muon bremsstrahlung near the array and is therefore detected in the vicinity of shower maximum. All features can be experimentally verified and incorporated in the triggers selecting horizontal air showers. Thus the measurement of high energy muon fluxes can undoubtedly provide useful data on the production of charm.

## II. CALCULATION

It is the purpose of this paper to illustrate how several existing installations can collect data relevant to charm production. Most importantly, the Akeno air shower detector in Japan has published in 1985 an upper limit on the flux of muon-poor horizontal air showers with energy in excess of 100 TeV [6]. It is straightforward to translate the upper limit into an upper limit on the charm cross section. The translation does however require knowledge of (i) the primary cosmic ray spectrum which is well measured in the energy region under consideration although the composition is a matter of debate, (ii) the energy loss of high energy muons, (iii) the structure of the electromagnetic shower radiated by the muon which is well understood, and (iv) the effective area and exposure time of the detector which has been published [6].

The calculations, which we have performed both analytically and by Monte Carlo, are routine. We briefly sketch them below; details can be found in Ref. [7]. The cascade equations for the production of charmed hadrons can be written as

$$\begin{aligned} \frac{d\phi_i(x, E_i, \theta)}{dx} = & -(1/\lambda_i)\phi_i(x, E_i, \theta) - \frac{E_{cr}^i}{E_i x} \phi_i(x, E_i, \theta) \\ & + \int_{E_i}^{\infty} (1/\lambda_N)\phi_N(x, E_N, \theta) \frac{dW^{iN}}{dE_i} dE_N, \end{aligned} \quad (1)$$

where  $\phi_i$  is the flux of particles of type  $i$  ( $= D^\pm, D^0, \Lambda_c$ ),  $\phi_N$  is the initial nucleon flux,  $x$  is the depth of atmosphere penetrated by the cascade, and  $\lambda_i$  is the nuclear interaction mean free path for particle  $i$ . Both  $x$  and  $\lambda_i$  are in units of  $\text{g cm}^{-2}$ . The critical energy  $E_{cr}^i$  corresponds (modulo the ratio  $x/\lambda_i$ ) to the energy at which the probability that the charm particle decays equals the probability of a nuclear interaction,

$$E_{cr}^i(\theta, x) = \frac{m_i c^2}{c\tau_i} \frac{x}{\rho(h)}. \quad (2)$$

Here  $m_i$  and  $\tau_i$  are the mass and lifetime of particle  $i$  and  $\rho(h)$  is the atmospheric density at altitude  $h$ . For an ideal isothermal atmosphere the ratio  $x/\rho(h)$  is essentially a function of zenith angle, independent of vertical depth. The first and second terms in (1) represent the depletion in the cascade of particles of type  $i$  as a result of their interaction in the air and decay, respectively. The last term takes into account the production of charm particles of type  $i$  in nuclear interactions in the cascade. This production rate is given by

$$\frac{1}{\lambda_N} \frac{dW^{iN}}{dE_i} = \frac{\sigma_{NA}^i}{\lambda_N \sigma_{in}^{NA}} \frac{df_i}{dE_i}. \quad (3)$$

Here  $\sigma_{in}^{NA}$  is the inelastic nucleon-air ( $A = 14.4$ ) cross section,  $\sigma_{NA}^i$  is the cross section, and  $df_i/dE_i$  the energy spectrum for the production of particles of type  $i$ .

The differential equation is solved by using the approximate solution for the nucleon flux,

$$\phi_N(x, E_N, \theta) = N E_N^{-(\gamma+1)} e^{-x/\Lambda_N}. \quad (4)$$

Equation (1) becomes

$$\frac{d\phi_i}{dx} = -\frac{1}{\lambda_i} \phi_i - \frac{E_{cr}^i}{E_i x} \phi_i + K_i e^{-x/\Lambda_N}, \quad (5)$$

where  $K_i$  describes the production of charmed particles at energy  $E_i$ ,

$$K_i = \int_{E_i}^{\infty} (N/\lambda_N) E_N^{-(\gamma+1)} \frac{dW^{iN}}{dE_i} dE_N. \quad (6)$$

The solution can be expressed as a combination of incomplete  $\Gamma$  functions which can be expanded as a power series in  $x$ , but for numerical calculations it is more convenient to express it as an integral:

$$\phi_i(x, E_i, \theta) = x^{-\frac{E_{cr}^i(\theta)}{E_i}} K_i e^{-x/\Lambda_N} \int_0^x dx x^{\frac{E_{cr}^i(\theta)}{E_i}} e^{x(\frac{1}{\lambda_i} - \frac{1}{\Lambda_N})}. \quad (7)$$

Here  $\lambda_i$ ,  $\Lambda_N$  are respectively the interaction length for particle  $i$  and the attenuation length for nucleons [7].

We are now ready to calculate the flux of muons of energy  $E_\mu$ , zenith angle  $\theta$  at depth  $x$ :

$$G_i^\mu(x, E_\mu, \theta) = \int_{E_i^{\min}}^{E_i^{\max}} dE_i \frac{E_{cr}^i(\theta)}{E_i x} B_i \frac{df^\mu}{dE_\mu} \phi_i(x, E_i, \theta), \quad (8)$$

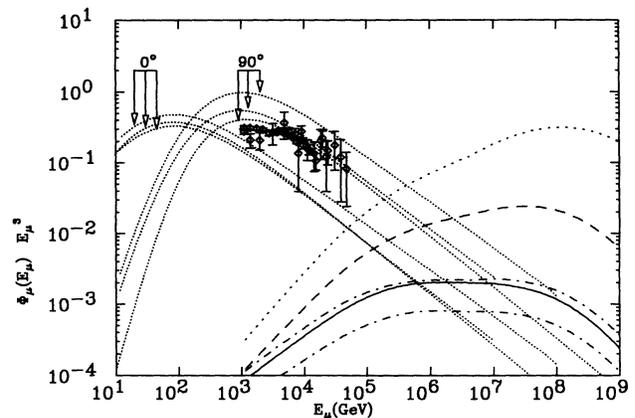


FIG. 1. Differential energy spectrum (times  $E_\mu^3$ ) for muons from pion decay (dotted lines showing the flux at  $90^\circ$  and  $0^\circ$  zenith angles) and from charm (all other). For pion-induced muons the higher flux is predicted by the QGSM, the two others are taken from Refs. [8, 9]. The dotted line is obtained using the charm production model of Ref. [12], the dashed and solid lines are the perturbative QCD predictions using KMRSD<sup>-</sup> and KMRSD<sup>0</sup> structure functions, and the dot-dashed lines are the QGSM result. Also shown are the experimental data from Ref. [16].

which incorporates the three-body decay of the charmed particles  $D \rightarrow K\mu\nu$  and  $\Lambda_c \rightarrow \Lambda_0\mu\nu$ .  $B_i$  is the branching ratio and  $df^\mu/dE_\mu$  is a three-body phase space integral. All the integrals can be evaluated using nonstatistical numerical methods. In addition we included variation of the critical energy with angle and depth. We took into account the variation of atmosphere density with height using a standard parametrization. In calculating the depth of atmosphere near the horizon it is essential to take into account the Earth's curvature. The muon energy loss in the atmosphere was calculated in the standard approximation. In Fig. 1 we show the muon flux for muons from the semileptonic decay of pions (at  $0^\circ$  and  $90^\circ$ ) and charm particles. The fluxes of conventional muons from pion decay have been calculated by several groups and are in agreement with the data [8, 9]. The upper curve, calculated from the quark-gluon string model (QGSM) [10], ostensibly disagrees with the data; we return to this further on. The flux of muons of charm origin are evaluated for a variety of models [11, 12, 10]. At the energies considered charmed mesons decay in their first interaction length. As a result muon production is independent of zenith angle. Pions and kaons, on the contrary, produce a muon flux which behaves roughly like  $\sec\theta$ .

High energy muons propagate through the atmosphere and lose energy by radiating bremsstrahlung photons of energy  $E_\gamma (= yE_\mu)$ . The photon initiates an electromagnetic cascade. The number of electrons in the shower peaks at a depth  $x \sim \ln(E_\gamma/E_c)$  g cm $^{-2}$  and decreases exponentially after shower maximum. Here  $E_c$  is the critical electromagnetic shower energy of 74 MeV. The number of electrons as a function of the depth and the initial photon energy is well parametrized by the Greisen function:

$$\phi_{\text{sh}}(N_e) = \int_0^{x_{\text{max}}} dx \int_0^1 \frac{dy}{y} \phi_\mu \left( E_\mu = \frac{E_\gamma(N_e, x)}{y} \right) \frac{d\sigma}{dy} \frac{dE}{dN_e}. \quad (10)$$

$\phi_{\text{sh}}(N_e)$  is the differential number of showers per unit time area and solid angle for fixed number of electrons  $N_e$ .  $\phi_\mu(E_\mu)$  is the differential flux of muons at energy  $E_\mu$  and  $d\sigma/dy$  is the differential bremsstrahlung cross section for muons in air. For more details see Ref. [8]. Our results are shown in Fig. 2 where we plot the integral shower rate produced by muons from pions (at  $0^\circ$  and  $90^\circ$ ) and from charm as a function of the shower size.

Finally, the number of observed showers above a given energy and angle is given by

$$N_{\text{sh}}(N_e > N_{e0}, \theta > \theta_0) = T \int_{N_{e0}}^{\infty} dN_e A(N_e) \int_0^{\Omega_0} d\Omega \phi_{\text{sh}}(N_e, \theta), \quad (11)$$

where  $A(N_e)$  is the effective area of the array and  $T$  is the observation time.

### III. RESULTS

We now study the implications for the charm cross section from the fact that the Akeno air shower array failed to observe horizontal air showers generated by muons of charm origin above an energy threshold  $N_e = 10^5$ – $10^6$ . It is easy to estimate [13] that the data is sensitive to initial proton energies one order of magnitude larger than the

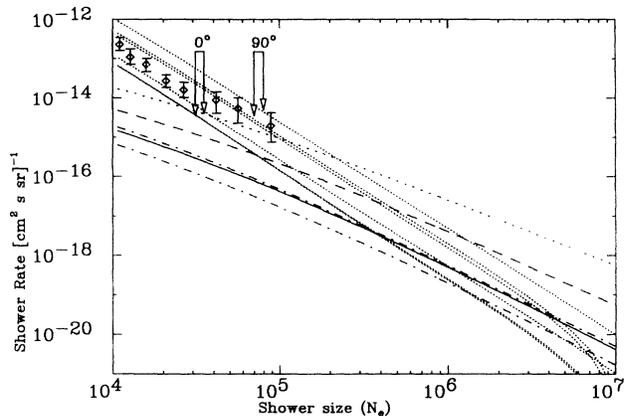


FIG. 2. Integral shower rate as a function of the shower size from pions (dotted line) and from charm (same notation as in Fig. 1) Also shown are data from a Tokyo University measurement [17].

$$N_e(E, x) = \frac{0.31}{\sqrt{\ln(E/E_c)}} e^{x(1 - \frac{3}{2} \ln \frac{2n+3x}{x+21\ln(E/E_c)})}. \quad (9)$$

In cosmic ray experiments it is customary to quote the observed number  $N_e$  of electrons and positrons in the horizontal shower rather than the energy of the photon which is given by inversion of the above equation. To a good approximation the energy of the photon, in GeV units, is given by  $2N_e$ .

The rates of horizontal cascades with fixed shower size  $N_e$  are obtained by integration of the parent muon flux, the  $y$ -differential cross section of the corresponding interaction and the atmospheric depth  $x$ :

muon energy. Our detailed simulation reproduces this result and we can therefore establish a bound on the charm cross section at energies of order  $E_{\text{lab}} \approx 10^3$ – $10^4$  TeV ( $\sqrt{s} \approx 1$ – $5$  TeV). Our conclusions are summarized in Figs. 3 and 4. Figure 3 shows our final result as an upper bound on the charm cross section obtained by comparing a variety of extrapolations of the accelerator data [14] with the published bound on horizontal air showers. We fitted the accelerator data in Fig. 3 with a function that reproduces the charm cross section up to some energy  $E_{\text{cut}}$ ,  $10^2 \lesssim E_{\text{cut}} \lesssim 10^3$  GeV. Above this energy we

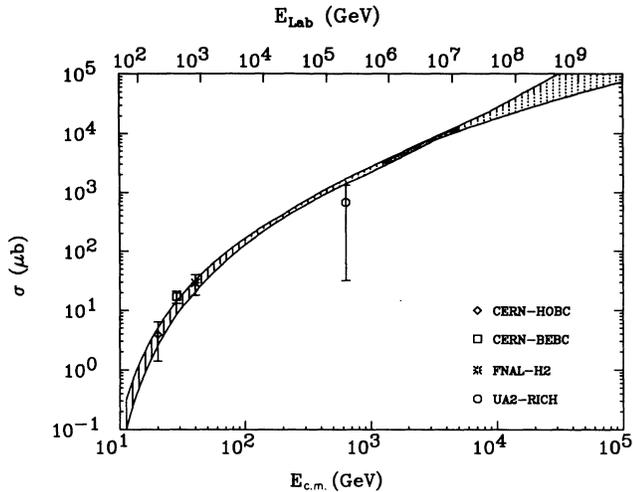


FIG. 3. Total charm quark production cross section vs c.m. and laboratory energy. The shaded band represents the upper bound on the cross section obtained by requiring that extrapolations of the accelerator data accommodate the failure of the Akeno cosmic ray experiment to detect muon decay of charm particles. The width of the band reflects the use of a variety of  $x_F$  distributions in the derivation of the bounds. The upper (lower) edge of the bands correspond to the steepest (flattest) assumption for the  $x_F$  distribution.

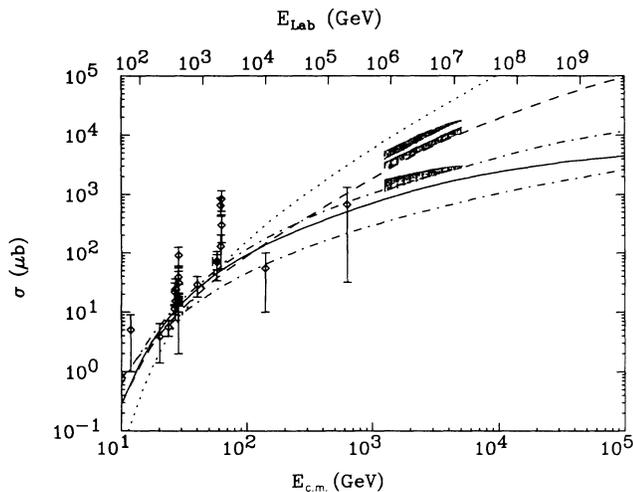


FIG. 4. Total charm quark production cross section vs the c.m. and laboratory energy for several models. The solid (dashed) line is the prediction from perturbative QCD to order  $O(\alpha_s^3)$  with KMRSD0 (KMRSD<sup>-</sup>) structure functions and  $m_c = 1.3$  GeV. The dotted line represents the result of the same calculation using the structure functions from Ref. [12] and  $m_c = 1.6$  GeV. In all cases the scale of both  $\alpha_s$  and the structure functions is  $Q^2 = 9$  GeV<sup>2</sup>. Dot-dashed lines are the nonperturbative results from the quark-gluon string model for two parameters values  $\alpha_\psi = -2.2$  (upper) and  $\alpha_\psi = 0$  (lower). Also shown is a compilation of experimental data from Ref. [14] and our bounds (shaded bands). Upper and lower band correspond to the limit using a  $x_F$  distribution  $d\sigma/dx_F \sim (1-x_F)^n$  for  $n = 40, 10$  respectively. The middle band represents the limit obtained with the more realistic  $x_F$  distribution from perturbative QCD and the quark-gluon string model. The bands span the sensitivity range of the Akeno data.

extrapolated the cross section using a variety of asymptotic parametrizations of the generic form  $a \ln^n(s) + b$  and  $as^n + b$  where  $a$  and  $b$  are chosen to ensure the continuity of the cross section and the slope at  $E = E_{\text{cut}}$ . The transition energy and the asymptotic behavior were varied as independent parameters. We verified that the result is insensitive to the parametrization.

The derivation does, however, require an assumption for the  $x_F$  dependence of the inclusive charm cross section. The bands representing the bounds in Figs. 3 and 4 reflect this ambiguity, which has been modeled using the  $x_F$  distributions for  $D$  mesons from extreme predictions ranging from the quark-gluon string model to perturbative QCD. These  $x_F$  distributions are compiled in Fig. 5. From the figure we conclude that the bounds are dominated by charm production in the  $x_F \sim 0.1$  region. In terms of a  $(1-x_F)^n$  parametrization our range of assumptions for the  $x_F$  distribution includes  $n = 10-40$ , thus generously covering the ambiguities associated with the gluon structure function in the relevant energy range; see Fig. 5. In the end the ambiguity is less than a factor of 2. Deriving the bound with a flatter  $x_F$  distribution will strengthen it as seen in Fig. 4. A flatter distribution would fall outside the expectations of perturbative QCD but could, for instance, be associated with the forward production of  $\Lambda_c$ ; see, e.g., Appel in Ref. [14]. The forward baryon is more effective at producing high energy muons and as a consequence the absence of a positive signal can only be accommodated with a smaller production cross section. This trend is, however, offset by a reduction of the  $\Lambda_c$  semileptonic branching ratio by a factor of 3.

The actual prediction for the integral number of horizontal air showers at  $\theta > 60^\circ$  corresponding to the bounds on the charm cross section shown in Fig. 3, is shown in Fig. 6 along with predictions from a variety of models for charm production. Also shown is the present

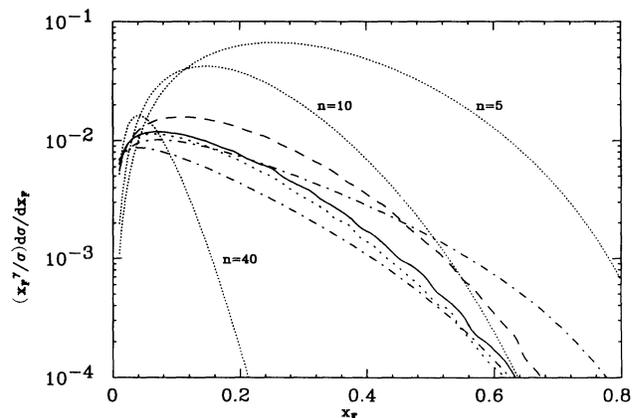


FIG. 5.  $x_F$  distribution of charm particles produced by a cosmic ray flux with a power law spectrum for a c.m. energy of  $E_{\text{c.m.}} = 1$  TeV. The dotted lines, shown for comparison, represent the Feynman- $x$  distribution  $(x_F^\gamma/\sigma)d\sigma/dx_F$  for the simple parametrization  $(1/\sigma)d\sigma/dx_F = (1-x_F)^n$  ( $n = 5, 10, 40$ ). Dashed and solid lines are the KMRSD<sup>-</sup> and KMRSD0 prediction respectively. Dotted line is the prediction from Ref. [12]. Dot-dashed lines are the result for the QGSM.

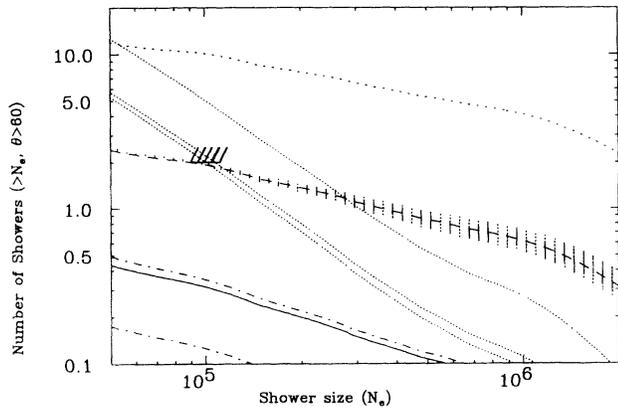


FIG. 6. Integral number of horizontal showers (shower size  $> N_e$ ) for zenith angles greater than  $60^\circ$  as a function of shower size. The shaded area corresponds to the minimum number of showers initiated by prompt muons from charm decay for the cross sections shown in Fig. 3. The other curves are the predictions for the different models in Fig. 4 (using the same notation and value of the parameters). Also shown is the limit from the Akeno air shower experiment for  $N_e = 10^5$ .

experimental upper limit on the horizontal air shower rates. This limit is established on the basis of less than two detected showers with  $N_e > 10^5$  at  $\theta > 60^\circ$  in  $T = 1.07 \times 10^8$  s. The bounds should be treated with caution as it is subject to the usual statistics of small numbers.

It can be seen from Fig. 6 that our calculation predicts two showers above  $N_e = 10^5$  from conventional pion decay origin. The experiment actually observed two. With a larger event sample one can differentiate the pion or charm origin of the horizontal showers by their contrasting zenith angle distributions. This is not possible with two events and therefore, in establishing a bound, we therefore allowed for the possibility that they are actually initiated by charm particles. Note that the predictions of the quark-gluon string model for both muons and horizontal showers from pion decay exceed the experimental observations; see Figs. 1 and 6. The model can be made consistent with observation by assuming that a significant fraction of the primaries are heavy nuclei rather than protons. Dividing the primary energy into approximately  $A$  nucleon showers of energy  $E/A$  shifts the predicted fluxes to lower muon(shower) energy thus avoiding the conflict with the data. This illustrates how the arguments in this paper can be turned around. In the presence of a measurement of the inclusive pion or charm cross sections at future colliders, the data in Figs. 1 and 6 can be used to determine the unknown chemical composition of the cosmic rays in this energy range.

From a theoretical point of view our results, though qualitative, are interesting. As shown in Fig. 4 the bound is saturated by a straightforward perturbative calculation of the charm cross section to order  $O(\alpha_s^3)$  using the Kwiecinski-Martin-Roberts-Stirling set  $D^-$  (KMRS $D^-$ ) structure functions [11]. The bound does not allow for

large enhancements in the cross section or in the growth of the gluon structure function at small values of  $x$  beyond what is implied by the scaling  $1/x^{1.5}$  prediction assumed in the KMRS $D^-$  parametrization. Both had been widely predicted in the literature. Also, the fact that the next-to-leading-order contribution to the cross section is of the same order as the leading-order result led to speculation of further enhancements associated with the resummation of large logarithms  $\ln(\sqrt{s}/m_c)$ . For illustration we have plotted the prediction for the charm cross section obtained from perturbative QCD to order  $O(\alpha_s^3)$  with structure functions growing as  $1/x$  (KMRS set  $D0$ ), and  $1/x^{1.5}$  (KMRS $D^-$  and Nason [12]) together with the nonperturbative result from the quark-gluon string model for two sets of parameters ( $\alpha_\Psi = 0, -2.18$ ) [10].

Is it possible to relax this bound? In deriving our bound we assumed that all high energy cosmic rays are protons. It is experimentally known that the cosmic ray spectrum is dominated by protons up to 100 TeV. Above this energy the composition is unknown. Introducing heavy primaries in the spectrum has, to a first approximation, the effect of replacing protons of energy  $E$  by  $A$  protons of energy  $E/A$  as already discussed above. These are less efficient at producing muons above the fixed threshold set by the experiment and our bound on the production cross section will be relaxed. We illustrate the sensitivity to composition introducing an extreme assumption. In order to accommodate cosmic ray observations without having to claim the appearance of “anomalies” such as the celebrated Centauro phenomena, the Fuji [15] group has assumed that 40% of the cosmic rays are heavy nuclei. We will follow this lead and make the further extreme assumption that all nuclei are iron. The net effect is to relax the bound in Fig. 3 by a factor 1.6–2 depending on the energy, allowing also a faster growth.

In summary, while we do not know of any credible way to weaken the bound we presented by more than “a factor,” it should hold in a qualitative sense. The production of forward particles will, on the contrary, considerably strengthen it. Evidence for forward production of charmed baryons and mesons has been presented in several experiments. Especially a component of forward  $D$  mesons with a harder energy spectrum should significantly strengthen the upper limit on charm hadroproduction we derived from horizontal air showers.

Consequences of relevance to future experiments both in collider physics and high energy astrophysics can be derived from our result. Our bound translates into a charm cross section of the order  $\sigma \approx 4\text{--}10$  mb at CERN Large Hadron Collider (LHC) and SSC energies. As pointed out the perspectives for the direct observation of  $\tau$  neutrinos, predominantly produced via the leptonic decay of strange charmed  $D$  mesons, depend crucially on the value of this cross section [1]. An estimate yields a number of neutrino interactions of the order 1000 (600) at LHC (SSC) per year.

The limit is consistent with the charm cross section derived from the observation of a penetrating component in cosmic-ray-induced showers, the so-called long-flying

component. Several other cosmic ray experiments have found indirect evidence for very large charm cross sections in the 10–100 TeV energy range. Our results do not support these claims [18].

Finally, the production of high energy neutrinos by charmed particles produced in the atmosphere has been identified as a potential background in the high energy neutrino telescopes presently under construction. The results derived in this paper are directly relevant and reassuring. Our results clearly imply that the observation of charm is within easy reach of air shower arrays having good acceptance at large angles. Also, existing underground experiments are in principle sensitive to the high energy muons discussed here [8].

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- [1] A. De Rújula, E. Fernández, and J.J. Gómez-Cadenas, *Nucl. Phys.* **B405**, 80 (1993).
  - [2] J.C. Collins and R.K. Ellis, *Nucl. Phys.* **B360**, 3 (1991).
  - [3] P. Nason, S. Dawson, and R.K. Ellis, *Nucl. Phys.* **B303**, 607 (1988).
  - [4] M. Nagano *et al.*, *J. Phys. Soc. Jpn.* **30**, 33 (1971); S. Mikamo *et al.*, *Lett. Nuovo Cimento* **34** (10), 273 (1982).
  - [5] O.C. Alkoffer *et al.*, in *Proceedings of the 17th International Cosmic Ray Conference*, Paris, France, 1981 (Centre d’Etudes Nucleaires, Saclay, 1981), Vol. 10, p. 321; Y. Muraki *et al.*, *Phys. Rev. D* **28**, 40 (1983); S. Matsuno *et al.*, *ibid.* **29**, 1 (1984).
  - [6] M. Nagano *et al.*, *J. Phys. G* **12**, 69 (1986); M. Nagano (private communication).
  - [7] T.K. Gaisser, *Cosmic Rays and Particle Physics* (Cambridge University Press, Cambridge, England, 1990).
  - [8] E. Zas, F. Halzen, and R.A. Vázquez, *Astropart. Phys.* **1**, 297 (1993); F. Halzen and E. Zas, *Phys. Lett. B* **289**, 184 (1992).
  - [9] L.V. Volkova, *Yad. Fiz.* **31**, 1510 (1980) [*Sov. J. Nucl. Phys.* **31**, 784 (1980)].
  - [10] A.B. Kaidalov and O.I. Piskunova, *Yad. Fiz.* **43**, 1545 (1986) [*Sov. J. Nucl. Phys.* **43**, 994 (1986)], and references therein. We use the parameters given by G.I. Lykasov and M.N. Sergeenko, *Z. Phys. C* **56**, 697 (1992).
  - [11] A.D. Martin, W.J. Stirling, and R.G. Roberts, *Phys. Rev. D* **47**, 867 (1993).
  - [12] J.P. Guillet, P. Nason, and H. Plothow-Besch, in *Proceedings of the ECFA Large Hadron Collider Workshop*, Aachen, Germany, 1990, edited by G. Jarlskog and D. Rein (CERN Report No. 90-10, Geneva, Switzerland, 1990), Vol. II, p. 116.
  - [13] E. Zas, F. Halzen, and R.A. Vázquez, in *Proceedings of the XXII International Symposium on Multiparticle Dynamics*, Santiago de Compostela, Spain, 1992, edited by C. Pajares (World Scientific, Singapore, in press).
  - [14] O. Botner *et al.*, *Phys. Lett. B* **236**, 488 (1990). For reviews on experimental data see S.P.K. Tavernier, *Rep. Prog. Phys.* **50**, 1439 (1987); J.A. Appel, *Annu. Rev. Nucl. Part. Sci.* **42**, 367 (1992).
  - [15] T. Yuda, in *Proceedings of the 22nd International Cosmic Ray Conference*, Dublin, Ireland, 1991, edited by M. Cawley *et al.* (Dublin Institute for Advanced Studies, Dublin, 1992).
  - [16] T.N. Afanasieva *et al.*, in *Proceedings of the 20th International Cosmic Ray Conference*, Moscow, USSR, 1987, edited by V.A. Kozyarivsky *et al.* (Nauka, Moscow, 1987), Vol. 9, p. 161.
  - [17] M. Nagano *et al.*, *J. Phys. Soc. Jpn.* **30**, 33 (1971).
  - [18] V. I. Yakovlev, in *Proceedings of the VIIth International Symposium on Very High Energy Cosmic Ray Interactions*, edited by L.W. Jones, AIP Conf. Proc. No. 276 (AIP, New York, 1992).