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Geometrical two-chain model of soft hadronic interactions: Multiplicity characteristics

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The geometrical two-chain model of soft hadronic interactions is developed to describe the main characteristics of low-energy hadronic interactions. It is a compound of the geometrical interaction picture based on the impact parameter eikonal formalism and the parton concept of hadrons as composite objects. In this paper the foundations of the model are described and the results on the multiplicity characteristics are presented and discussed.

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I. INTRODUCTION

The importance of making a fast and accurate event generator is an essential point of the cosmic-ray extensive air shower Monte Carlo programs. Most of the existing codes are irrelevant to this because they satisfy only one of those requirements: they are either fast or accurate enough, never both. The progress in our knowledge of high energy physics, on the one hand, and in the cosmic-ray experimental and computational techniques made so far and planned in the near future, on the other, force us to look for a compromise. In this paper we wish to present a part of the solution which looks promising. It is known that the interaction picture below $\sqrt{s} \sim 100$ GeV is much less complicated than the one for higher energies. The phenomena such as, for example, minijets, jets and all the features connected with high four-momentum transfer are not very important in the low energy region. On the other hand, kinematics starts to play an important role if the energy decreases. All leads to the conclusion that it would be reasonable to develop a Monte Carlo event generator only for the low energy region. The restrictions accepted, as has been said before, are low calculation time consumption and accuracy in comparison with other (not as fast) generators. The narrowing of the energy range allows us to look for a theoretical solution of the problem in a much broader class of interaction models which are connected with the so-called "soft" interactions. We have called the model presented in our paper the geometrical two-chain model (G2C). It is based on two well-known foundations appearing separately in high energy physics. A geometrical picture has been developed for elastic hadron scattering in [1] and introduced for inelastic interactions in [2] with the scaling properties given in [3]. It treats the interaction in a similar way as a nucleus in the optical model and parametrizes its properties as a function of the impact parameter of colliding particles by the use of the eikonal formalism developed and established for nuclear interactions. This "old-fashioned" treatment has recently been replaced by a more "modern" approach taken for the old Feynman idea [4] of the hadron constituents—partons. The undoubtful progress in understanding the multiparticle production processes using that concept makes the geometrical picture of less interest in high energy physics. Yet the still existing problems with low momentum transfer (soft) interactions make us try to combine the geometrical and parton picture in one self-consistent model for low energy soft interaction.

In the proposed G2C model the geometrical concepts are connected with the first stage of interaction between hadron constituents ("parton scattering"). Then the hadronization of outgoing colored fragments is described at the partonic level. The momentum and energy transfer at the first (geometrical) state can be parametrized as a function of scaled impact parameter as, e.g., was assumed in [5]. The eikonal functions of colliding hadrons are obtained from the low-energy elastic scattering data with the geometrical assumption of a simple exponential partonic matter distribution in colliding hadrons. It is known that the elastic data can be reproduced better with the more complicated density functions but the needed accuracy of the description of multiparticle production process properties allows us to use the simplest form.

The transfer of energy and momentum from the colliding hadrons to the objects appearing as the results of the "parton scattering" is *a priori* unknown. They can be found on the basis of QCD from the leptonic and high inelastic collisions data assuming that the parton distribution remains unchanged. That way is used in a well-established QCD based interaction model as such as the programs HIJING [6] and LUND [7] or the duals parton model (DPM) [8]. It works very well in high transverse momentum events but needs some additional assumptions when it is to be used for a complete description of soft interactions in which the perturbative QCD does not work. Another method is to assume the way in which the energy and momentum are shared between the "scattering" products, as it was suggested in [5]. We have employed the latter.

After "parton scattering" two intermediate objects called hereafter chains (forward and backward) are created. The portion of momentum and energy given to each chain is parameterized as a function of the geometrically scaled impact parameter.

The decay of each chain is considered on the parton level. The most popular models of the chain hadronization process are based on the old Feynman branching

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idea [9]. It works very well in the Lund model [7] or in the DPM [8] and also in other branching models, as shown in [5,6,10].

For the low energy particle production, kinematics plays an important role. The branching processes close to the kinematical edges become more complicated and the conservation of all values which have to be conserved makes the interaction picture vague. The phase-space effects are expected to be dominant and then the branching process starts to look less branching but more statistical.

We decided to decay the chains in a different way. They are fragmenting mainly due to available phase space. The dynamics of fragmentation is added to this mainly as the transverse momentum cutoff. Some of the less crucial correlations in the phase space will not be discussed in this paper as they do not effect the multiplicity characteristics of the hadronic interaction.

The connections between e^+e^- annihilation into hadrons and hadron-hadron multiparticle production reactions seen in multiplicity distributions and also in the dependence of the mean multiplicity on incoming energy are well known [11] and have to be taken into account in the construction of any rational high-energy interaction model. In our geometrical picture they become natural as the result of an impact parameter interaction picture.

The produced hadron flavors are created due to the common quark flavor generator with the strangeness and other suppression factors of the similar values as used in the well-established LUND model adjusted to reproduce the data on p-p interaction at the laboratory momentum of 400 GeV/c presented in [12]. All the resonances are created and decayed to the stable and long lived particles by using the LUND decay scheme [7].

II. DETAILED MODEL DESCRIPTION

A. Geometrical scaling approach

The idea of using the well-known eikonal formalism for inelastic hadron collisions is founded on the accomplishment of such a way of treating nucleus-nucleus interaction picture. The general idea of the geometrization of the interaction can be described by the equation

$$f(s,t)=\int_0^\infty J_0(b\sqrt{-t})[1-S(b)]b\,db\ , \tag{1}$$

which shows how to introduce the impact parameter b into the amplitude of the interaction process. In this equation s is the c.m. system (c.m.s.) energy of the interaction, t is the four-momentum transfer, J_0 is the Bessel function, and S is connected with the eikonal function Ω by the definition

$$S(s,b)=\exp[-\Omega(s,b)]\ . \tag{2}$$

The eikonal function is, in general, defined by the form of an interaction potential as a complex function. According to the similarity to the nucleus-nucleus collisions the Ω in hadronic interactions can be assumed equal to the geometrical opacity of interacting particles defined as

$$\Omega(b)=k_{AB}\int\int\int\rho_A(x-b_x,y-b_y,z)\rho_B x,y,z)dx\,dy\,dz\ , \tag{3}$$

where ρ_A is a "matter" density in the hadron A.

The ρ function can be found from the data on elastic scattering. It is also associated with the electric or magnetic hadron form factors [2]. In our calculations we have used the form suggested by those connections:

$$\rho_A=\frac{m_A^3}{8\pi}\exp(-m_A r)\ . \tag{4}$$

The parameters were fitted to the data on p-p, π-p, and K-p elastic scattering by using the dependence

$$\frac{d\sigma_{\rm el}}{dt}=\pi b_0^{4(s)}|\langle 1-S(R)\rangle|^2\ , \tag{5}$$

where the angular brackets describe the two-dimensional Fourier transform in the impact parameter plane.

The form used [Eq. (4)] with some well-known corrections due to the imaginary part of elastic amplitude seen in the experiments describes the data quite well (see Fig. 1). In the literature there obviously are much better formulas, especially for the high t values. Our model, by definition, is not to describe such high p_t events. It has ben built to depict low p_t ("soft") particle production, observed most commonly in the low energy collisions and for this the formula we use seems to be good enough.

The idea of geometrical scaling [3] was suggested to establish a simple connection of the interaction pictures for different energies. It can be expressed as

$$\Omega(s,b)=\Omega(R)\ , \tag{6}$$

where

$$R(s)=\frac{b}{b_0(s)}\ ,\quad \pi b_0^2=\sigma_{\rm inel}(s)\ . \tag{7}$$

Obviously, the unitarity is automatically satisfied:

$$\begin{aligned}\sigma_{\rm el}&=\pi b_0^2\int_0^\infty[1-\exp(-\Omega(R))]^2 dR^2\ ,\\ \sigma_{\rm inel}&=\pi b_0^2\int_0^\infty[1-\exp(-2\Omega(R))]dR^2\ .\end{aligned} \tag{8}$$

The accuracy of fitting of the opacity function can be seen in Table I, where the calculated values of elastic and inelastic cross sections are compared with the measurement results.

B. The first (scattering) stage of interaction

For the two colliding hadron the impact parameter b distribution can be obtained from

$$d\sigma_{\rm inel}=2\pi[1-S(b)S^*(b)]b\,db\ , \tag{9}$$

where S is given by Eq. (2). In the final state of that "first stage" process the two intermediate objects of the masses M_1 and M_2 are created with the momenta $p_1=-p_2$ (in the colliding hadron c.m.s.). The invariant masses M_i are the only variables to be determined at the first stage of in-

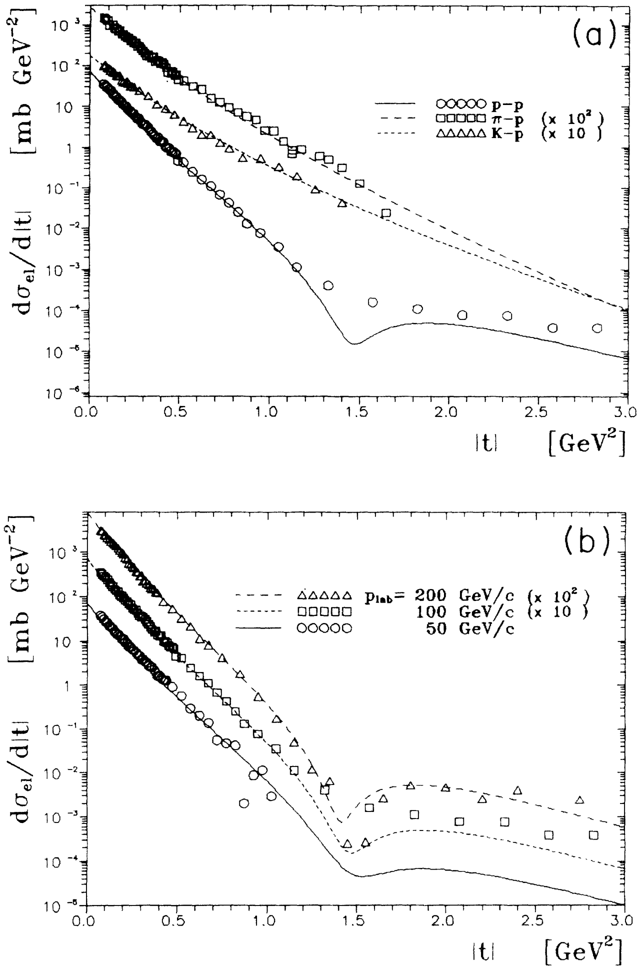

FIG. 1. The four-momentum transfer distribution in elastic scattering (a) for the p-p, π-p, and K-p collisions at the laboratory momentum of 100 GeV/c (the model-predicted curves and the experimental pints are shifted by the factor of 10, respectively) and (b) for the p-p scattering at the laboratory momenta of 50, 100, and 200 GeV/c (the calculated curves and the data points are shifted by the factor of 10, respectively). The data are from [23].

teraction. The four-momentum transfer between two chains does not affect multiplicity characteristics so it will not be discussed in the present paper. As has been said, in our model each mass M_i depends on the opacity calculated for the interacting hadrons. We have chosen the simplest power-law form of this dependence [5]:

TABLE I. The comparison of the calculated elastic and inelastic cross sections with the measurements at the laboratory momentum of 250 GeV/c [13].

	σ_{inel} (mb)		σ_{el} (mb)	
	Expt.	Calc.	Expt.	Calc.
p-p	32.40	32.2	6.89	6.97
π-p	20.94	20.9	3.22	3.19
K-p	17.72	17.6	2.55	2.53

$$M_i=(M_{i\,\max}-M_{i\,\min})\left[\frac{\Omega(b)}{\Omega(0)}\right]^{\lambda}+M_{i\,\min}\ , \qquad (10)$$

where $M_{i\,\min}$ is the mass of the lightest hadron which can be created from the quarks located at the chain ends and $M_{i\,\max}$ is the c.m.s. energy of the incoming particle.

The decomposition of interacting particles to two colored components [mesons to (q-$\bar{q}$), baryons to (q-qq)] is performed by the standard scheme used, e.g., in the LUND model [7]. For the interacting baryon the diquark always forms the most energetic end of the chain (in the c.m.s.); for kaons the strange quark is the most energetic one in 80% of all evens.

C. The second (hadronization) stage of interaction

After the scattering stage of the two intermediate objects have to hadronize into colorless hadrons. The hadronization is performed by the creation of (q-$\bar{q}$) and (qq-$\bar{q}\,\bar{q}$) pairs inside the chain. As has been said before, in the G2C model no branching processes were used to do so. Our general assumption is that there is no causal connection between the breakups of the chain, so only the number of breakups (created pairs) should describe the general characteristics of the chain hadronization. This number can, of course, depend on the invariant mass of the chain (the energy available for the hadron creation), but also dependence on the impact parameter of interacting particles is expected (the angular momentum of the created chain).

First we find the chain mass dependence. This can be done by using the data on e^+-e^- annihilation into hadrons. In the e^+-e^- annihilation, the total angular momentum of the virtual boson is 1. This experimental fact provides the assumption [14] that in geometrical approach the decaying chain can be treated as a chain produced with the impact parameter b equal 0. The simple $\ln(s)$ "soft" dependence has to be improved to describe the extremely low-energy data where other (resonant) processes of hadron production could play an important role, which, in general, is beyond the domain of the G2C model.

In our model calculations we used the following formula to obtain the mean number of chain breakups in the e^+-e^- annihilation:

$$\begin{aligned}\bar{n}_1&=\alpha_1+\alpha_2\ln(M_{\text{av}})|_{\geq 0}\ ,\\ \bar{n}_2&=\alpha_3+\alpha_4\ln(M_{\text{av}})|_{\geq 0}\ ,\\ \bar{n}_{qq}|_{b=0}&=\ln(e^{\bar{n}_1}+e^{\bar{n}_2})\ ,\end{aligned} \qquad (11)$$

where M_{av} is the chain mass reduced by the mass of the lightest particle which can be created by combining both ends of the chain. $\bar{n}_1$ fits the data above $\sqrt{s}\sim 10$ GeV while $\bar{n}_2$ is the low-mass correction. The form of $n_{qq}|_{b=0}$ was chosen to smooth the break in the energy region where the dependence changes from $\bar{n}_1$ to $\bar{n}_2$. In the low-energy region (below $\sqrt{s}\sim 5$ GeV) the multiplicities are strongly affected by energy conservation, so the number of the created particles are sensitive to the details of transverse momentum and rapidity distributions. The values of parameters used in this paper should be treated as the first crude estimation. The obtained results do not strongly depend on their choice. $n_{qq}|_{b=0}$ is the mean number of breakup points in the chain of the mass M created with $b=0$. Obviously, the number of created pairs in each individual event can be different. Its distribution has been found by using the data on charged particle multiplicity in e^+-e^- annihilation into hadrons. It is not very surprising that the Poissonian distribution

$$p(n_{qq},\bar{n}_{qq})=\frac{(\bar{n}_{qq})^{n_{qq}}}{n_{qq}!}e^{-\bar{n}_{qq}} \tag{12}$$

fits this data rather well [15].

The dependence of the mean number of q-$\bar{q}$ (qq-$\bar{q}\,\bar{q}$) pairs on the impact parameter of colliding hadrons has been found by using the low energy p-p multiplicity data. The form of this dependence is assumed to be [16]

$$\bar{n}_{qq}(b)=\bar{n}_{qq}|_{b=0}h(b)\ , \quad h(b)=h_0\left[\frac{\Omega(b)}{\Omega(0)}\right]^{\chi} . \tag{13}$$

The flavors of the generated q-$\bar{q}$ (qq-$\bar{q}\,\bar{q}$) pairs are obtained in the standard way. The reproduction of the resonance cross sections measured in [12] at the laboratory momentum of 400 GeV/c is achieved by slight changes of some suppression factors in the LUND scheme. Both constituents of the generated pair are given transverse momenta relative to the chain axis of the equal values and opposite signs so that the transverse momentum is conserved in each chain breakup point. If the created transverse masses exceed the available energy the momenta are generated again up to the satisfaction of this condition. The details of the particle creation "in the transverse direction" will not be discussed in this paper as they do not affect the multiplicity characteristics of interaction above $\sqrt{s}\sim 10$ GeV.

If the sum of the masses of particles to be created is going to be higher than the mass available in the interaction, the flavors are generated several times for the generated number of chain breakups. If not successful, the number of the created $q\bar{q}$ pairs is generated again. This is the only requirement of the energy and momenta conservation laws which affect the multiplicity characteristics in our G2C model event generator.

If the masses and transverse momenta are created, there are only the longitudinal momenta needed to complete the description of the final state of the interaction. That is done by means of the method described in [17], which places randomly all particles in the cylindrical phase space with the additional condition from the G2C model that the particle sequence in the rapidity space refers to the rank of the particle during the flavor generation as the chains have been broken.

III. THE G2C MODEL CALCULATION RESULTS

The Monte Carlo even generator based on the principles described above has been established and the results concerning the interaction multiplicity characteristics are presented below.

The model consists of a few parameters, which are fitted to the data. Their values are listed in Table II. The ones commonly know from other generators (such as the suppression factors) and those of no importance to the subject (details of transverse momentum distributions and dynamic correlations) are not included. There are, in general, very few free parameters in the G2C model; however, it can still describe a large number of experimentally observed features, as will be shown below.

TABLE II. The values of the parameters in geometrical two-chain model used in the present calculations.

Parameter	Fitted value	Equation
α_1	-3.4	(11)
α_2	4.1	(11)
α_3	0.5	(11)
α_4	2.6	(11)
h_0	0.8	(13)
χ	0.33	(13)
λ	0.2	(10)

The first set of parameters $\alpha_{1,\ldots,4}$ was fitted to the available energy dependence of the number of breakups of the chain at the fixed impact parameter. The fit form used in Eq. (11) is only one of a few possibilities. The same accuracy of the data reproduction can be achieved with the $\ln^2 M$ [18] dependence with one parameter less than the one in Eq. (11). The values of $\alpha_{1,\ldots,4}$ were fitted to the e^+e^- annihilation into hadrons data. The exactness of the fit is presented in Fig. 2. The divergence appearing at about $\sqrt{s}\sim 50$ GeV is expected according to the multijet events seen in the experiments. These phenomena cannot be described, by definition, in the concept of the geometrical two-chain model.

To confirm the assumption we made in Eq. (12) the multiplicity distributions in e^+e^- two-jet events calculated by the G2C generator have to be compared with the data from [19,20]. The comparison is presented in Fig. 3(a). To show the energy behavior of these distribution

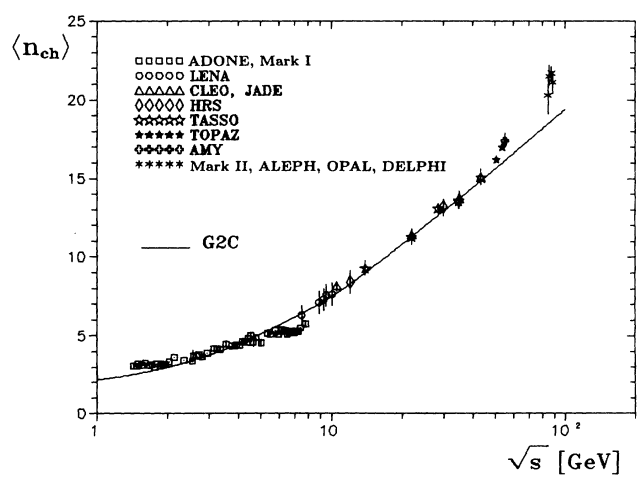

FIG. 2. The mean multiplicity of charged particles in e^+-e^- annihilation into hadrons calculated in the G2C model in comparison with the data compiled in [24].

shapes the first four normalized momenta are given and compared with the experimental results in Fig. 3(b). The divergence above $\sqrt{s} \sim 50$ GeV is clear again. The reason why it occurs seems to be the same as discussed for discrepancy in Fig. 2.

The parameters in Eq. (13) play a dominant role in the hadronic interaction multiplicity. The index χ is responsible mainly for the width of the multiplicity distribution and the normalization h_0 for the mean value of the multiplicity.

The parameter λ in Eq. (10) is correlated with those from Eq. (13). Its value is obtained by mainly using the data on the rapidity distribution width.

It is interesting to note the difference between the baryon- and meson-induced interaction multiplicities. It has been proved by experiments [13] that the mean multiplicity of the π-p at a laboratory momentum of 250 GeV/c is a little higher than that of p-p interaction. That difference appears in the G2C model as well. The kinematics of the chains, asymmetry in energy sharing, and the difference in the eikonal function Ω are some of the reasons, but the mean difference due to the smaller mass of the leading particle in the meson hemisphere leading to the increase of M_{av} in Eq. (11).

The accuracy of the mean multiplicity reproduction by the G2C generator is presented in Fig. 4(a). The treatment of meson interaction described above is compared with the data in Fig. 4(b) where the data from p-p and π-p collisions are compared with the model predictions. The shapes of multiplicity distributions are presented in Fig. 5 for different energies. The highest plotted energy reaches the limits of the G2C model applicability, but the accuracy is still satisfactory. The comparison of the multiplicity distribution for the p-p, π-p, and K^+-p interactions at the laboratory momentum of 250 GeV/c is given in Fig. 6. The behavior of the fist normalized momenta in the $p-p$ interaction as a function of the c.m.s. energy is presented in Fig. 7. It can be seen that the Koba-Nielsen-Olesen (KNO) scaling is (in contradiction with e^+-e^- annihilation events) valid approximately in the whole energy reign. The smoothness of the distributions (seen especially in the highest moment) at energies above $\sqrt{s} \sim 50$ GeV

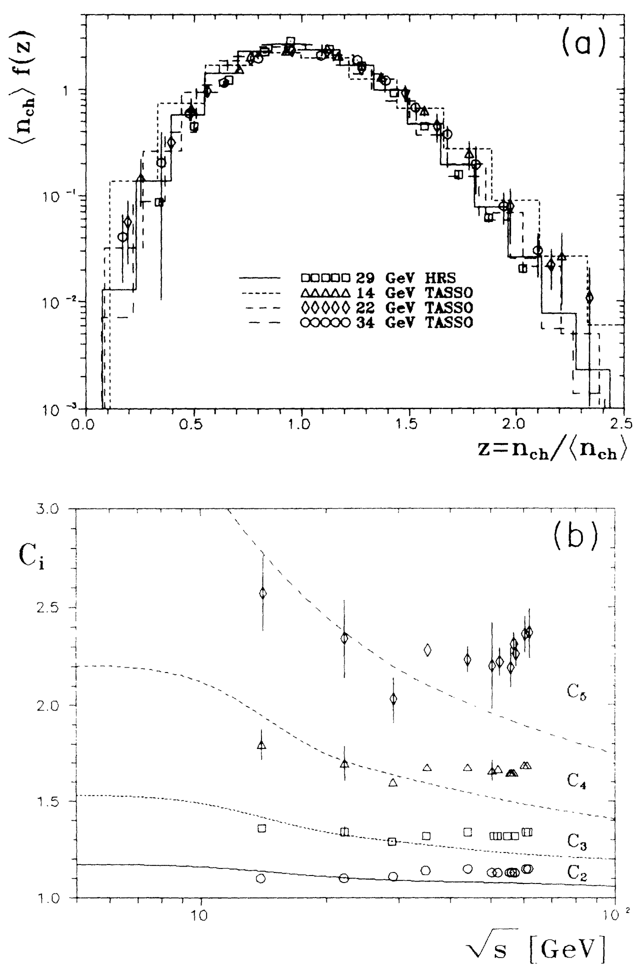

FIG. 3. (a) The multiplicity distributions in e^+-e^- annihilation for different energies. (b) The first four normalized moments ($C_i = \langle n^i\rangle / \langle n\rangle^i$) of the multiplicity distribution in e^+-e^- annihilation into hadrons. The data are from [19,20,24].

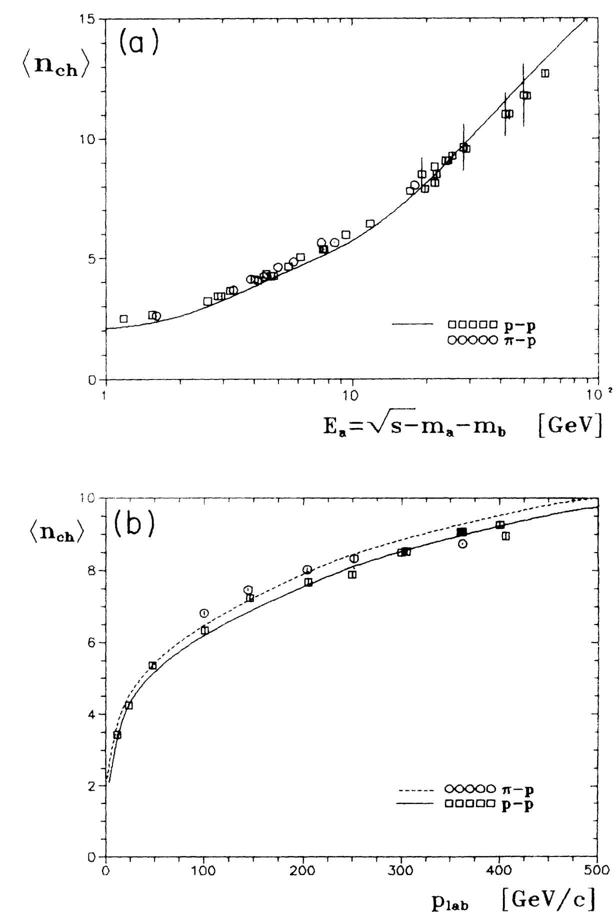

FIG. 4. (a) The mean multiplicity in p-p interaction as a function of the energy available for particle creation calculated in the G2C model compared with the data from [25]. (b) The comparison of the mean multiplicity in p-p interaction with the mean multiplicity in π-p interaction as a function of the incoming particle laboratory momentum. The data points from [26].

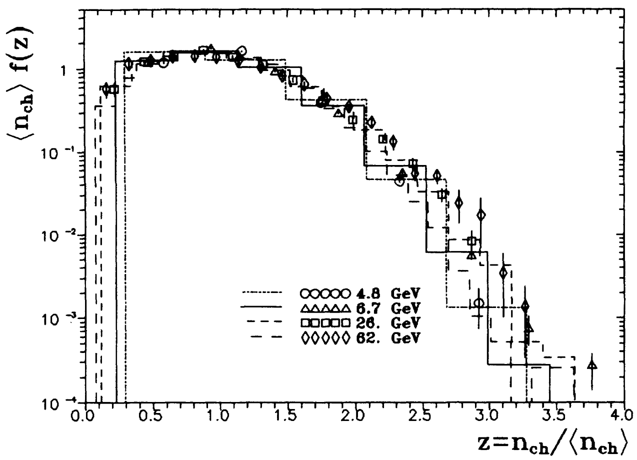

FIG. 5. The multiplicity distributions in *p-p* interaction for different c.m.s. energies. The data are from [26–28].

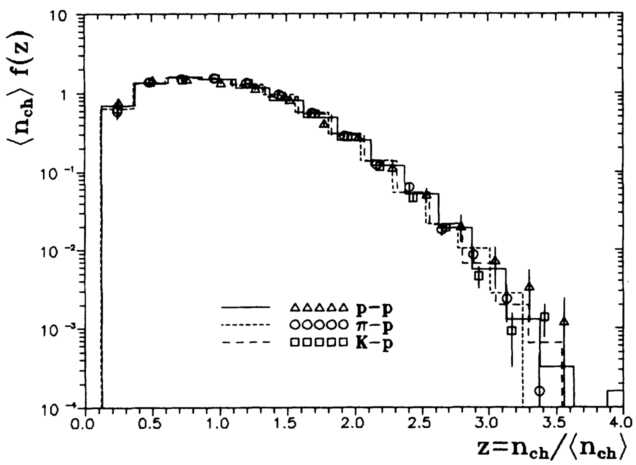

FIG. 6. The comparison of the multiplicity distributions in *p-p*, π^+ *-p*, and K^+*-p* interaction sat the laboratory momentum of 250 GeV/*c*. The data from [13].

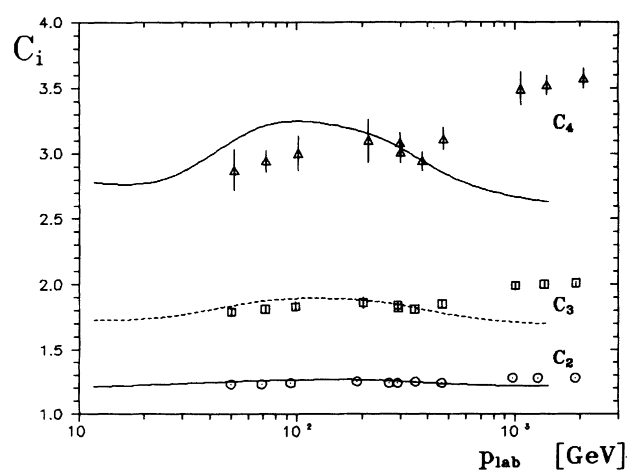

FIG. 7. The first there normalized momenta of the *p-p* multiplicity distribution as a function of the c.m.s. energy. The data from [26].

is expected. It takes place when the Poissonian fluctuation at a fixed (mean) value of the impact parameter exceeds the fluctuations resulting from the impact parameter distribution width.

The experimental result of some importance for a concept of the multiparticle production mechanism is a long-range (forward-backward) multiplicity correlation. It is well known that the dependence of the mean backward multiplicity measured at a fixed multiplicity in the forward hemisphere is rather weak for the e^+-e^- annihilation into hadrons in contradiction with the same dependence for the hadronic collisions. That fact is one of the phenomenological bases of the impact parameter, geometrical, interpretation of multiparticle production processes [11,21]. The G2C model gives the natural explanation of that feature as well. The result of the model calculations are presented in Fig. 8 and the good agreement is clearly seen both for the e^+-e^- annihilation and the *p-p* interactions.

IV. CONCLUSIONS

The geometrical two-chain model developed to describe the soft hadronic production in low-energy col-

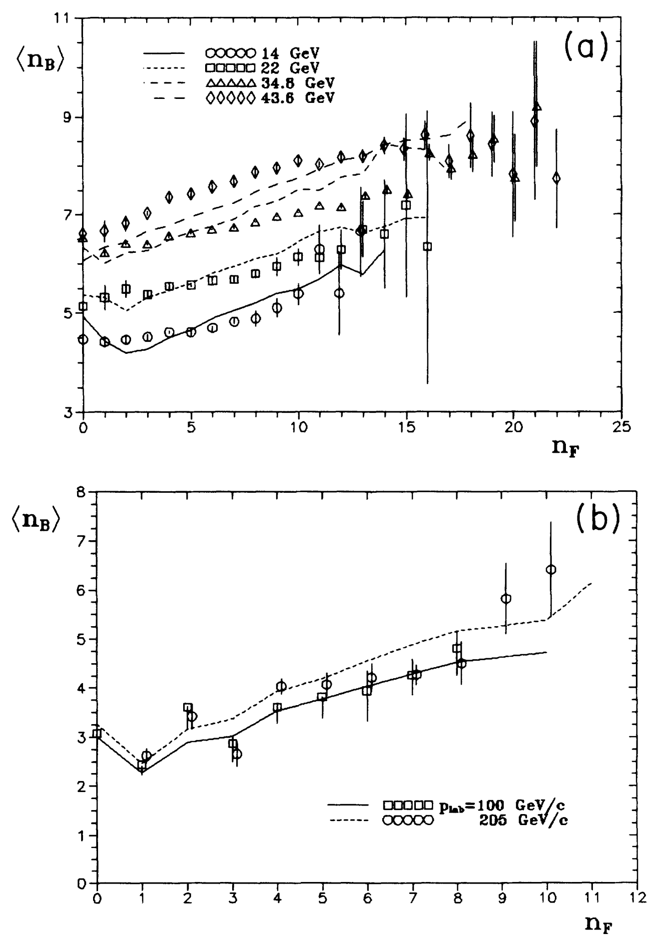

FIG. 8. The dependence of mean backward multiplicity measured at fixed multiplicity in the forward hemisphere for the e^+-e^- annihilation into hadrons (a) and for *p-p* interactions to different energies (b). The data points are from [19,25,29].

lisions is able to reproduce the main multiplicity characteristics of meson and baryon interactions in the $\sqrt{s}$ region of about two decades. The connections with the $e^+ - e^-$ annihilation into hadrons and the elastic scattering processes become natural in the proposed interaction scheme, which makes the interaction picture more complete and physically interesting. A very scant number of model parameters is the result of some scaling assumptions we made, which are confirmed by the comparison with the experimental results. The other interaction properties not discussed in this paper will be presented elsewhere, but even the results achieved for the multiplicity characteristics seem to make the idea of the geometrical two-chain model attractive. The description of the diffractive events and the resonance production in the fragmentation region is at present the main aim of our interest. The way of introducing them to the geometrical interaction picture is established and the results will be presented soon. This will improve the description of energetic properties of the produced and leading particles in the hadronic interaction. The results obtained so far allow us to suppose that the G2C model will be able to reproduce them correctly.

The G2C picture can easily be introduced into the Monte Carlo code of the event generator. All the model predications presented in this paper have been obtained in this way. The advantage of the G2C code is its natural way of satisfying all the requirements of the conservation laws (transverse and longitudinal momenta, energy, charge, baryon number, strangeness), which leads to a small number of generated-event rejections due to some conversation law violations. This, together with the simplification made to avoid including the high p_t events, allows us to create a very fast and effective soft event generator for hadron-hadron collisions.

The way of extending the G2C model to describe the interactions of nuclei was presented in [22] and will be used to build the G2C generator for the cosmic-ray extensive air shower simulation programs. As has been said, the other Monte Carlo event generators which become the standards in high energy physics, such as LUND of the DPM, were built not for such use. The same can be said about the models such as FRITIOF, HIJING, VENUS, or ECCO, made for the study of heavy nucleus interactions. The accuracy of the G2C model is not much worse and sometimes even better in its region of applicability. The possibility of studying a soft background of high energy nucleus interactions in the search for a quark-gluon plasma in the planned experiments by means of the G2C is another field in which the model can be used. The accuracy of the reproduction of low-energy interaction characteristics is an important point in studying the cascading effects of slow secondaries traversing heavy nuclei. The low calculation time consumption rate of the G2C code is its advantage for this purpose as well.

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