Sea contributions and nucleon structure

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We suggest a general formalism to treat a baryon as a composite system of three quarks and a "sea." In this formalism, the sea is a cluster which can consist of gluons and quark-antiquark pairs. The hadron wave function with a sea component is given. The magnetic moments, related sum rules, and axial vector weak coupling constants are obtained. The data seem to favor a vector sea rather than a scalar sea. The quark spin distributions in the nucleon are also discussed.

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I. INTRODUCTION

Historically, the static SU(6) quark model provided a good description of hadrons: Baryons (mesons) are color-singlet combinations of three quarks (quark-antiquark pairs) in the appropriate flavor and spin combination. The space-time part of a hadron wave function can be determined by using a specific model of confinement, e.g., the bag model [1,2] simple harmonic oscillator model [3–5], or other phenomenological models [6]. Although the naive SU(6) quark model works successfully in explaining various properties of hadrons, departures from the naive SU(6) results have been observed. The naive *valence* picture of hadron structure is a simplification or a first order approximation to the real system. Within the framework of QCD, quarks interact through color forces mediated by vector gluons. The QCD interaction Hamiltonian $H_I(x)=g\bar{\psi}(x)\gamma^{\mu}(\lambda^a/2)\psi(x)A_{\mu}^{a}(x)$ has several consequences. First of all, spin-dependent forces (e.g., color-hyperfine interactions [7]) between the quarks due to one (or multi) gluon exchange lift the SU(6) mass degeneracy and explain the basic pattern of baryon and meson spectroscopies. The spin dependent forces also cause different space-time distributions for different quark flavors and provide a good description of baryon magnetic moments and form factors [8,9]. Second, the existence of a quark-gluon interaction implies that quark-antiquark ($q\bar{q}$) pairs can be created by the virtual gluons emitted from valence quarks. These $q\bar{q}$ pairs are the so called sea quarks. Usually, the "sea" means a combination of the virtual gluons and sea quark-antiquark pairs. Although deep inelastic muon nucleon scattering shows that the sea components ($q\bar{q}$ pairs and gluons) indeed exist and play a very important role (e.g., gluons carry about one-half of the nucleon momentum and the sea dominates small-x behavior of structure functions), it is commonly believed that in the low energy regime, static properties of hadrons are dominated by their valence components. However, it has been shown [10,11] that the sea contributions may change the structure of hadrons and modify their low energy properties. Using the QCD interaction Hamiltonian and the MIT bag model, Donoghue and Golowich (DG) [10] (comments see cf [11]) calculated the probabilities of different sea quark components in the proton. Several models [12–15] have been suggested to study the gluon component in hadrons. In these models, a mixing of q^3 and q^3+gluon, in which a color $\mathbf{8}_c$ gluon coupled to a $\mathbf{8}_c$ q^3 state to form a color singlet, has been discussed. However, the "sea" could be a gluon (as discussed in [12–15]) or a quark-antiquark pair (as discussed in [10,11]), or even more complicated, for instance a multigluon state, multi-($q\bar{q}$) pairs or gluon(s) plus ($q\bar{q}$) pair(s). In this paper, we study the sea contributions in a more general formalism and treat the "sea" as a cluster which can consist of two-gluon and a gluon plus a (q-$\bar{q}$) pair or some admixture of both (which may be described by the generic term "flotsam"). Since the baryon should be colorless and a q^3 state can be in color states $\mathbf{1}_c$, $\mathbf{8}_c$, and $\mathbf{10}_c$, the "sea" should also be in corresponding color states to form a color singlet baryon. In addition, the "sea" spin is not required to be one (as in the single-gluon case). Furthermore, if the sea is in an S-wave state relative to the q^3 system, conservation of the angular momentum restricts that sea spin can only be 0, 1, or 2 to give a spin-1/2 baryon. If the sea is in a P-wave state, then its spin could be 0, 1, 2, or 3. In this paper, we only discuss the S-wave case. In Sec. II, a more general wave function of the baryon, which consists of q^3 and a "sea," is given. In Sec. III, the magnetic moments and related sum rules are derived and compared with the data. In Sec. IV, axial weak coupling constants and first moments of nucleon spin structure functions are calculated. A discussion of the sea contribution, numerical results, and several conclusions are given in Secs. V, VI, and VII, respectively.

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II. HADRON WAVE FUNCTION WITH A SEA COMPONENT

The three (valence) quark wave function of the baryon can be written as

$$\Psi=\Phi(|\phi\rangle|\chi\rangle|\psi\rangle)(|\xi\rangle) \quad (2.1)$$

where $|\phi\rangle$, $|\chi\rangle$, $|\psi\rangle$, and $|\xi\rangle$ denote flavor, spin, color, and space-time q^3 wave functions. For the lowest-lying hadrons, quarks appear to be in S-wave states and the space-time q^3 wave function $|\xi\rangle$ is total symmetric under permutation of any two quarks. Hence the flavor-spin-color part Φ should be total antisymmetric under $q_i \leftrightarrow q_j$. In the conventional quark model, the color wave function ψ is taken to be total antisymmetric, i.e., a color singlet. But in general this is not necessary if baryon is considered to have a sea component in addition to the q^3. Let superscripts S and A denote total permutation symmetry and antisymmetry, and λ,ρ denote symmetry and antisymmetry under quark permutation $q_1 \leftrightarrow q_2$. Then the q^3 wave functions for a flavor octet baryon are

$$\Phi_1^{(1/2)}\equiv\Phi(8,1/2,1_c)=F_S\psi_1^A , \quad (2.2)$$

$$\Phi_8^{(1/2)}\equiv\Phi(8,1/2,8_c)=\frac{1}{\sqrt{2}}(F_{MS}\psi_8^\rho-F_{MA}\psi_8^\lambda) , \quad (2.3)$$

$$\Phi_{10}^{(1/2)}\equiv\Phi(8,1/2,10_c)=F_A\psi_{10}^S , \quad (2.4)$$

$$\Phi_8^{(3/2)}\equiv\Phi(8,3/2,8_c)=F'_A\chi^{(3/2)} , \quad (2.5)$$

where

$$F_S=\frac{1}{\sqrt{2}}(\phi^\lambda\chi^\lambda+\phi^\rho\chi^\rho) , \quad (2.6)$$

$$F_{MS}=\frac{1}{\sqrt{2}}(\phi^\rho\chi^\rho-\phi^\lambda\chi^\lambda) , \quad (2.7)$$

$$F_{MA}=\frac{1}{\sqrt{2}}(\phi^\rho\chi^\lambda+\phi^\lambda\chi^\rho) , \quad (2.8)$$

$$F_A=\frac{1}{\sqrt{2}}(\phi^\lambda\chi^\rho-\phi^\rho\chi^\lambda) , \quad (2.9)$$

and

$$F'_A=\frac{1}{\sqrt{2}}(\phi^\lambda\psi_8^\rho-\phi^\rho\psi_8^\lambda) , \quad (2.10)$$

where the detail expressions for ϕ^λ, ϕ^ρ, χ^λ, and χ^ρ can be found in Ref. [16], and $\chi^{(3/2)}$ is the totally symmetric q^3 spin wave function with spin 3/2.

We note that $\Phi_1^{(1/2)}$ in (2.2) is the standard q^3 wave function which transforms as **56** of SU(6) and was denoted by $|N_0\rangle$ in Ref. [15]. Our $\Phi_8^{(1/2)}$ and $\Phi_8^{(3/2)}$ correspond to the notation $|^2N_g\rangle$ and $|^4N_g\rangle$ in Ref. [15] respectively, they transform as **70** of SU(6). There is no $\Phi_{10}^{(1/2)}$ term in previous works.

We consider a flavorless sea, which has spin (0,1,2, if we assume sea is in a S-wave state) and color ($\mathbf{1}_c$, $\mathbf{8}_c$, and $\overline{\mathbf{10}}_c$). Let $H_{0,1,2}$ and $G_{1,8,\overline{10}}$ denote spin and color sea wave functions, which satisfy

$$\langle H_i|H_j\rangle=\delta_{ij}, \quad \langle G_k|G_l\rangle=\delta_{kl} . \quad (2.11)$$

The possible combinations of q^3 and sea wave functions, which can give a spin 1/2, flavor octet, color singlet state, are

$$\Phi_1^{(1/2)}H_0G_1, \quad \Phi_8^{(1/2)}H_0G_8, \quad \Phi_{10}^{(1/2)}H_0G_{\overline{10}} , \quad (2.12)$$

$$\Phi_1^{(1/2)}H_1G_1, \quad \Phi_8^{(1/2)}H_1G_8, \quad \Phi_{10}^{(1/2)}H_1G_{\overline{10}} , \quad (2.13)$$

and

$$\Phi_8^{(3/2)}H_1G_8, \quad \Phi_8^{(3/2)}H_2G_8 . \quad (2.14)$$

The total flavor-spin-color wave function of a spin up baryon which consists of three-valence quarks and a sea component can be written as

$$|\Phi_{1/2}^{(\uparrow)}\rangle=\frac{1}{N}[\Phi_1^{(1/2\uparrow)}H_0G_1+a_8\Phi_8^{(1/2\uparrow)}H_0G_8+a_{10}\Phi_{10}^{(1/2\uparrow)}H_0G_{\overline{10}}+b_1(\Phi_1^{(1/2)}\otimes H_1)^\uparrow G_1$$
$$+b_8(\Phi_8^{(1/2)}\otimes H_1)^\uparrow G_8+b_{10}(\Phi_{10}^{(1/2)}\otimes H_1)^\uparrow G_{\overline{10}}+c_8(\Phi_8^{(3/2)}\otimes H_1)^\uparrow G_8+d_8(\Phi_8^{(3/2)}\otimes H_2)^\uparrow G_8] , \quad (2.15)$$

where

$$N^2=1+a_8^2+a_{10}^2+b_1^2+b_8^2+b_{10}^2+c_8^2+d_8^2 . \quad (2.16)$$

Although there are seven correction terms in (2.15), they are not equally important. Some arguments are given in Sec. V to show that the main modifications come from the vector sea, in particular b_8, b_1 and c_8 terms, and minor contributions come from the scalar sea, e.g., a_{10} term.

The first three terms in (2.15) come from a spin $1/2$ q^3 state coupled to a spin 0 (scalar) sea. The next three terms in (2.15) come from spin $1/2$ $q^3\otimes$spin 1 (vector) sea and in more detail we have

$$(\Phi_1^{(1/2)}\otimes H_1)^\uparrow\equiv\Phi_{b1}^{(1/2\uparrow)}\psi_1^A , \quad (2.17)$$

$$(\Phi_8^{(1/2)}\otimes H_1)^\uparrow\equiv\Phi_{b8}^{(1/2\uparrow)} , \quad (2.18)$$

$$(\Phi_{10}^{(1/2)}\otimes H_1)^\uparrow\equiv\Phi_{b10}^{(1/2\uparrow)}\psi_{10}^S , \quad (2.19)$$

where

$$\Phi_{b1}^{(1/2\uparrow)}=\sqrt{\tfrac{2}{3}}H_{1,1}F_S^{(1/2\downarrow)}-\sqrt{\tfrac{1}{3}}H_{1,0}F_S^{(1/2\uparrow)} , \quad (2.20)$$

$$\Phi_{b8}^{(1/2\uparrow)}=\sqrt{\tfrac{1}{2}}[\Phi_{b8S}^{(1/2\uparrow)}\psi_8^\rho-\Phi_{b8A}^{(1/2\uparrow)}\psi_8^\lambda] , \quad (2.21)$$

$$\Phi_{b10}^{(1/2\uparrow)}=\sqrt{\tfrac{2}{3}}H_{1,1}F_A^{(1/2\downarrow)}-\sqrt{\tfrac{1}{3}}H_{1,0}F_A^{(1/2\uparrow)} . \quad (2.22)$$

In (2.21), $\Phi_{b8S}^{(1/2\uparrow)}$ and $\Phi_{b8A}^{(1/2\uparrow)}$ are

$$\Phi_{b8S}^{(1/2\uparrow)}=\sqrt{\tfrac{2}{3}}H_{1,1}F_{MS}^{(1/2\downarrow)}-\sqrt{\tfrac{1}{3}}H_{1,0}F_{MS}^{(1/2\uparrow)}\,, \tag{2.23}$$

$$\Phi_{b8A}^{(1/2\uparrow)}=\sqrt{\tfrac{2}{3}}H_{1,1}F_{MA}^{(1/2\downarrow)}-\sqrt{\tfrac{1}{3}}H_{1,0}F_{MA}^{(1/2\uparrow)}\,. \tag{2.24}$$

The final two (c_8,d_8) terms in Eq. (2.15) come from spin 3/2 (q^3)$\otimes$ spin 1 (vector sea) and spin 3/2 (q^3)$\otimes$spin 2 (tensor sea), respectively. Their expressions are

$$(\Phi_8^{(3/2)}\otimes H_1)^{\uparrow}\equiv\Phi_{c8}^{(1/2\uparrow)}\,, \tag{2.25}$$

$$(\Phi_8^{(3/2)}\otimes H_2)^{\uparrow}\equiv\Phi_{d8}^{(1/2\uparrow)}\,, \tag{2.26}$$

where

$$\Phi_{c8}^{(1/2\uparrow)}=\left[\frac{1}{\sqrt{2}}H_{1,-1}\chi_{3/2}^{(3/2)}-\frac{1}{\sqrt{3}}H_{1,0}\chi_{1/2}^{(3/2)}+\frac{1}{\sqrt{6}}H_{1,1}\chi_{-1/2}^{(3/2)}\right]F'_A\,, \tag{2.27}$$

$$\Phi_{d8}^{(1/2\uparrow)}=\left[\sqrt{\tfrac{2}{5}}H_{2,2}\chi_{-3/2}^{(3/2)}-\sqrt{\tfrac{3}{10}}H_{2,1}\chi_{-1/2}^{(3/2)}+\sqrt{\tfrac{1}{5}}H_{2,0}\chi_{1/2}^{(3/2)}-\sqrt{\tfrac{1}{10}}H_{2,-1}\chi_{3/2}^{(3/2)}\right]F'_A\,. \tag{2.28}$$

The wave function used in Ref. [15] (see Eq. (3.9) in [15]) can be obtained from (2.15) by taking $a_{8,10}=b_{1,10}=d_8=0$ and $b_8=c_8=-\delta$. However, we would not like to restrict ourselves to this special case.

III. MAGNETIC MOMENTS AND RELATED SUM RULES

For any operator $\hat{O}$ which only depends on quark flavor and spin and does not depend on the color and space-time, we have

$$\langle\Phi_{1/2}^{(\uparrow)}|\hat{O}|\Phi_{1/2}^{(\uparrow)}\rangle=\frac{1}{N^2}\Bigg[\langle\Phi_1^{(1/2\uparrow)}|\hat{O}|\Phi_1^{(1/2\uparrow)}\rangle+\sum_{i=8,10}a_i^2\langle\Phi_i^{(1/2\uparrow)}|\hat{O}|\Phi_i^{(1/2\uparrow)}\rangle+\sum_{i=1,8,10}b_i^2\langle\Phi_{bi}^{(1/2\uparrow)}|\hat{O}|\Phi_{bi}^{(1/2\uparrow)}\rangle$$
$$+2b_8c_8\langle\Phi_{b8}^{(1/2\uparrow)}|\hat{O}|\Phi_{c8}^{(1/2\uparrow)}\rangle+c_8^2\langle\Phi_{c8}^{(1/2\uparrow)}|\hat{O}|\Phi_{c8}^{(1/2\uparrow)}\rangle+d_8^2\langle\Phi_{d8}^{(1/2\uparrow)}|\hat{O}|\Phi_{d8}^{(1/2\uparrow)}\rangle\Bigg]\,. \tag{3.1}$$

The first term is the conventional quark model result. The a_8,a_{10} terms are the corrections coming from the scalar sea, $b_{1,8,10}$, c_8 and b_8c_8 terms are from the vector sea, and the d_8 term is from the tensor sea.

If operator $\hat{O}$ has a form such as $\hat{O}=\sum_i\hat{O}_f^i\sigma_z^i$ where $\hat{O}_f^i$ depends only on the flavor of the ith quark and σ_z^i is the spin projection (z direction) operator of ith quark, from(3.1) we obtain

$$\langle\Phi_{1/2}^{(\uparrow)}|\hat{O}|\Phi_{1/2}^{(\uparrow)}\rangle=\frac{1}{N^2}\Bigg[a\sum_i[\langle O_f^i\rangle^{\lambda\lambda}\langle\sigma_z^i\rangle^{\lambda\uparrow\lambda\uparrow}+\langle O_f^i\rangle^{\rho\rho}\langle\sigma_z^i\rangle^{\rho\uparrow\rho\uparrow}+2\langle O_f^i\rangle^{\lambda\rho}\langle\sigma_z^i\rangle^{\lambda\uparrow\rho\uparrow}]$$
$$+b\sum_i(\langle O_f^i\rangle^{\lambda\lambda}+\langle O_f^i\rangle^{\rho\rho})(\langle\sigma_z^i\rangle^{\lambda\uparrow\lambda\uparrow}+\langle\sigma_z^i\rangle^{\rho\uparrow\rho\uparrow})$$
$$+c\sum_i[\langle O_f^i\rangle^{\lambda\lambda}\langle\sigma_z^i\rangle^{\rho\uparrow\rho\uparrow}+\langle O_f^i\rangle^{\rho\rho}\langle\sigma_z^i\rangle^{\lambda\uparrow\lambda\uparrow}-2\langle O_f^i\rangle^{\lambda\rho}\langle\sigma_z^i\rangle^{\lambda\uparrow\rho\uparrow}]$$
$$+d\left[\sum_i\langle O_f^i\rangle^{\lambda\lambda}+\sum_i\langle O_f^i\rangle^{\rho\rho}\right]+e\left[\sum_i(\langle O_f^i\rangle^{\rho\rho}-\langle O_f^i\rangle^{\lambda\lambda})\langle\sigma_z^i\rangle^{\lambda\uparrow 3/2\uparrow}+2\sum_i\langle O_f^i\rangle^{\lambda\rho}\langle\sigma_z^i\rangle^{\rho\uparrow 3/2\uparrow}\right]\Bigg]\,. \tag{3.2}$$

There are only five combinations of seven parameters appearing in (3.2):

$$a=\frac{1}{2}\left[1-\frac{b_1^2}{3}\right],\quad b=\frac{1}{4}\left[a_8^2-\frac{b_8^2}{3}\right],\quad c=\frac{1}{2}\left[a_{10}^2-\frac{b_{10}^2}{3}\right]\,, \tag{3.3}$$

$$d=\tfrac{1}{18}(5c_8^2-3d_8^2),\quad e=\frac{\sqrt{2}}{3}b_8c_8 \tag{3.4}$$

and $\langle O_f^i\rangle^{\lambda\lambda}\equiv\langle\phi^\lambda|O_f^i|\phi^\lambda\rangle$, $\langle\sigma_z^i\rangle^{\lambda\uparrow\lambda\uparrow}\equiv\langle\chi^{\lambda\uparrow}|\sigma_z^i|\chi^{\lambda\uparrow}\rangle$. Similar notation is used for $\langle O_f^i\rangle^{\rho\rho}$, $\langle\sigma_z^i\rangle^{\rho\uparrow\rho\uparrow}$ etc. All matrix elements for octet baryons are listed in Appendix A.

For magnetic moments, $O_f^i=e^i/2m_i$ ($i=u,d,s$). The baryon magnetic moments can be expressed in terms of the quark magnetic moments (μ_u,μ_d,μ_s) and two parameters α and β as

$$\mu_p=3(\mu_u\alpha-\mu_d\beta),\quad \mu_n=3(\mu_d\alpha-\mu_u\beta)\,, \tag{3.5}$$

$$\mu_\Lambda=\tfrac{1}{2}(\alpha-4\beta)(\mu_u+\mu_d)+(2\alpha+\beta)\mu_s\,, \tag{3.6}$$

$$\mu_{\Sigma^+}=3(\mu_u\alpha-\mu_s\beta),\quad \mu_{\Sigma^-}=3(\mu_d\alpha-\mu_s\beta),\quad \mu_{\Sigma^0}=\tfrac{1}{2}(\mu_{\Sigma^+}+\mu_{\Sigma^-})\,, \tag{3.7}$$

$$\mu_{\Xi^0}=3(\mu_s\alpha-\mu_u\beta),\quad \mu_{\Xi^-}=3(\mu_s\alpha-\mu_d\beta)\,. \tag{3.8}$$

Also, the transition moment

$$\mu_{\Sigma\Lambda}=-\frac{\sqrt{3}}{2}(\alpha+2\beta)(\mu_u-\mu_d)\ , \tag{3.9}$$

where $\mu_q=e/2m_q$ $(q=u,d,s)$ and

$$\alpha=\frac{1}{N^2}\left[\frac{4}{9}\right](2a+2b+3d+\sqrt{2}e)\ , \tag{3.10}$$

$$\beta=\frac{1}{N^2}\left[\frac{1}{9}\right](2a-4b-6c-6d+4\sqrt{2}e)\ . \tag{3.11}$$

One may ask why the seven parameters (a_i, b_i, etc.) in the wave function contribute only through the combinations given by α and β. The physical reason is that α and β are connected with the number of spin-up $[n(q_\uparrow)]$ and spin-down $[n(q_\downarrow)]$ quarks in the spin-up proton. If, $\Delta q\equiv n(q_\uparrow)-n(q_\downarrow)+n(\bar{q}_\uparrow)-n(\bar{q}_\downarrow)$, $q=u,d,s$ then $\Delta u=3\alpha$ and $\Delta d=-3\beta$. This can be directly checked from the wave function given in (2.15). Also, as there are no explicit antiquarks or s quarks in the wave function, $n(\bar{q}_\uparrow)-n(\bar{q}_\downarrow)=0$ and $\Delta s=0$. Further, because of in-built flavor SU(3) symmetry in the wave function, α and β determine the other magnetic moments. If there is no sea contribution, $2a=1$ and $b=c=d=e=0$, then $\alpha=4/9$ and $\beta=1/9$, and the simplest quark model result is reproduced [17]. A class of models [18,19] have been recently considered in which the magnetic moments have been expressed in terms of μ_q and Δq $(q=u,d,s)$ without giving an explicit wave function. Their expressions reduce to ours on putting $\Delta u=3\alpha$, $\Delta d=-3\beta$, and $\Delta s=0$ (see Refs. [18,19]).

At first sight, (3.5)–(3.9) seem to contain five parameters μ_q $(q=u,d,s)$, α and β. However, as these always appear as products there are only four effective parameters which we take to be $\tilde{U}\equiv 3\alpha\mu_d$, $\tilde{D}\equiv-3\beta\mu_d$, $2p\equiv-\mu_u/\mu_d>0$, and $r\equiv\mu_s/\mu_d>0$. The numerical results for this four-parameter fit to the magnetic moments are discussed later. Here we note the following four relations or sum rules between the eight magnetic moments:

$$(4.71)\mu_p-\mu_n=\mu_{\Sigma^+}-\mu_{\Sigma^-}-(\mu_{\Xi^0}-\mu_{\Xi^-})(4.15\pm0.07)\ , \tag{3.12}$$

$$(3.68\pm0.02)-6\mu_\Lambda=(\mu_{\Sigma^+}+\mu_{\Sigma^-})-2(\mu_p+\mu_n+\mu_{\Xi^0}+\mu_{\Xi^-})(3.36\pm0.09)\ , \tag{3.13}$$

$$(3.42\pm0.26)(\mu_{\Sigma^+}^2-\mu_{\Sigma^-}^2)-(\mu_{\Xi^0}^2-\mu_{\Xi^-}^2)=\mu_p^2-\mu_n^2(4.14)\ , \tag{3.14}$$

$$(5.58\pm0.28)-2\sqrt{3}\mu_{\Sigma\Lambda}=2(\mu_p-\mu_n)-(\mu_{\Sigma^+}-\mu_{\Sigma^-})(5.83\pm0.06)\ . \tag{3.15}$$

The values of the two sides taken from the data [17] are shown in parentheses. The three sum rules in (3.12)–(3.14) are not new and hold in the class of models with $\Delta s\neq0$ referred to above. A discussion of why they are poorly satisfied can be found in Ref. [18]. The sum rule in (3.15), a consequence of the four-parameter model, is surprisingly well satisfied.

The simpler case with three effective parameters $\mu_0\alpha$, $\mu_0\beta$, and r $(\mu_0\equiv e/2m_u)$ is of interest since it makes the natural assumption $m_u=m_d$ or $\mu_u=-2\mu_d$. This implies an additional sum rule [apart from the (3.12)–(3.15)]

$$(1.61\pm0.08)\mu_{\Sigma\Lambda}=\frac{\sqrt{3}}{2}\mu_n(1.66)\ , \tag{3.16}$$

which is quite well satisfied [20,21].

The important point to note is that because of the sea contribution, α and β are free parameters and not restricted to the simple quark model value. Finally, we note that the failure of the data to satisfy the relations (3.12)–(3.14) implies that one can only obtain, at best, an approximate fit to the magnetic moment data in all the above cases.

IV. WEAK DECAY CONSTANTS AND SPIN DISTRIBUTIONS

For the weak decay constant (g_A/g_V), $O_f^i=2I_3^i$ and we obtain

$$(g_A/g_V)_{n\to p}=\frac{1}{N^2}\left[\frac{5}{3}\right]\left[2a+\frac{4}{5}b-\frac{6}{5}c+\frac{6}{5}d+\frac{8}{5}\sqrt{2}e\right]$$
$$=3(\alpha+\beta)\ , \tag{4.1}$$

$$(g_A/g_V)_{\Xi^-\to\Xi^0}=\frac{1}{N^2}\left[-\frac{1}{3}\right](2a-4b-6c-6d+4\sqrt{2}e)$$
$$=-3\beta\ . \tag{4.2}$$

Using (3.5) and (4.1), (4.2) we obtain

$$(\mu_{\Xi^0}-\mu_{\Xi^-})/(\mu_p-\mu_n)=(g_A/g_V)_{\Xi^-\to\Xi^0}/(g_A/g_V)_{n\to p}\ . \tag{4.3}$$

Note that the relation Eq. (4.3) continues to hold in models [18,19] with $\Delta s\neq0$ mentioned above. For the three-parameter model (i.e., with $\mu_u=-2\mu_d$) in addition to (4.3), one obtains

$$(\mu_p+2\mu_n)/(\mu_p-\mu_n)=(g_A/g_V)_{\Xi^-\to\Xi^0}/(g_A/g_V)_{n\to p}\ . \tag{4.4}$$

The relations (4.3) and (4.4) cannot be checked directly with data as $(g_A/g_V)_{\Xi^-\to\Xi^0}$ is not measured. However, we can predict (see Table I) the (g_A/g_V) for various semi-leptonic decays since they can be expressed, using flavor SU(3) symmetry, in terms of F and D or α and β.

In fact $(g_A/g_V)_{n\to p}=F+D$ and $(g_A/g_V)_{\Xi^-\to\Xi^0}=F-D$, from (4.1) and (4.2) we have

$$F=3\alpha/2,\quad D=3(\alpha+2\beta)/2,\quad F/D=\alpha/(\alpha+2\beta)\ . \tag{4.5}$$

It is easy to verify that when there is no sea contribution (i.e., $a_{8,10}=b_{1,8,10}=c_8=d_8=0$) and $\mu_u=-2\mu_d$, the standard SU(6) quark model results, e.g., $\mu_n/\mu_p=-2/3$, $(g_A/g_V)_{n\to p}=5/3$, and $F/D=2/3$ follow.

For spin distributions in the proton and neutron, we have

$$I_1^p=\frac{1}{2}\left\langle\sum_i e_i^2\sigma_z^i\right\rangle_n=\frac{1}{3N^2}\left[\frac{5}{3}a+2b+\frac{c}{3}+3d+\frac{2}{3}\sqrt{2}e\right], \tag{4.6}$$

$$I_1^n=\frac{1}{2}\left\langle\sum_i e_i^2\sigma_z^i\right\rangle_n=\frac{1}{3N^2}\left[\frac{4}{3}b+\frac{4}{3}c+2d-\frac{2}{3}\sqrt{2}e\right], \tag{4.7}$$

where $I_1^p\equiv\int g_1^p(x)dx$ etc. Similarly, one can obtain

$$I_1^\Lambda=\frac{1}{2}\left\langle\sum_i e_i^2\sigma_z^i\right\rangle_\Lambda=\frac{1}{3N^2}\left[\frac{1}{3}a+\frac{4}{3}b+c+2d-\frac{2}{3}\sqrt{2}e\right]. \tag{4.8}$$

Using the parameters α and β, they are

$$I_1^p=\tfrac{1}{6}(4\alpha-\beta),\quad I_1^n=\tfrac{1}{6}(\alpha-4\beta),\quad I_1^\Lambda=\tfrac{1}{4}(\alpha-2\beta)\ . \tag{4.9}$$

One can see that the standard SU(6) result gives $\int g_1^p(x)dx=5/18$, $\int g_1^n(x)dx=0$, and $\int g_1^\Lambda(x)dx=1/18$. Including the sea contributions, however, $\int g_1^p(x)dx$, $\int g_1^\Lambda(x)dx$ could be different from their SU(6) value, and also $\int g_1^n(x)dx$ could be nonzero. One can verify, however, that the Bjorken sum rule is still satisfied:

$$\int[g_1^p(x)-g_1^n(x)]dx=\tfrac{1}{6}(g_A/g_V)_{n\to p}\ . \tag{4.10}$$

In addition, we have

$$\int[g_1^p(x)-g_1^\Lambda(x)]dx=\tfrac{1}{12}[(g_A/g_V)_{n\to p}+(g_A/g_V)_{\Lambda\to p}]\ . \tag{4.11}$$

In our model, $\int_0^1 g_1^\Lambda(x)dx$ will be less than its SU(6) value if sea contributions are taken into account (see Table I). It is interesting to note that an experiment to measure the spin structure function of the Λ particle has been suggested recently [22].

V. DISCUSSION OF THE SEA CONTRIBUTION

For simplicity, we consider the case when the magnetic moments are given by three parameters α, β, and r [i.e., put $\mu_u=-2\mu_d$ in (3.5)–(3.9)]. The discussion for the case when $\mu_u\neq-2\mu_d$ can be carried out on similar lines and suggests that $-\mu_u/2\mu_d<1$ for both pure scalar and vector sea.

A. Scalar sea

If sea spin is zero, $a_8\neq0$ and $a_{10}\neq0$, but $b_1=b_8=b_{10}=c_8=d_8=0$, one obtains

TABLE I. Comparison of the calculated magnetic moments and axial vector coupling constants of baryons with data and other models.

Baryon	Data [16]	SQM[a]	Set I[b]	Set II[c]
P	2.7928	2.793[d]	2.7928[d]	2.793[d]
n	−1.9130	−1.913[d]	−1.913[d]	−1.917[d]
Λ	−0.613±0.004	−0.613[d]	−0.613[d]	−0.613[d]
$\Sigma^0\Lambda$	−1.61±0.08	−1.63	−1.61[d]	−1.66
Σ^+	2.42±0.05	2.674	2.678	2.664
Σ^0		0.791	0.761	0.830
Σ^-	−1.160±0.025	−1.092	−1.156	−1.004
Ξ^0	−1.250±0.014	−1.435	−1.408	−1.463
Ξ^-	−0.6507±0.0025	−0.493	−0.537	−0.421
$(g_A/g_V)_{n\to p}$	1.2573±0.0028	1.666	1.2571	1.2573
$(g_A/g_V)_{\Lambda\to p}$	0.718±0.015	1.000	0.7605	0.7455
$(g_A/g_V)_{\Sigma^-\to n}$	−0.340±0.017	−0.333	−0.2325	−0.2781
$(g_A/g_V)_{\Xi^-\to\Lambda}$	0.25±0.05	0.333	0.2640	0.2337
$(g_A/g_V)_{\Xi^-\to\Xi^0}$		−0.333	−0.2325	−0.2781
$\int g_1^p(x)dx$	0.126±0.010±0.015	0.278	0.2147	0.202
$\int g_1^n(x)dx$	−0.08±0.06[e]	0.0	−0.0052	−0.007
$\int g_1^\Lambda(x)dx$		0.0556	0.0466	0.0353

[a]Standard quark model result, e.g., see Ref. [16], VIII. 59.
[b]Four-parameter fit.
[c]Three-parameter fit.
[d]Input.
[e]Ref. [24].

$$\mu_n/\mu_p=(-\tfrac{2}{3})\frac{1-a_{10}^2}{1+\frac{1}{3}(a_8^2-a_{10}^2)}\simeq(-\tfrac{2}{3})(1-\tfrac{1}{3}a_8^2a_{10}^2+\cdots) \tag{5.1}$$

since $0\le a_8^2, a_{10}^2\le 1$. It is obvious that the contribution from the scalar sea leads to a wrong correction to the ratio of neutron and proton magnetic moments. For F/D ratio, one obtains

$$F/D=\tfrac{2}{3}(1+\tfrac{1}{2}a_8^2+a_{10}^2+\cdots)\ , \tag{5.2}$$

which also disagrees with the data. Furthermore, for the scalar sea, the first moment of the neutron spin structure function,

$$\int g_1^n(x)dx=\frac{1}{9N^2}(a_8^2+2a_{10}^2)\ , \tag{5.3}$$

is positive which seems to contradict the negative value indicated by the earlier analysis of the European Muon Collaboration (EMC) result [23] and the latest data given by the Spin Muon Collaboration (SMC) [24].

B. Vector sea

We first look at a special vector sea as discussed in Ref. [15]. Assuming $a_{8,10}=b_{1,10}=d_8=0$ and $b_8=c_8=-\delta$, it is easy to see from (3.5) that

$$\mu_p=\mu_0\frac{1+\frac{4}{3}\delta^2}{1+2\delta^2},\quad \mu_n=\mu_0\left[\frac{2}{3}\right]\frac{1+\frac{4}{3}\delta^2}{1+2\delta^2}\ , \tag{5.4}$$

where $\mu_0=e/2m_u$. Hence the relation $\mu_n/\mu_p=-2/3$ is preserved as given in [15]. Similarly, from (4.1) and (4.2), we have

$$(g_A/g_V)_{n\to p}=\left[\frac{5}{3}\right]\frac{1+\frac{4}{3}\delta^2}{1+2\delta^2}\ , \tag{5.5}$$

$$(g_A/g_V)_{\Xi^-\to\Xi^0}=\left[-\frac{1}{3}\right]\frac{1+\frac{4}{3}\delta^2}{1+2\delta^2}\ ; \tag{5.6}$$

one can see that the conventional SU(6) result $(g_A/g_V)_{n\to p}/(g_A/g_V)_{\Xi^-\to\Xi^0}=-5$ is also preserved. However, using the parameter $\delta=-0.35$ given in [15], we obtain $(g_A/g_V)_{n\to p}=1.727$, which is inconsistent with the data [17] $(g_A/g_V)_{n\to p}=1.257\pm0.003$. This disagreement is not unexpected. Because the perturbative calculation of the mixing parameters and its result $b_8=c_8=-\delta$ are questionable. It is obvious that the nonperturbative effects, which are dominant in the low energy region, would change the relative weight of these mixing parameters significantly. Therefore, we prefer to discuss a more general vector sea and to see if there is another appropriate parameter set, in which the nonperturbative and perturbative effects are taken into account, that can lead to a better agreement with the low energy baryon properties. We will show below that this parameter set not only gives a right modification to the ratio μ_n/μ_p but also gives a very good result for axial vector coupling constants.

As we mentioned above, the mixing parameters basically come from the nonperturbative interactions between quarks and gluons. Hence we do not attempt to calculate these parameters, but rather estimate them by the required agreement with the low energy data. Before doing this, we give some arguments as motivations for choosing the parameters. Since the sea basically comes from the emission of virtual gluons, the b_8 term would be dominant and we would expect

$$b_1^2, b_{10}^2(\text{two-gluon sea})\ll b_8^2(\text{one-gluon sea})\ . \tag{5.7}$$

The c_8 term is expected to be small due to another reason

$$c_8^2(\text{quark spin flip})\ll b_8^2(\text{quark spin nonflip})\ . \tag{5.8}$$

The scalar sea a_8 and a_{10} terms are expected to be also small because they can only come from the two-gluon sea. The tensor sea (d_8) term comes from two-gluon sea and quark spin-flip processes; hence, it should be highly suppressed. Assuming no scalar and tensor sea contribution and neglecting the c_8^2 term (since $c_8^2\ll b_8^2$), we have

$$\mu_n/\mu_p=(-2/3)\frac{1-\frac{1}{3}b_1^2}{1-\frac{1}{3}b_1^2-\frac{1}{9}b_8^2}\simeq(-\tfrac{2}{3})(1+\tfrac{1}{9}b_8^2)\ , \tag{5.9}$$

thus, the sea contribution gives a correction in the right direction.

VI. NUMERICAL RESULTS

To obtain numerical results, we use the data on magnetic moments and weak decay coupling constants to determine the parameters. In particular, the values of α and β so obtained should be reproducible by choice of the seven basic parameters a_8, a_{10}, b_1, b_8, etc. which determine the sea contribution. It is clear from (3.10) and (3.11) that there are many ways of choosing a_8, etc. to give the same α and β. However, guided by the qualitative discussion of Sec. V, we will assume the sea is mainly vector with a small scalar component. The tensor sea is neglected ($d_8=0$). We shall see that the parameters (b_8, c_8, etc.), which determine the contribution of such a sea to the baryon structure, can be chosen to give the α and β determined from the data.

A. Four-parameter fit

The magnetic moments in (3.5)–(3.9) are given in terms of four effective parameters $\tilde{U}\equiv3\alpha\mu_d$, $\tilde{D}\equiv-3\beta\mu_d$, $2p\equiv-\mu_u/\mu_d>0$, and $r\equiv\mu_s/\mu_d>0$. Using μ_p, μ_n, $\mu_{\Sigma,\Lambda}$ as input one can directly determine $\tilde{U}=-1.348$, $\tilde{D}=0.306$, and $p=0.922$ as these do not involve the parameter r. The value of μ_Λ is used as input to fix $r=0.6255$. Knowledge of the ratio $\alpha/\beta=4.406$ immediately predicts [see (4.5)]

$$F/D=0.6878\ . \tag{6.1}$$

A more realistic model with a small $\Delta s\neq0$ could easily modify this value. Note that in the models of [19] and [18] with extra parameter (Δs) they obtain 0.726 and 0.585 for this ratio. To separate out the parameters α

and β, we use the axial coupling constant data to obtain

$$\alpha=0.3415, \quad \beta=0.0775 . \tag{6.2}$$

The values obtained for the quark magnetic moments (in nuclear magnetons μ_N) are

$$\mu_u=2.428, \quad \mu_d=-1.316, \quad \mu_s=-0.823 . \tag{6.3}$$

A choice of sea parameters which reproduce the parameters α and β given in (6.2) are $b_1^2=0.0039$, $b_8^2=0.22$, $c_8^2=0.027$ (for vector sea), and $a_{10}^2=0.0975$ (for scalar sea) with $b_8c_8>0$.

The values obtained for magnetic moments and other quantities are displayed in column 4 of Table I. It can be seen that the fit to the magnetic moments and the axial coupling constants is quite reasonable except for μ_{Σ^+}. For the quark spin distributions our calculation suggests a small nonzero negative value for $\int_0^1 g_1^n(x)dx$, however, the result for $\int_0^1 g_1^p(x)dx=0.2147$ is much larger than the experimental value [23] of 0.126 ± 0.018. One must note, however, that the EMC experiment gives this value for $\langle Q^2\rangle=10.7$ $(\mathrm{GeV}/c)^2$ and this can be very different from the very low Q^2 result predicted by our q^3+sea model.

B. Three-parameter fit

The natural assumption $m_u=m_d$ implies the relation $\mu_u=-2\mu_d$. Implementing relation in (3.5)–(3.9) gives $\mu_p=\mu_0(2\alpha+\beta)$, $\mu_n=-\mu_0(\alpha+2\beta)$ etc., where $\mu_0 \equiv e/2m_u$. The magnetic moments are given in terms of three effective parameters $\mu_0\alpha$, $\mu_0\beta$, and r. Guided by a four-parameter fit we choose sea parameters similar to that case, namely $b_1^2=0.1$, $b_8^2=0.22$, $c_8^2=0.027$, and $a_{10}^2=0.02$ with $b_8c_8>0$. Basically we have enhanced the vector sea with a larger value of b_1 and reduced the scalar sea with a smaller value of a_{10}. This choice immediately gives $\alpha=0.3264$ and $\beta=0.0927$. Using μ_p and μ_Λ as inputs then determines $\mu_0=3.7465\mu_N$ and $r=0.6286$. The results of magnetic moments etc. are listed in column 5 of Table I. Since the ratio $\alpha/\beta=3.521$ one obtains

$$F/D=0.6380 , \tag{6.4}$$

which is fairly close to the experimental value [25]. Since F/D increases monotonically with increasing α/β, for the simple quark model ($\alpha/\beta=4$) the value of $F/D=2/3$ lies between those in (6.1) and (6.4). The results for quark spin distributions are similar to the four-parameter case.

For comparison, in column 3 of Table I the results for the simple quark model are given. In this case there is no sea contribution, and the baryons are given by standard q^3 wave function which fixes $\alpha=4/9$ and $\beta=1/9$. The magnetic moments are given in terms of three-parameters μ_u, μ_d, and μ_s. This fit with μ_p, μ_n, and μ_Λ as inputs gives $\mu_u/(-2\mu_d)=p=0.953$ and $r=\mu_s/\mu_u=0.63$. We have used the same inputs in all three cases for a meaningful comparison. From Table I one can see that the four-parameter gives a somewhat better overall fit.

VII. SUMMARY

In summary, we have suggested a general formalism to treat a baryon as a composite system of q^3 plus a flavorless sea. The modifications of the different properties of spin 1/2 baryon, by the sea, are given. Numerical fits to the individual magnetic moments, $\Sigma\Lambda$-transition moment and axial weak coupling constants for the baryon octet have been obtained. These results seem to favor a dominantly vector sea.

It should be noted that our results and conclusions are subject to the following points: (i) the sea and the three-quarks are considered to be in a relative S state, possible higher angular momentum states have been neglected; (ii) the sea is assumed to be flavorless and has been specified only by its total quantum numbers; (iii) further, modification of the baryon wave function is needed to have nonzero Δs in the nucleon; (iv) relativistic corrections have been neglected although the internal motion of the light quarks in the baryon is expected to be relativistic; (v) all calculations have been performed in the baryon rest frame. This may be reasonable for the magnetic moments and the weak decay constants, but may not be appropriate for comparing the spin distribution calculated by us (at low Q^2 scale) with the EMC data at much high momentum transfer. All these points need to be considered in future work to fully understand baryon structure.

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APPENDIX A: MATRIX ELEMENTS FOR DIFFERENT OPERATORS

1. Spin projection operator

$$\langle\sigma_z^{(1)}\rangle^{\lambda\uparrow\lambda\uparrow}=\langle\sigma_z^{(2)}\rangle^{\lambda\uparrow\lambda\uparrow}=2/3, \quad \langle\sigma_z^{(3)}\rangle^{\lambda\uparrow\lambda\uparrow}=-1/3 , \tag{A1}$$

$$\langle\sigma_z^{(1)}\rangle^{\rho\uparrow\rho\uparrow}=\langle\sigma_z^{(2)}\rangle^{\rho\uparrow\rho\uparrow}=0, \quad \langle\sigma_z^{(3)}\rangle^{\rho\uparrow\rho\uparrow}=1 , \tag{A2}$$

$$\langle\sigma_z^{(1)}\rangle^{\lambda\uparrow\rho\uparrow}=-\langle\sigma_z^{(2)}\rangle^{\lambda\uparrow\rho\uparrow}=1/\sqrt{3}, \quad \langle\sigma_z^{(3)}\rangle^{\lambda\uparrow\rho\uparrow}=0 . \tag{A3}$$

It is easy to see that the matrix elements in (1) and (2) satisfy

$$\sum_{i=1}^{3}\langle\sigma_z^{(i)}\rangle^{\lambda\uparrow\lambda\uparrow}=\sum_{i=1}^{3}\langle\sigma_z^{(i)}\rangle^{\rho\uparrow\rho\uparrow}=1 . \tag{A4}$$

In addition,

$$\begin{aligned}&\langle\sigma_z^{(1)}\rangle^{\lambda\uparrow 3/2\uparrow}=\langle\sigma_z^{(2)}\rangle^{\lambda\uparrow 3/2\uparrow}=-\sqrt{2}/3 ,\\ &\langle\sigma_z^{(3)}\rangle^{\lambda\uparrow 3/2\uparrow}=2\sqrt{2}/3 ,\end{aligned} \tag{A5}$$

$$\begin{aligned}&\langle\sigma_z^{(1)}\rangle^{\rho\uparrow 3/2\uparrow}=-\langle\sigma_z^{(2)}\rangle^{\rho\uparrow 3/2\uparrow}=\sqrt{2/3} ,\\ &\langle\sigma_z^{(3)}\rangle^{\rho\uparrow 3/2\uparrow}=0 .\end{aligned} \tag{A6}$$

The matrix elements in (3), (5), and (6) satisfy

$$\sum_{i=1}^{3}\langle\sigma_z^{(i)}\rangle^{\lambda\uparrow\rho\uparrow}=\sum_{i=1}^{3}\langle\sigma_z^{(i)}\rangle^{\lambda\uparrow 3/2\uparrow}=\sum_{i=1}^{3}\langle\sigma_z^{(i)}\rangle^{\rho\uparrow 3/2\uparrow}=0\ . \tag{A7}$$

2. Isospin projection operator

For the proton, we have

$$\langle I_3^{(1)}\rangle_p^{\lambda\lambda}=\langle I_3^{(2)}\rangle_p^{\lambda\lambda}=1/3,\quad \langle I_3^{(3)}\rangle_p^{\lambda\lambda}=-1/6\ , \tag{A8}$$

$$\langle I_3^{(1)}\rangle_p^{\rho\rho}=\langle I_3^{(2)}\rangle_p^{\rho\rho}=0,\quad \langle I_3^{(3)}\rangle_p^{\rho\rho}=1/2\ , \tag{A9}$$

$$\langle I_3^{(1)}\rangle_p^{\lambda\rho}=-\langle I_3^{(2)}\rangle_p^{\lambda\rho}=1/2\sqrt{3},\quad \langle I_3^{(3)}\rangle_p^{\lambda\rho}=0\ ; \tag{A10}$$

for the neutron, all matrix elements get an opposite sign. For the Σ^+ hyperon, we have

$$\langle I_3^{(1)}\rangle_{\Sigma^+}^{\lambda\lambda}=\langle I_3^{(2)}\rangle_{\Sigma^+}^{\lambda\lambda}=5/12,\quad \langle I_3^{(3)}\rangle_{\Sigma^+}^{\lambda\lambda}=1/6\ , \tag{A11}$$

$$\langle I_3^{(1)}\rangle_{\Sigma^+}^{\rho\rho}=\langle I_3^{(2)}\rangle_{\Sigma^+}^{\rho\rho}=1/4,\quad \langle I_3^{(3)}\rangle_{\Sigma^+}^{\rho\rho}=1/2\ , \tag{A12}$$

$$\langle I_3^{(1)}\rangle_{\Sigma^+}^{\lambda\rho}=-\langle I_3^{(2)}\rangle_{\Sigma^+}^{\lambda\rho}=1/4\sqrt{3},\quad \langle I_3^{(3)}\rangle_{\Sigma^+}^{\lambda\rho}=0\ ; \tag{A13}$$

similarly, for Σ^- the matrix elements reverse their signs. For the Ξ^0 hyperon, we have

$$\langle I_3^{(1)}\rangle_{\Xi^0}^{\lambda\lambda}=\langle I_3^{(2)}\rangle_{\Xi^0}^{\lambda\lambda}=1/12,\quad \langle I_3^{(3)}\rangle_{\Xi^0}^{\lambda\lambda}=1.3\ , \tag{A14}$$

$$\langle I_3^{(1)}\rangle_{\Xi^0}^{\rho\rho}=\langle I_3^{(2)}\rangle_{\Xi^0}^{\rho\rho}=1/4,\quad \langle I_3^{(3)}\rangle_{\Xi^0}^{\rho\rho}=0\ , \tag{A15}$$

$$\langle I_3^{(1)}\rangle_{\Xi^0}^{\lambda\rho}=-\langle I_3^{(2)}\rangle_{\Xi^0}^{\lambda\rho}=-1/4\sqrt{3},\quad \langle I_3^{(3)}\rangle_{\Xi^0}^{\lambda\rho}=0\ , \tag{A16}$$

for Ξ^-, all matrix elements reverse their signs. Finally, all isospin matrix elements for Λ and Σ^0 hyperons are zero.

3. Charge operator with symmetry breaking effect

For the proton, we have

$$\left\langle e^{(1)}\frac{m}{m_1}\right\rangle_p^{\lambda\lambda}=\left\langle e^{(2)}\frac{m}{m_2}\right\rangle_p^{\lambda\lambda}=1/2,\quad \left\langle e^{(3)}\frac{m}{m_3}\right\rangle_p^{\lambda\lambda}=0\ , \tag{A17}$$

$$\left\langle e^{(1)}\frac{m}{m_1}\right\rangle_p^{\rho\rho}=\left\langle e^{(2)}\frac{m}{m_2}\right\rangle_p^{\rho\rho}=1/6,\quad \left\langle e^{(3)}\frac{m}{m_3}\right\rangle_p^{\rho\rho}=2/3\ , \tag{A18}$$

$$\left\langle e^{(1)}\frac{m}{m_1}\right\rangle_p^{\lambda\rho}=-\left\langle e^{(2)}\frac{m}{m_2}\right\rangle_p^{\lambda\rho}=1/2\sqrt{3}\ ,\qquad \left\langle e^{(3)}\frac{m}{m_3}\right\rangle_p^{\lambda\rho}=0\ , \tag{A19}$$

where $m=m_u=m_d$. We note that the matrix element $\langle e^{(3)}m/m_3\rangle^{\lambda\rho}$ vanishes for all octet baryons.

For the neutron, the matrix elements in (A19) reverse their signs. But in (A18) the first two matrix elements *do not* change the sign, i.e., $\langle e^{(i)}m/m_i\rangle_n^{\rho\rho}=\langle e^{(i)}m/m_i\rangle_p^{\rho\rho}$ $(i=1,2)$ and the third one becomes $\langle e^{(3)}m/m_3\rangle_n^{\rho\rho}=-1/3$. For the neutron matrix elements in (A17), we have

$$\left\langle e^{(1)}\frac{m}{m_1}\right\rangle_n^{\lambda\lambda}=\left\langle e^{(2)}\frac{m}{m_2}\right\rangle_n^{\lambda\lambda}=-1/6\ ,\qquad \left\langle e^{(3)}\frac{m}{m_3}\right\rangle_n^{\lambda\lambda}=1/3\ . \tag{A20}$$

For Σ^+, we obtain

$$\left\langle e^{(1)}\frac{m}{m_1}\right\rangle_{\Sigma^+}^{\lambda\lambda}=\left\langle e^{(2)}\frac{m}{m_2}\right\rangle_{\Sigma^+}^{\lambda\lambda}=(10-r)/18,\quad \left\langle e^{(3)}\frac{m}{m_3}\right\rangle_{\Sigma^+}^{\lambda\lambda}=2(1-r)/9\ , \tag{A21}$$

$$\left\langle e^{(1)}\frac{m}{m_1}\right\rangle_{\Sigma^+}^{\rho\rho}=\left\langle e^{(2)}\frac{m}{m_2}\right\rangle_{\Sigma^+}^{\rho\rho}=(2-r)/6,\quad \left\langle e^{(3)}\frac{m}{m_3}\right\rangle_{\Sigma^+}^{\rho\rho}=2/3\ , \tag{A22}$$

$$\left\langle e^{(1)}\frac{m}{m_1}\right\rangle_{\Sigma^+}^{\lambda\rho}=-\left\langle e^{(2)}\frac{m}{m_2}\right\rangle_{\Sigma^+}^{\lambda\rho}=(2+r)/6\sqrt{3}\ ; \tag{A23}$$

while for Σ^- we have

$$\left\langle e^{(1)}\frac{m}{m_1}\right\rangle_{\Sigma^-}^{\lambda\lambda}=\left\langle e^{(2)}\frac{m}{m_2}\right\rangle_{\Sigma^-}^{\lambda\lambda}=-(5+r)/18,\quad \left\langle e^{(3)}\frac{m}{m_3}\right\rangle_{\Sigma^-}^{\lambda\lambda}=-(1+2r)/9\ , \tag{A24}$$

$$\left\langle e^{(1)}\frac{m}{m_1}\right\rangle_{\Sigma^-}^{\rho\rho}=\left\langle e^{(2)}\frac{m}{m_2}\right\rangle_{\Sigma^-}^{\rho\rho}=-(1+r)/6,\quad \left\langle e^{(3)}\frac{m}{m_3}\right\rangle_{\Sigma^-}^{\rho\rho}=-1/3\ , \tag{A25}$$

$$\left\langle e^{(1)}\frac{m}{m_1}\right\rangle_{\Sigma^-}^{\lambda\rho}=-\left\langle e^{(2)}\frac{m}{m_2}\right\rangle_{\Sigma^-}^{\lambda\rho}=-(1-r)/6\sqrt{3}\ . \tag{A26}$$

For Ξ^0, we have

$$\left\langle e^{(1)}\frac{m}{m_1}\right\rangle_{\Xi^0}^{\lambda\lambda}=\left\langle e^{(2)}\frac{m}{m_2}\right\rangle_{\Xi^0}^{\lambda\lambda}=(2-5r)/18,\quad \left\langle e^{(3)}\frac{m}{m_3}\right\rangle_{\Xi^0}^{\lambda\lambda}=(4-r)/9\ , \tag{A27}$$

$$\left\langle e^{(1)}\frac{m}{m_1}\right\rangle_{\Xi^0}^{\rho\rho}=\left\langle e^{(2)}\frac{m}{m_2}\right\rangle_{\Xi^0}^{\rho\rho}=(2-r)/6,\quad \left\langle e^{(3)}\frac{m}{m_3}\right\rangle_{\Xi_0}^{\rho\rho}=-r/3\ , \tag{A28}$$

$$\left\langle e^{(1)}\frac{m}{m_1}\right\rangle^{\lambda\rho}_{\Xi^0}=-\left\langle e^{(2)}\frac{m}{m_2}\right\rangle^{\lambda\rho}_{\Xi^0}=-(2+r)/6\sqrt{3}\ ; \tag{A29}$$

and, for Ξ^-,

$$\left\langle e^{(1)}\frac{m}{m_1}\right\rangle^{\lambda\lambda}_{\Xi^-}=\left\langle e^{(2)}\frac{m}{m_2}\right\rangle^{\lambda\lambda}_{\Xi^-}=-(1+5r)/18,\quad \left\langle e^{(3)}\frac{m}{m_3}\right\rangle^{\lambda\lambda}_{\Xi^-}=-(2+r)/9\ , \tag{A30}$$

$$\left\langle e^{(1)}\frac{m}{m_1}\right\rangle^{\rho\rho}_{\Xi^-}=\left\langle e^{(2)}\frac{m}{m_2}\right\rangle^{\rho\rho}_{\Xi^-}=-(1+r)/6,\quad \left\langle e^{(3)}\frac{m}{m_3}\right\rangle^{\rho\rho}_{\Xi^-}=-r/3\ , \tag{A31}$$

$$\left\langle e^{(1)}\frac{m}{m_1}\right\rangle^{\lambda\rho}_{\Xi^-}=-\left\langle e^{(2)}\frac{m}{m_2}\right\rangle^{\lambda\rho}_{\Xi^-}=(1-r)/6\sqrt{3}\ . \tag{A32}$$

For Λ^0, we obtain

$$\left\langle e^{(1)}\frac{m}{m_1}\right\rangle^{\lambda\lambda}_{\Lambda^0}=\left\langle e^{(2)}\frac{m}{m_2}\right\rangle^{\lambda\lambda}_{\Lambda^0}=(1-2r)/12,\quad \left\langle e^{(3)}\frac{m}{m_3}\right\rangle^{\lambda\lambda}_{\Lambda^0}=1/6\ , \tag{A33}$$

$$\left\langle e^{(1)}\frac{m}{m_1}\right\rangle^{\rho\rho}_{\Lambda^0}=\left\langle e^{(2)}\frac{m}{m_2}\right\rangle^{\rho\rho}_{\Lambda^0}=(5-2r)/36,\quad \left\langle e^{(3)}\frac{m}{m_3}\right\rangle^{\rho\rho}_{\Lambda^0}=(1-4r)/18\ , \tag{A34}$$

$$\left\langle e^{(1)}\frac{m}{m_1}\right\rangle^{\lambda\rho}_{\Lambda^0}=-\left\langle e^{(2)}\frac{m}{m_2}\right\rangle^{\lambda\rho}_{\Lambda^0}=-(1+2r)/12\sqrt{3}\ , \tag{A35}$$

and for Σ^0 one obtains

$$\left\langle e^{(1)}\frac{m}{m_1}\right\rangle^{\lambda\lambda}_{\Sigma^0}=\left\langle e^{(2)}\frac{m}{m_2}\right\rangle^{\lambda\lambda}_{\Sigma^0}=(5-2r)/36,\quad \left\langle e^{(3)}\frac{m}{m_3}\right\rangle^{\lambda\lambda}_{\Sigma^0}=(1-4r)/18\ , \tag{A36}$$

$$\left\langle e^{(1)}\frac{m}{m_1}\right\rangle^{\rho\rho}_{\Sigma^0}=\left\langle e^{(2)}\frac{m}{m_2}\right\rangle^{\rho\rho}_{\Sigma^0}=(1-2r)/12,\quad \left\langle e^{(3)}\frac{m}{m_3}\right\rangle^{\rho\rho}_{\Sigma^0}=1/6\ , \tag{A37}$$

$$\left\langle e^{(1)}\frac{m}{m_1}\right\rangle^{\lambda\rho}_{\Sigma^0}=-\left\langle e^{(2)}\frac{m}{m_2}\right\rangle^{\lambda\rho}_{\Sigma^0}=(1+2r)/12\sqrt{3}\ . \tag{A38}$$

Finally, for $\Sigma^0\rightarrow\Lambda^0$ transition elements we have

$$\left\langle e^{(1)}\frac{m}{m_1}\right\rangle^{\lambda\lambda}_{\Sigma^0\rightarrow\Lambda^0}=\left\langle e^{(2)}\frac{m}{m_2}\right\rangle^{\lambda\lambda}_{\Sigma^0\rightarrow\Lambda^0}=-1/4\sqrt{3},\quad \left\langle e^{(3)}\frac{m}{m_3}\right\rangle^{\lambda\lambda}_{\Sigma^0\rightarrow\Lambda^0}=1/2\sqrt{3}\ , \tag{A39}$$

$$\left\langle e^{(1)}\frac{m}{m_1}\right\rangle^{\rho\rho}_{\Sigma^0\rightarrow\Lambda^0}=\left\langle e^{(2)}\frac{m}{m_2}\right\rangle^{\rho\rho}_{\Sigma^0\rightarrow\Lambda^0}=1/4\sqrt{3},\quad \left\langle e^{(3)}\frac{m}{m_3}\right\rangle^{\rho\rho}_{\Sigma^0\rightarrow\Lambda^0}=-1/2\sqrt{3}\ , \tag{A40}$$

$$\left\langle e^{(1)}\frac{m}{m_1}\right\rangle^{\lambda\rho}_{\Sigma^0\rightarrow\Lambda^0}=-\left\langle e^{(2)}\frac{m}{m_2}\right\rangle^{\lambda\rho}_{\Sigma^0\rightarrow\Lambda^0}=-1/4\ . \tag{A41}$$

4. Charge square operator

We only discuss the nucleon case: for the proton we obtain

$$\langle e^{(1)^2}\rangle^{\lambda\lambda}_p=\langle e^{(2)^2}\rangle^{\lambda\lambda}_p=7/18,\quad \langle e^{(3)^2}\rangle^{\lambda\lambda}_p=2/9\ , \tag{A42}$$

$$\langle e^{(1)^2}\rangle^{\rho\rho}_p=\langle e^{(2)^2}\rangle^{\rho\rho}_p=5/18,\quad \langle e^{(3)^2}\rangle^{\rho\rho}_p=4/9\ , \tag{A43}$$

$$\langle e^{(1)^2}\rangle^{\lambda\rho}_p=-\langle e^{(2)^2}\rangle^{\lambda\rho}_p=1/6\sqrt{3},\quad \langle e^{(3)^2}\rangle^{\lambda\rho}_p=0\ ; \tag{A44}$$

while for the neutron one obtains

$$\langle e^{(1)^2}\rangle^{\lambda\lambda}_n=\langle e^{(2)^2}\rangle^{\lambda\lambda}_n=1/6,\quad \langle e^{(3)^2}\rangle^{\lambda\lambda}_n=1/3\ , \tag{A45}$$

$$\langle e^{(1)^2}\rangle^{\rho\rho}_n=\langle e^{(2)^2}\rangle^{\rho\rho}_n=5/18,\quad \langle e^{(3)^2}\rangle^{\rho\rho}_n=1/9\ , \tag{A46}$$

$$\langle e^{(1)^2}\rangle^{\lambda\rho}_n=-\langle e^{(2)^2}\rangle^{\lambda\rho}_n=-1/6\sqrt{3},\quad \langle e^{(3)^2}\rangle^{\lambda\rho}_n=0\ . \tag{A47}$$

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