(4,0) super-Beltrami parametrization and super-operator-product expansions

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The superconformal structure of the (4,0) supergeometry of two-dimensional harmonic superspace is characterized by the super-Beltrami differentials. The BRST algebra including super-Beltrami variables and matter superfields of arbitrary U(1) charges is constructed. The (4,0) locally supersymmetric matter action in the super-Beltrami parametrization is constructed and the super-stress-energy tensors are obtained. Furthermore, the (4,0) superconformal anomaly is given and the super-operator-product expansion of the (4,0) super-stress-energy tensor $J(Z)$ is derived.

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I. INTRODUCTION

Riemann surfaces are the basic geometric objects which appear in conformal field theory and perturbative string theory [1,2]. The Beltrami differentials are considered as the proper tool for describing the Riemann surfaces [3] and the Beltrami parametrization makes the so-called factorization property manifest at all levels [1,4]. The generalization of these tools to super-Riemann surfaces proves equally important for superstring theory [5,6].

The super-Riemann surfaces can be described geometrically by the introduction of two-dimensional supergravity geometry which is subject to torsion constraints on the superfield strengths [7,4]. The supersymmetric version of the Beltrami parametrization is presented for $N=1$ and 2 cases [8–10,4]. In Ref. [10] the Becchi-Rouet-Stora-Tyutin (BRST) quantization is discussed for (2,0) supersymmetry and the super-stress-energy tensors are obtained. In this paper we study the (4,0) super-Beltrami parametrization by using the techniques of harmonic superspace. These latters have been used successfully in two-dimensional (4,4) and (4,0) supersymmetric [11] and supergravity [12] theories. This is achieved by simplifying the $SO(4)\approx SU(2)\times SU(2)$ tensor calculus at the level of the Grassmann variables $\theta^{\mu I}\sim\theta^{\mu s i}\,(\mu=+)$ and projecting one of the SU(2)'s down to its Cartan-Weyl subgroup U(1) as follows:

$$\theta^{\mu s\pm}\equiv\theta^{\hat{\mu}\pm}=\theta^{\hat{\mu}i}U_i^{\pm}\ .$$

The resulting (4,0) curved harmonic superspace is $z^M=(x^{\pm\pm},\theta^{\hat{\mu}\pm},U_i^{\pm})$ called the central basis and the corresponding curved analytic basis $(z_A,\theta_A^{\hat{\mu}-})$ with $z_A=(x_A^m,x_A^{\overline{m}},\theta_A^{\hat{\mu}+},U_i^{\pm})$, $m=++$, $\overline{m}=--$ is obtained from the central basis by a coordinate change [12]. In the analytic superspace geometry we include the harmonic differentials $d\eta^{\pm\pm}$, $d\eta^0$ which are necessary in order to put in evidence the general coordinate transformations of the harmonic variables

$$\delta U_i^+=\lambda^{++}U_i^-\ ,$$
$$\delta U_i^-=0\ ,$$

generating the superconformal symmetry [13]. Furthermore, we introduce the i_ξ contraction operator along the vector ghost superfields [14] where

$$i_\xi(d\eta^{++})=\xi^{++}\ ,$$
$$i_\xi(d\eta^{--})=0=i_\xi(d\eta^0)\ ,$$
$$d\eta^{\pm\pm}=U^{\pm i}dU_i^{\pm}\ ,$$
$$d\eta^0=\tfrac{1}{2}(U^{+i}dU_i^- - U_i^- dU_i^+)\ .$$

With this structure the general supercoordinate transformations and the SU(2) symmetry are both contained in the superdiffeomorphisms of the analytic superspace. The frames are parametrized with respect to the rigid (4,0) harmonic superspace frames and the Becchi-Rouet-Stora-Tyutin (BRST) algebra including super-Beltrami variables, matter superfields of arbitrary conformal weights, and U(1) charges have been constructed. These developments allow us to derive the (4,0) superconformal anomaly and the super-operator-product expansion of the super-stress-energy tensors.

The organization of this paper is as follows. In Sec. II, we formulate our background geometry leading to the horizontality constraints on the super-Beltrami differentials. These are independent of the Lorentz super-Weyl connections and the superfield strengths, but the U(1) Kac-Moody subgroup of the SU(2) local symmetry is connection dependent. The BRST transformations of the super-Beltrami variables and their corresponding ghost superfields are presented by parametrizing the frames with respect to the rigid (4,0) superspace frames. Some super-Beltrami variables have been fixed in order to recover the harmonic analyticity conditions of the ghost superfields $\Sigma^{\overline{a}}$, $\Sigma^{\hat{\alpha}+}$, and $\Sigma^{\pm\pm}$ corresponding to superdiffeomorphisms. Furthermore, we determine the BRST transformations of the conformal factors and the superconformal spinorial connections $\lambda_a^{\hat{\alpha}\pm}$. In Sec. III, we present the conformal analytic superfields deduced from analytic superfields of arbitrary spins, Weyl weights, and U(1) charges. The superconformal covariant derivatives are derived from the horizontality conditions on the matter superfields. Therefore, we have constructed the

(4,0) locally supersymmetric matter action in the super-Beltrami parametrization in harmonic superspace leading by the following to the derivation of the super-stress-energy tensors. In Sec. IV, the (4,0) superconformal anomaly is given, leading to the derivation of the anomalous Ward identities. Therewith, the (4,0) super-operator-product expansions of the supercurrent $J(Z)$ are obtained. Finally, we make some concluding remarks in Sec. V and some technical details are given in the Appendixes.

II. (4,0) SUPER-BELTRAMI DIFFERENTIALS

We recall that the frames in two-dimensional (4,0) harmonic superspace with analytic local coordinates $z^M=(z_A,\theta_A^{\hat{\mu}-})$ are defined by the super-one forms

$$E^A=dz^M E_M^A(z)\ , \tag{2.1}$$

with $A=(a,\bar{a},\hat{\alpha}\pm,\pm\pm,0)$. The two-form torsion is given by

$$T^A=dE^A+E^B(\Omega+A+B)_B^A\ , \tag{2.2}$$

where Ω and A are Lorentz and Weyl superconnections, respectively, and B is the U(1) Kac-Moody superconnection (see Appendix A). The introduction of the latter is in accordance with the analytic line integrals defined globally on super-Riemann surfaces as in the $N=2$ case [6]. The situation is such that $T_{\bar{a}\hat{\alpha}}^{\pm}=0$; then $\nabla_{\bar{a}}$ and $\nabla_{\hat{\alpha}}^{\pm}$ commute with each other on scalar superfields. In fact, when we define the analytic integral of the superfields $\phi_{\hat{\alpha}}^{\pm}$ with

$$\nabla_{\bar{a}}\phi_{\hat{\alpha}}^{\pm}=0\ , \tag{2.3}$$

one can always find a scalar superfield ϕ such that

$$\begin{aligned}&\phi_{\hat{\alpha}}^{+}=\nabla_{\hat{\alpha}}^{+}\phi\ ,\\ &\phi_{\hat{\alpha}}^{-}=\nabla_{\hat{\alpha}}^{-}\phi\ ,\\ &\nabla_{\bar{a}}\phi=0\ .\end{aligned} \tag{2.4}$$

Therefore, the (4,0) supergravity constraints can be summarized as

$$\begin{aligned}&[\nabla_{\hat{\alpha}}^{+},\nabla_{\hat{\beta}}^{-}\}=2\delta_{\hat{\alpha}\hat{\beta}}^{a}\nabla_a\ ,\\ &[\nabla_{\hat{\alpha}}^{+},\nabla_{\bar{a}}\}=R_{\hat{\alpha}\bar{a}}^{+}\ ,\\ &[\nabla_{\hat{\alpha}}^{-},\nabla_{\bar{a}}\}=R_{\hat{\alpha}\bar{a}}^{-}\ ,\\ &[\nabla_a,\nabla_{\bar{a}}\}=X^{\hat{\alpha}+}\nabla_{\hat{\alpha}}^{-}+X^{\hat{\alpha}-}\nabla_{\hat{\alpha}}^{+}+R_{a\bar{a}}\ ,\end{aligned} \tag{2.5}$$

and the harmonic constraints are given by

$$\begin{aligned}&[\nabla^{\pm\pm},\nabla_{\hat{\alpha}}^{\mp}]=\nabla_{\hat{\alpha}}^{\pm}\ ,\\ &[\nabla^{\pm\pm},\nabla_{\hat{\alpha}}^{\pm}]=0\ ,\\ &[\nabla^{\pm\pm},\nabla_a]=0=[\nabla^{\pm\pm},\nabla_{\bar{a}}]\ ,\\ &[\nabla^{0},\nabla^{\pm\pm}]=\pm 2\nabla^{\pm\pm}\ ,\\ &[\nabla^{++},\nabla^{--}]=\nabla^{0}\ ,\end{aligned} \tag{2.6}$$

where $\nabla^{\pm\pm}$ are the harmonic covariant derivatives and ∇^0 is the counting $U^C(1)$ Cartan-Weyl charge operator.

The torsion constraints of the two-dimensional (4,0) harmonic superspace geometry can be written as

$$dE^a+E^a\Omega^R=2E^{\hat{\alpha}+}E^{\hat{\beta}-}\delta_{\hat{\alpha}\hat{\beta}}^{a}\ , \tag{2.7a}$$

$$dE^{\hat{\alpha}+}+\tfrac{1}{2}E^{\hat{\alpha}+}\Omega^R+E^{\hat{\alpha}+}B=-E^aE^{\bar{a}}X^{\hat{\alpha}+}+E^{\hat{\alpha}-}E^{++}-E^{\hat{\alpha}+}E^0\ , \tag{2.7b}$$

$$dE^{\hat{\alpha}-}+\tfrac{1}{2}E^{\hat{\alpha}-}\Omega^R-E^{\hat{\alpha}-}B=-E^aE^{\bar{a}}X^{\hat{\alpha}-}+E^{\hat{\alpha}+}E^{--}-E^{\hat{\alpha}-}E^0\ , \tag{2.7c}$$

$$dE^{\bar{a}}+E^{\bar{a}}\Omega^L=0\ , \tag{2.7d}$$

$$dE^{\pm\pm}\pm 2E^{\pm\pm}B=\pm 2E^0E^{\pm\pm}\ , \tag{2.7e}$$

$$dE^0=E^{--}E^{++}\ , \tag{2.7f}$$

with

$$\Omega^{R,L}=\Omega\pm A\ .$$

The (4,0) supersymmetric horizontality constraints on the super-Beltrami variables are easily obtained by generalizing the exterior derivative and the frames in (2.7). These are given by

$$\tilde{E}^a\tilde{d}\tilde{E}^a-2\tilde{E}^a\tilde{E}^{\hat{\alpha}+}\tilde{E}^{\hat{\alpha}-}=0\ , \tag{2.8a}$$

$$\tilde{E}^a\tilde{d}\tilde{E}^{\hat{\alpha}\pm}+\tfrac{1}{2}\tilde{E}^{\hat{\alpha}\pm}\tilde{d}\tilde{E}^a-\tilde{E}^{\hat{\alpha}\pm}\tilde{E}^{\hat{\beta}+}\tilde{E}^{\hat{\beta}-}\pm\tilde{E}^a\tilde{E}^{\hat{\alpha}\pm}\tilde{B}-\tilde{E}^a(\tilde{E}^{\hat{\alpha}\mp}\tilde{E}^{\pm\pm}\mp\tilde{E}^{\hat{\alpha}\pm}\tilde{E}^0)=0\ , \tag{2.8b}$$

$$\tilde{E}^{\pm\pm}\tilde{d}\tilde{E}^{\pm\pm}=0\ , \tag{2.8c}$$

$$4\tilde{E}^0\tilde{d}\tilde{E}^0+\tilde{E}^{--}\tilde{d}\tilde{E}^{++}+\tilde{E}^{++}\tilde{d}\tilde{E}^{--}=0\ , \tag{2.8d}$$

$$\tilde{E}^{\bar{a}}\tilde{d}\tilde{E}^{\bar{a}}=0\ , \tag{2.8e}$$

with

$$\begin{aligned}&\tilde{d}=d+s\ ,\\ &\tilde{E}^A=E^A+i_\xi E^A\ ,\\ &\tilde{\Omega}^{R,L}=\Omega^{R,L}+C^{R,L}\ ,\\ &\tilde{B}=B+b\ ,\end{aligned} \tag{2.9}$$

and where we have combined the generalized equations of (2.7) in order to eliminate the connections and superfield strengths. i_ξ is the contraction operator along the ghost vector $\xi^M=(\xi^m,\xi^{\bar{m}},\xi^{\hat{\mu}\pm},\xi^{\pm\pm})$ of the superdiffeomorphism transformations where $\xi^m,\xi^{\bar{m}},\xi^{\hat{\mu}+},\xi^{++}$ satisfy the harmonic analyticity condition ($D_{\hat{\alpha}}^{+}\xi^M=0$) and $\xi^{\hat{\mu}-}$ is unconstrained [12]. $C^{R,L}$ and b are the ghost superfields for the right, left, and U(1) transformations.

On the other hand, the horizontality constraints (2.8) remain invariant under the redefinitions

$$\begin{aligned}&\tilde{M}^a=\tilde{E}^a\Lambda\ ,\\ &\tilde{M}^{\hat{\alpha}\pm}=\tilde{E}^{\hat{\alpha}\pm}\Lambda^{1/2}+\tilde{E}^a\Lambda_a^{\hat{\alpha}\pm}\ ,\\ &\tilde{M}^{\pm\pm,0}=\tilde{E}^{\pm\pm,0}\ ,\\ &\tilde{W}=\tilde{B}+\Lambda^{-1/2}(\tilde{E}^{\hat{\alpha}+}\Lambda_a^{\hat{\alpha}-}-\tilde{E}^{\hat{\alpha}-}\Lambda_a^{\hat{\alpha}+})+\tilde{E}^a\Lambda_a^0\ ,\\ &\tilde{M}^{\bar{a}}=\tilde{E}^{\bar{a}}\Lambda'\ ,\end{aligned} \tag{2.10}$$

where Λ, Λ', and $\Lambda_a^{\hat{\alpha}\pm}$ are independent local superparameters. We note that these redefinitions are equivalent to those given by Thümmel [15] for the $N=4$ supergravity theory in ordinary superspace. Consequently, in terms of $\tilde{M}^A$ and $\tilde{W}$ variables, the horizontality constraints (2.8) on the (4,0) super-Beltrami variables become

$$\begin{aligned}
&\tilde{M}^a\tilde{d}\tilde{M}^a-2\tilde{M}^a\tilde{M}^{\hat{\alpha}+}\tilde{M}^{\hat{\alpha}-}=0\ ,\\
&\tilde{M}^a(\tilde{d}\pm\tilde{W})\tilde{M}^{\hat{\alpha}\pm}+\tfrac{1}{2}\tilde{M}^{\hat{\alpha}\pm}\tilde{d}\tilde{M}^a-\tilde{M}^{\hat{\alpha}\pm}\tilde{M}^{\hat{\beta}+}\tilde{M}^{\hat{\beta}-}\\
&\qquad-\tilde{M}^a(\tilde{M}^{\hat{\alpha}\mp}\tilde{M}^{\pm\pm}\mp\tilde{M}^{\hat{\alpha}\pm}\tilde{M}^0)=0\ ,\\
&\tilde{M}^{\bar{a}}\tilde{d}\tilde{M}^{\bar{a}}=0\ , \qquad (2.11)\\
&\tilde{M}^{\pm\pm}\tilde{d}\tilde{M}^{\pm\pm}=0\ ,\\
&4\tilde{M}^0\tilde{d}\tilde{M}^0+\tilde{M}^{--}\tilde{d}\tilde{M}^{++}+\tilde{M}^{++}\tilde{d}\tilde{M}^{--}=0\ ,
\end{aligned}$$

with

$$\begin{aligned}
&\tilde{M}^A=M^A+i_\xi M^A=M^A+\Sigma^A\ ,\\
&\tilde{W}=W+\eta\ ,\\
&i_\xi M^0=0\ ,\\
&M^0=d\eta\ .
\end{aligned}$$

The super-Beltrami differentials M^A are invariant under Lorentz and super-Weyl symmetries. Their BRST transformations corresponding to (4,0) superdiffeomorphisms can be derived from the horizontal conditions (2.11) (see Appendix B). The last constraint of (2.11) expresses the dependence of the harmonic super-Beltrami differentials M^{++} and M^{--}. Furthermore, it is known that the $N=4$ SU(2) conformal structure can be solved by the harmonic analyticity condition which leads to the formulation of the superconformal field theory on the analytic subspace [13]. In order to establish this result at the level of the ghost superfields $\Sigma^{\bar{a}}$, $\Sigma^{\hat{\alpha}+}$, and $\Sigma^{\pm\pm}$ we make the choices

$$\begin{aligned}
&M_{\hat{\beta}}{}^{+\hat{\alpha}+}=0=M_{\hat{\beta}}^{-\hat{\alpha}-}\ ,\\
&M_{\hat{\beta}}{}^{+\hat{\alpha}-}=\delta_{\hat{\beta}}^{\hat{\alpha}}=M_{\hat{\beta}}^{-\hat{\alpha}+}\ ,\\
&M_{\hat{\beta}}{}^{\pm a}=0=M_{\hat{\beta}}{}^{+\bar{a}}\ ,\\
&M_{\hat{\beta}}{}^{+++}=0=M_{\hat{\beta}}{}^{+--}\ ,
\end{aligned}\qquad (2.12)$$

leading to

$$D_{\hat{\beta}}^+\Sigma^{\hat{\alpha}+}=0=D_{\hat{\beta}}^+\Sigma^{\pm\pm}\ , \qquad (2.13a)$$

$$D_{\hat{\beta}}^+\Sigma^{\bar{a}}=0\ , \qquad (2.13b)$$

$$D_{\hat{\beta}}^{\pm}\Sigma^{a}=2\Sigma^{\hat{\beta}\pm}\ . \qquad (2.13c)$$

On the other hand, the degrees of freedom carried by $\Sigma^{\pm\pm}$ can be fixed by the harmonic constraint equations

$$D^{++}\Sigma^{++}=0=D^{--}\Sigma^{--} \qquad (2.14)$$

which are obtained by setting $M^{4+}=0=M^{4-}$. The ghost number zero equations of (2.11) show that the super-Beltrami variables are not all independent. For instance, the first equation of (2.11) by the use of the choices (2.12) allows one to obtain

$$D_{\hat{\alpha}}^{\pm}M_{\bar{a}}^{a}=2M_{\bar{a}}^{\hat{\alpha}\pm}\ , \qquad (2.15a)$$

$$D^{\pm\pm}M_{\bar{a}}^{a}+(\partial_{\bar{a}}-M_{\bar{a}}^{a}\partial_a-\partial_aM_{\bar{a}}^{a})M^{\pm\pm a}-M^{\pm\pm\hat{\alpha}+}D_{\hat{\alpha}}^{-}M_{\bar{a}}^{a}-M^{\pm\pm\hat{\alpha}-}D_{\hat{\alpha}}^{+}M_{\bar{a}}^{a}=0\ . \qquad (2.15b)$$

Moreover, the second and third equations in Eq. (2.11) with (2.12)–(2.14) and the convenient choices

$$M^{0\hat{\alpha}\pm}=\theta^{\hat{\alpha}\pm} \qquad (2.16)$$

lead to

$$\Sigma^{\pm\pm}=D^{\pm\pm}\partial_a\Sigma^a\ . \qquad (2.17)$$

The horizontality conditions (2.11) can be rewritten under other compact forms which are useful for the knowledge of the BRST operator action on the superconformal factors: namely,

$$\begin{aligned}
&\tilde{d}\tilde{M}^a+\tilde{M}^a\tilde{\Gamma}_a^a-2\tilde{M}^{\hat{\alpha}+}\tilde{M}^{\hat{\alpha}-}=0\ ,\\
&(\tilde{d}\pm\tilde{W})\tilde{M}^{\hat{\alpha}\pm}+\tfrac{1}{2}\tilde{M}^{\hat{\alpha}\pm}\tilde{\Gamma}_a^a+\tilde{M}^a\tilde{\Gamma}_a^{\hat{\alpha}\pm}\\
&\qquad-\tilde{M}^{\hat{\alpha}\mp}\tilde{M}^{\pm\pm}\pm\tilde{M}^{\hat{\alpha}\pm}\tilde{M}^0=0\ , \qquad (2.18)\\
&\tilde{d}\tilde{M}^{\pm\pm}+\tilde{M}^{\pm\pm}\tilde{\Gamma}^{\mp\mp\pm\pm}=0\ ,\\
&\tilde{d}\tilde{M}^{\bar{a}}+\tilde{M}^{\bar{a}}\tilde{\Gamma}_{\bar{a}}^{\bar{a}}=0\ ,
\end{aligned}$$

where

$$\begin{aligned}
&\tilde{\Gamma}_a^a=-E^{(0)A}\partial_aM_A^a-\partial_a\Sigma^a\ ,\\
&\tilde{\Gamma}_a^{\hat{\alpha}\pm}=-E^{(0)A}\partial_aM_A^{\hat{\alpha}\pm}-\partial_a\Sigma^{\hat{\alpha}\pm}\ ,\\
&\tilde{\Gamma}^{\pm\pm\mp\mp}=-E^{(0)A}D^{\pm\pm}M_A^{\mp\mp}-D^{\pm\pm}\Sigma^{\mp\mp}\ ,\\
&\tilde{\Gamma}_{\bar{a}}^{\bar{a}}=-E^{(0)A}\partial_{\bar{a}}M_A^{\bar{a}}-\partial_{\bar{a}}\Sigma^{\bar{a}}\ .
\end{aligned}\qquad (2.19)$$

The BRST transformations of the superconformal factors can be obtained by combining (2.18), (2.8), and the decomposition (C1)–(C6). This leads, in a first way, to

$$\tilde{\Omega}^R+\tilde{d}\,\ln\Delta_a^a-\tilde{\Gamma}_a^a=\tilde{M}^a\chi_a+2\left[\tilde{M}^{\hat{\alpha}+}\lambda_a^{\hat{\alpha}-}+\tilde{M}^{\hat{\alpha}-}\lambda_a^{\hat{\alpha}+}\right]\ , \qquad (2.20)$$

where χ_a is a function of the allowed superfields which can be obtained from the ghost number zero equation of (2.20) as

$$\Omega_a^R+\partial_a\ln\Delta_a^a=\chi_a\ , \qquad (2.21a)$$

$$\Omega_{\bar{a}}^R+\partial_{\bar{a}}\ln\Delta_a^a+\partial_aM_{\bar{a}}^a=M_{\bar{a}}^a\chi_a+2\left[M_{\bar{a}}^{\hat{\alpha}+}\lambda_a^{\hat{\alpha}-}+M_{\bar{a}}^{\hat{\alpha}-}\lambda_a^{\hat{\alpha}+}\right]\ , \qquad (2.21b)$$

$$\Omega_{\hat{\alpha}}^{R\pm}+D_{\hat{\alpha}}^{\pm}\ln\Delta_a^a=2\lambda_a^{\hat{\alpha}\pm}\ , \qquad (2.21c)$$

$$D^{\pm\pm}\ln\Delta_a^a+\partial_aM^{\pm\pm a}=M^{\pm\pm a}\chi_a+2\left[M^{\pm\pm\hat{\alpha}+}\lambda_a^{\hat{\alpha}-}+M^{\pm\pm\hat{\alpha}-}\lambda_a^{\hat{\alpha}+}\right]\ . \qquad (2.21d)$$

We note that the Eqs. (2.21a)–(2.21c) are already given in Ref. [15] for $N=4$ supergravity with SU(2)×SU(2) symmetry. In our formalism we see that aside from (2.21a)–(2.21c) additional equations (2.21d) appear showing the dependence of the superconformal factors in harmonic variables. Thereafter, the ghost number one equation of (2.20) gives the BRST transformation of the superconformal factor Δ_a^a:

$$s\ln\Delta_a^a=-C^R-\partial_a\Sigma^a+\Sigma^a\chi_a+2(\Sigma^{\hat{a}+}\lambda_a^{\hat{a}-}+\Sigma^{\hat{a}-}\lambda_a^{\hat{a}+}) . \tag{2.22}$$

On the other hand, we obtain in a second way,

$$\tilde{\Omega}^L+\tilde{d}\ln\Delta_{\bar{a}}^{\bar{a}}-\tilde{\Gamma}_{\bar{a}}^{\bar{a}}=\tilde{M}^{\bar{a}}\chi_{\bar{a}} , \tag{2.23}$$

which leads in the same way to

$$\begin{aligned}&\Omega_{\bar{a}}^L+\partial_{\bar{a}}\ln\Delta_{\bar{a}}^{\bar{a}}=\chi_{\bar{a}} ,\\&\Omega_a^L+\partial_a\ln\Delta_{\bar{a}}^{\bar{a}}+\partial_{\bar{a}}M_a^{\bar{a}}=M_a^{\bar{a}}\chi_{\bar{a}} ,\\&\Omega_{\hat{\beta}}^{\pm L}+D_{\hat{\beta}}^{\pm}\ln\Delta_{\bar{a}}^{\bar{a}}+\partial_{\bar{a}}M_{\hat{\beta}}^{\pm\bar{a}}=M_{\hat{\beta}}^{\pm\bar{a}}\chi_{\bar{a}} ,\\&D^{\pm\pm}\ln\Delta_{\bar{a}}^{\bar{a}}+\partial_{\bar{a}}M^{\pm\pm\bar{a}}=M^{\pm\pm\bar{a}}\chi_{\bar{a}} ,\end{aligned} \tag{2.24}$$

and to the BRST transformation of the superconformal factor $\Delta_{\bar{a}}^{\bar{a}}$: namely,

$$s\ln\Delta_{\bar{a}}^{\bar{a}}=-C^L-\partial_{\bar{a}}\Sigma^{\bar{a}}+\Sigma^{\bar{a}}\chi_{\bar{a}} . \tag{2.25}$$

Finally, Eqs. (2.8b) and (2.8c) with the use of Eqs. (2.7b) and (2.7c) allow us to obtain the equations

$$\begin{aligned}(\tilde{d}\pm\tilde{W})\lambda_a^{\hat{a}\pm}-\tfrac{1}{2}\lambda_a^{\hat{a}\pm}\tilde{\Gamma}_a^a+\tilde{\Gamma}_a^{\hat{a}\pm}&\\=-\tilde{M}^aG_{aa}^{\hat{a}\pm}+\tfrac{1}{2}\tilde{M}^{\hat{a}\pm}\chi_a+\lambda_a^{\hat{a}\mp}\tilde{M}^{\pm\pm}&\\-\tfrac{1}{2}\lambda_a^{\hat{a}\pm}(\tilde{M}^a\chi_a+2\tilde{M}^0)&\\-\tilde{M}^{\bar{a}}S^{\hat{a}\pm}-2\lambda_a^{\hat{a}\pm}\lambda_a^{\hat{\beta}\mp}\tilde{M}^{\hat{\beta}\pm} ,&\end{aligned} \tag{2.26}$$

with

$$\begin{aligned}&G_{aa}^{\hat{a}\pm}=-\partial_a\lambda_a^{\hat{a}\pm}-\tfrac{1}{2}\lambda_a^{\hat{a}\pm}\chi_a-M_a^{\bar{a}}S^{\hat{a}\pm} ,\\&S^{\hat{a}\pm}=(\Delta_a^a)^{1/2}\Delta_{\bar{a}}^{\bar{a}}X^{\hat{a}\pm} ,\end{aligned} \tag{2.27}$$

giving at ghost number one the BRST transformations of the spinorial superfields $\lambda_a^{\hat{a}\pm}$: namely,

$$\begin{aligned}S\lambda_a^{\hat{a}\pm}=\partial_a\Sigma^{\hat{a}\pm}&-\tfrac{1}{2}\lambda_a^{\hat{a}\pm}(\partial_a\Sigma^a+\Sigma^a\chi_a)+\lambda_a^{\hat{a}\mp}\Sigma^{\pm\pm}+\tfrac{1}{2}\Sigma^{\hat{a}\pm}\chi_a\\&\mp\eta\lambda_a^{\hat{a}\pm}-2\lambda_a^{\hat{a}\pm}\lambda_a^{\hat{\beta}\mp}\Sigma^{\hat{\beta}\pm}-\Sigma^{\bar{a}}S^{\hat{a}\pm}\\&+\Sigma^a(\partial_a\lambda_a^{\hat{a}\pm}+\tfrac{1}{2}\lambda_a^{\hat{a}\pm}\chi_a+M_a^{\bar{a}}S^{\hat{a}\pm}) .\end{aligned} \tag{2.28}$$

We note that in these developments we have used the decomposition (C1) where the matrix Λ in (C4) is rewritten as

$$\Lambda_C^A=\lambda_C^Df_D^A \tag{2.29}$$

with f_D^A the superconformal factors given by

$$\begin{aligned}&f_a^a=\Delta_a^a,\quad f_{\bar{a}}^{\bar{a}}=\Delta_{\bar{a}}^{\bar{a}} ,\\&f_{\hat{a}}^{+\hat{\beta}-}=\sqrt{\Delta_a^a}\,\delta_{\hat{a}}^{\hat{\beta}}=f_{\hat{a}}^{-\hat{\beta}+} ,\\&f^{\pm\pm\mp\mp}=1=f^{00} .\end{aligned} \tag{2.30}$$

What remains is the BRST transformation of the U(1) Kac-Moody connection W. For that purpose we use the horizontal constraint imposed on the gauge superfield B: namely,

$$\tilde{d}\tilde{B}=dB=\tilde{d}(\tilde{W}-\tilde{M}^{\hat{a}-}\lambda_a^{\hat{a}+}+\tilde{M}^{\hat{a}+}\lambda_a^{\hat{a}-}) . \tag{2.31}$$

The formalism developed earlier allows us to obtain the constraint

$$\tilde{d}\tilde{W}+\tilde{M}^{\hat{a}-}\Gamma_a^{\hat{a}+}-\tilde{M}^{\hat{a}+}\tilde{\Gamma}_a^{\hat{a}-}+\tilde{M}^a\tilde{H}_a=0 , \tag{2.32}$$

with

$$\tilde{H}_a=-\partial_aW_-\partial_a\eta .$$

Consequently,

$$\begin{aligned}sW=-d\eta+\Sigma^{\hat{a}-}\partial_aM^{\hat{a}+}+M^{\hat{a}-}\partial_a\Sigma^{\hat{a}+}-\Sigma^{\hat{a}+}\partial_aM^{\hat{a}-}&\\-M^{\hat{a}+}\partial_a\Sigma^{\hat{a}-}+\Sigma^a\partial_aW+M^a\partial_a\eta&\end{aligned} \tag{2.33}$$

and

$$s\eta=\Sigma^{\hat{a}-}\partial_a\Sigma^{\hat{a}+}-\Sigma^{\hat{a}+}\partial_a\Sigma^{\hat{a}-}+\Sigma^a\partial_a\eta . \tag{2.34}$$

Furthermore, the information contained in the extended equations of (2.7f) with the decomposition (C1)–(C6) lead to the equations

$$\tilde{W}\pm\tfrac{1}{2}\tilde{\Gamma}^{\pm\pm\mp\mp}-(\tilde{M}^{\hat{a}-}\lambda_a^{\hat{a}+}-\tilde{M}^{\hat{a}+}\lambda_a^{\hat{a}-})+\tilde{M}^0=\tilde{M}^{\mp\mp}\chi^{\pm\pm} \tag{2.35}$$

and the resulting equation

$$\tfrac{1}{2}(\tilde{\Gamma}^{++--}+\tilde{\Gamma}^{--++})=\tilde{M}^{--}\chi^{++}-\tilde{M}^{++}\chi^{--} . \tag{2.36}$$

This latter equation and the choices (2.14) allow us to fix the values of the functions $\chi^{\pm\pm}$ to zero. Therefore, the ghost number one equations of (2.35) and (2.36) imply

$$\begin{aligned}&\eta-\tfrac{1}{2}D^{++}\Sigma^{--}-(\Sigma^{\hat{a}-}\lambda_a^{\hat{a}+}-\Sigma^{\hat{a}+}\lambda_a^{\hat{a}-})=0 ,\\&D^{++}\Sigma^{--}+D^{--}\Sigma^{++}=0 ,\end{aligned}$$

where we have taken (see Appendix B)

$$\begin{aligned}&M^{++--}=1=M^{--++} ,\\&M^0=d\eta^0 .\end{aligned}$$

III. (4,0) SUPER-STRESS-ENERGY TENSORS

In this section we will examine if it is possible to describe the (4,0) locally supersymmetric matter action in the super-Beltrami parametrization. For this reason we consider a matter superfield φ^q, carrying a U(1) charge q and of weights (r,l) with respect to right and left transformations in the (4,0) super-Beltrami parametrization, and we try to define the (4,0) superconformally covariant derivatives. However, from the horizontal condition

$$(\tilde{d}+r\,\tilde{\Omega}^R+l\tilde{\Omega}^L+q\tilde{B})\varphi^q=\tilde{E}^A\nabla_A\varphi^q \tag{3.1}$$

we deduce the BRST transformation of the matter superfield φ^q: namely,

$$s\varphi^q=\Sigma^A\nabla_A\varphi^q-rC^R\varphi^q-lC^L\varphi^q-qb\varphi^q \ . \tag{3.2}$$

In order to obtain (4,0) superconformally invariant theories, one has to express the covariant derivatives in terms of the super-Beltrami variables. Thereby, we redefine the φ^q superfield by

$$\varphi^q=\phi^q(\Delta_a^a)^r(\Delta_{\bar a}^{\bar a})^l \ , \tag{3.3}$$

where ϕ^q is a superfield of U(1) charge q, carrying zero weight, and is inert under Lorentz and super-Weyl transformations. We note that if the superfield φ^q is defined in the analytic subspace, i.e., $\nabla_{\hat a}^+\varphi^q=0$, then the second member of (3.3) must be analytic. This fact restricts the local superparameters Λ and Λ' and consequently the superconformal factors Δ_a^a and $\Delta_{\bar a}^{\bar a}$ to be analytic. However, with the change (3.3) the horizontality condition (3.1) becomes

$$\begin{aligned}\tilde d\phi^q+(r\tilde\Gamma_a^a+l\tilde\Gamma_{\bar a}^{\bar a}+q\tilde W)\phi^q
&=\tilde M^a(\mathcal{D}_a+\lambda_a^{\hat a+}\mathcal{D}_{\hat a}^-+\lambda_a^{\hat a-}\mathcal{D}_{\hat a}^+-r\chi_a)\phi^q+\tilde M^{\hat a+}[\mathcal{D}_{\hat a}^--(2r+q)\lambda_a^{\hat a-}]\phi^q+\tilde M^{\hat a-}[\mathcal{D}_{\hat a}^+-(2r-q)\lambda_a^{\hat a+}]\phi^q\\
&\quad+\tilde M^{++}\mathcal{D}^{--}\phi^q+\tilde M^{--}\mathcal{D}^{++}\phi^q+\tilde M^0\mathcal{D}^0\phi^q+\tilde M^{\bar a}(\mathcal{D}_{\bar a}-l\chi_{\bar a})\phi^q \ ,\end{aligned} \tag{3.4}$$

where

$$\begin{aligned}&\mathcal{D}_a\phi^q=(\Delta_a^a)^{1-r}(\Delta_{\bar a}^{\bar a})^{-l}\nabla_a\varphi^q \ ,\\
&\mathcal{D}_{\bar a}\phi^q=(\Delta_a^a)^{-r}(\Delta_{\bar a}^{\bar a})^{1-l}\nabla_{\bar a}\varphi^q \ ,\\
&\mathcal{D}_{\hat a}^{\pm}\phi^q=(\Delta_a^a)^{1/2-r}(\Delta_{\bar a}^{\bar a})^{-l}\nabla_{\hat a}^{\pm}\varphi^q \ ,\\
&\mathcal{D}^{\pm\pm}\phi^q=(\Delta_a^a)^{-r}(\Delta_{\bar a}^{\bar a})^{-l}\nabla^{\pm\pm}\varphi^q \ ,\\
&\mathcal{D}^0\phi^q=(\Delta_a^a)^{-r}(\Delta_{\bar a}^{\bar a})^{-l}\nabla^0\varphi^q\end{aligned} \tag{3.5}$$

are the (4,0) superconformally covariant derivatives of ϕ^q. Thereafter, Eq. (3.4) at ghost number zero allows us to express the (4,0) superconformally covariant derivatives as

$$\begin{aligned}&\mathcal{D}_a\phi^q=(L_a+r\chi_a)\phi^q \ ,\\
&\mathcal{D}_{\bar a}\phi^q=(L_{\bar a}+l\chi_{\bar a})\phi^q \ ,\\
&\mathcal{D}_{\hat a}^-\phi^q=[L_{\hat a}^-+2(r+q/2)\lambda_a^{\hat a-}]\phi^q \ ,\\
&\mathcal{D}^{\pm\pm}\phi^q=L^{\pm\pm}\phi^q \ ,\\
&\mathcal{D}^0\phi^q=(L^0-M^{0\hat a+}L_{\hat a}^--M^{0\hat a-}L_{\hat a}^+)\phi^q \ ,\\
&\mathcal{D}_{\hat a}^+\phi^q=(L_{\hat a}^++2r\lambda_a^{\hat a+})\phi^q\equiv(D_{\hat a}^++2r\lambda_a^{\hat a+})\phi^q \ .\end{aligned} \tag{3.6}$$

We note that the covariant superconformally analytic condition is then given by

$$\mathcal{D}_{\hat a}^+\phi^q=0 \tag{3.7}$$

and the derivation of the operators L_a, $L_{\bar a}$, $L_{\hat a}^-$, and $L^{\pm\pm}$ can be simplified (see Appendix D) if we take the following restriction of the geometry:

$$M^{--\hat a+}=0=M^{--a} \ , \tag{3.8a}$$

$$M^{++a}=0=M^{++\hat a+} \ . \tag{3.8b}$$

The ghost number one equation of (3.4) gives the BRST transformation of the conformal superfield ϕ^q, which we assume to be analytic: namely,

$$\begin{aligned}s\phi^q=&\Sigma^a\partial_a\phi^q+\Sigma^{\bar a}\partial_{\bar a}\phi^q+(\Sigma^a\lambda_a^{\hat a+}+\Sigma^{\hat a+})L_{\hat a}^-\phi^q\\
&+(\Sigma^{++}L^{--}+\Sigma^{--}L^{++})\phi^q-(2r-q)\Sigma^{\hat a-}\lambda_a^{\hat a+}\phi^q\\
&+\phi^q(r\partial_a\Sigma^a+l\partial_{\bar a}\Sigma^{\bar a}-q\eta) \ .\end{aligned} \tag{3.9}$$

Finally, let us recall that the free two-dimensional (4,0) supersymmetric matter action can be written in the full (4,0) harmonic superspace [16] as

$$I=\frac{1}{2}\int dz_{aa}\left[\overset{*}{\varphi}{}^+\frac{D^{--}(\partial_{\bar a})^2}{\Box}\varphi^+\right] \tag{3.10}$$

with

$$dz_{aa}=d^2x\,d^2\theta_-^+d^2\theta_-^-dU$$

and φ^+ is the analytic complex matter hypermultiplet of weight (0,0). The (4,0) locally supersymmetric matter action can be obtained from (3.10) by covariantizing the derivatives and integrating over the invariant measure $dz_{aa}E^{-1}$:

$$I=\frac{1}{2}\int dz_{aa}E^{-1}\left[\overset{*}{\varphi}{}^+\frac{\nabla^{--}(\nabla_{\bar a})^2}{\Box}\varphi^+\right] \ . \tag{3.11}$$

This latter can be expressed in terms of the superconformally covariant derivatives. In fact, with the choice (3.8) and those fixed in the previous section, we have

$$E^{-1}=\mathrm{s\,det}E_M^A=\Delta(\Delta_{\bar a}^{\bar a})$$

with

$$\begin{aligned}\Delta=1&-M_{\bar a}^aM_a^{\bar a}-M_{\bar a}^{\hat a+}M_{\hat a}^{-\bar a}\\
&-(M_{\bar a}^{++}-M_{\bar a}^{\hat a+}M_{\hat a}^{-++})M^{--\bar a}\\
&-(M_{\bar a}^{--}-M_{\bar a}^{\hat a+}M_{\hat a}^{---})M^{++\bar a}\end{aligned}$$

and the redefinitions (3.3) and (3.4) allow us to obtain the action

$$I=\frac{1}{2}\int dz_{aa}\Delta[\overset{*}{\Phi}{}^+\mathcal{D}^{--}(\mathcal{D}_{\bar a})^2\Phi^+ +\text{terms in }\Phi^+\text{ and }\ln\Delta_{\bar a}^{\bar a}] \ , \tag{3.12}$$

where

$$\Phi^+=\frac{1}{\Delta_{\bar{a}}^{\bar{a}}(\Box)^{1/2}}\phi^+$$

is a nonanalytic superfield. Therefore, the classical super-stress-energy tensors are obtained by differentiating (3.12) with respect to the super-Beltrami variables. The couplings of the latter variables, which are assumed to be nonanalytic, to the nonanalytic super-stress-energy tensors deduced from (3.12) without fixing the choice (3.8a) are given by

$$\begin{aligned}I=\int dZ_{aa}(&M^{--\bar{a}}J^{++}_{\bar{a}\bar{a}\bar{a}}+M^{--a}J^{++}_{\bar{a}}+M^{--\hat{a}+}J^{+}_{\bar{a}\bar{a}\hat{a}}\\&+M^{--\hat{a}-}J^{+++}_{\bar{a}\bar{a}\hat{a}}+M^{a}_{\bar{a}}J+M^{\hat{a}+}_{\bar{a}}J^{-}_{\bar{a}\hat{a}}+M^{\hat{a}-}_{\bar{a}}J^{+}_{\bar{a}\hat{a}}\\&+M^{++}_{\bar{a}}K^{--}_{\bar{a}}+M^{--}_{\bar{a}}K^{++}_{\bar{a}})\ ,\end{aligned}\tag{3.13}$$

with

$$\begin{aligned}J^{++}_{\bar{a}\bar{a}\bar{a}}&=-\tfrac{1}{2}\overset{*}{\Phi}{}^{+}(\partial_{\bar{a}})^3\Phi^+\ ,\\J^{++}_{\bar{a}}&=-\tfrac{1}{2}\overset{*}{\Phi}{}^{+}\partial_a(\partial_{\bar{a}})^2\Phi^+\ ,\\J^{+++}_{\bar{a}\bar{a}\hat{a}}&=-\tfrac{1}{2}\overset{*}{\Phi}{}^{+}D^{+}_{\hat{a}}(\partial_{\bar{a}})^2\Phi^+\ ,\\J^{+}_{\bar{a}\bar{a}\hat{a}}&=-\tfrac{1}{2}\overset{*}{\Phi}{}^{+}D^{-}_{\hat{a}}(\partial_{\bar{a}})^2\Phi^+\ ,\\J&=-\tfrac{1}{2}\overset{*}{\Phi}{}^{+}D^{--}\partial_a\partial_{\bar{a}}\Phi^+\ ,\\J^{-}_{\bar{a}\hat{a}}&=-\tfrac{1}{2}\overset{*}{\Phi}{}^{+}D^{--}D^{-}_{\hat{a}}\partial_{\bar{a}}\Phi^+\ ,\\J^{+}_{\bar{a}\hat{a}}&=-\tfrac{1}{2}\overset{*}{\Phi}{}^{+}D^{--}D^{+}_{\hat{a}}\partial_{\bar{a}}\Phi^+\ ,\\K^{--}_{\bar{a}}&=-\tfrac{1}{2}\overset{*}{\Phi}{}^{+}(D^{--})^2\partial_{\bar{a}}\Phi^+\ ,\\K^{++}_{\bar{a}}&=-\tfrac{1}{2}\overset{*}{\Phi}{}^{+}D^{--}D^{++}\partial_{\bar{a}}\Phi^+\ ,\end{aligned}\tag{3.14}$$

where $(J^{++}_{\bar{a}\bar{a}\bar{a}},J^{++}_{\bar{a}})$, $(J^{+++}_{\bar{a}\bar{a}\hat{a}},J^{+}_{\bar{a}\bar{a}\hat{a}})$, J, $J^{\pm}_{\bar{a}\hat{a}}$, and $K^{\pm\pm}_{\bar{a}}$ are of dimensions $+1$, $+\frac{1}{2}$, 0, $-\frac{1}{2}$, and -1, respectively. The analytic super-stress-energy tensors are obtained from (3.14) by applying the $(D^{+}_{\hat{a}})^2$ derivatives: namely,

$$\begin{aligned}&T^{4+}_{\bar{a}\bar{a}}=(D^{+}_{\hat{a}})^2J^{++}_{\bar{a}\bar{a}\bar{a}},\quad T^{5+}_{\bar{a}\hat{a}}=(D^{+}_{\hat{a}})^2J^{3+}_{\bar{a}\bar{a}\hat{a}}\ ,\\&T^{4+}=(D^{+}_{\hat{a}})^2J^{++}_{\bar{a}},\quad T^{3+}_{\bar{a}\hat{a}}=(D^{+}_{\hat{a}})^2J^{+}_{\bar{a}\bar{a}\hat{a}}\ ,\\&T^{++}_{a}=(D^{+}_{\hat{a}})^2J\ ,\\&T^{+}_{\hat{a}}=(D^{+}_{\hat{a}})^2J^{-}_{\bar{a}\hat{a}},\quad N=(D^{+}_{\hat{a}})^2K^{--}_{\bar{a}}\ ,\\&T^{3+}_{\hat{a}}=(D^{+}_{\hat{a}})^2J^{+}_{\bar{a}\hat{a}},\quad N^{4+}=(D^{+}_{\hat{a}})^2K^{++}_{\bar{a}}\ ,\end{aligned}\tag{3.15}$$

where $(T^{4+}_{\bar{a}\bar{a}},T^{4+})$, $(T^{5+}_{\bar{a}\hat{a}},T^{3+}_{\bar{a}\hat{a}})$, T, $(T^{+}_{\hat{a}},T^{3+}_{\hat{a}})$, and (N,N^{4+}) are of dimensions $+2$, $+\frac{3}{2}$, $+1$, $+\frac{1}{2}$, and 0, respectively.

Note that other analytic super-stress-energy tensors with the same dimensions can be obtained from (3.12) but they are dependent on the super-Beltrami variables. Then when we differentiate with respect to the super-Beltrami variables and set it equal to zero as in (3.14) all these super-stress-energy tensors cancel. As is already known, the superconformal properties of a general theory are determined by properties of the super-stress-energy tensors. These tensors have to be completed by the contribution from the ghost superfields which appear in the gauge fixing of the world-sheet variables. Moreover, the properties of the (4,0) supersymmetric BRST current algebra may also be studied by localizing the BRST differential algebra. On the other hand, with the use of these techniques, the derivation of the (4,0) superdiffeomorphism anomaly and by the following the obtention of the Wess-Zumino action associated with the (4,0) superdiffeomorphism group can be achieved in the same way as the (4,4) superconformal case [17].

IV. (4,0) SUPERSYMMETRIC OPERATOR-PRODUCT EXPANSION

In this section, we derive the anomalous Ward identities generated by supercoordinate transformations leading to the super-operator-product expansion for (4,0) super-stress-energy tensors. Before we proceed any further, let us recall that the (4,0) superdiffeomorphism anomaly [17] is given in the full (4,0) harmonic superspace by the expression

$$\mathcal{A}^{(4,0)}_{\text{diff}}=\alpha\int dz_{aa}(\Sigma^aD^{++}D^{--}\partial_aM^{a}_{\bar{a}})\ ,\tag{4.1}$$

where $M^{a}_{\bar{a}}$, which is considered to be nonanalytic, contains the Beltrami variable $\mu^{a}_{\bar{a}}$ as a zero θ_- component: namely,

$$M^{a}_{\bar{a}}=\mu^{a}_{\bar{a}}+\theta^{+\hat{a}}V^{-\hat{a}}_{\bar{a}}+\theta^{-\hat{a}}V^{+\hat{a}}_{\bar{a}}+\cdots\ .$$

Therefore, in terms of component fields, the expression (4.1) of the (4,0) superdiffeomorphism anomaly becomes

$$\begin{aligned}\mathcal{A}^{(4,0)}_{\text{diff}}=\alpha\int d^2x\,du(&\Sigma^a\partial^3_a\mu^{a}_{\bar{a}}+2\Sigma^{\hat{a}-}\partial^2_a\partial^{++}V^{-\hat{a}}_{\bar{a}}\\&+2\Sigma^{\hat{a}+}\partial^2_a\partial^{--}V^{+\hat{a}}_{\bar{a}})\ ,\end{aligned}\tag{4.2}$$

where

$$\Sigma^a|_{\theta=0}=\Sigma^a,\quad \Sigma^{\hat{a}\pm}|_{\theta=0}=\Sigma^{\hat{a}\pm}\ .$$

Note that we can add the supersymmetric partners of (4.1): namely,

$$\int dz_{aa}(\Sigma^{\hat{a}+}D^{--}M^{\hat{a}+}_{\bar{a}}+\Sigma^{\hat{a}-}D^{++}M^{\hat{a}-}_{\bar{a}})\ .\tag{4.3}$$

We will see at the end of this section that these terms are necessary in order to recover the full $N=4$ superconformal algebra [18]. But for the moment let us consider the first term (4.1) of the anomaly.

The $N=4$ super-operator-product expansion can be derived by following the procedure used in Ref. [19] for the bosonic theory and extended to the $N=1$ and 2 supersymmetric cases [20,21]. As we have seen, the interesting (4,0) super-Beltrami variable which contains the necessary component fields is $M^{a}_{\bar{a}}$ and its coupling to the corresponding super-stress-energy tensor is given by

$$\int dz_{aa}(M^{a}_{\bar{a}}J)\ .\tag{4.4}$$

Therefore, the Ward-identity operator acting on the generating functional Z_c is given by

$$\mathcal{B}(\zeta\partial)=\int dz_{aa}\left[\delta_\zeta M^{a}_{\bar{a}}\frac{\delta}{\delta M^{a}_{\bar{a}}}+\cdots\right]\ ,\tag{4.5}$$

where the functional derivative of the generating functional Z_c with respect to $M^a_{\bar{a}}$ is given by

$$\left.\frac{\delta Z_c}{\delta M^a_{\bar{a}}}\right|_{M=0}=J \tag{4.6}$$

and $\delta_\zeta M^a_{\bar{a}}$ is nothing but the BRST variation [the first equation in (B4)] where the ghost superfield Σ^a is replaced by the superparameter ζ^a. Therefore, the anomalous Ward identity

$$\mathcal{B}(\zeta\partial)Z_c \alpha \int dz_{aa}(\zeta^a D^{++}D^{--}\partial_a M^a_{\bar{a}}) \tag{4.7}$$

is explicitly given by

$$[\partial_{\bar{a}}-M^a_{\bar{a}}\partial_a-4\partial_a M^a_{\bar{a}}-(D^+_{\hat{a}}M^a_{\bar{a}})D^-_{\hat{a}}-(D^-_{\hat{a}}M^a_{\bar{a}})D^+_{\hat{a}}]\frac{\delta Z_c}{\delta M^a_{\bar{a}}}=kD^{++}D^{--}\partial_a M^a_{\bar{a}}\ , \tag{4.8}$$

where we have used the relations (2.13c) and (2.15a) and k is an arbitrary constant. Now we introduce a Green's function $G_{\bar{a}}(z_1,z_2)$, which satisfies the differential equation

$$\partial_{\bar{a}}G_{\bar{a}}(z_1,z_2)=\pi\delta^6_{\bar{a}\bar{a}}(z_1-z_2)\delta^{(2,-2)}(U_1,U_2)\ , \tag{4.9}$$

where

$$\delta^6_{\bar{a}\bar{a}}(z_1-z_2)=\delta^2(x_1-x_2)\delta^2(\theta^+_{-1}-\theta^+_{-2})\times\delta^2(\theta^-_{-1}-\theta^-_{-2})$$

is the (4,0) δ function [22] and $\delta^{(2,-2)}(U_1,U_2)$ is such that

$$\int dU_2\delta^{(2,-2)}(U_1,U_2)\phi^q(U_2)=\delta^{q2}\phi^q(U_1)\ . \tag{4.10}$$

The solution of (4.9) is then given by

$$G_{\bar{a}}(z_1,z_2)=\frac{(\theta^+_{-12})^2(\theta^-_{-12})^2}{z_{12}}\delta^{(2,-2)}(U_1,U_2) \tag{4.11}$$

with

$$z_{12}=x_{--1}-x_{--2}-\theta^-_{-2}\theta^+_{-1}-\theta^+_{-2}\theta^-_{-1}+\theta^+_{-1}\theta^-_{-1}-\theta^+_{-2}\theta^-_{-2}\ ,$$

$$\theta^\pm_{12}=\theta^\pm_{-1}-\theta^\pm_{-2}\ .$$

Multiplying (4.8) by (4.11) and integrating on the (4,0) harmonic superspace by using Eq. (4.9), the anomalous Ward identity (4.8) is transposed to the final result

$$\pi\frac{\delta Z_c}{\delta M^a_{\bar{a}}(z_2)}=\int dz_{1aa}M^a_{\bar{a}}(z_1)\left|-kD_1^{++}D_1^{--}\partial_a G_{\bar{a}}(z_1,z_2)+[4\partial_a G_{\bar{a}}(z_1,z_2)+5G_{\bar{a}}(z_1,z_2)\partial_a+D^+_{1\hat{a}}G_{\bar{a}}(z_1,z_2)D^-_{1\hat{a}}+D^-_{1\hat{a}}G_{\bar{a}}(z_1,z_2)D^+_{1\hat{a}}]\frac{\delta z_c}{\delta M^a_{\bar{a}}(z_1)}\right|\ , \tag{4.12}$$

which becomes, after a second variation with respect to $M^a_{\bar{a}}$,

$$\left.\pi\frac{\delta^2 Z_c}{\delta M^a_{\bar{a}}(z_1)\delta M^a_{\bar{a}}(z_2)}\right|_{M=0}=-k\partial_a D_1^{--}D_1^{++}G_{\bar{a}}(z_1,z_2)+[4\partial_a G_{\bar{a}}(z_1,z_2)+5G_{\bar{a}}(z_1,z_2)\partial_a+D^+_{1\hat{a}}G_{\bar{a}}(z_1,z_2)D^-_{1\hat{a}}+D^-_{1\hat{a}}G_{\bar{a}}(z_1,z_2)D^+_{1\hat{a}}]\left.\frac{\delta Z_c}{\delta M^a_{\bar{a}}(z_1)}\right|_{M=0}\ . \tag{4.13}$$

Consequently, the use of (4.6) leads to the super operator product expansion of the super-stress-energy tensor J which reads

$$\begin{aligned}J(z_1,U_1)J(z_2,U_2)=&C\frac{(\theta^+_{-12})^2(\theta^-_{-12})^2}{z^2_{12}}D_1^{--}D_1^{++}\delta^{(2,-2)}(U_1,U_2)-4\frac{(\theta^+_{-12})^2(\theta^-_{-12})^2}{z^2_{12}}\delta^{(2,-2)}(U_1,U_2)J(z_2,U_2)\\&+5\frac{(\theta^+_{-12})^2(\theta^-_{-12})^2}{z_{12}}\delta^{(2,-2)}(U_1,U_2)\partial_a J(z_2,U_2)+2\frac{(\theta^-_{-12})(\theta^+_{-12})^2}{z_{12}}\delta^{(2,-2)}(U_1,U_2)D^-_{2\hat{a}}J(z_2,U_2)\\&+2\frac{(\theta^-_{-12})^2(\theta^+_{-12})}{z_{12}}\delta^{(2,-2)}(U_1,U_2)D^+_{2\hat{a}}J(z_2,U_2)\ .\end{aligned} \tag{4.14}$$

The space-time ordinary-product expansions of the different currents constituting the zero dimension super-stress-energy tensor, J, namely,

$$J=j+\theta^{+\hat{a}}j^-_{\hat{a}}+\theta^{-\hat{a}}j^+_{\hat{a}}+\theta^{+\hat{a}}\theta^{+\hat{a}}j_a^{--}+\theta^{-\hat{a}}\theta^{-\hat{a}}j_a^{++}+\theta^{+\hat{a}}\theta^{-\hat{a}}j_a+\theta^{+\hat{a}}\theta^{+\hat{a}}\theta^{-\hat{\beta}}j^-_{a\hat{\beta}}+\theta^{-\hat{a}}\theta^{-\hat{a}}\theta^{+\hat{\beta}}j^+_{a\hat{\beta}}+(\theta^{+\hat{a}})^2(\theta^{-\hat{\beta}})^2 j_{aa}\ ,$$

where the currents j, $j^\pm_{\hat{a}}$, $(j^{\pm\pm}_a, j_a)$, $j^\pm_{a\hat{a}}$, and j_{aa} are of dimensions 0, $\frac{1}{2}$, 1, $\frac{3}{2}$, and 2, respectively, and can be deduced in a straightforward way from (4.14). More specifically, the expression (4.14) contains all the known operator-product expansions of the operators which make up the $N=4$ superconformal algebra given in Refs. [18] with the exception of that having $\frac{1}{2}$ dimensions. However, these can be recovered by adding the expression (4.3) which corresponds to the supersymmetric partners of (4.1). Therefore, the anomalous Ward identity (4.12) is rewritten under a more complete shape: namely,

$$\pi\frac{\delta Z_c}{\delta M^a_{\bar a}(z_2)}=\int dz_{1aa}M^a_{\bar a}(z_1)\Big[-kD_1^{++}D_1^{--}\partial_a G_{\bar a}(z_1,z_2)+\frac{k}{4}D^+_{1\hat a}D_1^{--}D^+_{1\hat a}G_{\bar a}(z_1,z_2)+\frac{k}{4}D^-_{1\hat a}D_1^{++}D^-_{1\hat a}G_{\bar a}(z_1,z_2)$$
$$+[4\partial_a G_{\bar a}(z_1,z_2)+5G_{\bar a}(z_1,z_2)\partial_a+D^+_{1\hat a}G_{\bar a}(z_1,z_2)D^-_{1\hat a}$$
$$+D^-_{1\hat a}G_{\bar a}(z_1,z_2)D^+_{1\hat a}]\frac{\delta Z_c}{\delta M^a_{\bar a}(z_1)}\Big], \tag{4.15}$$

where we have used relations (2.13c) and (2.15a). Therefore, the final expression of the super-operator-product expansion is given by

$$J(z_1,U_1)J(z_2,U_2)=C\Big\{\frac{(\theta^+_{-12})^2(\theta^-_{-12})^2}{z_{12}^2}(D_1^{--}D_1^{++}-1)+\frac{2(\theta^+_{-12})(\theta^-_{-12})-\theta^+_{-2}\theta^-_{-1}+\theta^+_{-1}\theta^-_{-2}}{z_{12}}$$
$$+\frac{1}{2}\frac{(\theta^-_{-12})^2D_1^{++}-(\theta^+_{-12})^2D_1^{--}}{z_{12}}\Big\}\delta^{(2,-2)}(U_1,U_2)$$
$$-4\frac{(\theta^+_{-12})^2(\theta^-_{-12})^2}{z_{12}^2}\delta^{(2,-2)}(U_1,U_2)J(z_2,U_2)+5\frac{(\theta^+_{-12})^2(\theta^-_{-12})^2}{z_{12}}\delta^{(2,-2)}(U_1,U_2)\partial_a J(z_2,U_2)$$
$$+2\frac{(\theta^-_{-12})(\theta^+_{-12})^2}{z_{12}}\delta^{(2,-2)}(U_1,U_2)D^-_{2\hat a}J(z_2,U_2)$$
$$+2\frac{(\theta^-_{-12})^2(\theta^+_{-12})}{z_{12}}\delta^{(2,-2)}(U_1,U_2)D^+_{2\hat a}J(z_2,U_2)\,. \tag{4.16}$$

The expansions in terms of component fields will be presented in more details elsewhere.

V. CONCLUSION

The (4,0) super-Beltrami parametrization has been studied by using the techniques of harmonic superspace. This study has been done by considering the general supercoordinate transformations and the SU(2) symmetry as contained in the superdiffeomorphisms of the analytic superspace. The constraints on the torsions of (4,0) supergravity supplemented by the constraints of the harmonic covariant derivatives which represent a super-Beltrami parametrization in harmonic superspace have been modified by the introduction of the U(1) connection. This is compatible with the analytic line integrals defined globally on super-Riemann surfaces. Such formalism allows us to obtain in addition to the super-Beltrami differentials $(M^a,\Sigma^a),(M^{\hat a\pm},\Sigma^{\hat a\pm})$, the harmonic super-Beltrami differentials $(M^{\pm\pm},\Sigma^{\pm\pm})$. The transformations of (4,0) super-Beltrami variables are obtained by parametrizing the frames with respect to the rigid (4,0) harmonic superspace frames. Some suitable choices for the components of the super-Beltrami differentials have been done in order to recover the harmonic analyticity condition leading to the formulation of the superconformal field theory on the analytic subspace. Furthermore, the frames are rewritten as the product of a super-Beltrami form and a matrix scale factor which can be decomposed in terms of the conformal factors. These latter are shown to be dependent on the harmonic variables. Therefore, the (4,0) locally supersymmetric matter action is constructed in the super-Beltrami parametrization and the super-stress-energy tensors are derived. We note that some of these super-stress-energy tensors are nonlocal and we hope that this nonlocality disappears at the level of field components. On the other hand, the anomalous Ward identity is derived and the super-operator-product expansion of the (4,0) super-stress-energy tensor $J(Z,U)$, which contains all the necessary operators constituting the $N=4$ superconformal algebra [18], is obtained. Finally, we note that our formalism of the super-Beltrami parametrization in two-dimensional (4,0) harmonic superspace is fundamental since it allows us in a more convenient way to know all the $N=4$ super-stress-energy tensors which are characterized by conformal dimensions and U(1) charges. The knowledge of the super-operator-product expansions of the supercurrent $J(Z,U)$ generating the $N=4$ superconformal algebra [18] can be used in order to generate an $N=4$ super W_3 algebra. This subject is under study.

APPENDIX A: CONVENTIONS

A covariant derivative on the harmonic superspace is given by

$$\nabla_M=\partial_M+\Omega_M\mathcal{M}+A_M\mathcal{D}+B_MY \tag{A1}$$

with $M=(m,\bar m,\hat\mu\pm,\pm\pm,0)$. It is convenient to convert the world index of the covariant derivatives into the Lorentz index by using the inverse of the zweibein:

$$\nabla_A = E_A^M \nabla_M = \partial_A + \Omega_A \mathcal{M} + A_A \mathcal{D} + B_A Y \tag{A2}$$

with $A=(a,\bar{a},\hat{\alpha}\pm,\pm\pm,0)$. $\mathcal{M}$ is the Lorentz generator which has the diagonal form

$$\begin{aligned}&[\mathcal{M},\psi_a]=\psi_a\ ,\\&[\mathcal{M},\psi_{\bar{a}}]=\psi_{\bar{a}}\ ,\\&[\mathcal{M},\psi_{\hat{\alpha}}^{\pm}]=\tfrac{1}{2}\psi_{\hat{\alpha}}^{\pm}\ ,\\&[\mathcal{M},\psi^q]=0\ ,\end{aligned}\tag{A3}$$

where q is the U(1) charge. This latter is usually taken in the upper position and Y is the U(1) generator counting the Cartan-Weyl charge:

$$\begin{aligned}&[Y,\psi_{\hat{\alpha}}^{\pm}]=\pm\psi_{\hat{\alpha}}^{\pm}\ ,\\&[Y,\psi^q]=q\psi^q\ .\end{aligned}\tag{A4}$$

$\mathcal{D}$ is the dilatation generator acting on the indices $(a,\bar{a},\hat{\alpha}\pm)$:

$$\begin{aligned}&[\mathcal{D},\psi_a]=\psi_a,\quad [\mathcal{D},\psi_{\bar{a}}]=\psi_{\bar{a}}\ ,\\&[\mathcal{D},\psi_{\hat{\alpha}}^{\pm}]=\tfrac{1}{2}\psi_{\hat{\alpha}}^{\pm},\quad [\mathcal{D},\psi^q]=0\ .\end{aligned}\tag{A5}$$

APPENDIX B: BRST TRANSFORMATIONS

The parametrization of the frames E^A with respect to the rigid (4,0) superspace frames $E^{(0)B}$ is

$$E^A = E^{(0)B}\Delta_B^A\ . \tag{B1}$$

The substitution (2.10) allows us to obtain the local superparameters Λ, $\Lambda_a^{\hat{\alpha}\pm}$, Λ_a^0, and Λ' in terms of the superfields Δ_B^A:

$$\begin{aligned}&\Lambda=(\Delta_a^a)^{-1}\ ,\\&\Lambda_a^{\hat{\alpha}\pm}=-(\Delta_a^a)^{-3/2}\Delta_a^{\hat{\alpha}\pm}\ ,\\&\Lambda_a^0=-2(\Delta_a^a)^{-2}\Delta_a^{\hat{\alpha}-}\Delta_a^{\hat{\alpha}+}\ ,\\&\Lambda'=(\Delta_{\bar{a}}^{\bar{a}})^{-1}\ ,\end{aligned}\tag{B2}$$

with the choices

$$\begin{aligned}&M_a^a=1=M_{\bar{a}}^{\bar{a}},\quad M^{++--}=1=M^{--++}\ ,\\&M_a^{\hat{\alpha}\pm}=0=M_a^{\pm\pm},\quad M^0=d\eta^0\ ,\\&W_a=0=B_a\ ,\end{aligned}\tag{B3}$$

suitable for the derivation of the BRST transformations of the (4,0) super-Beltrami variables and their corresponding ghost superfields. These are obtained by the expansion of the constraints (2.11) at ghost number one and two:

$$\begin{aligned}&SM^a=-(d-M^a\partial_a)\Sigma^a-\Sigma^a\partial_aM^a\\&\qquad+2(M^{\hat{\alpha}+}\Sigma^{\hat{\alpha}-}+M^{\hat{\alpha}-}\Sigma^{\hat{\alpha}+})\ ,\\&SM^{\hat{\alpha}\pm}=-(d\pm W-M^a\partial_a)\Sigma^{\hat{\alpha}\pm}-\Sigma^a\partial_aM^{\hat{\alpha}\pm}+\tfrac{1}{2}M^{\hat{\alpha}\pm}\partial_a\Sigma^a\\&\qquad-\tfrac{1}{2}\Sigma^{\hat{\alpha}\pm}\partial_aM^a+\Sigma^{\hat{\alpha}\mp}M^{\pm\pm}+\Sigma^{\pm\pm}M^{\hat{\alpha}\mp}\mp\eta M^{\hat{\alpha}\pm}\ ,\\&SM^{\pm\pm}=-(d-M^{\pm\pm}D^{\mp\mp})\Sigma^{\pm\pm}-\Sigma^{\pm\pm}D^{\mp\mp}M^{\pm\pm}\ ,\\&SM^{\bar{a}}=-(d-M^{\bar{a}}\partial_{\bar{a}})\Sigma^{\bar{a}}-\Sigma^{\bar{a}}\partial_{\bar{a}}M^{\bar{a}}\ ,\\&S\Sigma^a=-\Sigma^a\partial_a\Sigma^a+2\Sigma^{\hat{\alpha}+}\Sigma^{\hat{\alpha}-}\ ,\\&S\Sigma^{\hat{\alpha}\pm}=-\Sigma^a\partial_a\Sigma^{\hat{\alpha}\pm}-\tfrac{1}{2}\Sigma^{\hat{\alpha}\pm}\partial_a\Sigma^a\\&\qquad+\Sigma^{\hat{\alpha}\mp}\Sigma^{\pm\pm}\mp\eta\Sigma^{\hat{\alpha}\pm}\ ,\\&S\Sigma^{\pm\pm}=-\Sigma^{\pm\pm}D^{\mp\mp}\Sigma^{\pm\pm}\ ,\\&S\Sigma^{\bar{a}}=-\Sigma^{\bar{a}}\partial_{\bar{a}}\Sigma^{\bar{a}}\ .\end{aligned}\tag{B4}$$

APPENDIX C: CONVENIENT DECOMPOSITION

The parametrization (B1), which can be rewritten as

$$E^A=E^{(0)B}M_B^C\Lambda_C^A\ , \tag{C1}$$

allows, with the choices (B3) and (2.12)–(2.14), to identify

$$\begin{aligned}&\Delta_{\hat{\beta}}^{-\hat{\alpha}+}=(\Delta_a^a)^{1/2}\delta_{\hat{\beta}}{}^{\hat{\alpha}}=\Delta_{\hat{\beta}}{}^{+\hat{\alpha}-}\ ,\\&\Delta^{--++}=1=\Delta^{++--}\ ,\\&\Delta^{00}=1\ ,\end{aligned}\tag{C2}$$

and

$$\begin{aligned}&M_B^a=\frac{\Delta_B^a}{\Delta_a^a},\quad M_B^{\hat{\alpha}\pm}=\frac{\Delta_B^{\hat{\alpha}\pm}-M_B^a\Delta_a^{\hat{\alpha}\pm}}{(\Delta_a^a)^{1/2}}\ ,\\&M_B^{\bar{a}}=\frac{\Delta_B^{\bar{a}}}{\Delta_{\bar{a}}^{\bar{a}}},\quad M_B^{\pm\pm}=\Delta_B^{\pm\pm}\ .\end{aligned}\tag{C3}$$

The matrix Λ which constitutes the parametrization around the rigid (4,0) harmonic superspace is given by

$$\Lambda=\begin{bmatrix}\Delta_a^a & 0 & \Delta_a^{\hat{\alpha}+} & \Delta_a^{\hat{\alpha}-} & 0\\ 0 & \Delta_{\bar{a}}^{\bar{a}} & 0 & 0 & 0\\ 0 & 0 & (\Delta_a^a)^{1/2}\delta_{\hat{\alpha}}^{\hat{\beta}} & 0 & 0\\ 0 & 0 & 0 & (\Delta_a^a)^{1/2}\delta_{\hat{\alpha}}^{\hat{\beta}} & 0\\ 0 & 0 & 0 & 0 & \mathbb{1}\end{bmatrix}\ , \tag{C4}$$

where $\mathbb{1}$ is the identity in the sector of harmonic super-Beltrami variables. Furthermore, it is convenient to decompose (C4) as follows:

$$\Lambda_C^A=\lambda_C^Df_D^A\ , \tag{C5}$$

where f_D^A are the superconformal factors given in (2.30) and the matrix λ is then expressed as

$$\lambda=\begin{bmatrix}1 & 0 & \lambda_a^{\hat{\alpha}+} & \lambda_a^{\hat{\alpha}-} & 0\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & \delta_{\hat{\alpha}}^{\hat{\beta}} & 0 & 0\\ 0 & 0 & 0 & \delta_{\hat{\alpha}}^{\hat{\beta}} & 0\\ 0 & 0 & 0 & 0 & \mathbb{1}\end{bmatrix}\ . \tag{C6}$$

APPENDIX D: OPERATORS $L_a, L_{\bar a}, L_{\hat\alpha}^-, L^{\pm\pm}, L^0$

These operators are expressed, with the restriction (3.8), as

$$\begin{aligned}
L_a=\frac{1}{\Delta}\{&[1-M_{\bar a}^{++}M^{--\bar a}-M_{\bar a}^{--}M^{++\bar a}-(M_{\bar a}^{\hat\alpha +}+\lambda_a^{\hat\alpha +}M_{\bar a}^{a})(M_{\hat\alpha}^{-\bar a}-M_{\hat\alpha}^{-++}M^{--\bar a}-M_{\hat\alpha}^{---}M^{++\bar a})](\partial_a-l\partial_{\bar a}M_a^{\bar a})\\
&-[M_a^{\bar a}-\lambda_a^{\hat\alpha +}(M_{\hat\alpha}^{-\bar a}-M_{\hat\alpha}^{-++}M^{--\bar a}-M_{\hat\alpha}^{---}M^{++\bar a})](\partial_{\bar a}+qW_{\bar a}-r\partial_a M_{\bar a}^{a})\\
&-[\lambda_a^{\hat\alpha +}(1-M_{\bar a}^{++}M^{--\bar a}-M_{\bar a}^{--}M^{++\bar a})-M_a^{\bar a}(M_{\bar a}^{\hat\alpha +}+M_{\bar a}^{a}\lambda_a^{\hat\alpha +})](D_{\hat\alpha}^{-}+qW_{\hat\alpha}^{-}-l\partial_{\bar a}M_{\hat\alpha}^{-\bar a})\\
&+[M_a^{\bar a}(M_{\bar a}^{++}-M_{\bar a}^{\hat\alpha +}M_{\hat\alpha}^{-++})-\lambda_a^{\hat\alpha +}(M_{\hat\alpha}^{-\bar a}-M_{\hat\alpha}^{---}M^{++\bar a})M_{\bar a}^{++}-\lambda_a^{\hat\alpha +}M_{\hat\alpha}^{-++}(1-M_{\bar a}^{a}M_a^{\bar a}-M_{\bar a}^{--}M^{++\bar a})]\\
&\quad\times(D^{--}+qW^{--}-l\partial_{\bar a}M^{--\bar a})\ ,\\
&\quad+[M_a^{\bar a}(M_{\bar a}^{--}-M_{\bar a}^{\hat\alpha +}M_{\hat\alpha}^{---})-\lambda_a^{\hat\alpha +}(M_{\hat\alpha}^{-\bar a}-M_{\hat\alpha}^{-++}M^{--\bar a})M_{\bar a}^{--}-\lambda_a^{\hat\alpha +}M_{\hat\alpha}^{---}(1-M_{\bar a}^{a}M_a^{\bar a}-M_{\bar a}^{++}M^{--\bar a})]\\
&\quad\times(D_{+}^{++}qW^{++}-l\partial_{\bar a}M^{++\bar a})\}\ ,
\end{aligned}\tag{D1}$$

$$\begin{aligned}
L_{\bar a}=\frac{1}{\Delta}\{&[(\partial_{\bar a}+qW_{\bar a}-r\partial_a M_{\bar a}^{a})-M_{\bar a}^{a}(\partial_a-l\partial_{\bar a}M_a^{\bar a})-M_{\bar a}^{\hat\alpha +}(D_{\hat\alpha}^{-}+qW_{\hat\alpha}^{-}-l\partial_{\bar a}M_{\hat\alpha}^{-\bar a})\\
&-(M_{\bar a}^{++}-M_{\bar a}^{\hat\alpha +}M_{\hat\alpha}^{-++})(D^{--}+qW^{--}-l\partial_{\bar a}M^{--\bar a})-(M_{\bar a}^{--}-M_{\bar a}^{\hat\alpha +}M_{\hat\alpha}^{---})(D^{++}+qW^{++}-l\partial_{\bar a}M^{++\bar a})]\},
\end{aligned}\tag{D2}$$

$$\begin{aligned}
L_{\hat\alpha}^{-}=\frac{1}{\Delta}\{&(1-M_{\bar a}^{a}M_a^{\bar a}-M_{\bar a}^{++}M^{--\bar a}-M_{\bar a}^{--}M^{++\bar a})(D_{\hat\alpha}^{-}+qW_{\hat\alpha}^{-}-l\partial_{\bar a}M_{\hat\alpha}^{-\bar a})\\
&-(M_{\hat\alpha}^{-\bar a}-M_{\hat\alpha}^{-++}M^{--\bar a}-M_{\hat\alpha}^{---}M^{++\bar a})[(\partial_{\bar a}+qW_{\bar a}-r\partial_a M_{\bar a}^{a})-M_{\bar a}^{a}(\partial_a-l\partial_{\bar a}M_a^{\bar a})]\\
&+[(M_{\hat\alpha}^{-\bar a}-M_{\hat\alpha}^{---}M^{++\bar a})M_{\bar a}^{++}-M_{\hat\alpha}^{-++}(1-M_{\bar a}^{a}M_a^{\bar a}-M_{\bar a}^{--}M^{++\bar a})](D^{--}+qW^{--}-l\partial_{\bar a}M^{--\bar a})\\
&+[(M_{\hat\alpha}^{-\bar a}-M_{\hat\alpha}^{-++}M^{--\bar a})M_{\bar a}^{--}-M_{\hat\alpha}^{---}(1-M_{\bar a}^{a}M_a^{\bar a}-M_{\bar a}^{++}M^{--\bar a})](D^{++}+qW^{++}-l\partial_{\bar a}M^{++\bar a})\}\ ,
\end{aligned}\tag{D3}$$

$$\begin{aligned}
L^{\pm\pm}=\frac{1}{\Delta}\{&[1-M_{\bar a}^{a}M_a^{\bar a}-M_{\bar a}^{\hat\alpha +}M_{\hat\alpha}^{-\bar a}-(M_{\bar a}^{\pm\pm}-M_{\bar a}^{\hat\alpha +}M_{\hat\alpha}^{-\pm\pm})M^{\mp\mp\bar a}](D^{\pm\pm}+qW^{\pm\pm}-l\partial_{\bar a}M^{\pm\pm\bar a})\\
&-M^{\pm\pm\bar a}(\partial_{\bar a}+qW_{\bar a}-r\partial_a M_{\bar a}^{a})+M^{\pm\pm\bar a}M_{\bar a}^{a}(\partial_a-l\partial_{\bar a}M_a^{\bar a})+M^{\pm\pm\bar a}M_{\bar a}^{\hat\alpha +}(D_{\hat\alpha}^{-}+qW_{\hat\alpha}^{-}-l\partial_{\bar a}M_{\hat\alpha}^{-\bar a})\\
&+M^{\pm\pm\bar a}(M_{\bar a}^{\pm\pm}-M_{\bar a}^{\hat\alpha +}M_{\hat\alpha}^{-\pm\pm})(D^{\mp\mp}+qW^{\mp\mp}-l\partial_{\bar a}M^{\mp\mp\bar a})\}\ ,
\end{aligned}\tag{D4}$$

$$L^0=(D^0+qW^0)\ ,\tag{D5}$$

with

$$\Delta=1-M_{\bar a}^{a}M_a^{\bar a}-M_{\bar a}^{\hat\alpha +}M_{\hat\alpha}^{-\bar a}-(M_{\bar a}^{++}-M_{\bar a}^{\hat\alpha +}M_{\hat\alpha}^{-++})M^{--\bar a}-(M_{\bar a}^{--}-M_{\bar a}^{\hat\alpha +}M_{\hat\alpha}^{---})M^{++\bar a}\ .\tag{D6}$$

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[1] D. Friedan, S. Shenker, and E. Martinec, Nucl. Phys. **B271**, 93 (1986); A. A. Belavin and V. G. Knizhnik, Phys. Lett. **168B**, 201 (1986); G. Moore, J. Harris, P. Nelson, and I. Singer, Phys. Lett. B **178**, 167 (1986); E. Martinec, Nucl. Phys. **B281**, 157 (1987).

[2] S. Randjbar-Daemi and J. Strathdee, in *Introductory Lectures on Conformal Field Theory and Strings,* Lectures Given in Summer School in High Energy Physics and Cosmology (ICTP, Trieste, 1989).

[3] F. Gieres, Gen. Relativ. Gravit. **22**, 889 (1990); L. Crane and J. M. Rabin, Commun. Math. Phys. **113**, 601 (1988); P. Nelson, *ibid.* **115**, 167 (1988); O. Lehto, *Univalent Functions and Teichmüller Spaces,* Graduate texts in Mathematics, Vol. 109 (Springer, Berlin, 1987).

[4] S. J. Gates and F. Gieres, Nucl. Phys. **B320**, 310 (1989); L. Baulieu, M. Bellon, and R. Grimm, Phys. Lett. B **198**, 343 (1987).

[5] J. D. Cohn, Nucl. Phys. **B284**, 349 (1987); **B306**, 239 (1988); J. M. Rabin, talk presented at the 18th International Conference on Differential Geometric Methods in Physics, Tahoe City, California, 1989 (unpublished); J. M. Rabin, Phys. Lett. B **190**, 40 (1987).

[6] H. Kanno and Y. S. Myung, Phys. Rev. D **40**, 1974 (1989).

[7] M. T. Grisaru and R. M. Xu, Phys. Lett. B **205**, 486 (1988); J. Grundberg and R. Nakayama, "On 2*d* (super) gravity in the Light cone gauge," Nordita Report No.

88.14P (unpublished); A. M. Polyakov and A. B. Zamolodchikov, Mod. Phys. Lett. A **3**, 1213 (1988).
[8] R. Grimm, Ann. Phys. (N.Y.) **200**, 49 (1990); L. Baulieu, M. Bellon, and R. Grimm, Nucl. Phys. **B321**, 697 (1989).
[9] O. Thümmel, Class. Quantum Grav. **7**, 639 (1990).
[10] M. Kachkachi and T. Lhallabi, J. Math. Phys. **34**, 844 (1993).
[11] T. Lhallabi and E. H. Saidi, Int. J. Mod. Phys. A **3**, 187 (1988); **3**, 419 (1988).
[12] T. Lhallabi and E. H. Saidi, Nucl. Phys. **B335**, 689 (1990).
[13] E. H. Saidi and M. Zakkari, Int. J. Mod. Phys. A **18**, 3175 (1991).
[14] M. Kachkachi and T. Lhallabi, J. Math. Phys. **33**, 725 (1992).
[15] O. Thümmel, Dipl. Phys. thesis, 1991.
[16] T. Lhallabi, Phys. Rev. D **43**, 2649 (1991).
[17] T. Lhallabi, Class. Quantum Grav. **10**, 1 (1993).
[18] A. Sevrin, W. Troost, and A. Van Proeyen, Phys. Lett. B **208**, 447 (1988); M. Günaydin, J. L. Petersen, A. Taormina, and A. Van Proeyen, Nucl. Phys. **B322**, 402 (1989); S. Matsuda, Phys. Lett. B **282**, 56 (1992).
[19] C. Becchi, Nucl. Phys. **B304**, 513 (1988).
[20] F. Delduc and F. Gieres, Int. J. Mod. Phys. A **7**, 1685 (1992).
[21] T. Lhallabi, "(2,0) superconformal anomaly," Internal report Trieste IC/92/182 (unpublished).
[22] T. Lhallabi, Class. Quantum Grav. **10**, 439 (1993).