

Equivalence of Hamiltonian and Lagrangian path integral quantization: Effective gauge theories

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The equivalence of correct Hamiltonian and naive Lagrangian (Faddeev-Popov) path integral quantization (Matthews's theorem) is proven for gauge theories with arbitrary effective interaction terms. Effective gauge-boson self-interactions and effective interactions with scalar and fermion fields are considered. This result becomes extended to effective gauge theories with higher derivatives of the fields.

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I. INTRODUCTION

Gauge theories with arbitrary (non-Yang-Mills) effective interaction terms have been examined in order to parametrize possible deviations of the self-interactions of the electroweak gauge bosons [1,2] and of the gluons [3] from the standard model predictions with respect to experimental tests of these couplings. Such effective Lagrangians usually are quantized within the Faddeev-Popov formalism [4], which yields the generating functional [path integral (PI)]

$$Z[J] = \int \mathcal{D}\Phi \exp \left\{ i \int d^4x [\mathcal{L} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{ghost}} + J\Phi] \right\}, \quad (1.1)$$

where \mathcal{L} is the effective Lagrangian, \mathcal{L}_{GF} and $\mathcal{L}_{\text{ghost}}$ are the gauge-fixing (GF) term and the ghost term which are obtained in the standard manner. (Φ is a shorthand notation for all fields in the quantized Lagrangian $\mathcal{L} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{ghost}}$.) The generating functional (1.1) is very convenient for practical calculations because it is manifestly covariant (if a covariant gauge is chosen), it does not involve the generalized momenta of the fields, and it directly implies the Feynman rules (i.e., the quadratic terms in $\mathcal{L} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{ghost}}$ yield the propagators and the other terms yield the vertices in the usual way). However, (1.1) is derived from a naive Lagrangian PI ansatz [4], while correct quantization has to be performed within the more elaborate Hamiltonian PI formalism [5–8]. Thus, to justify the (Lagrangian) Faddeev-Popov PI (1.1) for effective gauge theories one has to derive it within the Hamiltonian PI formalism; i.e., one has to prove the equivalence of Hamiltonian and Lagrangian PI quantization, which is known as Matthews's theorem.¹

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¹Originally, the name "Matthews's theorem" simply denotes the statement that the Feynman rules directly follow from the effective Lagrangian in the usual way [9]. (Of course for a gauge-invariant Lagrangian \mathcal{L} , the Feynman rules do not follow from \mathcal{L} alone but from $\mathcal{L} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{ghost}}$.) Reformulated within the PI formalism, however, this means that an arbitrary Lagrangian can be quantized by using the naive Lagrangian PI ansatz [10–12].

Matthews's theorem has been proven for Yang-Mills theories *without* additional effective interaction terms by Faddeev [5] and for massive (and thus gauge noninvariant) Yang-Mills theories without effective interaction terms by Senjanovic [6]. For *arbitrary* interactions of scalar fields, this theorem has been derived by Bernard and Duncan [10] and for arbitrary interactions of massive vector fields by myself [11]. In [12] I have generalized these results to effective interactions which also involve higher derivatives of the fields. In this article I will complete the proof of Matthews's theorem for arbitrary interactions of the physically most important types of particles by considering effective Lagrangians with massless vector fields and with fermion fields.

Massless vector fields necessarily have to be understood as gauge fields. A Lagrangian with massless vector fields but gauge noninvariant interactions of these would make no physical sense because without a gauge fixing term, which only becomes introduced for gauge invariant Lagrangians (within the Hamiltonian PI as well as within the Lagrangian PI), the operator occurring in the quadratic part of the Lagrangian has no inverse and therefore it is impossible to obtain a propagator for the vector fields. Thus I will prove Matthews's theorem for gauge theories with additional arbitrary (non-Yang-Mills) self-interactions of the gauge fields, with arbitrary couplings of the gauge fields to scalar fields and to fermion fields and with arbitrary interactions among the scalar and fermion fields. All effective interaction terms are assumed to be gauge invariant. The proof also applies to the case of spontaneously broken gauge theories (SBGT's), i.e., gauge theories with massive gauge fields, because one can assume that the scalar fields that are coupled to the gauge fields have a nonvanishing vacuum expectation value. Matthews's theorem for SBGT's in which *all* gauge bosons are massive has already been derived in [11,12]. There, an SBGT was rewritten as a gauge-noninvariant model by applying the Stueckelberg formalism and then Matthews's theorem for gauge-noninvariant Lagrangians was used. In this article I will present a more direct proof of this theorem that does not use the Stueckelberg formalism and that also applies to SBGT's in which not all gauge bosons are massive (like electroweak models).

Lagrangians with gauge fields and with fermion fields

are singular. The presence of gauge fields implies first class constraints, and the presence of fermion fields implies second class constraints. Therefore, to prove Matthews's theorem one has to take into account the formalism of quantization of constrained systems which goes back to Dirac [13] and which has been formulated in the PI formalism by Faddeev [5] and Senjanovic [6]. (Extensive treatises on this subject can be found in [7,8].) Within this formalism, a gauge theory cannot be *directly* quantized in the Lorentz-gauge or, for SGBT's, in the R_ξ gauge (which are the most convenient gauges for practical calculations) because the corresponding GF conditions cannot be written as relations among the fields and the conjugate fields alone and thus they are not GF conditions within the Hamiltonian framework. Therefore, I will first derive the generating functional (1.1) in the Coulomb gauge and then use the equivalence of all gauges, i.e., the independence of the S -matrix elements from the choice of the gauge in the Faddeev-Popov formalism [14,15], in order to generalize this result to any other gauge.

To complete the proof of Matthews's theorem, one has to take into account effective gauge theories with higher derivatives of the fields, which also have been investigated for phenomenological reasons [16]. Actually, all unphysical effects that are connected with Lagrangians with higher derivatives (higher-order Lagrangians) [10,17,18], namely, additional degrees of freedom, unbound energy from below, etc., are absent within the effective-Lagrangian formalism [12] because an effective Lagrangian is assumed to be the low-energy approximation of well-behaved "new physics"; i.e., it parametrizes the low-energy effects of a renormalizable theory with heavy particles in which no higher derivatives occur. In fact, all higher time derivatives of the fields can be eliminated from the effective Lagrangian by applying the equations of motion (EOM) to the effective interaction term (upon neglecting higher powers of the effective coupling constant). The (in general forbidden) use of the EOM is correct because one can find field transformations which have the same effect as the application of the EOM to the effective interaction term [12,19,20]; these transformations involve derivatives of the fields. In [12] it has been shown that Lagrangians which are related by such field transformations are physically equivalent (at the classical and at the quantum level) because these become canonical transformations within the Hamiltonian treatment of higher-order Lagrangians (Ostrogradsky formalism [17]). Thus, each effective higher-order Lagrangian can be reduced to an equivalent Lagrangian without higher time derivatives. Since the use of the EOM does not affect the gauge invariance of a Lagrangian, Matthews's theorem for effective gauge theories with higher derivatives can be proven by using this reduction and by applying Matthews's theorem for effective gauge theories with at most first time derivatives. Especially the treatment of fermion fields can be simplified very much because the EOM for these fields only depend on first time derivatives. Therefore one can eliminate not only higher but also first time derivatives of the fermion fields from the effective interaction term and thus

the proof of Matthews's theorem can be reduced to the case of effective interactions in which no time derivatives of these fields occur.

In this article I will assume that the effective interactions, which are only the *deviations* from the standard interactions (i.e., from the Yang-Mills self-interactions of the gauge fields, minimal gauge couplings of these to the scalar and fermion fields, Yukawa couplings, and derivative-free scalar self-interactions), are proportional to a coupling constant ϵ with $\epsilon \ll 1$. This is justified for phenomenologically motivated effective Lagrangians because these are studied in order to parametrize small deviations from the standard model [1-3]. When deriving Matthews's theorem, I will, according to [10-12], neglect higher powers of ϵ and, besides, terms proportional to $\delta^4(0)$ which become zero if dimensional regularization is applied.

This paper is organized as follows. In Sec. II, I derive the Faddeev-Popov path integral for effective gauge theories (without higher derivatives) by using the Hamiltonian path integral formalism. In Sec. III, I generalize this proof of Matthews's theorem to effective gauge theories with higher derivatives by applying the equations of motion in order to remove all higher time derivatives from the effective interaction term. Section IV contains the summary of my results.

II. MATTHEWS'S THEOREM FOR EFFECTIVE GAUGE THEORIES

In this section I quantize a gauge theory with an additional arbitrary effective interaction term in the Hamiltonian PI formalism [5-8] in order to derive the Faddeev-Popov PI (1.1).

The effective Lagrangian is given by

$$\begin{aligned} \mathcal{L} = \mathcal{L}_0 + \epsilon \mathcal{L}_I = & -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a + i \bar{\psi}_a \gamma^\mu D_\mu \psi_a \\ & + (D^\mu \varphi_a^\dagger) (D_\mu \varphi_a) - V(\psi_a, \bar{\psi}_a, \varphi_a, \varphi_a^\dagger) \\ & + \epsilon \mathcal{L}_I. \end{aligned} \quad (2.1)$$

The field strength tensor and the covariant derivatives are

$$F_{\mu\nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f_{abc} A_\mu^b A_\nu^c, \quad (2.2)$$

$$D_\lambda F_{\mu\nu}^a \equiv \partial_\lambda F_{\mu\nu}^a - g f_{abc} A_\lambda^b F_{\mu\nu}^c, \quad (2.3)$$

$$D_\mu \psi_a \equiv \partial_\mu \psi_a + i g A_\mu^c t_c^{ab} \psi_b, \quad (2.4)$$

$$D_\mu \bar{\psi}_a \equiv \overline{(D_\mu \psi_a)}, \quad (2.5)$$

$$D_\mu \varphi_a \equiv \partial_\mu \varphi_a + i g A_\mu^c \bar{t}_c^{ab} \varphi_b, \quad (2.6)$$

$$D_\mu \varphi_a^\dagger \equiv (D_\mu \varphi_a)^\dagger. \quad (2.7)$$

(Higher covariant derivatives are defined analogously.) g is the gauge coupling constant, f_{abc} are the structure constants, and t_c^{ab} and \bar{t}_c^{ab} are the generators of the gauge group in its representation in the fermion sector and in the scalar sector respectively. $V(\psi_a, \bar{\psi}_a, \varphi_a, \varphi_a^\dagger)$ contains derivative-free interactions of the fermion and scalar fields, viz., Yukawa couplings and scalar self-interactions.

The effective interaction term $\epsilon\mathcal{L}_I$, which parametrizes the deviations from the minimal gauge theory, contains arbitrary interactions of the fields which are governed by the effective coupling constant ϵ with $\epsilon \ll 1$. As pointed out in the Introduction, an effective Lagrangian such as (2.1) only has a physical meaning if the effective interaction term is gauge invariant. This means that *the gauge fields A_μ^a do not occur arbitrarily in \mathcal{L}_I but only through the field strength tensor and through covariant derivatives*. Furthermore, in this section I assume that \mathcal{L}_I does depend neither on higher time derivatives of the fields nor on first time derivatives² of the A_0^a and of the fermion fields ψ_a and $\bar{\psi}_a$. The case of interactions with higher derivatives will be treated in the next section.

From (2.1) one finds the conjugate fields (generalized momenta)

$$\pi_0^a = \frac{\partial\mathcal{L}}{\partial\dot{A}_0^a} = 0, \quad (2.8)$$

$$\begin{aligned} \pi_i^a &= \frac{\partial\mathcal{L}}{\partial\dot{A}_i^a} = F_{i0}^a + \epsilon \frac{\partial\mathcal{L}_I}{\partial\dot{A}_i^a} \\ &= \dot{A}_i^a + \partial_i A_0^a - gf_{abc}A_i^b A_0^c + \epsilon \frac{\partial\mathcal{L}_I}{\partial\dot{A}_i^a}, \end{aligned} \quad (2.9)$$

$$\pi_\psi^a = \frac{\partial\mathcal{L}}{\partial\dot{\psi}_a} = i\bar{\psi}_a\gamma^0, \quad (2.10)$$

$$\pi_{\bar{\psi}}^a = \frac{\partial\mathcal{L}}{\partial\dot{\bar{\psi}}_a} = 0, \quad (2.11)$$

$$\pi_\varphi^a = \frac{\partial\mathcal{L}}{\partial\dot{\varphi}_a} = D_0\varphi_a^\dagger + \epsilon \frac{\partial\mathcal{L}_I}{\partial\dot{\varphi}_a} = \dot{\varphi}_a^\dagger - igA_0^c \bar{t}_c^{ba} \varphi_b^\dagger + \epsilon \frac{\partial\mathcal{L}_I}{\partial\dot{\varphi}_a}, \quad (2.12)$$

$$\begin{aligned} \mathcal{H} &= \pi_\mu^a \dot{A}_\mu^a + \pi_\psi^a \dot{\psi}_a + \bar{\psi}_a \pi_{\bar{\psi}}^a + \pi_\varphi^a \dot{\varphi}_a + \pi_{\varphi^\dagger}^a \dot{\varphi}_a^\dagger - \mathcal{L} \\ &= \frac{1}{2} \pi_i^a \pi_i^a - \pi_i^a \partial_i A_0^a + gf_{abc} \pi_i^a A_i^b A_0^c + \frac{1}{4} F_{ij}^a F_{ij}^a - igA_0^c t_c^{ab} (\pi_\psi^a \psi_b - \bar{\psi}_a \pi_\psi^b) + i\bar{\psi}_a \gamma_i D_i \psi_a \\ &\quad + \pi_{\varphi^\dagger}^a \pi_\varphi^a - igA_0^c \bar{t}_c^{ab} (\pi_\varphi^a \varphi_b - \varphi_a^\dagger \pi_{\varphi^\dagger}^b) + (D_i \varphi_a^\dagger)(D_i \varphi_a) + V - \epsilon \bar{\mathcal{L}}_I(A_i^a, \psi_a, \bar{\psi}_a, \varphi_a, \varphi_a^\dagger, \pi_i^a, \pi_\varphi^a, \pi_{\varphi^\dagger}^a) + O(\epsilon^2), \end{aligned} \quad (2.17)$$

with

$$\bar{\mathcal{L}}_I(A_i^a, \psi_a, \bar{\psi}_a, \varphi_a, \varphi_a^\dagger, \pi_i^a, \pi_\varphi^a, \pi_{\varphi^\dagger}^a) \equiv \mathcal{L}_I \Big|_{\substack{F_{i0}^a \rightarrow \pi_i^a \\ D_0 \varphi_a^\dagger \rightarrow \pi_{\varphi^\dagger}^a \\ D_0 \varphi_a \rightarrow \pi_\varphi^a}}. \quad (2.18)$$

One can use the identities

$$[D_\mu, D_\nu] \psi_a = ig F_{\mu\nu}^c t_c^{ab} \psi_b, \quad (2.19)$$

$$[D_\mu, D_\nu] \varphi_a = ig F_{\mu\nu}^c \bar{t}_c^{ab} \varphi_b, \quad (2.20)$$

$$[D_\mu, D_\nu] F_{\kappa\lambda}^a = -gf_{abc} F_{\mu\nu}^b F_{\kappa\lambda}^c \quad (2.21)$$

(and the corresponding relations for $\bar{\psi}_a$ and φ_a^\dagger), in order

²Actually, the absence of \dot{A}_0^a already follows from the gauge invariance and the requirement that no higher time derivatives occur in \mathcal{L}_I .

$$\pi_{\varphi^\dagger}^a = \frac{\partial\mathcal{L}}{\partial\dot{\varphi}_a^\dagger} = D_0\varphi_a + \epsilon \frac{\partial\mathcal{L}_I}{\partial\dot{\varphi}_a^\dagger} = \dot{\varphi}_a + igA_0^c \bar{t}_c^{ab} \varphi_b + \epsilon \frac{\partial\mathcal{L}_I}{\partial\dot{\varphi}_a^\dagger}. \quad (2.13)$$

The relations (2.8), (2.10), and (2.11) do not contain ϵ terms due to the assumption that \mathcal{L}_I does not depend on \dot{A}_0^a , $\dot{\psi}_a$, and $\dot{\bar{\psi}}_a$. These relations cannot be solved for the velocities; they are constraints. The remaining of the above equations can be solved for the velocities; they become (in the first order of ϵ)

$$\dot{A}_i^a = \pi_i^a - \partial_i A_0^a + gf_{abc} A_i^b A_0^c - \epsilon \frac{\partial\mathcal{L}_I}{\partial\dot{A}_i^a} \Big|_{\substack{F_{i0}^a \rightarrow \pi_i^a \\ D_0 \varphi_a^\dagger \rightarrow \pi_{\varphi^\dagger}^a \\ D_0 \varphi_a \rightarrow \pi_\varphi^a}} + O(\epsilon^2), \quad (2.14)$$

$$\dot{\varphi}_a^\dagger = \pi_{\varphi^\dagger}^a + igA_0^c \bar{t}_c^{ba} \varphi_b^\dagger - \epsilon \frac{\partial\mathcal{L}_I}{\partial\dot{\varphi}_a^\dagger} \Big|_{\substack{F_{i0}^a \rightarrow \pi_i^a \\ D_0 \varphi_a^\dagger \rightarrow \pi_{\varphi^\dagger}^a \\ D_0 \varphi_a \rightarrow \pi_\varphi^a}} + O(\epsilon^2), \quad (2.15)$$

$$\dot{\varphi}_a = \pi_\varphi^a - igA_0^c \bar{t}_c^{ab} \varphi_b - \epsilon \frac{\partial\mathcal{L}_I}{\partial\dot{\varphi}_a} \Big|_{\substack{F_{i0}^a \rightarrow \pi_i^a \\ D_0 \varphi_a^\dagger \rightarrow \pi_{\varphi^\dagger}^a \\ D_0 \varphi_a \rightarrow \pi_\varphi^a}} + O(\epsilon^2). \quad (2.16)$$

One obtains the Hamiltonian

to rewrite those expressions in \mathcal{L}_I , in which time and spatial covariant derivatives act on the fields, such that the time derivatives are applied first. Remembering the discussion of the paragraph preceding Eq. (2.8) one can then easily see that \mathcal{L}_I depends on the A_0^a only through the expressions

$$F_{i0}^a, \quad D_0 F_{ij}^a, \quad D_0 \varphi_a, \quad D_0 \varphi_a^\dagger. \quad (2.22)$$

Using the relation

$$D_\lambda F_{\mu\nu}^a + D_\mu F_{\nu\lambda}^a + D_\nu F_{\lambda\mu}^a = 0 \quad (2.23)$$

in order to rewrite $D_0 F_{ij}^a$ as

$$D_0 F_{ij}^a = D_j F_{i0}^a - D_i F_{j0}^a \quad (2.24)$$

and the definition (2.18), one finds that $\bar{\mathcal{L}}_I$ does not depend on the A_0^a . Thus, the gauge invariance and the absence of higher time derivatives (and of first time derivatives of A_0^a , ψ_a , and $\bar{\psi}_a$) in \mathcal{L}_I yield

$$\frac{\partial \bar{\mathcal{L}}_I}{\partial A_0^a} = 0. \quad (2.25)$$

As mentioned, the relations (2.8), (2.10), and (2.11) imply the primary constraints

$$\phi_1^a = \pi_0^a = 0, \quad (2.26)$$

$$\phi_\psi^a = \pi_\psi^a - i\bar{\psi}_a \gamma^0 = 0, \quad (2.27)$$

$$\phi_{\bar{\psi}}^a = \pi_{\bar{\psi}}^a = 0. \quad (2.28)$$

The requirement that these primary constraints have to be consistent with the EOM, i.e., the demand

$$\dot{\phi}_a^{(1)} = \{\phi_a^{(1)}, H^{(1)}\} = 0, \quad \text{with } \mathcal{H}^{(1)} = \mathcal{H} + \lambda_a^{(1)} \phi_a^{(1)} \quad (2.29)$$

(where the $\phi_a^{(1)}$ are the primary constraints; and the $\lambda_a^{(1)}$ are Lagrange multipliers), yields secondary constraints. Actually, (2.27) and (2.28) do not imply secondary constraints, the relation (2.29) only determines the Lagrange multipliers corresponding to these constraints [8]. The ϕ_1^a (2.26) imply the secondary constraints

$$\begin{aligned} \phi_2^a &= \partial_i \pi_i^a + g f_{abc} \pi_i^b A_i^c - i g t_a^{bc} (\pi_\psi^b \psi_c - \bar{\psi}_b \pi_{\bar{\psi}}^c) \\ &\quad - i g t_a^{bc} (\pi_{\bar{\psi}}^b \varphi_c - \varphi_b^\dagger \pi_{\bar{\psi}}^c) = 0. \end{aligned} \quad (2.30)$$

Because of (2.25), these secondary constraints do not contain $O(\epsilon)$ terms; i.e., they are independent of the form of the effective interaction term \mathcal{L}_I (in the first order of ϵ). There are no tertiary, etc., constraints. One can easily check that the constraints ϕ_ψ^a (2.27) and $\phi_{\bar{\psi}}^a$ (2.28) are second class and that the constraints ϕ_1^a (2.26) and ϕ_2^a (2.30) are first class.

Because of the presence of the first class constraints, the solutions of the Hamiltonian EOM contain undeter-

mined Lagrange multipliers. To remove these ambiguities, one has to introduce additional gauge-fixing conditions so that constraints and GF conditions together form a set of second class constraints which is consistent with the EOM [5–8]. As mentioned in the Introduction, the usual Lorentz GF conditions

$$\chi_1^a = \partial^\mu A_\mu^a - C^a = 0 \quad (2.31)$$

(and also the R_ξ GF conditions for SBGT's) are not GF conditions within the Hamiltonian formalism [5–8] because they are not relations among the fields and the conjugate fields alone due to the presence of the velocities A_0^a in (2.31), which cannot be expressed in terms of the momenta. Therefore I quantize the effective gauge theory within the Coulomb gauge, i.e., by choosing the primary GF conditions

$$\chi_1^a = \partial^i A_i^a - C^a = 0. \quad (2.32)$$

(Instead of the Coulomb gauge, one can alternatively choose the axial gauge or, for SBGT's, the unitary gauge [11].) Next, one has to construct secondary GF conditions χ_2^a by demanding

$$\{\chi_1^a, H\} = 0, \quad (2.33)$$

which ensures the consistency with the EOM [7,8]. One finds³

$$\chi_2^a = \Delta A_0^a - \partial_i \pi_i^a - g f_{abc} \partial_i (A_i^b A_0^c) + \epsilon \partial_i \frac{\partial \bar{\mathcal{L}}_I}{\partial \pi_i^a} = 0. \quad (2.34)$$

The Hamiltonian path integral [5–8] for this system is given by⁴

$$\begin{aligned} Z &= \int \mathcal{D}A_\mu^a \mathcal{D}\psi_a \mathcal{D}\bar{\psi}_a \mathcal{D}\varphi_a \mathcal{D}\varphi_a^\dagger \mathcal{D}\pi_\mu^a \mathcal{D}\pi_\psi^a \mathcal{D}\pi_{\bar{\psi}}^a \mathcal{D}\pi_\varphi^a \mathcal{D}\pi_{\varphi^\dagger}^a \\ &\quad \times \exp \left\{ i \int d^4x \left[\pi_\mu^a \dot{A}_\mu^a + \pi_\psi^a \dot{\psi}_a + \bar{\psi}_a \dot{\pi}_{\bar{\psi}}^a + \pi_\varphi^a \dot{\varphi}_a + \pi_{\varphi^\dagger}^a \dot{\varphi}_a^\dagger - \mathcal{H} \right] \right\} \delta(\phi_\psi^a) \delta(\phi_{\bar{\psi}}^a) \delta(\phi_1^a) \delta(\phi_2^a) \delta(\chi_1^a) \delta(\chi_2^a) \\ &\quad \times \text{Det}^{1/2} [\{\Phi_{2\text{nd}}^a(\mathbf{x}), \Phi_{2\text{nd}}^b(\mathbf{y})\} \delta(x^0 - y^0)] \text{Det} [\{\Phi_{1\text{st}}^a(\mathbf{x}), X^b(\mathbf{y})\} \delta(x^0 - y^0)], \end{aligned} \quad (2.35)$$

where $\Phi_{2\text{nd}}^a$, $\Phi_{1\text{st}}^a$, and X^a denote *all* second class constraints, first class constraints, and GF conditions respectively. First let me consider the determinants⁵ occurring in (2.35). The fundamental Poisson brackets immediately imply

$$\text{Det}^{1/2} [\{\Phi_{2\text{nd}}^a(\mathbf{x}), \Phi_{2\text{nd}}^b(\mathbf{y})\} \delta(x^0 - y^0)] = \text{const}. \quad (2.36)$$

Therefore, this term can be neglected in the PI. Furthermore, one finds

$$\{\phi_1^a(\mathbf{x}), \chi_1^b(\mathbf{y})\} = 0, \quad (2.37)$$

$$\begin{aligned} -\{\phi_1^a(\mathbf{x}), \chi_2^b(\mathbf{y})\} &= \{\phi_2^a(\mathbf{x}), \chi_1^b(\mathbf{y})\} \\ &= [\delta_{ab} \Delta + g f_{abc} (\partial_i A_i^c) \\ &\quad + g f_{abc} A_i^c \partial_i] \delta^3(\mathbf{x} - \mathbf{y}). \end{aligned} \quad (2.38)$$

³The GF conditions (2.32) and (2.34) do not fulfill the condition $\{\chi_1^a, \chi_2^b\} = 0$ required in [5,6]. However, this demand is unnecessary [7,8,21].

⁴For convenience, I will introduce the source terms in the PI after all manipulations will have been done. (The source terms for the ghost fields have to be introduced later, anyway.) Actually, if the source terms would be considered from the beginning, the subsequent procedure would not leave them unchanged. However, a change in the source terms does not effect the S -matrix elements [15].

⁵The factors $\delta(x^0 - y^0)$ in the arguments of the determinants are missing in [5–8]. However, they necessarily have to be present because $\text{Det} \{\Phi_{1\text{st}}^a(\mathbf{x}), X^b(\mathbf{y})\}$ [where $\Phi_{1\text{st}}^a(\mathbf{x})$ and $X^b(\mathbf{y})$ are taken at equal times] has to be introduced for *all* times and $\text{Det} [\{\Phi_{1\text{st}}^a(\mathbf{x}), X^b(\mathbf{y})\} \delta(x^0 - y^0)]$ is the “product” of this expression over all times. [The same is true for the other determinant in (2.35).]

The absence of $O(\epsilon)$ terms in (2.38) is again a consequence of (2.25). This yields

$$\begin{aligned} \text{Det} \{ \{ \Phi_{1st}^a(\mathbf{x}), X^b(\mathbf{y}) \} \delta(x^0 - y^0) \} \\ = \text{Det}^2 \{ \{ \delta_{ab} \Delta + g f_{abc} (\partial_i A_i^c) \\ + g f_{abc} A_i^c \partial_i \} \delta^4(x - y) \}. \end{aligned} \quad (2.39)$$

The following steps are very similar to those made ei-

ther within the Hamiltonian PI quantization of a Yang-Mills theory without effective interaction terms in [7,8] or within the treatment of gauge noninvariant effective Lagrangians in [10,11]. Therefore, I will discuss them only very briefly. First one observes that \mathcal{H} (2.17) contains a term $A_0^a \phi_2^a$. Because of the presence of $\delta(\phi_2^a)$ in the PI, this term can be omitted. Then one integrates over π_0^a , π_ψ^a , and π_φ^a and finds

$$\begin{aligned} Z = \int \mathcal{D}A_\mu^a \mathcal{D}\psi_a \mathcal{D}\bar{\psi}_a \mathcal{D}\varphi_a \mathcal{D}\varphi_a^\dagger \mathcal{D}\pi_i^a \mathcal{D}\pi_\varphi^a \mathcal{D}\pi_{\varphi^\dagger}^a \\ \times \exp \left\{ i \int d^4x \left[-\frac{1}{2} \pi_i^a \pi_i^a + \pi_i^a \dot{A}_i^a - \frac{1}{4} F_{ij}^a F_{ij}^a + i \bar{\psi}_a \gamma^0 \dot{\psi}_a - i \psi_a \gamma_i D_i \psi_a - \pi_\varphi^a \pi_{\varphi^\dagger}^a + \pi_\varphi^a \dot{\varphi}_a + \pi_{\varphi^\dagger}^a \dot{\varphi}_a^\dagger \right. \right. \\ \left. \left. - (D_i \varphi_a^\dagger)(D_i \varphi_a) - V + \epsilon \bar{\mathcal{L}}_I(A_i^a, \psi_a, \bar{\psi}_a, \varphi_a, \varphi_a^\dagger, \pi_i^a, \pi_\varphi^a, \pi_{\varphi^\dagger}^a) \right] \right\} \\ \times \delta(\bar{\phi}_2^a) \delta(\chi_1^a) \delta(\chi_2^a) \text{Det} \{ \{ \Phi_{1st}^a(\mathbf{x}), X^b(\mathbf{y}) \} \delta(x^0 - y^0) \}, \end{aligned} \quad (2.40)$$

with

$$\begin{aligned} \bar{\phi}_2^a = \partial_i \pi_i^a + g f_{abc} \pi_i^b A_i^c + g \bar{\psi}^b \gamma^0 t_a^{bc} \psi_c \\ - i g \bar{t}_a^{bc} (\pi_\varphi^b \varphi_c - \varphi_b^\dagger \pi_{\varphi^\dagger}^c). \end{aligned} \quad (2.41)$$

After rewriting

$$\begin{aligned} \delta(\chi_2^a) = \delta(A_0^a - \bar{A}_0^a) \text{Det}^{-1} \{ \{ \delta_{ab} \Delta + g f_{abc} (\partial_i A_i^c) \\ + g f_{abc} A_i^c \partial_i \} \delta^4(x - y) \} \end{aligned} \quad (2.42)$$

[where \bar{A}_0^a is the solution of the differential equation (2.34) with the boundary condition that \bar{A}_0^a vanishes for $|\mathbf{x}| \rightarrow \infty$], one can also integrate over A_0^a . Because of (2.25), the argument of the determinant in (2.42) also does not contain $O(\epsilon)$ terms and, besides, the integration over A_0^a does not affect $\bar{\mathcal{L}}_I$. Next one reintroduces the variables A_0^a by using

$$\delta(\bar{\phi}_2^a) = \int \mathcal{D}A_0^a \exp \left\{ -i \int d^4x A_0^a \bar{\phi}_2^a \right\} \quad (2.43)$$

and gets

$$\begin{aligned} Z = \int \mathcal{D}A_\mu^a \mathcal{D}\psi_a \mathcal{D}\bar{\psi}_a \mathcal{D}\varphi_a \mathcal{D}\varphi_a^\dagger \mathcal{D}\pi_i^a \mathcal{D}\pi_\varphi^a \mathcal{D}\pi_{\varphi^\dagger}^a \\ \times \exp \left\{ i \int d^4x \left[-\frac{1}{2} \pi_i^a \pi_i^a + \pi_i^a F_{i0}^a - \frac{1}{4} F_{ij}^a F_{ij}^a + i \psi_a \gamma^\mu D_\mu \psi_a - \pi_\varphi^a \pi_{\varphi^\dagger}^a + \pi_\varphi^a D_0 \varphi_a + \pi_{\varphi^\dagger}^a D_0 \varphi_a^\dagger \right. \right. \\ \left. \left. - (D_i \varphi_a^\dagger)(D_i \varphi_a) - V + \epsilon \bar{\mathcal{L}}_I(A_i^a, \psi_a, \bar{\psi}_a, \varphi_a, \varphi_a^\dagger, \pi_i^a, \pi_\varphi^a, \pi_{\varphi^\dagger}^a) \right] \right\} \\ \times \delta(\partial^i A_i^a - C^a) \text{Det} \{ \{ \delta_{ab} \Delta + g f_{abc} (\partial_i A_i^c) + g f_{abc} A_i^c \partial_i \} \delta^4(x - y) \}. \end{aligned} \quad (2.44)$$

In order to obtain expressions quadratic in the momenta, one rewrites this as

$$\begin{aligned} Z = \int \mathcal{D}A_\mu^a \mathcal{D}\psi_a \mathcal{D}\bar{\psi}_a \mathcal{D}\varphi_a \mathcal{D}\varphi_a^\dagger \exp \left\{ i \epsilon \int d^4x \bar{\mathcal{L}}_I \left(A_i^a, \psi_a, \bar{\psi}_a, \varphi_a, \varphi_a^\dagger, \frac{\delta}{i \delta K_i^a}, \frac{\delta}{i \delta K_\varphi^a}, \frac{\delta}{i \delta K_{\varphi^\dagger}^a} \right) \right\} \\ \times \int \mathcal{D}\pi_i^a \mathcal{D}\pi_\varphi^a \mathcal{D}\pi_{\varphi^\dagger}^a \exp \left\{ i \int d^4x \left[-\frac{1}{2} \pi_i^a \pi_i^a + \pi_i^a F_{i0}^a - \frac{1}{4} F_{ij}^a F_{ij}^a \right. \right. \\ \left. \left. + i \psi_a \gamma^\mu D_\mu \psi_a - \pi_\varphi^a \pi_{\varphi^\dagger}^a + \pi_\varphi^a D_0 \varphi_a + \pi_{\varphi^\dagger}^a D_0 \varphi_a^\dagger - (D_i \varphi_a^\dagger)(D_i \varphi_a) - V \right. \right. \\ \left. \left. + K_i^a \pi_i^a + K_\varphi^a \pi_\varphi^a + K_{\varphi^\dagger}^a \pi_{\varphi^\dagger}^a \right] \right\} \Big|_{K_i^a = K_\varphi^a = K_{\varphi^\dagger}^a = 0} \\ \times \delta(\partial^i A_i^a - C^a) \text{Det} \{ \{ \delta_{ab} \Delta + g f_{abc} (\partial_i A_i^c) + g f_{abc} A_i^c \partial_i \} \delta^4(x - y) \}. \end{aligned} \quad (2.45)$$

Now one can do the Gaussian integrations over the momenta. With \mathcal{L}_0 given in (2.1) one finds

$$\begin{aligned}
Z = & \int \mathcal{D}A_\mu^\alpha \mathcal{D}\psi_a \mathcal{D}\bar{\psi}_a \mathcal{D}\varphi_a \mathcal{D}\varphi_a^\dagger \exp \left\{ i \int d^4x \mathcal{L}_0 \right\} \\
& \times \exp \left\{ i\epsilon \int d^4x \tilde{\mathcal{L}}_I \left(A_i^\alpha, \psi_a, \bar{\psi}_a, \varphi_a, \varphi_a^\dagger, \frac{\delta}{i\delta K_i^\alpha}, \frac{\delta}{i\delta K_\varphi^\alpha}, \frac{\delta}{i\delta K_{\varphi^\dagger}^\alpha} \right) \right\} \\
& \times \exp \left\{ i \int d^4x \left[\frac{1}{2} K_i^\alpha K_i^\alpha + K_\varphi^\alpha K_{\varphi^\dagger}^\alpha + K_i^\alpha F_{i0}^\alpha + K_\varphi^\alpha D_0 \varphi_a^\dagger + K_{\varphi^\dagger}^\alpha D_0 \varphi_a \right] \right\} \Bigg|_{K_i^\alpha = K_\varphi^\alpha = K_{\varphi^\dagger}^\alpha = 0} \\
& \times \delta(\partial^i A_i^\alpha - C^\alpha) \text{Det} \{ [\delta_{ab} \Delta + g f_{abc} (\partial_i A_i^c) + g f_{abc} A_i^c \partial_i] \delta^4(x-y) \} .
\end{aligned} \tag{2.46}$$

This expression can be simplified in complete analogy to the procedure described in [10,11]. Thus I only present the result which is found after some calculations [by neglecting $O(\epsilon^2)$ and $\delta^4(0)$ terms], viz.,

$$\begin{aligned}
Z = & \int \mathcal{D}A_\mu^\alpha \mathcal{D}\psi_a \mathcal{D}\bar{\psi}_a \mathcal{D}\varphi_a \mathcal{D}\varphi_a^\dagger \exp \left\{ i \int d^4x [\mathcal{L}_0 + \epsilon \tilde{\mathcal{L}}_I] \right\} \\
& \times \delta(\partial^i A_i^\alpha - C^\alpha) \text{Det} \{ [\delta_{ab} \Delta + g f_{abc} (\partial_i A_i^c) \\
& + g f_{abc} A_i^c \partial_i] \delta^4(x-y) \} ,
\end{aligned} \tag{2.47}$$

where $\tilde{\mathcal{L}}_I$ turns out to be

$$\tilde{\mathcal{L}}_I = \tilde{\mathcal{L}}_I \Bigg|_{\substack{\pi_i^\alpha \rightarrow F_{i0}^\alpha \\ \pi_\varphi^\alpha \rightarrow D_0 \varphi_a^\dagger \\ \pi_{\varphi^\dagger}^\alpha \rightarrow D_0 \varphi_a}} = \mathcal{L}_I. \tag{2.48}$$

(2.47) with (2.48) is identical to the result obtained in the Faddeev-Popov formalism by choosing the (Coulomb) GF conditions χ_1^α (2.32) because the change of χ_1^α under infinitesimal variations of the gauge parameter α_b is

$$\frac{\delta \chi_1^\alpha(x)}{\delta \alpha_b(y)} = [\delta_{ab} \Delta + g f_{abc} (\partial_i A_i^c) + g f_{abc} A_i^c \partial_i] \delta^4(x-y). \tag{2.49}$$

To derive the form (1.1) of the generating functional one has, as usual, to construct the GF term by using the δ function and to rewrite the determinant as a ghost term. Finally the source terms have to be added. It is essential for the derivation of this result that, due to (2.25), no $O(\epsilon)$ terms occur in the argument of the determinant in (2.47). Thus the ghost term is independent of the form of the effective interaction term as in the Faddeev-Popov formalism. Because of the equivalence of all gauges [14,15] the result (1.1) can be rewritten in any other gauge which can be derived within the Faddeev-Popov formalism, e.g., in the Lorentz gauge or in the R_ξ gauge (for SBT's).

The gauge theory given by (2.1) is spontaneously broken if the vacuum expectation value of the scalar fields (implied by the scalar self-interactions in V) is nonzero; this does not affect the above proof. Actually, this proof holds for both SBT's with a *linearly* realized scalar sector, which contain (a) physical Higgs boson(s) [2], and gauged nonlinear σ models, i.e., SBT's with a *nonlinearly* realized scalar sector and without physical Higgs bosons [1], because the latter ones can be obtained

from the first ones by making a point transformation (which does not affect the Hamiltonian PI [5-8]) in order to rewrite the scalar sector nonlinearly [11,12,14,22,23] and then taking the limit $M_H \rightarrow \infty$ [1,11,12,23]. Thus, for an arbitrary effective gauge theory (without higher derivatives) the simple Faddeev-Popov PI can be derived within the correct Hamiltonian PI formalism.

III. EFFECTIVE GAUGE THEORIES WITH HIGHER DERIVATIVES

In this section I generalize the results of the preceding one to effective gauge theories with higher time derivatives.

Each effective Lagrangian like (2.1) can be reduced to a Lagrangian without higher time derivatives (and also without first time derivatives of the A_0^α , ψ_a , and $\bar{\psi}_a$) because *the equations of motion following from \mathcal{L}_0 in (2.1) can be applied in order to convert the effective interaction term \mathcal{L}_I [12] (upon neglecting higher powers of ϵ). This statement is nontrivial because, in general, the EOM must not be inserted into the Lagrangian. However, one can find field transformations which have the same effect as the application of the EOM to the effective interaction term [12,19,20]. Actually, a field transformation*

$$\Phi \rightarrow \Phi + \epsilon T \tag{3.1}$$

(where Φ may represent any field occurring in \mathcal{L} and T is an arbitrary function of the fields and their derivatives) applied to (2.1) yields an extra term

$$\epsilon T \left(\frac{\partial \mathcal{L}_0}{\partial \Phi} - \partial^\mu \frac{\partial \mathcal{L}_0}{\partial (\partial^\mu \Phi)} \right) + O(\epsilon^2) \tag{3.2}$$

to the effective Lagrangian. Lagrangians that are related by field transformations such as (3.1) are physically equivalent (at the classical and at the quantum level) [12] although these transformations involve derivatives of the fields (contained in T) because they become point transformations (and thus canonical transformations) within the Hamiltonian formalism for Lagrangians with higher derivatives (Ostrogradsky formalism [17]). The reason for this is that in the Ostrogradsky formalism for an N th-order Lagrangian all derivatives of the fields up to the order $N-1$ are treated as independent coordinates, and the order N can be chosen arbitrarily high without affecting the physical content of the theory [8,12].

An arbitrary effective gauge theory can be reduced

to one of the type considered in the previous section as follows: Because of gauge invariance, derivatives of the fields occur in the effective interaction term only as covariant derivatives or through the field strength tensor. Using the identities (2.19), (2.20), and (2.21) (and the corresponding relations for $\bar{\psi}_a$ and φ_a^\dagger) one again rewrites all expressions in \mathcal{L}_I such that the covariant time derivatives are applied to the fields before the covariant spatial derivatives. Then, higher time derivatives (and first time derivatives of A_0^a , ψ_a , and $\bar{\psi}_a$) occur in \mathcal{L}_I only through the expressions

$$\begin{aligned} D_0 F_{0i}^a, \quad D_0 D_0 F_{ij}^a, \quad D_0 \psi_a, \\ D_0 \bar{\psi}_a, \quad D_0 D_0 \varphi_a, \quad D_0 D_0 \varphi_a^\dagger, \end{aligned} \quad (3.3)$$

and even higher derivatives of these terms. After using (2.21) and (2.23) in order to rewrite $D_0 D_0 F_{ij}^a$ as

$$D_0 D_0 F_{ij}^a = D_i D_0 F_{0j}^a - D_j D_0 F_{0i}^a - 2g f_{abc} F_{i0}^b F_{j0}^c, \quad (3.4)$$

one can convert the terms (3.3) to terms without higher time derivatives (and without first time derivatives of A_0^a , ψ_a , and $\bar{\psi}_a$) by using the EOM following from \mathcal{L}_0 , viz.,

$$\begin{aligned} D_0 F_{0i}^a &= D_j F_{ji}^a + g \bar{\psi}_b \gamma_i t_a^{bc} \psi_c \\ &\quad - i g t_a^{bc} [(D_i \varphi_b^\dagger) \varphi_c - \varphi_b^\dagger (D_i \varphi_c)], \end{aligned} \quad (3.5)$$

$$D_0 \psi_a = \gamma^0 \left(\gamma_i D_i \psi_a - i \frac{\partial V}{\partial \psi_a} \right), \quad (3.6)$$

$$D_0 D_0 \varphi_a = D_i D_i \varphi_a - \frac{\partial V}{\partial \varphi_a^\dagger} \quad (3.7)$$

(and the corresponding equations for $\bar{\psi}_a$ and φ_a^\dagger). By repeated application of the EOM one can eliminate all higher time derivatives from \mathcal{L}_I . The fact that the EOM do not contain second time derivatives of A_0^a , ψ_a , and $\bar{\psi}_a$ makes it possible to eliminate not only higher but also first time derivatives of these fields. The Lagrangian obtained by applying the EOM is gauge invariant, too, because the form of the EOM is invariant under gauge transformations.

Now Matthews's theorem for effective gauge theories with higher time derivatives can be proven as follows.

(1) Given an arbitrary gauge invariant effective Lagrangian \mathcal{L} , this can be reduced to an equivalent gauge-invariant Lagrangian \mathcal{L}_{red} without higher time derivatives (and without first time derivatives of A_0^a , ψ_a , and $\bar{\psi}_a$) by applying the EOM, i.e., actually by making field transformations such as (3.1). This does not affect the Hamiltonian PI [12].

(2) \mathcal{L}_{red} can be quantized within the Hamiltonian PI formalism by applying Matthews's theorem for first-order Lagrangians derived in Sec. II. This yields the PI

$$Z = \int \mathcal{D}\Phi \exp \left\{ i \int d^4x [\mathcal{L}_{\text{red}} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{ghost}}] \right\}. \quad (3.8)$$

(3) Going reversely through the Faddeev-Popov procedure one can rewrite (3.8) (after introducing an infinite constant into the PI) as

$$Z = \int \mathcal{D}\Phi \exp \left\{ i \int d^4x \mathcal{L}_{\text{red}} \right\}. \quad (3.9)$$

(4) Within the Lagrangian PI (3.9) the field transformations (3.1) applied in step (1) are done inversely in order to reconstruct the primordial effective Lagrangian.⁶ One obtains

$$Z = \int \mathcal{D}\Phi \exp \left\{ i \int d^4x \mathcal{L} \right\}. \quad (3.10)$$

The Jacobian determinant implied by the change of the functional integration measure corresponding to these transformations only yields extra $\delta^4(0)$ terms [20] which are neglected here.

(5) Applying the Faddeev-Popov formalism⁷ to (3.10) and adding the source terms one finally finds (1.1) in an arbitrary gauge.

This completes the proof of Matthews's theorem for effective gauge theories.

IV. SUMMARY

In this article I have completed the proof of Matthews's theorem for arbitrary interactions of the physically most important types of particles. I have shown that a gauge theory with an arbitrary effective interaction term can be quantized by using the convenient (Lagrangian) Faddeev-Popov path integral because this can be derived from the correct Hamiltonian path integral. Thus Hamiltonian and Lagrangian path integral quantization are equivalent. This means that the Feynman rules can be obtained in the usual way from the effective Lagrangian.

Matthews's theorem also applies to effective gauge theories with higher derivatives of the fields. Each effective gauge theory can be reduced to a gauge theory with at most first time derivatives by applying the equations of motion to the effective interaction term. Thus, an effective higher-order Lagrangian can formally be treated in the same way as a first-order one; all unphysical effects,

⁶Note that the use of the transformations (3.1) in (3.8) would result in an application of the EOM following from $\mathcal{L}_0 + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{ghost}}$ (and not from \mathcal{L}_0 alone) which would not yield the desired result.

⁷In distinction from naive Lagrangian PI quantization, (3.10) is not taken as an ansatz here but it has been derived from the Hamiltonian PI.

which normally occur when dealing with higher-order Lagrangians, are absent because an effective Lagrangian is assumed to parametrize the low-energy effects of well-behaved “new physics.”

Actually, these results justify the straightforward treatment of effective gauge theories in the phenomenological literature.

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- [1] T. Appelquist and C. Bernard, *Phys. Rev. D* **22**, 200 (1980); A. C. Longhitano, *Nucl. Phys.* **B188**, 118 (1981); A. Dobado, D. Espriu, and M. J. Herrero, *Phys. Lett. B* **225**, 405 (1991); B. Holdom, *ibid.* **258**, 156 (1991); A. Falk, M. Luke, and E. Simmons, *Nucl. Phys.* **B365**, 523 (1991); D. Espriu and M. J. Herrero, *ibid.* **B373**, 117 (1991); J. Bagger, S. Dawson, and G. Valencia, *ibid.* **B399**, 364 (1993); T. Appelquist and G.-H. Wu, *Phys. Rev. D* **48**, 3235 (1993).
- [2] C. N. Leung, S. T. Love, and S. Rao, *Z. Phys. C* **31**, 433 (1986); W. Buchmüller and D. Wyler, *Nucl. Phys. Lett. B* **265**, 326 (1991); A. de Rújula, M. B. Gavela, P. Hernández, and E. Massó, *Nucl. Phys.* **B384**, 3 (1992); K. Hagiwara, S. Ishihara, R. Szalapski, and D. Zeppenfeld, *Phys. Lett. B* **283**, 353 (1992); G. J. Gounaris and F. M. Renard, *Z. Phys. C* **59**, 133 (1993); P. Hernández and F. J. Vegas, *Phys. Lett. B* **307**, 116 (1993); M. Bilenky, J. L. Kneur, F. M. Renard, and D. Schildknecht, *Nucl. Phys.* **B409**, 22 (1993); K. Hagiwara, S. Ishihara, R. Szalapski, and D. Zeppenfeld, *Phys. Rev. D* **48**, 2182 (1993); C. Grosse-Knetter, I. Kuss, and D. Schildknecht, *Z. Phys. C* **60**, 375 (1993).
- [3] Leung, Love, and Rao [2]; Buchmüller and Wyler [2]; E. H. Simmons, *Phys. Lett. B* **226**, 132 (1989); **246**, 471 (1990); H. Dreiner, A. Duff, and D. Zeppenfeld, *ibid.* **282**, 441 (1992).
- [4] L. D. Faddeev and V. N. Popov, *Phys. Lett.* **25B**, 29 (1967).
- [5] L. D. Faddeev, *Teor. Mat. Fiz.* **1**, 3 (1969) [*Theor. Math. Phys.* **1**, 1 (1970)].
- [6] P. Senjanovic, *Ann. Phys. (N.Y.)* **100**, 227 (1976).
- [7] K. Sundermeyer, *Constrained Dynamics* (Springer, Berlin, 1982).
- [8] D. M. Gitman and I. V. Tyutin, *Quantization of Fields with Constraints* (Springer, Berlin, 1990).
- [9] P. T. Matthews, *Phys. Rev.* **76**, 684 (1949).
- [10] C. Bernard and A. Duncan, *Phys. Rev. D* **11**, 848 (1975).
- [11] C. Grosse-Knetter, *Phys. Rev. D* **48**, 2854 (1993).
- [12] C. Grosse-Knetter, Bielefeld Report No. BI-TP 93/29, hep-ph/9306321, 1993 (unpublished).
- [13] P. A. M. Dirac, *Can. J. Math.* **2**, 129 (1950); *Lectures on Quantum Mechanics* (Belfer Graduate School, Yeshiva University, New York, 1964).
- [14] B. W. Lee and J. Zinn-Justin, *Phys. Rev. D* **5**, 3121 (1972); **5**, 3137 (1972); **5**, 3155 (1972); **7**, 1049 (1973).
- [15] E. S. Abers and B. W. Lee, *Phys. Rep.* **9**, 1 (1973).
- [16] Appelquist and Bernard [1]; Longhitano [1]; Leung, Love, and Rao [2]; Grinstein and Wise [2]; Simmons [3]; Gounaris and Renard [2]; Hagiwara, Ishihara, Szalapski, and Zeppenfeld [2].
- [17] M. V. Ostrogradsky, *Mem. Acad. St. Petersburg* **6**, 385 (1850).
- [18] A. Pais and G. Uhlenbeck, *Phys. Rev.* **79**, 145 (1950); S. W. Hawking, in *Quantum Field Theory and Quantum Statistics*, edited by I. A. Batalin, C. J. Isham, and C. A. Vilkovisky (Hilger, Bristol, 1987), p. 129; D. A. Eliezer and R. P. Woodard, *Nucl. Phys.* **B325**, 389 (1989); J. Z. Simon, *Phys. Rev. D* **41**, 3720 (1990).
- [19] D. Barua and S. N. Gupta, *Phys. Rev. D* **16**, 413 (1977); G. Schäfer, *Phys. Lett.* **100A**, 128 (1984); T. Damour and G. Schäfer, *J. Math. Phys.* **32**, 127 (1991); H. Georgi, *Nucl. Phys.* **B361**, 339 (1991); H. Leutwyler, Bern Report No. BUTP-91/26, 1991 (unpublished).
- [20] H. D. Politzer, *Nucl. Phys.* **B172**, 349 (1980); C. Arzt, Michigan Report No. UM-TH-92-28, hep-ph/9304230, 1992 (unpublished).
- [21] S. Kaptanoğlu, *Phys. Lett.* **98B**, 77 (1981); P. Ditsas, *Ann. Phys. (N.Y.)* **167**, 36 (1986).
- [22] J. M. Cornwall, D. N. Levin, and G. Tiktopoulos, *Phys. Rev. D* **10**, 1145 (1974).
- [23] C. Grosse-Knetter and R. Kögerler, *Phys. Rev. D* **48**, 2865 (1993).