Supergravity coupled to chiral matter at one loop

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We extend earlier calculations of the one-loop contributions to the effective Bose Lagrangian in supergravity coupled to chiral matter. We evaluate all logarithmically divergent contributions for arbitrary background scalar fields and space-time metrics. We show that, with a judicious choice of gauge fixing and of the definition of the action expansion, much of the result can be absorbed into a redefinition of the metric and a renormalization of the Kähler potential. Most of the remaining terms depend on the curvature of the Kähler metric. Further simplification occurs in models obtained from superstrings in which the Kähler Riemann tensor is covariantly constant.

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I. INTRODUCTION

Considerable progress has recently been made in understanding Yang-Mills couplings at the quantum level [1, 2] in effective supergravity theories obtained from superstrings. Specifically, it is understood how to cancel the modular anomaly that arises at the quantum level of the effective field theory. From the field theory point of view, the modular anomaly is equivalent to the standard chiral and conformal anomalies of Yang-Mills theories. In particular, the conformal anomaly enters through the dependence of the effective cutoff on the moduli fields [2, 3]. In a general field theory the conformal anomaly entails all operators that have logarithmically divergent coefficients at the quantum level. Understanding the structure of the divergences in the full effective supergravity theory is a necessary step in determining what counterterms are needed to fully restore modular invariance. The determination of these loop corrections may also provide a guide to the construction of an effective theory for a composite chiral multiplet that is a bound state of strongly coupled Yang-Mills superfields, which in turn could shed light on gaugino condensation as a mechanism for supersymmetry breaking.

In Refs. [4, 5] we identified the divergent one-loop contributions to the effective Bose Lagrangian, with a flat space-time background metric, in a general $N = 1$ supergravity theory, with specialization to the no-scale form suggested by superstrings. Here we present the full results for a general supergravity theory coupled to chiral matter with an arbitrary background space-time metric and arbitrary background scalar fields. Partial results for a curved-space time metric have been given in [6, 7], and particularly in [8], where it was shown how to recast the Einstein term in canonical form by a redefinition of the background metric. However, the results are gauge dependent [9], and therefore not very meaningful unless one can isolate those terms that actually contribute to the S matrix. This is the purpose of the present paper. We choose a gauge-fixing prescription which, together with a redefinition of the expansion of the action, enhances supersymmetry cancellations between boson and fermion loop contributions. With these choices, all operators of dimension six or less, and most of those of dimension eight, that do not depend on the Kähler curvature can be either absorbed by field redefinitions or interpreted as renormalizing the Kähler potential. By an operator of dimension d we mean a Kähler invariant operator whose term of lowest dimension is d, where scalar fields are assigned the canonical dimension of unity. In many effective theories from superstrings, such as the untwisted sector in many orbifold compactifications, the Kähler Riemann tensor is covariantly constant; in this case the results simplify further.

In order to complete the program of determining one-loop supergravity, the Yang-Mills sector must be included. We will present the full results in a subsequent paper [10], where we will also consider the parity odd operators that arise from integration over fermionic degrees of freedom. As mentioned above, the effective cutoff of effective theories derived from superstrings is field dependent; moreover the field dependence is different for loop corrections arising from different sectors of the theory [2]. Here we use a single cutoff and neglect its derivatives. The latter does not represent a loss of generality, since terms involving derivatives of the cutoff have a different dependence on the moduli and must be considered together with terms that are one-loop finite. Our results, some of which are collected in an appendix, will be presented in such a way that the contributions from different sectors can be isolated and the correct cutoffs included.

In Sec. II, we discuss gauge fixing and describe the prescription used here. The results of our calculation are presented in a succinct form in Sec. III; further simplifications arising in models from string theory are pointed out in Sec. IV. In Appendix A we define our conventions and give that part of the tree-level Lagrangian that is needed to perform our calculations. In Appendix B we list the operators that appear in the quantum action as defined by our gauge fixing and expansion prescriptions, as well as the traces of products of these operators that

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determine the divergent terms in the effective one-loop action. In a final appendix we list corrections to and misprints in [4, 5].

II. GAUGE FIXING AND THE EXPANSION OF THE ACTION

The S matrix is independent of gauge fixing and also of shifts in the propagators that are proportional to $\mathcal{L}_A = \partial\mathcal{L}/\partial\phi^A$ where ϕ^A is any field. However, certain choices can lead to an effective Lagrangian that better displays the symmetries of the theory. For example, we expand the action S in terms of normal scalar coordinates [11, 12] $\hat{z}^I$,

$$S = S(z) + D_I S\big|_z \hat{z}^I + \tfrac{1}{2} D_I D_J S\big|_z \hat{z}^I \hat{z}^J + \cdots, \qquad (2.1)$$

where D_I is the field redefinition covariant derivative defined in Appendix A, and interpret the determinant of the second term in (2.1) as the one-loop effective action for a scalar theory. The result differs from that of a standard Taylor expansion by terms of the form $F^{JL}(z)\Gamma(z)^I_{JK}(D_I S)_z$, where Γ^I_{JK} is the connection associated with the covariant derivative D_I, and F^{JL} is an arbitrary matrix-valued function of the background scalar fields. Such terms vanish when the classical equations of motion for the background fields z are satisfied. The expansion (2.1) yields an effective action that is manifestly field redefinition invariant. It therefore preserves nonlinear symmetries among the scalar fields, up to quantum anomalies.

Supersymmetry is also a nonlinear symmetry in supergravity theories, even when auxiliary fields are used. We have no formal argument by which we can determine the gauge fixing and expansion prescription so as to yield an effective action that is manifestly supersymmetric.[1] Instead, we adopt a pragmatic approach, and use prescriptions that give the most boson-fermion cancellations, and/or simplify the calculation. We find that with our prescription the operators of dimension six or less can be interpreted as renormalizations of the tree Lagrangian, except for those that depend on the scalar curvature tensor. Additional operators of dimension eight can be isolated into terms of the form $F^{JL}\Gamma^I_{JK} D_I S|_z$, which do not contribute to the S matrix. It turns out that the gauge-fixing prescription that satisfies these properties yields an effective quantum Lagrangian that is of a particularly simple form: all the propagators are the same as those of standard scalar or spin-$\frac{1}{2}$ fermions. It is possible that this feature contributes to the enhanced cancellations. We first discuss the case of flat supersymmetric (SUSY) Yang-Mills theory, where a similar gauge fixing dependence arises [15], and where a "supersymmetric gauge" can be found.

[1]Since we set background fermions to zero, our effective action cannot be manifestly supersymmetric. However supersymmetry constrains [13, 14] the bosonic part of the action; by "manifest supersymmetry" we are referring to these constraints.

A. Supersymmetric Yang-Mills theory

In background field calculations of the effective one-loop action, the Landau gauge fixing condition $\mathcal{D}^\mu \hat{A}_\mu = 0$ has frequently been used [4–6]. In the absence of a superpotential, the dimension four operators of the resulting supergravity Lagrangian for the gauge nonsinglet scalars can be interpreted in terms of two renormalizations. The first is a renormalization of the matrix-valued function $x^a_b(z,\bar{z}) = \mathrm{Re} f(z)^a_b$ that normalizes the Yang-Mills kinetic term $-\frac{1}{4}x^a_b F^b_{\mu\nu}F_{a\mu\nu}$. The second is a renormalization of the Kähler potential $K(z,\bar{z})$, where $z = (\bar{z})^\dagger$ is a complex scalar field. Here (and throughout) we consider the case $x^a_b = \delta^a_b x$ at the tree level, for which the results are

$$\delta K = \frac{\ln\Lambda^2}{32\pi^2}\left[-\frac{2}{x}K_{\bar{m}j}(T_a\bar{z})^{\bar{m}}(T^a z)^j\right] + \text{higher dimension terms}, \qquad (2.2)$$

where T^a represents the gauge group on the scalar fields $z^n = (\bar{z}^{\bar{n}})^\dagger$, and

$$\delta x^b_a = \frac{\ln\Lambda^2}{32\pi^2}\left[2D_i(T_a z)^j D_j(T^b z)^i - 6C^{(a)}_G \delta^b_a\right] + \text{higher dimension terms}, \qquad (2.3)$$

where $C^{(a)}_G$ is the Casimir of the adjoint representation and the field redefinition covariant scalar derivative D_i is defined in Appendix A. The fact that (2.3) is not the real part of a holomorphic function has been discussed elsewhere in the literature (see, e.g., [1]). In the flat SUSY limit $x \to$ const, $K_{i\bar{m}} \to \delta_{im}$, and the renormalizations reduce to constants that depend on the Casimirs of the matter representations R:

$$\delta K_{i\bar{m}} \to -\frac{\ln\Lambda^2}{16\pi^2 x}\sum_a (T_a)^2_{i\bar{m}} = -\delta_{im}\frac{\ln\Lambda^2}{16\pi^2 x}\sum_a C^a_2(R_i),$$

$$\delta x^b_a = \delta^a_b \frac{\ln\Lambda^2}{16\pi^2 x}\mathrm{Tr}(T_a)^2 = \delta^a_b\frac{\ln\Lambda^2}{16\pi^2 x}\sum_R C^a_R.$$

When a superpotential is included, the results obtained in the Landau gauge can no longer be interpreted in terms of these renormalizations. This is similar to the result found in [15]. However, if we use a smeared gauge-fixing prescription defined by

$$\mathcal{L} \to \mathcal{L} - \frac{x}{2}C_a C^a,$$

$$C^a = \mathcal{D}^\mu \hat{A}^a_\mu + \frac{i}{x}\left[(T^a\bar{z})^{\bar{m}}\hat{z}^i - (T^a\bar{z})^i\hat{z}^{\bar{m}}\right]K_{i\bar{m}}, \qquad (2.4)$$

the results can once again be interpreted as above, with, instead of (2.2),

$$\delta K = \frac{\ln\Lambda^2}{32\pi^2}\left(-\frac{4}{x}K_{\bar{m}j}(T_a\bar{z})^{\bar{m}}(T^a z)^j + e^{-K}A_{ij}\bar{A}^{ij}\right) + \text{higher dimension terms}, \qquad (2.5)$$

where A_{ij} is defined in Appendix A; in the flat SUSY limit it reduces to the second derivative of the superpotential W:

$$e^{-K}A_{ij}\bar{A}^{ij} \to e^{K}W_{ij}\overline{W}^{ij}.$$

Note that the gauge-dependent term in (2.5) differs by a factor of 2 from that in (2.2). The result (2.5) agrees with the chiral matter wave function renormalization found in [15] and in a recent string loop calculation [16].

Unlike the Landau gauge, the smeared gauge fixing (2.4) gives a quantum Lagrangian of the simple form (3.1) below. The field-dependent masses have the correct poles for unitarity when evaluated at the ground state configuration for the background fields, i.e., $\mathcal{D}_\mu z = A_\mu = \partial_i V = 0$, where V is the scalar potential. We will use gauge-fixing prescriptions for supergravity that share this feature. In addition, the transformation laws for supergravity are nonlinear even when auxiliary fields are used.[2] This suggests that it may be necessary to redefine [9] the expansion in a manner analogous to (2.1), in order to obtain a manifestly SUSY result.

B. Gauge fixing the gravity supermultiplet

We set background fermions to zero, and use unhatted symbols for quantum fermion fields (ψ,χ,λ).

The commonly used gauge fixing[3] for the graviton [18, 19, 4, 8], when generalized to include the YM sector, is defined by

$$\mathcal{L} \to \mathcal{L} + \frac{1}{2}C_\mu C^\mu,$$

$$C_\mu = \frac{1}{\sqrt{2}}\left(\nabla^\nu h_{\mu\nu} - \frac{1}{2}\nabla_\mu h^\nu_\nu - 2\mathcal{D}_\mu z^I Z_{IJ}\hat{z}^J + xF^a_{\mu\nu}\hat{A}^\nu_a\right), \qquad (2.6)$$

where $Z_{IJ}(z,\bar{z})$ is the scalar metric, $\hat{z}$, $\hat{A}$ are the quantum scalar and gauge fields, and the symmetric tensor $h_{\mu\nu}$ is the quantum part of the gravitational field. Like the smeared Yang-Mills gauge fixing (2.4), this leads to a Lagrangian of the form (3.1).

For the gravitino, two types of gauge fixing have been used: the Landau gauge [20, 4] $\gamma\cdot\psi = 0$, which is implemented with the aid of an auxiliary field, and the smeared gauge-fixing [8] $\mathcal{L} \to \mathcal{L} - \bar{F}\mathcal{M}F$, $F = \gamma\cdot\psi$, $\mathcal{M} = \frac{1}{4}\left(i\not{\mathcal{D}} + 2M_\psi\right)$, which requires Nielsen-Kallosh ghosts. Neither of these has the feature that the quantum Lagrangian reduces to the simple form (3.1). In addition, while the Landau gauge propagators have the correct poles for constant background fields, the smeared gauge fixing propagators do not. Here we adopt an unsmeared gauge which satisfies both requirements.

In a supergravity theory in which the Yang-Mills normalization function satisfies $\mathrm{Re}f_{ab} = \delta_{ab}x$, the part of the Lagrangian that depends on the gravitino ψ_μ is [13, 14]

$$\mathcal{L}_\psi = \frac{1}{4}\bar{\psi}_\mu\gamma^\nu(i\not{\mathcal{D}} + M)\gamma^\mu\psi_\nu - \frac{1}{4}\bar{\psi}_\mu\gamma^\mu(i\not{\mathcal{D}} + M)\gamma^\nu\psi_\nu + \left[\frac{x}{8}\bar{\psi}_\mu\sigma^{\nu\rho}\gamma^\mu\lambda_a F^a_{\nu\rho} - \bar{\psi}_\mu\not{\mathcal{D}}\bar{z}^{\bar{m}}K_{i\bar{m}}\gamma^\mu L\chi^i + \frac{1}{4}\bar{\psi}_\mu\gamma^\mu\gamma_5\lambda^a\mathcal{D}_a - i\bar{\psi}_\mu\gamma^\mu L\chi^i m_i + \text{H.c.}\right] + \text{four-fermion terms}, \qquad (2.7)$$

where

$$\bar{M} = (M)^\dagger = e^{K/2}\left(WR + \overline{W}L\right), \quad R,L = \frac{1}{2}\left(1\pm\gamma_5\right), \quad m_n = (\bar{m}_{\bar{n}})^\dagger = e^{-K/2}D_n(e^K W), \quad \mathcal{D}_a = K_i(T_a z)^i. \qquad (2.8)$$

We take the Landau gauge condition $G = 0$, where

$$G = -\gamma^\nu(i\not{\mathcal{D}} - \bar{M})\psi_\nu - \frac{x}{2}\sigma^{\nu\rho}\lambda_a F^a_{\nu\rho} - 2(\not{\mathcal{D}}z^i K_{i\bar{m}}R\chi^{\bar{m}} + \not{\mathcal{D}}\bar{z}^{\bar{m}}K_{i\bar{m}}L\chi^i) + 2im_I\chi^I - \gamma_5\mathcal{D}_a\lambda^a, \qquad (2.9)$$

which we implement by inserting a δ function in the functional integral over $\hat{f}$. Writing

$$\delta(G) = \int d\alpha \exp\left(i\alpha G\right),$$

and defining

$$\psi' = \psi + \gamma\alpha, \quad \bar{\psi}' = \bar{\psi} + \bar{\alpha}\gamma,$$

we obtain

[2]Once the auxiliary fields have been eliminated, the transformation laws for fermions in flat supersymmetry are also nonlinear. However it is easy to show that eliminating these fields before or after functional integration gives the same one-loop effective action.

[3]The gauge fixing of supergravity using superfields is considered in [17], where it is necessary to introduce "ghosts of ghosts" because the Faddeev-Popov action has itself a gauge invariance, as well as so-called "hidden" ghosts because the gauge smearing parameters are constrained. The component action gauge fixing we describe here has no such proliferation of ghosts.

$$\begin{aligned}\mathcal{L} &= -\frac{1}{2}\bar{\psi}'^{\mu}(i\not{D}-\bar{M})\psi'_{\mu} + \frac{1}{2}\bar{\alpha}\gamma^{\mu}(i\not{D}-\bar{M})\gamma_{\mu}\alpha + \text{matter terms} \\ &= -\frac{1}{2}\bar{\psi}'^{\mu}(i\not{D}-\bar{M})\psi'_{\mu} - \bar{\alpha}(i\not{D}+2M)\alpha + \bar{\alpha}\left(\frac{x}{2}\sigma^{\nu\rho}\lambda_a F^a_{\nu\rho} + 2im_I\chi^I - \gamma_5\mathcal{D}_a\lambda^a\right) \\ &\quad -ix\bar{\psi}'_{\mu}\not{F}^{\mu}_{a}\lambda^a - 2\bar{\psi}'_{\mu}(\mathcal{D}^{\mu}\bar{z}^{\bar{m}}K_{i\bar{m}}L\chi^i + \mathcal{D}^{\mu}z^iK_{i\bar{m}}R\chi^{\bar{m}}).\end{aligned} \tag{2.10}$$

Note that ψ is C even: $\psi = C\bar{\psi}^T$, then $\psi' = C\bar{\psi}'^T$ requires $\alpha = -C\bar{\alpha}^T$, i.e., α is C odd; note also that α has negative metric.[4] All the terms remaining in the Lagrangian (2.10) are of the form of either a mass or a connection; that is, (2.10) is of the form (3.1).

To obtain the ghostino determinant we use the supersymmetry transformations [13]

$$\begin{aligned} i\delta\psi_{\mu} &= (iD_{\mu} - \tfrac{1}{2}\gamma_{\mu}M)\epsilon, \quad i\delta\chi^i = \frac{1}{2}(\not{D}z^iR - i\bar{m}^iL)\epsilon, \quad i\delta\chi^{\bar{m}} = \left[\frac{1}{2}(\not{D}\bar{z}^{\bar{m}}L - im^{\bar{m}}R)\right]\epsilon, \\ \bar{m}^i &= K^{i\bar{m}}\bar{m}_{\bar{m}}, \quad m^{\bar{m}} = K^{i\bar{m}}m_i, \quad i\delta\lambda^a = \left[-\frac{i}{4}\gamma^{\mu}\gamma^{\nu}F^a_{\mu\nu} - \frac{1}{2x}\mathcal{D}^a\right]\epsilon, \end{aligned} \tag{2.11}$$

to obtain

$$\begin{aligned}\frac{\partial\delta G}{\partial\epsilon} &= D^{\mu}D_{\mu} - \frac{1}{2}\gamma^{\mu}\gamma^{\nu}[D_{\mu},D_{\nu}] - i[\not{D},M] - 2M\bar{M} + \bar{m}^im_i + \mathcal{D} + 2i\bar{m}_{\bar{m}}\not{D}\bar{z}^{\bar{m}}L + 2im_i\not{D}z^iR \\ &\quad + \frac{x}{2}\sigma_{\sigma\rho}F^{\sigma\rho}_a\left(\frac{1}{4}\sigma^{\mu\nu}F^a_{\mu\nu} + \frac{1}{x}\gamma_5\mathcal{D}^a\right) - \mathcal{D}_{\mu}z^iK_{i\bar{m}}\mathcal{D}^{\mu}\bar{z}^{\bar{m}} + \frac{1}{2}\gamma_5[\gamma^{\mu},\gamma^{\nu}]\mathcal{D}_{\mu}\bar{z}^{\bar{m}}K_{i\bar{m}}\mathcal{D}_{\nu}z^i.\end{aligned} \tag{2.12}$$

For constant background fields the ghostino propagator becomes

$$D^{\mu}D_{\mu} - 2M\bar{M} + \bar{m}^im_i + \mathcal{D} = D^{\mu}D_{\mu} + M\bar{M} + V, \tag{2.13}$$

where V is the potential. When we evaluate this at a ground state with a flat background metric, the vacuum energy necessarily vanishes: $V = 0$, so the (fourfold) ghostino pole is at $p^2 = -D^2 = M^2$. If the cosmological constant is nonzero the curvature is also, and there are additional terms in all the masses.

Now the goldstino is unmixed with the gravitino, but instead mixes with α. The normalized (left-handed) goldstino field χ_L is

$$\chi_L = \left(m_i\chi^i_L - \frac{i}{2}\mathcal{D}_a\lambda^a_L\right)\Big/\sqrt{\frac{1}{2}(m_i\bar{m}^i + \mathcal{D})}, \tag{2.14}$$

and its mass is

$$m_{\chi} = e^{-K}\left(e^{-K}A_{ij}\bar{A}^i\bar{A}^j + 4\mathcal{D}\bar{A} - \frac{1}{2x}\mathcal{D}f_i\bar{A}^i\right)\Big/\left(e^{-K}A_i\bar{A}^i + \mathcal{D}\right), \tag{2.15}$$

where λ is a gaugino, χ^i is the left-handed superpartner of z^i, and

$$A_{ij} = D_iA_j = D_iD_jA, \quad A = e^KW = \bar{M}.$$

At the ground state

$$\begin{aligned} V_i = 0 = \bar{A}^iV_i &= e^{-K}A_{ij}\bar{A}^i\bar{A}^j + 2\mathcal{D}\bar{A} \\ &\quad - \frac{1}{2x}\mathcal{D}f_i\bar{A}^i - 2e^{-K}\bar{A}^iA_i\bar{A}. \end{aligned} \tag{2.16}$$

Using this gives $m_{\chi} = 2M$.

Here we show that unitarity is satisfied in the case where there are no gauge couplings: $\mathcal{D}_a = 0$; the argument goes through in the same way when gauge couplings are included [10]. The normalized [4] (α,χ) mass matrix is

$$\bar{M}_{\frac{1}{2}} = \begin{pmatrix} M^i_{\bar{m}} & M^i_{\alpha} \\ M^{\alpha}_{\bar{m}} & M^{\alpha}_{\alpha}\end{pmatrix} = \begin{pmatrix} \bar{\mu}^i_{\bar{m}} & i\bar{m}^i \\ i\bar{m}_{\bar{m}} & -2\bar{M}\end{pmatrix}, \tag{2.17}$$

where

$$\mu_{ij} = (\bar{\mu}_{\bar{i}\bar{j}})^{\dagger} = e^{-K/2}A_{ij}, \quad \bar{\mu}^{ij} = K^{i\bar{m}}K^{j\bar{n}}\bar{\mu}_{\bar{m}\bar{n}},$$

is the normalized mass matrix for left-handed chiral fermions. In the traces used to evaluate the one-loop effective action (see Sec. III) this gets multiplied by

$$M_{\frac{1}{2}} = \begin{pmatrix} M^{\bar{m}}_j & M^{\bar{m}}_{\alpha} \\ M^{\alpha}_j & M^{\alpha}_{\alpha}\end{pmatrix} = \begin{pmatrix} \mu^{\bar{m}}_j & im^{\bar{m}} \\ im_j & -2M\end{pmatrix}, \tag{2.18}$$

so

$$\bar{M}_{\frac{1}{2}}M_{\frac{1}{2}} = \begin{pmatrix} \bar{\mu}^{ik}\mu_{kj} - \bar{m}^im_j & i\bar{\mu}^{ik}m_k - 2iM\bar{m}^i \\ i\bar{m}^k\mu_{kj} - 2i\bar{M}m_j & 4M\bar{M} - \bar{m}^km_k\end{pmatrix}. \tag{2.19}$$

For the goldstino at a ground state with vanishing cosmo-

[4] In the notation of (3.1), $Z_{\alpha\alpha} = -2$; including the contribution proportional to $\mathrm{Det}Z_{\alpha\alpha}$ we get a quartically divergent term proportional to $\ln 2$ which cancels a similar contribution from the graviton ghost [4].

logical constant [see (2.13) and (2.16)], $\mu_{ij} \to 2M$, $m_i \to \sqrt{3}M$, so the α-goldstino squared mass matrix reduces to

$$\bar{M}_{\frac{1}{2}} M_{\frac{1}{2}} \to M\bar{M} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

The ghostino determinant removes four poles at $p^2 = M^2$ and the unphysical fields χ and α restore two of them, so the singularities are correct. Note that because α is C odd while χ is C even, $M_{\frac{1}{2}}$, which operates on left-handed fermions, is not the Hermitian conjugate of $\bar{M}_{\frac{1}{2}}$, which operates on right-handed fermions.

C. Modification of the graviton propagator

The S matrix is unchanged if we add terms proportional to $\mathcal{L}_A$ to the propagators, as in (2.1). Consider the graviton-scalar sector. We have

$$\begin{aligned}
\mathcal{L}_i &= -\left(K_{i\bar{m}} D^\mu \mathcal{D}_\mu \bar{z}^{\bar{m}} + V_i\right), \\
\mathcal{L}_{\mu\nu} &= \frac{1}{2} g_{\mu\nu} \left(\frac{r}{2} - V + K_{i\bar{m}} \mathcal{D}_\rho z^i \mathcal{D}^\rho \bar{z}^{\bar{m}}\right) - \frac{1}{2} r_{\mu\nu} \\
&\quad - \frac{1}{2}\left(K_{i\bar{m}} \mathcal{D}_\mu z^i \mathcal{D}_\nu \bar{z}^{\bar{m}} + K_{i\bar{m}} \mathcal{D}_\nu z^i \mathcal{D}_\mu \bar{z}^{\bar{m}}\right), \\
\mathcal{L}^\mu_\mu &= \frac{r}{2} - 2V + K_{i\bar{m}} \mathcal{D}_\rho z^i \mathcal{D}^\rho \bar{z}^{\bar{m}},
\end{aligned}$$

where $g_{\mu\nu}$ is the background metric. We can redefine the graviton propagator by

$$\Delta^{-1}_{I\mu\nu} = \frac{1}{\sqrt{g}} \left(D_I D_{\mu\nu} S - \frac{1}{2} g_{\mu\nu} D_I S\right), \tag{2.20}$$

and

$$\begin{aligned}
\Delta^{-1}_{\mu\nu,\rho\sigma} &\to \Delta^{-1}_{\mu\nu,\rho\sigma} - P_{\mu\nu,\rho\sigma} \mathcal{L}^\lambda_\lambda - \frac{1}{2}\left[g_{\mu\nu}\mathcal{L}_{\rho\sigma} + g_{\rho\sigma}\mathcal{L}_{\mu\nu}\right] \\
&\quad + \frac{1}{2}\left[g_{\mu\rho}\mathcal{L}_{\nu\sigma} + g_{\nu\rho}\mathcal{L}_{\mu\sigma} + g_{\mu\sigma}\mathcal{L}_{\nu\rho} + g_{\nu\sigma}\mathcal{L}_{\mu\rho}\right] \\
&\equiv -\left(P\nabla^2 + X\right)_{\mu\nu,\rho\sigma},
\end{aligned} \tag{2.21}$$

where the spin-2 projection operator P is defined in (B2), and

$$\mathcal{L}_{\mu\nu} = g_{\mu\rho} g_{\nu\sigma} \frac{\partial \mathcal{L}}{\partial g_{\rho\sigma}}.$$

The unmodified propagators have been evaluated[5] elsewhere [18, 19]; using these results in the above we get

$$X_{\mu\nu,\rho\sigma} = -2P_{\mu\nu,\rho\sigma} V - \frac{1}{4}\left[r_{\mu\rho\nu\sigma} + r_{\nu\rho\mu\sigma}\right]. \tag{2.22}$$

Evaluating the determinants in (3.2) below gives an effective Lagrangian including terms linear and quadratic in the space-time curvature:

$$\mathcal{L}_1 \ni \mathcal{L}_r = \frac{1}{2}\sqrt{g}\Bigg[\epsilon_0(z,\bar{z}) r + H_{\mu\nu}\left(\mathcal{D}_\rho z, \mathcal{D}_\rho \bar{z}, F_{\rho\sigma}\right) r^{\mu\nu} + \alpha r^2 + \beta r^{\mu\nu} r_{\mu\nu}\Bigg]. \tag{2.23}$$

The Einstein term can be put in canonical form by a redefinition of the metric [8]

$$\begin{aligned}
g_{\mu\nu} &= (1-\epsilon) g^R_{\mu\nu} + \epsilon_{\mu\nu}, \\
\epsilon &= \epsilon' + \alpha r, \quad \epsilon_{\mu\nu} = \mathcal{H}_{\mu\nu} + \beta r_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu}(\mathcal{H}^\lambda_\lambda + \beta r), \\
\epsilon' &= \epsilon_0 + (4\alpha + \beta) V, \\
\mathcal{H}_{\mu\nu} &= H_{\mu\nu} - \beta g_{\mu\nu} \frac{x}{4} F_{\sigma\rho} F^{\sigma\rho} - 2\alpha g_{\mu\nu} \mathcal{D}_\rho z^i \mathcal{D}^\rho \bar{z}^{\bar{m}} K_{i\bar{m}} \\
&\quad + \beta\Bigg(-\mathcal{D}_\mu z^i \mathcal{D}_\nu \bar{z}^{\bar{m}} K_{i\bar{m}} \\
&\qquad - \mathcal{D}_\nu z^i \mathcal{D}_\mu \bar{z}^{\bar{m}} K_{i\bar{m}} + x F_{\mu\rho} F_\nu{}^\rho\Bigg).
\end{aligned} \tag{2.24}$$

This induces additional matter terms:

$$\begin{aligned}
\mathcal{L}(g) + \mathcal{L}_1(g) &= \mathcal{L}(g_R) + \mathcal{L}_1 - \mathcal{L}_r \\
&\quad + \sqrt{g}\Bigg(2\epsilon' V - \epsilon' \mathcal{D}_\mu z^i \mathcal{D}^\mu \bar{z}^{\bar{m}} K_{i\bar{m}} + \frac{1}{2}\mathcal{H}^\mu_\mu V \\
&\quad - \mathcal{H}^{\mu\nu} \mathcal{D}_\mu z^i \mathcal{D}_\nu \bar{z}^{\bar{m}} K_{i\bar{m}} + \frac{x}{2}\mathcal{H}^{\mu\nu} F_{\mu\rho} F_\nu{}^\rho \\
&\quad - \frac{x}{8}\mathcal{H}^\nu_\nu F_{\mu\rho} F^{\mu\rho}\Bigg),
\end{aligned} \tag{2.25}$$

where the tree Lagrangian $\mathcal{L}(g)$ is given in Appendix A. Note that any terms containing factors of $\mathcal{L}_{\mu\nu}$ that can appear in $\mathcal{L}_1$ are completely removed by this metric redefinition.

III. THE ONE-LOOP EFFECTIVE ACTION

In the absence of gauge fields, the quantum action obtained by the prescriptions defined in the preceding section takes the form

$$\begin{aligned}
\mathcal{L}_q = &-\frac{1}{2}\Phi^T Z_\Phi \left(D^2 + H_\Phi\right)\Phi + \frac{1}{2}\bar{\Theta} Z_\Theta \left(i\not{D} - M_\Theta\right)\Theta \\
&+ \mathcal{L}_{\rm gh} + \mathcal{L}_{\rm Gh}.
\end{aligned} \tag{3.1}$$

The last two terms are the ghost and ghostino terms, respectively, $\Phi = (h_{\mu\nu}, \hat{z}^i, \hat{\bar{z}}^{\bar{m}})$ is a $2N + 10$ component scalar, $\Theta = (\psi_\mu, \chi^I = L\chi^i + R\chi^{\bar{\imath}}, \alpha)$ is an $N + 5$ component Majorana fermion, where N is the number of chiral multiplets, and the matrix valued metrics Z_Φ and Z_Θ, as well as the matrix-valued covariant derivative D_μ, are defined in Appendix A. The one-loop contribution to the effective action is

$$\begin{aligned}
\mathcal{L}_1 = &\frac{i}{2}{\rm Tr}\ln(D^2 + H_\Phi) - \frac{i}{2}{\rm Tr}\ln(-i\not{D} + M_\Theta) \\
&+ i{\rm Tr}\ln(D^2 + M^2_{\rm Gh}) - i{\rm Tr}\ln(D^2 + M^2_{\rm gh}).
\end{aligned} \tag{3.2}$$

Because of the simple form of (3.1) we can immediately apply the general results obtained in [12, 6, 4] to evaluate the determinants:[6]

---

[5] As a check, we have also calculated the curvature-dependent terms using the unmodified propagators; we agree with the results of [18], but not with [8] for these terms.

[6] The expression for the logarithmically divergent term agrees with the one given in [21].

$$\frac{i}{2}\mathrm{Tr}\ln(D^2+H_\Phi)=\sqrt{g}\left\{\frac{\Lambda^2}{32\pi^2}\mathrm{Tr}\left(\frac{1}{6}r-H_\Phi\right)+\frac{\ln\Lambda^2}{32\pi^2}\mathrm{Tr}\left(\frac{1}{2}H_\Phi^2-\frac{1}{6}rH_\Phi+\frac{1}{12}G_{\mu\nu}G^{\mu\nu}+\frac{1}{120}\left[r^2+2r^{\mu\nu}r_{\mu\nu}\right]\right)\right\}, \tag{3.3}$$

and since

$$-\frac{i}{2}\mathrm{Tr}\ln(-i\not{D}+M_\Theta)=-\frac{i}{4}\mathrm{Tr}\ln[D^2+H_\Theta], H_\Theta=M_\Theta^2-i[\not{D},M_\Theta]+\tfrac{1}{4}[\gamma^\mu,\gamma^\nu]G_{\mu\nu}, \tag{3.4}$$

the fermion trace is $-\frac{1}{2}$ times (3.3) with the substitution $H_\Phi\to H_\Theta$, and the trace includes a trace over Dirac indices, so

$$\frac{1}{2}(\mathrm{Tr}\ 1)_\Theta=(\mathrm{Tr}\ 1)_\Phi=2N+10.$$

Similarly, the ghost and ghostino contributions are equivalent to, respectively, -2 and $+2$ times the contribution of a four-component scalar with the masses $M_{\rm gh}^2=H_{\rm gh}$ and connections as determined in Sec. II. The matrix elements of H and

$$G_{\mu\nu}=[D_\mu,D_\nu] \tag{3.5}$$

are given in Appendix B.

The traces in (3.3)–(3.5) are explicitly evaluated in Appendix B; here we simply state the result. If $\mathcal{L}(g,K)$ is the standard Lagrangian [13, 14] for $N=1$ supergravity coupled to matter with space-time metric $g_{\mu\nu}$ and Kähler potential K, then the logarithmically divergent part of the one-loop-corrected Lagrangian is

$$\begin{aligned}\mathcal{L}_{\rm eff}=\mathcal{L}\left(g_R,K_R\right)+\sqrt{g}\frac{\ln\Lambda^2}{32\pi^2}\Bigg\{&e^{-2K}\Big[A_i\bar{A}^kR^{m\ i}_{n\ k}R^{n\ p}_{m\ q}A_p\bar{A}^q-4R^{m\ i}_{n\ k}A_i\bar{A}^kA_m\bar{A}^n-\frac{2}{3}R^m_nA_m\bar{A}^nA_j\bar{A}^j\\&+(R^{j\ k}_{n\ i}A_{jk}\bar{A}^nA\bar{A}^i+\text{H.c.})-R^{j\ k}_{\ell\ i}A_{jk}\bar{A}^{\ell n}A_n\bar{A}^i-(D^\ell R^{j\ k}_{n\ i})A_{jk}\bar{A}^nA_\ell\bar{A}^i-R^{j\ k}_{n\ i}R^{\ell\ m}_{j\ k}A_\ell\bar{A}^nA_m\bar{A}^i\\&-R^{\ell\ m}_{j\ k}A_{i\ell}\bar{A}^{jk}A_m\bar{A}^i-(D_iR^{\ell\ m}_{j\ k})A_\ell\bar{A}^{jk}A_m\bar{A}^i\Big]+8\hat{V}^2+\frac{2}{3}(N+5)\hat{V}M_\psi^2+(N+5)M_\psi^4\\&+\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}^{\bar{m}}\Bigg(e^{-K}\Big[-\frac{2}{3}K_{i\bar{m}}R^k_nA_k\bar{A}^n+2R^k_{i\bar{m}j}R^{\ell\ j}_{n\ k}A_\ell\bar{A}^n-4R^k_{i\bar{m}j}A_k\bar{A}^j\\&\qquad-R^{j\ k}_{n\ i}A_{jk}\bar{A}^n_{\bar{m}}-(D_{\bar{m}}R^{j\ k}_{n\ i})A_{jk}\bar{A}^n-R^\ell_{j\bar{m}k}A_{i\ell}\bar{A}^{jk}-(D_iR^\ell_{j\bar{m}k})A_\ell\bar{A}^{jk}\Big]\\&\qquad+\left[\frac{1}{3}(N+29)\hat{V}+\frac{2}{3}(N+5)M_\psi^2\right]K_{i\bar{m}}-2R_{i\bar{m}}\left[\frac{1}{3}\hat{V}+M_\psi^2\right]\Bigg)\\&+e^{-K}\left(\mathcal{D}_\mu z^i\mathcal{D}^\mu z^j[A_{ik\ell}\bar{A}^{\bar{n}}R^{k\ \ell}_{n\ j}-R^{k\ \ell}_{j\ i}(A_{mk\ell}\bar{A}^m-A_{k\ell}\bar{A})]+\text{H.c.}\right)\\&-4\left(\mathcal{D}_\mu\bar{z}^{\bar{m}}\mathcal{D}^\mu z^iK_{i\bar{m}}\right)^2+\left(\frac{N}{6}+7\right)\mathcal{D}_\mu z^j\mathcal{D}^\mu z^i\mathcal{D}_\nu\bar{z}^{\bar{m}}\mathcal{D}^\nu\bar{z}^{\bar{n}}K_{i\bar{n}}K_{j\bar{m}}\\&+\frac{32}{3}\mathcal{D}_\mu\bar{z}^{\bar{m}}\mathcal{D}^\mu z^i\mathcal{D}_\nu\bar{z}^{\bar{n}}\mathcal{D}^\nu z^jK_{i\bar{n}}K_{j\bar{m}}-\frac{2}{3}\mathcal{D}_\rho z^i\mathcal{D}^\rho\bar{z}^{\bar{m}}K_{i\bar{m}}\mathcal{D}^\mu z^j\mathcal{D}_\mu\bar{z}^{\bar{n}}R_{j\bar{n}}\\&+\mathcal{D}_\mu z^j\mathcal{D}^\mu\bar{z}^{\bar{m}}R^k_{j\bar{m}i}\mathcal{D}_\nu z^\ell\mathcal{D}^\nu\bar{z}^{\bar{n}}R^i_{\ell\bar{n}k}+\mathcal{D}_\mu z^j\mathcal{D}^\mu z^iR^{k\ \ell}_{\ j\ i}\mathcal{D}_\nu\bar{z}^{\bar{n}}\mathcal{D}^\nu\bar{z}^{\bar{m}}R_{\bar{n}k\bar{m}\ell}\\&+4\mathcal{D}_\mu\bar{z}^{\bar{m}}\mathcal{D}^\mu\bar{z}^{\bar{n}}\mathcal{D}_\nu z^j\mathcal{D}^\nu z^iR_{\bar{n}j\bar{m}i}-4\mathcal{D}_\mu\bar{z}^{\bar{m}}\mathcal{D}^\mu z^i\mathcal{D}_\nu z^j\mathcal{D}^\nu\bar{z}^{\bar{n}}R_{\bar{m}j\bar{n}i}\\&+\frac{1}{2}\left[\mathcal{D}_\mu z^j\mathcal{D}_\nu\bar{z}^{\bar{m}}R^k_{i\bar{m}j}\mathcal{D}^\mu z^\ell\mathcal{D}^\nu\bar{z}^{\bar{n}}R^i_{k\bar{n}\ell}-\mathcal{D}_\mu z^j\mathcal{D}_\nu\bar{z}^{\bar{m}}R^k_{i\bar{m}j}\mathcal{D}^\nu z^\ell\mathcal{D}^\mu\bar{z}^{\bar{n}}R^i_{k\bar{n}\ell}\right]+4\left(\mathcal{L}_i\bar{A}^iAe^{-K}+\text{H.c.}\right)\Bigg\}.\end{aligned} \tag{3.6}$$

The classical Lagrangian $\mathcal{L}(g,K)$ is given in Appendix A. Since we are neglecting gauge couplings, the gauge covariant derivative $\mathcal{D}_\mu$ is here an ordinary derivative: $\mathcal{D}_\mu \to \partial_\mu$. The renormalized Kähler potential is

$$K_R = K + \frac{\ln\Lambda^2}{32\pi^2}e^{-K}\left[A_{ij}\bar{A}^{ij} - 2A_i\bar{A}^i - 4A\bar{A}\right], \quad A = e^K W = (\bar{A})^\dagger, \quad A_i = D_i A, \quad \bar{A}^i = K^{i\bar{m}}D_{\bar{m}}\bar{A}, \text{ etc.,} \tag{3.7}$$

and the field redefinition covariant derivative D_i is defined in Appendix A. The renormalized space-time metric is given by

$$\begin{aligned} g_{\mu\nu} &= (1-\epsilon)g^R_{\mu\nu} + \epsilon_{\mu\nu}, \\ \epsilon &= -\frac{\ln\Lambda^2}{32\pi^2}\left[e^{-K}\left(A_{ki}\bar{A}^{ik} - \frac{1}{3}R^k_n A_k\bar{A}^n\right) + \frac{N+17}{2}\hat{V} + \frac{2N+16}{3}M^2_\psi + \frac{2}{3}r\right], \\ \epsilon_{\mu\nu} &= \frac{\ln\Lambda^2}{32\pi^2}\Bigg\{\left[\frac{N-19}{6}g_{\mu\nu}\mathcal{D}_\rho z^i\mathcal{D}^\rho\bar{z}^{\bar{m}} - \frac{N+29}{6}\left(\mathcal{D}_\mu z^i\mathcal{D}_\nu\bar{z}^{\bar{m}} + \mathcal{D}_\nu z^i\mathcal{D}_\mu\bar{z}^{\bar{m}}\right)\right]K_{i\bar{m}} \\ &\quad - \frac{2}{3}R_{i\bar{m}}g_{\mu\nu}\mathcal{D}_\rho z^i\mathcal{D}^\rho\bar{z}^{\bar{m}} + \frac{1}{6}(N+5)r_{\mu\nu}\Bigg\}. \end{aligned} \tag{3.8}$$

The term in (3.6) proportional to $\mathcal{L}_I$ can be removed by a (nonholomorphic) scalar field redefinition:

$$z^i \to z^i + X^i, \quad \mathcal{L}(z) \to \mathcal{L}(z) + X^i\mathcal{L}_i,$$

$$X^i = -\frac{\ln\Lambda^2}{8\pi^2}\bar{A}^i A e^{-K}.$$

The quadratically divergent contributions are given by (B20) and (B21). However, as emphasized in [4], the relative coefficients of the quadratically divergent terms are unreliable as they depend on the explicit regularization procedure used [6, 22]. Therefore we have actually only identified all the ultraviolet divergent terms at one loop in the effective bosonic Lagrangian of supergravity theories, and determined the coefficients of the logarithmically divergent terms. The full quadratically divergent one-loop correction to the effective Lagrangian for a toy model [23] has been determined [22] for the leading term in the number $N_{\rm ns}$ of gauge nonsinglet chiral multiplets, for which the definition of a Pauli-Villars regularization scheme consistent with the requisite symmetries is straightforward; the effective cutoff in that scheme coincides with the one required [2] for consistency, within a supersymmetric theory, between the chiral and conformal anomalies under modular transformations in target space; the conformal anomaly is related to the choice of cutoff, while the axial anomaly is finite and unambiguously determined. Defining consistent regularization schemes for higher spin loops appears much more problematic. Moreover, in realistic theories the effective cutoffs appearing in different terms will not even have a uniform dependence on the scalar fields. The issue of removing the breaking of modular invariance induced by the quadratically divergent terms has yet to be addressed.

IV. STRING MODELS

We have shown that most of the Kähler curvature-independent terms that appear in the logarithmically divergent one-loop contributions to the effective supergravity action can be absorbed into field redefinitions or interpreted as a renormalization of the Kähler potential of the standard classical Lagrangian. The curvature-dependent terms vanish for models with a minimal kinetic term: $K_{i\bar{m}} = \delta_{im}$. More interesting for string phenomenology is a class of theories in which the Kähler potential separates into disconnected sectors that depend on different subsets α of chiral fields:

$$\begin{aligned} &K = \sum K^\alpha, \quad \partial_i\partial_j K^\alpha = n_\alpha K^\alpha_i K^\alpha_j, \\ &R^\alpha_{\bar{n}j\bar{m}i} = -n_\alpha(K^\alpha_{i\bar{n}}K^\alpha_{j\bar{m}} + K^\alpha_{j\bar{n}}K^\alpha_{i\bar{m}}), \quad D_i R_{\bar{n}j\bar{m}k} = 0, \end{aligned} \tag{4.1}$$

because the metric is covariantly constant. These results apply to Witten's toy model [23] and to the untwisted sector of orbifold compactifications [24]. In models with three matter generations in the untwisted sector, there is further simplification because [24, 25] $\partial_i\partial_m W = 0$ if $K_{i\bar{m}} \neq 0$ and also $n_\alpha = +1$ for all α. This is true for the three matter + moduli generations, as well as for the dilaton, which (neglecting nonperturbative effects) has no superpotential. In this case one finds, for the covariant derivatives,

$$A_{ij} = 0 \quad \text{if} \quad \alpha_i = \alpha_j. \tag{4.2}$$

(Here the notation $\alpha_i = \alpha_j$ means "if i and j belong to the same subset.") Then since

$$R^{i\ j}_{\ m\ n} = 0 \ \text{ if } \alpha_i \neq \alpha_j, \tag{4.3}$$

the result (3.6) reduces to

$$
\begin{aligned}
\mathcal{L}_{eff} = \mathcal{L}\left(g_R, K_R\right) + \sqrt{g}\frac{\ln\Lambda^2}{32\pi^2}\Bigg\{ & e^{-2K}\left[\sum_\alpha (N+7)\,(A_i\bar{A}^i)^2_\alpha - \frac{2}{3}A_i\bar{A}^i\sum_\alpha N_\alpha (A_j\bar{A}^j)_\alpha\right] \\
& +2\left(\frac{1}{3}V + M_\psi^2\right)e^{-K}\sum_\alpha \left(A_i\bar{A}^i\right)_\alpha + 2\hat{V}^2 + \frac{2}{3}\left(N-1\right)\hat{V}M_\psi^2 + (N-1)M_\psi^4 \\
& +\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}^{\bar{m}}e^{-K}\left[-\frac{2}{3}K_{i\bar{m}}\sum_\alpha N_\alpha (A_j\bar{A}^j)_\alpha + \sum_\alpha \left(2N_\alpha + 8\right)K^\alpha_{i\bar{m}}(A_j\bar{A}^j)_\alpha\right] \\
& +6e^{-K}\sum_\alpha \left(\mathcal{D}_\mu z^i A^i\right)_\alpha \left(\mathcal{D}^\mu \bar{z}^{\bar{m}}\bar{A}_{\bar{m}}\right)_\alpha \\
& +\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}^{\bar{m}}e^{-K}\left[\left(\frac{N+27}{3}\hat{V} + \frac{2}{3}(N+2)M_\psi^2\right)K_{i\bar{m}} - 2R_{i\bar{m}}\left(\frac{1}{3}\hat{V} + M_\psi^2\right)\right] \\
& -4\left(\mathcal{D}_\mu\bar{z}^{\bar{m}}\mathcal{D}^\mu z^i K_{i\bar{m}}\right)^2 + \left(\frac{N}{6}+7\right)\mathcal{D}_\mu z^j\mathcal{D}^\mu z^i\mathcal{D}_\nu\bar{z}^{\bar{m}}\mathcal{D}^\nu\bar{z}^{\bar{n}}K_{i\bar{n}}K_{j\bar{m}} \\
& +\frac{32}{3}\mathcal{D}_\mu\bar{z}^{\bar{m}}\mathcal{D}^\mu z^i\mathcal{D}_\nu\bar{z}^{\bar{n}}\mathcal{D}^\nu z^j K_{i\bar{n}}K_{j\bar{m}} + \frac{2}{3}\mathcal{D}_\rho z^i\mathcal{D}^\rho\bar{z}^{\bar{m}}K_{i\bar{m}}\sum_\alpha \left(N_\alpha+1\right)\mathcal{D}^\mu z^j\mathcal{D}_\mu\bar{z}^{\bar{n}}K^\alpha_{j\bar{n}} \\
& +\sum_\alpha\Bigg[\left(N_\alpha + \frac{11}{2}\right)\left(\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}^{\bar{m}}K^\alpha_{i\bar{m}}\right)^2 + \frac{1}{2}\left(N_\alpha - 5\right)\mathcal{D}_\mu z^j\mathcal{D}^\mu z^i\mathcal{D}_\nu\bar{z}^{\bar{n}}\mathcal{D}^\nu\bar{z}^{\bar{m}}K^\alpha_{i\bar{m}}K^\alpha_{j\bar{n}} \\
& \qquad -\frac{1}{2}\left(N_\alpha - 8\right)\mathcal{D}_\mu\bar{z}^{\bar{m}}\mathcal{D}^\mu z^i\mathcal{D}_\nu z^j\mathcal{D}^\nu\bar{z}^{\bar{n}}K^\alpha_{j\bar{m}}K^\alpha_{i\bar{n}}\Bigg]\Bigg\},
\end{aligned}
\tag{4.4}
$$

where

$$N = \sum_\alpha N_\alpha, \quad (A_iB^i)_\alpha = \sum_{i\in\alpha} A_iB^i,$$

and, for the dilaton s,

$$N_s = 1, \quad A_s\bar{A}^s = A\bar{A}. \tag{4.5}$$

Our results will be extended to include the gauge sector in [10].

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APPENDIX A: CONVENTIONS AND NOTATION

Our Dirac matrices and space-time metric signature $(+---)$ are those of Bjorken and Drell or Itzykson and Zuber. We use upper case notation (R, Γ) for derivatives of the Kähler metric, and lower case (r, γ) for those of the space-time metric. Our sign conventions for, respectively, the Riemann tensor, Ricci tensor, and curvature scalar are

$$
\begin{aligned}
& r^\mu_{\nu\rho\sigma} = g^{\mu\lambda}r_{\lambda\nu\rho\sigma} = \partial_\sigma\gamma^\mu_{\nu\rho} - \partial_\rho\gamma^\mu_{\nu\sigma} + \gamma^\mu_{\sigma\lambda}\gamma^\lambda_{\nu\rho} - \gamma^\mu_{\rho\lambda}\gamma^\lambda_{\nu\sigma}, \\
& r_{\mu\nu} = r^\rho_{\mu\rho\nu}, \quad r = g^{\mu\nu}r_{\mu\nu},
\end{aligned}
\tag{A1}
$$

and covariant differentiation is defined by

$$\nabla_\mu A_\nu = \partial_\mu A_\nu - \gamma^\rho_{\mu\nu}A_\rho, \quad \text{etc.} \tag{A2}$$

The scalar field redefinition covariant quantities are defined identically with

$$g_{\mu\nu} \to Z_{IJ}, \quad \gamma \to \Gamma, \quad r \to R, \quad \nabla_\mu \to D_I, \quad I = i, \bar{\imath}, \tag{A3}$$

where z^i, $\bar{z}^{\bar{m}} = (z^m)^\dagger$ are the scalar partners of left- and right-handed Weyl fermions, respectively. Because the scalar metric is Kähler, there is only one type of nonvanishing element of the Riemann tensor: namely,

$$
\begin{aligned}
R^i_{jk\bar{m}} &= \partial_{\bar{m}}\Gamma^i_{jk} = D_{\bar{m}}\Gamma^i_{jk} = -R^i_{j\bar{m}k}, \\
R_{\bar{n}jk\bar{m}} &= R_{\bar{n}kj\bar{m}} = R_{\bar{m}jk\bar{n}} = R_{\bar{m}kj\bar{n}} \\
&= -R_{\bar{n}j\bar{m}k} = -R_{\bar{n}k\bar{m}j} = -R_{\bar{m}j\bar{n}k} = -R_{\bar{m}k\bar{n}j}.
\end{aligned}
\tag{A4}
$$

Note that since $R^i_{jk\ell} = 0$, $[D_i, D_j] = 0$, and the tensors

$$A_{i_1\cdots i_n} = D_{i_1}\cdots D_{i_n}A, \quad \bar{A}^{i_1\cdots i_n} = D^{i_1}\cdots D^{i_n}\bar{A} \tag{A5}$$

are symmetric in all indices. It follows from the Bianchi identities that $D_iR^n_{j\bar{m}k}$ is totally symmetric in $\{ijk\}$.

We work in the Kähler covariant formalism [14], which differs from that of Cremmer *et al.* [13] by a phase transformation on the fermions that removes phases proportional to $\mathrm{Im}(W/\overline{W})$, where W is the superpotential. In this formalism the fermion U(1) Kähler connection is just

$$\Gamma_\mu = \frac{i}{4}\left(K_i\mathcal{D}_\mu z^i - K_{\bar{m}}\mathcal{D}_\mu\bar{z}^{\bar{m}}\right), \tag{A6}$$

where $\mathcal{D}_\mu$ is the gauge covariant derivative. It is convenient to introduce the notation

$$A = e^K W, \quad \bar{A} = e^K \overline{W}. \tag{A7}$$

Then the classical potential is $V = \hat{V} + \mathcal{D}$, where

$$\hat{V} = e^{-K}(A_i \bar{A}^i - 3A\bar{A}), \quad \mathcal{D} = \frac{1}{2x}\mathcal{D}_a \mathcal{D}^a,$$
$$\mathcal{D}_a = K_i(T^a z^i). \tag{A8}$$

With these conventions the tree-level Lagrangian [13, 14] for the case $f(z)_{ab} = \delta_{ab} f(z) = \delta_{ab}[x(z,\bar{z}) + iy(z,\bar{z})]$ is

$$\begin{aligned} g^{-\frac{1}{2}}\mathcal{L}(g,K,f) = {} & \frac{1}{2}r + K_{i\bar{m}}\mathcal{D}^\mu z^i \mathcal{D}_\mu \bar{z}^{\bar{m}} - \frac{x}{4}F_{\mu\nu}F^{\mu\nu} - \frac{y}{4}g^{-\frac{1}{2}}\tilde{F}_{\mu\nu}F^{\mu\nu} - V \\ & + \frac{ix}{2}\bar{\lambda}\not{D}\lambda + iK^{i\bar{m}}\left(\bar{\chi}_L^{\bar{m}}\not{D}\chi_L^i + \bar{\chi}_R^i \not{D}\chi_R^{\bar{m}}\right) + e^{-K/2}\left(\frac{1}{4}f_i \bar{A}^i \bar{\lambda}_R \lambda_L - A_{ij}\bar{\chi}_R^i \chi_L^j + \text{H.c.}\right) \\ & + \left(i\bar{\lambda}_R^a\left[2K_{i\bar{m}}(T_a \bar{z})^{\bar{m}} - \frac{1}{2x}f_i \mathcal{D}_a + \frac{1}{4}\sigma_{\mu\nu}F_a^{\mu\nu}f_i\right]\chi_L^i + \text{H.c.}\right) + \mathcal{L}_\psi + \text{ four-fermion terms,} \end{aligned} \tag{A9}$$

with $\mathcal{L}_\psi$ given in (2.7). In the notation of [4] [see Eq. (3.91)], the masses operating on the left-handed gravitino and chiral fermions are

$$m^\psi = e^{-K/2}\bar{A}, \quad m_{ij}^\chi = 2e^{-K/2}A_{ij}. \tag{A10}$$

These are related to the elements of M_Θ in (3.1) by (see [4])

$$\begin{aligned} M_\nu^\mu &= g_\nu^\mu e^{-K/2}\bar{A} = g_\nu^\mu M_\psi, \\ M_j^{\bar{m}} &= K^{\bar{m}i}e^{-K/2}A_{ij} = e^{-K/2}A_j^{\bar{m}}. \end{aligned} \tag{A11}$$

Note that the normalization of our chiral fermions is the same as in [13], which differs by a factor $\sqrt{2}$ from [14]. The covariant derivatives D_μ include the spin connection, the gauge connection, the Kähler connection (A6), the affine connection, and the field reparametrizaton connection for chiral fields. For fermions,

$$\begin{aligned} D_\mu \psi &= \left[\nabla_\mu + \frac{1}{4}\gamma_\nu\left(\nabla_\mu \gamma^\nu\right) + i\gamma_5 \Gamma_\mu\right]\psi, \\ D_\mu \chi^I &= \left[\mathcal{D}_\mu + \frac{1}{4}\gamma_\nu\left(\nabla_\mu \gamma^\nu\right) - i\gamma_5 \Gamma_\mu\right]\chi^I + \mathcal{D}_\mu z^J \Gamma_{JK}^I \chi^K. \end{aligned} \tag{A12}$$

[The gauginos have the same Kähler weight as the gravitino, and an additional connection which is given in (C7) below.] Operating on a function of scalar fields, $D_\mu = \mathcal{D}_\mu z^I D_I$, where $\mathcal{D}_\mu$ is gauge and general coordinate covariant.

APPENDIX B: OPERATORS AND TRACES

In this appendix we list the matrix elements of the operators appearing in Eqs. (3.1)–(3.5) and the traces needed to evaluate the divergent contributions to the one-loop effective action (3.2). We drop all total derivatives in the traces.

1. The bosonic sector

In the absence of the Yang-Mills sector, the operator H_Φ can be expressed as [18, 19, 12, 6, 4]

$$Z_\Phi H_\Phi = H + X + Y, \tag{B1}$$

with

$$Z_{i\mu\nu} = 0, \quad Z_{i\bar{m}} = K_{i\bar{m}}, \quad Z_{ij} = Z_{\bar{m}\bar{n}} = 0, Z_{\mu\nu,\rho\sigma} = P_{\mu\nu,\rho\sigma} = \tfrac{1}{8}\left(g_{\mu\rho}g_{\nu\sigma} + g_{\nu\rho}g_{\mu\sigma} - g_{\mu\nu}g_{\rho\sigma}\right) = \tfrac{1}{16}P^{-1}_{\mu\nu,\rho\sigma}. \tag{B2}$$

The nonvanishing elements of $Z_\Phi H_\Phi$ are H_{IJ}, $X_{\mu\nu}$, and $Y_{\mu I}$, with

$$\begin{aligned} & H_{IJ} = V_{IJ} + h_{IJ}, \quad h_{IJ} = \mathcal{R}_{IJ} + U_{IJ}, \\ & V_{IJ} = D_I D_J V, \quad \mathcal{R}_{IJ} = \mathcal{D}_\mu z^K \mathcal{D}_\nu z^L R_{IKLJ}, \\ & U_{IJ} = -2\mathcal{D}_\mu z^K \mathcal{D}_\nu \bar{z}^L Z_{IK} Z_{JL}\ , \\ & X_{\mu\nu,\rho\sigma} = -2P_{\mu\nu,\rho\sigma}V - \frac{1}{4}\left[r_{\mu\rho\nu\sigma} + r_{\nu\rho\mu\sigma}\right], Y_{\mu\nu I} = Y_{I\mu\nu} = -Z_{IJ}D_\mu \mathcal{D}_\nu z^J. \end{aligned} \tag{B3}$$

The contribution U_{IJ} to the scalar "squared mass" arises from the graviton gauge fixing term which is the same as in Refs. [18, 19, 4, 8]. The expressions for X and Y are simpler than in those references because of the propagator modification introduced in Sec. II. Using

$$\hat{V}_i = e^{-K}[A_{ji}\bar{A}^j - 2A_i\bar{A}], \quad \hat{V}_{ij} = e^{-K}[A_{jik}\bar{A}^k - A_{ij}\bar{A}],$$

$$\hat{V}_i^j = K^{j\bar{m}}\hat{V}_{i\bar{m}} = e^{-K}[A_{ki}\bar{A}^{jk} - A_i\bar{A}^j + \delta_i^j A_k\bar{A}^k - 2\delta_i^j A\bar{A} + R^{k\ j}_{\ n\ i}A_k\bar{A}^n], \tag{B4}$$

we obtain the traces (Lorentz indices are raised with $g^{\mu\nu}$ and scalar indices are raised with $K^{i\bar{m}}$)

$$\begin{aligned}
\mathrm{Tr}H &= 2H^i_i = 2e^{-K}[A_{ki}\bar{A}^{ik} - R^k_n A_k \bar{A}^n] + 2(N-1)\hat{V} + 2(N-3)M^2_\psi - \mathcal{D}^\mu z^i \mathcal{D}_\mu \bar{z}^{\bar{m}}(2R_{i\bar{m}} + 4K_{i\bar{m}}), \\
\frac{1}{2}\mathrm{Tr}H^2 &= H^i_j H^j_i + H_{ij}H^{ij} \\
&= e^{-2K}[A_{ij}\bar{A}^{jk}\bar{A}^{im}A_{mk} - 2A_{ij}\bar{A}^{jk}\bar{A}^i A_k + A_{ij}\bar{A}^{ij}(2\bar{A}^k A_k - 3A\bar{A}) \\
&\quad +2A_{ij}\bar{A}^{jk}R^{m\ i}_{n\ k}A_m\bar{A}^n + A_i\bar{A}^k R^{m\ i}_{n\ k}R^{n\ p}_{m\ q}A_p\bar{A}^q - 2R^{m\ i}_{n\ k}A_i\bar{A}^k A_m\bar{A}^n \\
&\quad -2R^m_n A_m\bar{A}^n\left(\hat{V} + M^2_\psi\right) + (N-1)\hat{V}^2 + 2(N-1)\hat{V}M^2_\psi + (N+3)M^4_\psi \\
&\quad +A_{kij}\bar{A}^{ijm}\bar{A}^k A_m - (A_{ijk}\bar{A}^{ik}\bar{A}^j A + \mathrm{H.c.})] \\
&\quad -2(\mathcal{D}_\mu z^j \mathcal{D}^\mu z^i e^{-K}[A_{jik}\bar{A}^k - A_{ij}\bar{A}] + \mathrm{H.c.}) \\
&\quad -\mathcal{D}_\mu \bar{z}^{\bar{m}}\mathcal{D}^\mu z^i\left\{4e^{-K}[A_{ij}\bar{A}^j_{\bar{m}} - A_i\bar{A}_{\bar{m}}] + (4K_{i\bar{m}} + 2R_{i\bar{m}})\left(\hat{V} + M^2_\psi\right)\right\} \\
&\quad +2\mathcal{D}_\mu z^j \mathcal{D}^\mu \bar{z}^{\bar{m}} R^k_{j\bar{m}i} e^{-K}[A_{k\ell}\bar{A}^{\ell i} - 3A_k\bar{A}^i + R^{\ell\ i}_{n\ k}A_\ell\bar{A}^n] \\
&\quad -(\mathcal{D}_\mu z^j \mathcal{D}^\mu z^i R^{k\ \ell}_{j\ i} e^{-K}[A_{mk\ell}\bar{A}^m - A_{k\ell}\bar{A}] + \mathrm{H.c.}) \\
&\quad +\mathcal{D}_\mu z^j \mathcal{D}^\mu \bar{z}^{\bar{m}} R^k_{j\bar{m}i}\mathcal{D}_\nu z^\ell \mathcal{D}^\nu \bar{z}^{\bar{n}} R^i_{\ell\bar{n}k} + \mathcal{D}_\mu z^j \mathcal{D}^\mu z^i R^{k\ \ \ell}_{\ j\ \ i}\mathcal{D}_\nu \bar{z}^{\bar{n}}\mathcal{D}^\nu \bar{z}^{\bar{m}} R_{\bar{n}k\bar{m}\ell} \\
&\quad +4\mathcal{D}_\mu z^j \mathcal{D}^\mu z^i \mathcal{D}_\nu \bar{z}^{\bar{m}}\mathcal{D}^\nu \bar{z}^{\bar{n}} K_{i\bar{n}}K_{j\bar{m}} + 4\mathcal{D}_\mu \bar{z}^{\bar{m}}\mathcal{D}^\mu z^i \mathcal{D}_\nu \bar{z}^{\bar{n}}\mathcal{D}^\nu z^j K_{i\bar{n}}K_{j\bar{m}} \\
&\quad +4\mathcal{D}_\mu \bar{z}^{\bar{m}}\mathcal{D}^\mu \bar{z}^{\bar{n}}\mathcal{D}_\nu z^j \mathcal{D}^\nu z^i R_{\bar{n}j\bar{m}i} - 4\mathcal{D}_\mu \bar{z}^{\bar{m}}\mathcal{D}^\mu z^i \mathcal{D}_\nu z^j \mathcal{D}^\nu \bar{z}^{\bar{n}} R_{\bar{m}j\bar{n}i},
\end{aligned} \tag{B5}$$

$$\begin{aligned}
\mathrm{Tr}X &= -20V + 2r, \\
\mathrm{Tr}X^2 &= 40V^2 - 8rV + 8r_{\mu\nu}r^{\mu\nu} - 2r^2 + \text{total derivative},
\end{aligned} \tag{B6}$$

and

$$\mathrm{Tr}Y^2 = 4\left(D_\mu \mathcal{D}_\nu z^i\right)\left(D^\mu \mathcal{D}^\nu z^{\bar{m}}\right)K_{i\bar{m}} + 4r^{\mu\nu}\mathcal{D}_\mu z^i \mathcal{D}_\nu z^{\bar{m}} K_{i\bar{m}}. \tag{B7}$$

Finally we need

$$\begin{aligned}
G_{\mu\nu} &= (G_z + G_G)_{\mu\nu}, \\
\left(G^z_{\mu\nu}\right)^I_J &= (R_{\mu\nu})^I_J = \mathcal{D}_\mu z^K \mathcal{D}_\nu z^L R^I_{JLK}, \\
\mathrm{Tr}R_{\mu\nu}R^{\mu\nu} &= 2\left[\mathcal{D}_\mu z^j \mathcal{D}_\nu \bar{z}^{\bar{m}} R^k_{i\bar{m}j}\mathcal{D}^\mu z^\ell \mathcal{D}^\nu \bar{z}^{\bar{n}} R^i_{k\bar{n}\ell} - \mathcal{D}_\mu z^j \mathcal{D}_\nu \bar{z}^{\bar{m}} R^k_{i\bar{m}j}\mathcal{D}^\nu z^\ell \mathcal{D}^\mu \bar{z}^{\bar{n}} R^i_{k\bar{n}\ell}\right], \\
\left(G^G_{\mu\nu}\right)_{\alpha\beta,\gamma\delta} &= \delta_{\alpha\beta,\rho\sigma}\left(r^\rho_{\gamma\mu\nu}g^\sigma_\delta + r^\rho_{\delta\mu\nu}g^\sigma_\gamma\right), \\
\mathrm{Tr}\left[G_{\mu\nu}G^{\mu\nu}\right]_G &= -6r^{\rho\sigma\mu\nu}r_{\rho\sigma\mu\nu} = 12\left(\frac{1}{2}r^2 - 2r^{\mu\nu}r_{\mu\nu}\right) + \text{total derivative}.
\end{aligned} \tag{B8}$$

2. The fermion sector

The metric is

$$\begin{aligned}
Z_{\chi^I\chi^J} &= 2Z_{IJ}, \quad Z_{\alpha\alpha} = -2, \quad Z_{\mu\nu} = -g_{\mu\nu}, \\
Z_{\chi\alpha} &= Z_{\chi\mu} = Z_{\alpha\mu} = 0.
\end{aligned} \tag{B9}$$

The matrix elements of M_Θ are given by (2.16), (2.17), (A.11), and

$$M^\mu_I = 2Z_{IJ}\mathcal{D}^\mu z^J, \quad M^I_\mu = -\mathcal{D}_\mu z^I. \tag{B10}$$

We also need their covariant derivatives, which have been defined in [4], with the difference that the Kähler connection is here given by (A6). In evaluating these derivatives it is useful to recall that the gaugino has opposite Kähler weight from the chiral fermions and the auxiliary field α. One finds, for the covariant derivatives of the matrix elements defined in (2.16), (2.17), and (A7),

$$\begin{aligned}
D_\mu \mu_{ij} &= e^{-K/2}(A_{ijk}\mathcal{D}_\mu z^k + \mathcal{D}_\mu \bar{z}^{\bar{m}}[A_i K_{j\bar{m}} + A_j K_{i\bar{m}} + A_n R^n_{i\bar{m}j}]), \\
D_\mu m_i &= e^{-K/2}\left(A_{ik}\mathcal{D}_\mu z^k + \mathcal{D}_\mu \bar{z}^{\bar{m}} A K_{i\bar{m}}\right), \\
D_\mu M &= e^{-K/2}A_k \mathcal{D}_\mu z^k.
\end{aligned} \tag{B11}$$

The nonvanishing matrix elements of $G_{\mu\nu}$ are

$$(G_{\mu\nu})^I_J = (R_{\mu\nu})^I_J + \delta^I_J\left(-\gamma_5\Gamma_{\mu\nu} + Z_{\mu\nu}\right),$$
$$(G_{\mu\nu})^\alpha_\alpha = -\gamma_5\Gamma_{\mu\nu} + Z_{\mu\nu},$$
$$(G_{\mu\nu})^\rho_\sigma = \delta^\rho_\sigma\left(\gamma_5\Gamma_{\mu\nu} + Z_{\mu\nu}\right) - \frac{1}{2}r^\rho_{\sigma\nu\mu}, \tag{B12}$$

where

$$\Gamma_{\mu\nu} = \frac{1}{2}\left(\mathcal{D}_\nu\bar{z}^{\bar{m}}K_{i\bar{m}}\mathcal{D}_\mu z^i - \mathcal{D}_\mu\bar{z}^{\bar{m}}K_{i\bar{m}}\mathcal{D}_\nu z^i\right), \quad Z_{\mu\nu} = \frac{1}{4}r_{\rho\sigma\nu\mu}\gamma^\rho\gamma^\sigma. \tag{B13}$$

Then we find the traces

$$\frac{1}{4}\mathrm{Tr}M^2_\Theta = e^{-K}\left[A_{ij}\bar{A}^{ij} - 2\hat{V} + 2M^2_\psi\right] - 4\mathcal{D}^\mu z^i\mathcal{D}_\mu\bar{z}^{\bar{m}}K_{i\bar{m}},$$
$$\begin{aligned}\frac{1}{4}\mathrm{Tr}M^4_\Theta = {}& e^{-2K}[A_{ij}\bar{A}^{jk}\bar{A}^{im}A_{km} - 4A_{ij}\bar{A}^{jk}A_k\bar{A}^i + 4(A\bar{A}^iA_{ik}\bar{A}^k + \mathrm{H.c.})]\\ &+2\hat{V}^2 - 4\hat{V}M^2_\psi - 10M^4_\psi - 8e^{-K}\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}^{\bar{m}}\left(A_{ij}\bar{A}^j_{\bar{m}} + K_{i\bar{m}}A\bar{A} - A_i\bar{A}_{\bar{m}}\right)\\ &-4e^{-K}(\mathcal{D}_\mu z^i\mathcal{D}^\mu z^jA_{ij}\bar{A} + \mathrm{H.c.}) + 8\mathcal{D}_\mu z^j\mathcal{D}^\nu z^i\mathcal{D}_\nu\bar{z}^{\bar{m}}\mathcal{D}^\mu\bar{z}^{\bar{n}}K_{i\bar{n}}K_{j\bar{m}},\end{aligned}$$
$$\begin{aligned}\frac{1}{4}\mathrm{Tr}|D_\mu M_\Theta|^2 = {}& e^{-K}\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}^{\bar{m}}[A_{ijk}\bar{A}^{jk}_{\bar{m}} + 10A_i\bar{A}_{\bar{m}} + 4R^k_{i\bar{m}j}A_k\bar{A}^j + R^\ell_{j\bar{m}\ n}R^{j\ n}_{\ k\ i}A_\ell\bar{A}^k - 2\bar{A}^j_{\bar{m}}A_{ij}]\\ &+2K_{i\bar{m}}\left(\hat{V} + 2M^2_\psi\right) + e^{-K}\{\mathcal{D}_\mu z^i\mathcal{D}^\mu z^j[2A_{ijk}\bar{A}^k + A_{ik\ell}\bar{A}^nR^{k\ \ell}_{\ n\ j} - 2\bar{A}A_{ij}] + \mathrm{H.c.}\}\\ &-4\left(D_\mu\mathcal{D}_\nu z^j\right)\left(D^\mu\mathcal{D}^\nu\bar{z}^{\bar{m}}\right)K_{j\bar{m}},\end{aligned}$$
$$\frac{1}{4}\mathrm{Tr}\left([\gamma^\mu,\gamma^\nu]G_{\mu\nu}\right) = (N+5)r,$$
$$\frac{1}{4}\mathrm{Tr}\left(M^2_\Theta[\gamma^\mu,\gamma^\nu]G_{\mu\nu}\right) = \frac{1}{4}r\mathrm{Tr}M^2_\Theta,$$
$$\frac{1}{16}\mathrm{Tr}\left([\gamma^\mu,\gamma^\nu]G_{\mu\nu}\right)^2 = -2(N+5)\Gamma_{\mu\nu}\Gamma^{\mu\nu} - [\mathrm{Tr}G_{\mu\nu}G^{\mu\nu}]_z + \frac{1}{4}(N-3)r^2 + 8r_{\mu\nu}r^{\mu\nu},$$
$$\mathrm{Tr}G_{\mu\nu}G^{\mu\nu} = 4(N+5)\Gamma_{\mu\nu}\Gamma^{\mu\nu} + 2\left[\mathrm{Tr}G_{\mu\nu}G^{\mu\nu}\right]_z + (N+13)\left(\frac{1}{2}r^2 - 2r_{\mu\nu}r_{\mu\nu}\right), \tag{B14}$$

where

$$\Gamma_{\mu\nu}\Gamma^{\mu\nu} = \frac{1}{2}\left(\mathcal{D}_\mu z^j\mathcal{D}^\mu z^i\mathcal{D}_\nu\bar{z}^{\bar{m}}\mathcal{D}^\nu\bar{z}^{\bar{n}}K_{i\bar{n}}K_{j\bar{m}} - \mathcal{D}_\mu\bar{z}^{\bar{m}}\mathcal{D}^\mu z^i\mathcal{D}_\nu\bar{z}^{\bar{n}}\mathcal{D}^\nu z^jK_{i\bar{n}}K_{j\bar{m}}\right). \tag{B15}$$

3. The ghost sector

For the graviton ghost we have [18, 19]

$$H^{\mu\nu}_{\mathrm{gh}} = -2\mathcal{D}^\mu z^I\mathcal{D}^\nu z^JZ_{IJ} - r^{\mu\nu}, \quad (G^{\mathrm{gh}}_{\mu\nu})^\alpha_\beta = r^\alpha_{\beta\mu\nu},$$
$$\mathrm{Tr}H_{\mathrm{gh}} = -4\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}^mK_{i\bar{m}} - r,$$
$$\mathrm{Tr}H^2_{\mathrm{gh}} = 8\mathcal{D}_\mu z^j\mathcal{D}^\mu z^i\mathcal{D}_\nu\bar{z}^{\bar{m}}\mathcal{D}^\nu\bar{z}^{\bar{n}}K_{i\bar{n}}K_{j\bar{m}} + 8\mathcal{D}_\mu\bar{z}^{\bar{m}}\mathcal{D}^\mu z^i\mathcal{D}_\nu\bar{z}^{\bar{n}}\mathcal{D}^\nu z^jK_{i\bar{n}}K_{j\bar{m}} + 8r_{\mu\nu}\mathcal{D}^\mu z^i\mathcal{D}^\nu\bar{z}^{\bar{m}}K_{i\bar{m}} + r_{\mu\nu}r^{\mu\nu},$$
$$\mathrm{Tr}G_{\mu\nu}G^{\mu\nu} = -r_{\alpha\beta\mu\nu}r^{\alpha\beta\mu\nu} = r^2 - 4r_{\mu\nu}r^{\mu\nu} + \text{total derivative}. \tag{B16}$$

For the gravitino ghost, $H_{\mathrm{Gh}} = M^2_{\mathrm{Gh}}$ is given by (2.12), and

$$[D_\mu, D_\nu] = G_{\mu\nu} = \gamma_5\Gamma_{\mu\nu} + Z_{\mu\nu}. \tag{B17}$$

We get

$$\mathrm{Tr}H_{\mathrm{Gh}} = 4\left(\hat{V} + M^2_\psi\right) - 4\mathcal{D}_\mu z^iK_{i\bar{m}}\mathcal{D}^\mu\bar{z}^{\bar{m}} - r$$
$$\begin{aligned}\mathrm{Tr}H^2_{\mathrm{Gh}} = {}& 4\left(\hat{V} + M^2_\psi\right)^2 + 4\left(\mathcal{D}_\mu z^iK_{i\bar{m}}\mathcal{D}^\mu\bar{z}^{\bar{m}}\right)^2 + \frac{1}{4}r^2 - \left(8\mathcal{D}_\mu z^iK_{i\bar{m}}\mathcal{D}^\mu\bar{z}^{\bar{m}} + 2r\right)\left(\hat{V} + M^2_\psi\right)\\ &-4\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}^{\bar{m}}A_i\bar{A}_{\bar{m}}e^{-K} + 2r\mathcal{D}_\mu z^iK_{i\bar{m}}\mathcal{D}^\mu\bar{z}^{\bar{m}} - 6\Gamma_{\mu\nu}\Gamma^{\mu\nu},\end{aligned}$$
$$\mathrm{Tr}G_{\mu\nu}G^{\mu\nu} = 4\Gamma_{\mu\nu}\Gamma^{\mu\nu} + \frac{1}{2}r^2 - 2r^{\mu\nu}r_{\mu\nu}. \tag{B18}$$

4. Supertraces

If we define

$$ S\mathrm{Tr}F = \mathrm{Tr}F_\Phi - \frac{1}{2}\mathrm{Tr}F_\Theta - 2\mathrm{Tr}F_{\rm gh} + 2\mathrm{Tr}F_{\rm Gh}, \tag{B19} $$

the effective Lagrangian (3.2) is

$$ g^{-\frac{1}{2}}\mathcal{L}_1 = -\frac{\Lambda^2}{32\pi^2}S\mathrm{Tr}H + \frac{\ln\Lambda^2}{32\pi^2}S\mathrm{Tr}\left(\frac{1}{2}H^2 - \frac{1}{6}rH + \frac{1}{12}G_{\mu\nu}G^{\mu\nu}\right), \tag{B20} $$

with

$$ \begin{aligned} -\frac{r}{6}S\mathrm{Tr}H =& \frac{N+1}{12}r^2 - \frac{N-5}{3}r\hat{V} - \frac{N-1}{3}rM_\psi^2 + \frac{1}{3}rR^i_jA_i\bar{A}^j + \frac{1}{3}r\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}^{\bar{m}}\left(R_{i\bar{m}} - 2K_{i\bar{m}}\right), \\ \frac{1}{2}S\mathrm{Tr}H^2 =& e^{-2K}[2A_{ij}\bar{A}^{jk}\bar{A}^iA_k + A_{ij}\bar{A}^{ij}\left(2\hat{V} + 3M_\psi^2\right) \\ &+2A_{ij}\bar{A}^{jk}R^{m\ i}_{n\ k}A_m\bar{A}^n + A_i\bar{A}^kR^{m\ i}_{n\ k}R^{n\ p}_{m\ q}A_p\bar{A}^q - 2R^{m\ i}_{n\ k}A_i\bar{A}^kA_m\bar{A}^n \\ &-2R^m_nA_m\bar{A}^n\left(\hat{V} + M_\psi^2\right) + A_{kij}\bar{A}^{ijm}\bar{A}^kA_m + (N+21)\hat{V}^2 + (N+17)M_\psi^4 + 2(N+5)\hat{V}M_\psi^2 \\ &-(A_{ijk}\bar{A}^{ik}\bar{A}^jA + 4A\bar{A}^i\bar{A}^jA_{ij} + \mathrm{H.c.})] + e^{-K}(\mathcal{D}_\mu z^j\mathcal{D}^\mu z^i[4A_{ij}\bar{A} + A_{ik\ell}\bar{A}^nR^{k\ \ \ell}_{\ n\ \ j}] + \mathrm{H.c.}) \\ &+\mathcal{D}_\mu\bar{z}^{\bar{m}}\mathcal{D}^\mu z^i\Big(e^{-K}[A_{ijk}\bar{A}^{jk}_{\bar{m}} + 2A_{ij}\bar{A}^j_{\bar{m}} + 2A_i\bar{A}_{\bar{m}} - 10K_{i\bar{m}}\hat{V} \\ &+R^\ell_{j\bar{m}n}R^{j\ \ n}_{\ k\ \ i}A_\ell\bar{A}^k] - 2R_{i\bar{m}}\left(\hat{V} + M_\psi^2\right)\Big) \\ &+2\mathcal{D}_\mu z^j\mathcal{D}^\mu\bar{z}^{\bar{m}}R^k_{j\bar{m}i}e^{-K}[A_{k\ell}\bar{A}^{\ell i} - A_k\bar{A}^i + R^{\ell\ i}_{n\ k}A_\ell\bar{A}^n] \\ &-(\mathcal{D}_\mu z^j\mathcal{D}^\mu z^iR^{k\ \ell}_{j\ i}e^{-K}[A_{mk\ell}\bar{A}^m - A_{k\ell}\bar{A}] + \mathrm{H.c.}) \\ &+\mathcal{D}_\mu z^j\mathcal{D}^\mu\bar{z}^{\bar{m}}R^k_{j\bar{m}i}\mathcal{D}_\nu z^\ell\mathcal{D}^\nu\bar{z}^{\bar{n}}R^i_{\ell\bar{n}k} + \mathcal{D}_\mu z^j\mathcal{D}^\mu z^iR^{k\ \ \ell}_{\ j\ \ i}\mathcal{D}_\nu\bar{z}^{\bar{n}}\mathcal{D}^\nu\bar{z}^{\bar{m}}R_{\bar{n}k\bar{m}\ell} \\ &+\frac{N+9}{4}\mathcal{D}_\mu z^j\mathcal{D}^\mu z^i\mathcal{D}_\nu\bar{z}^{\bar{m}}\mathcal{D}^\nu\bar{z}^{\bar{n}}K_{i\bar{n}}K_{j\bar{m}} - \frac{N-23}{4}\mathcal{D}_\mu\bar{z}^{\bar{m}}\mathcal{D}^\mu z^i\mathcal{D}_\nu\bar{z}^{\bar{n}}\mathcal{D}^\nu z^jK_{i\bar{n}}K_{j\bar{m}} \\ &+4\mathcal{D}_\mu\bar{z}^{\bar{m}}\mathcal{D}^\mu\bar{z}^{\bar{n}}\mathcal{D}_\nu z^j\mathcal{D}^\nu z^iR_{\bar{n}j\bar{m}i} - 4\mathcal{D}_\mu\bar{z}^{\bar{m}}\mathcal{D}^\mu z^i\mathcal{D}_\nu z^j\mathcal{D}^\nu\bar{z}^{\bar{n}}R_{\bar{m}j\bar{n}i} \\ &+\frac{1}{4}\mathrm{Tr}R_{\mu\nu}R^{\mu\nu} + 4\left(\mathcal{D}_\mu\bar{z}^{\bar{m}}\mathcal{D}^\mu z^iK_{i\bar{m}}\right)^2 - 5rV - 3rM_\psi^2 \\ &+4r\mathcal{D}_\mu\bar{z}^{\bar{m}}\mathcal{D}^\mu z^iK_{i\bar{m}} - 4r^{\mu\nu}\mathcal{D}_\mu\bar{z}^{\bar{m}}\mathcal{D}_\nu z^iK_{i\bar{m}} - \frac{N+9}{16}r^2 + r^{\mu\nu}r_{\mu\nu}, \end{aligned} $$

$$ \frac{1}{12}S\mathrm{Tr}G_{\mu\nu}G^{\mu\nu} = -\frac{N+1}{6}\Gamma_{\mu\nu}\Gamma^{\mu\nu} + \frac{7-N}{48}\left(r^2 - 4r^{\mu\nu}r_{\mu\nu}\right). \tag{B21} $$

Inserting these supertraces in (B20) gives a contribution of the form (2.23) with

$$ \begin{aligned} \alpha =& -\frac{2}{3}\frac{\ln\Lambda^2}{32\pi^2}, \quad \beta = \frac{N+5}{6}\frac{\ln\Lambda^2}{32\pi^2}, \quad \epsilon_0 = -\frac{\ln\Lambda^2}{32\pi^2}\left\{e^{-K}\left(A_{ij}\bar{A}^{ij} - \frac{2}{3}R^i_jA_i\bar{A}^j\right) + \frac{2N+20}{3}\hat{V} + \frac{2N+16}{3}M_\psi^2\right\}, \\ H_{\mu\nu} =& \frac{\ln\Lambda^2}{32\pi^2}\{g_{\mu\nu}\mathcal{D}_\rho z^i\mathcal{D}^\rho\bar{z}^{\bar{m}}[\tfrac{20}{3}K_{i\bar{m}} + \tfrac{2}{3}R_{i\bar{m}}] - 4\left[\mathcal{D}_\mu z^i\mathcal{D}_\nu\bar{z}^{\bar{m}}K_{i\bar{m}} + \mathcal{D}_\nu z^i\mathcal{D}_\mu\bar{z}^{\bar{m}}K_{i\bar{m}}\right]\}, \end{aligned} \tag{B22} $$

so the metric redefinition (2.24) gives (3.8), and we get a correction $\Delta_r\mathcal{L}$ given by the last term in (2.25):

$$ \begin{aligned} \frac{1}{\sqrt{g}}\Delta_r\mathcal{L} = \frac{\ln\Lambda^2}{32\pi^2}\Bigg[&\left\{-2e^{-K}\left(A_{ki}\bar{A}^{ik} - \frac{2}{3}R^k_nA_k\bar{A}^n\right) - (N+17)\hat{V} - \frac{4N+32}{3}M_\psi^2 - \frac{4}{3}r\right\}\hat{V} \\ &+\left[K_{i\bar{m}}\left\{\frac{N+59}{3}\hat{V} + e^{-K}\left(A_{ki}\bar{A}^{ik} - \frac{2}{3}R^k_nA_k\bar{A}^n\right) + \frac{2N+16}{3}M_\psi^2 + \frac{2}{3}r\right\} + \frac{4}{3}R_{i\bar{m}}\hat{V}\right]\mathcal{D}_\rho z^i\mathcal{D}^\rho\bar{z}^{\bar{m}} \\ &-\left\{\left(\frac{2}{3}R_{i\bar{m}} + 8K_{i\bar{m}}\right)\mathcal{D}_\rho z^i\mathcal{D}^\rho\bar{z}^{\bar{m}} + \frac{N+29}{6}\left(\mathcal{D}_\mu z^i\mathcal{D}_\nu\bar{z}^{\bar{m}} + \mathcal{D}_\nu z^i\mathcal{D}_\mu\bar{z}^{\bar{m}}\right)K_{i\bar{m}}\right\}\mathcal{D}_\mu z^j\mathcal{D}^\mu\bar{z}^{\bar{n}}K_{i\bar{n}}\Bigg]. \end{aligned} \tag{B23} $$

The Kähler potential redefinition (3.7) absorbs the contribution $\Delta_K\mathcal{L}$, where

$$\frac{1}{\sqrt{g}}\Delta_K\mathcal{L} = \frac{\ln\Lambda^2}{32\pi^2}\{e^{-2K}[-(A_{ijk}\bar{A}^{jk}A\bar{A}^i + R^{j\,k}_{\,n\,i}A_{jk}\bar{A}^n A\bar{A}^i + \text{H.c.}) + 3A_{ij}\bar{A}^{ij}A\bar{A}$$
$$+A_{ijk}\bar{A}^{jkn}A_n\bar{A}^i + 2A_{ij}\bar{A}^{jn}A_n\bar{A}^i + R^{j\,k}_{\,\ell\,i}A_{jk}\bar{A}^{\ell n}A_n\bar{A}^i + (D^\ell R^{j\,k}_{\,n\,i})A_{jk}\bar{A}^n A_\ell\bar{A}^i + R^{j\,k}_{\,n\,i}R^{\ell\,m}_{\,j\,k}A_\ell\bar{A}^n A_m\bar{A}^i$$
$$+2R^{\ell\,m}_{\,j\,i}A_{\ell n}\bar{A}^{jn}A_m\bar{A}^i + R^{\ell\,m}_{\,j\,k}A_{i\ell}\bar{A}^{jk}A_m\bar{A}^i + (D_iR^{\ell\,m}_{\,j\,k})A_\ell\bar{A}^{jk}A_m\bar{A}^i$$
$$+2R^{k\,\,j}_{\,n\,\,i}A_k\bar{A}^n A_j\bar{A}^i] - 4\hat{V}^2 - 20M_\psi^2\hat{V} - 36M_\psi^4$$
$$+e^{-K}[A_{ijk}\bar{A}^{jk}_{\bar{m}} + 2A_{ij}\bar{A}^{j}_{\bar{m}} + R^{j\,k}_{\,n\,i}A_{jk}\bar{A}^n_{\bar{m}} + (D_{\bar{m}}R^{j\,k}_{\,n\,i})A_{jk}\bar{A}^n$$
$$-6A_i\bar{A}_{\bar{m}} + K_{i\bar{m}}A_{jk}\bar{A}^{jk} + R^{j\,k}_{\,n\,i}R^\ell_{j\bar{m}k}A_\ell\bar{A}^n + 2R^\ell_{j\bar{m}i}A_{\ell n}\bar{A}^{jn} + R^\ell_{j\bar{m}k}A_{i\ell}\bar{A}^{jk} + (D_iR^\ell_{j\bar{m}k})A_\ell\bar{A}^{jk}$$
$$+2R^k_{n\bar{m}i}A_k\bar{A}^n]\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}^{\bar{m}} - 6M_\psi^2K_{i\bar{m}}\mathcal{D}_\mu z^i\mathcal{D}^\mu\bar{z}^{\bar{m}}\}. \tag{B24}$$

Finally,

$$4\mathcal{L}_i\bar{A}^iAe^{-K} + \text{H.c.} = 4\left(-e^{-2K}A_{ij}\bar{A}^i\bar{A}^jA + e^{-K}\mathcal{D}_\mu z^i\mathcal{D}^\mu z^jA_{ij}\bar{A} + \text{H.c.}\right)$$
$$+16e^{-2K}A_i\bar{A}^iA\bar{A} + 8e^{-K}\mathcal{D}^\mu z^i\mathcal{D}_\mu\bar{z}^{\bar{m}}(\bar{A}_iA_{\bar{m}} + K_{i\bar{m}}A\bar{A}). \tag{B25}$$

Then evaluating

$$\mathcal{L}_1 - \mathcal{L}_r + \Delta_r\mathcal{L} - \Delta_K\mathcal{L} - 4\left(\mathcal{L}_i\bar{A}^iAe^{-K} + \text{H.c.}\right)$$

yields the result given in (3.6).

APPENDIX C: ERRATA

In this appendix we list errata for Refs. [4, 5]. In both of these papers the term

$$\mathcal{L}_q \ni -\frac{x}{2}h^\rho_\rho F^{\mu\nu}\mathcal{D}_\mu\hat{A}_\nu + xh^\mu_\nu F_{\mu\rho}\left(\mathcal{D}^\nu\hat{A}^\rho - \mathcal{D}^\rho\hat{A}^\nu\right)$$

was inadvertently omitted from the quantum Lagrangian, and graviton-Yang-Mills ghost mixing was neglected; this will be corrected in [10].

1. Corrections to Ref. [4]

(a) The D term is missing from the tree-level bosonic Lagrangian [13] in Eq. (1.8):

$$\frac{1}{\sqrt{g}}\mathcal{L}_B \ni -\frac{1}{2}\text{Re}f^{-1}_{ab}\mathcal{G}_i(T^a)^i_jz^j\mathcal{G}_k(T^b)^k_\ell z^\ell. \tag{C1}$$

(b) A factor $e^{\mathcal{G}/2}$ is missing from the last term in the first line of (1.11). The signs of the last term in the fourth line and the second term of the last line of the same equation should be changed.

(c) Equations (2.33) and (2.34) should read, respectively,

$$d_\mu \to ip_\mu + \tilde{G}_{\mu\nu}\frac{\partial}{\partial p_\nu}$$
$$\equiv ip_\mu - \sum_{n=1}^{\infty}\frac{n}{(n+1)!}(d_{\mu_1}\cdots d_{\mu_{n-1}}G_{\mu_n\mu})$$
$$\times\frac{(-i)^n\partial^n}{\partial p_{\mu_1}\cdots\partial p_{\mu_n}}, \tag{C2}$$

$$F \to \hat{F} = \sum_{n=0}^{\infty}\frac{1}{n!}(d_{\mu_1}\cdots d_{\mu_n}F)\frac{(-i)^n\partial^n}{\partial p_{\mu_1}\cdots\partial p_{\mu_n}}. \tag{C3}$$

(d) Equation (2.46) should read

$$\mathcal{L}^{\text{aux}}_{\text{reg}} = \frac{1}{32\pi^2}\text{Tr}\left\{2\mu^2\tilde{M}^2\ln 2 + \frac{1}{2}\left[(\tilde{M}^4 + \tilde{\mathcal{D}}^2\tilde{M}^2)\right.\right.$$
$$\left.\left.-\frac{1}{2}(e^2S^2 + [K+Q]^2)\ln(2\mu_0^2/\mu^2)\right]\right\}. \tag{C4}$$

(e) The sign of the gauge connection in (2.48), (3.84), and (3.104) is incorrect.

(f) A term $8\tilde{D}^\mu z^i\tilde{D}_\mu z^{\bar{\jmath}}V_{i\bar{\jmath}} + 8\partial^\mu\partial_\mu\bar{s}V_{s\bar{s}}$ is missing from (2.67).

(g) Equation (2.79) should read

$$K^2 = 4\left(\tilde{D}_\mu\tilde{F}^{\mu\rho}\right)^a\left(\tilde{D}^\nu\tilde{F}_{\nu\rho}\right)_a + 2\left(\tilde{D}_\sigma\tilde{F}_{\mu\nu}\right)^a\left(\tilde{D}^\sigma\tilde{F}^{\mu\nu}\right)_a. \tag{C5}$$

(h) The sign of part of the gaugino connection is incorrect. Equation (3.84) should read

$$(d_\mu)^b_c = \delta^b_c(\partial_\mu + i\Gamma_\mu\gamma_5) + \epsilon^b_{ca}\tilde{A}^a_\mu - \frac{i}{2}(1/\text{Re}f)^{ba}\tilde{\mathcal{D}}_\mu(\text{Im}f_{ac})\gamma_5, \tag{C6}$$

and (3.89) should read

$$(L_\mu)^b_c = -\frac{1}{2}(1/\text{Re}f)^{ba}\tilde{\mathcal{D}}_\mu(\text{Im}f_{ac}). \tag{C7}$$

As a consequence of this sign error, there are errors in the $\partial_\mu s$ terms in the final equations (4.1) and (4.2) of [4]. The correct result will be given in [10].

(i) The right-hand sides of (3.106), (3.107), and (3.112) should be multiplied by $\frac{1}{2}$.

(j) The sign of the right-hand side of the last line of (3.91) is incorrect, and (3.120) should read

$$Z\tilde{M} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -(\tilde{M}^{\lambda\chi})_{\mu\nu} \\ 0 & (\tilde{M}^{\lambda\chi})_{\mu\nu} & 0 \end{pmatrix}. \tag{C8}$$

This sign error modifies the relative coefficients of (3.174) and (3.75) for the terms containing $\tilde{M}^{\chi\lambda}$.

(k) A term is missing from (3.133):

$$(F^{+}_{\mu\nu})^{\bar{\imath}}_{\bar{\jmath}}(F^{+\mu\nu})^{\bar{\jmath}}_{\bar{\imath}} \ni (f^{\mu\nu}f_{\mu\nu})^{\bar{\imath}}_{\bar{\imath}}. \tag{C9}$$

Corresponding terms that were omitted from the final equations of I will be given in [10].

(l) The paragraph after Eq. (3.145) should read "We also need to expand the remaining terms in (3.19). Only the second term, being of order M^2, will yield both quadratically and logarithmically divergent corrections. The other terms yield only logarithmically divergent terms. Again, after using ...."

(m) The signs of $\hat{M}_\psi$ in (3.141) and of M_ψ, N_ψ in (3.149)–(3.152) should be changed; this does not affect the final result.

(n) The subscripts a and μ on $\tilde{\mathcal{G}}^{\psi\lambda}$ in Eqs. (3.159) and (3.160) should be interchanged.

(o) The right-hand side of (3.165) should read

$$\ln(\gamma_\mu \Delta^{\mu\nu}_\theta \gamma_\nu) - \ln(-2/\not{p}), \tag{C10}$$

and the second line of the same equation should read

$$-\frac{1}{2}M_\psi\{\cdots . \tag{C11}$$

(p) In Eqs. (3.174)–(3.176), $G^{\chi\lambda}$ should be replaced everywhere by $G^{\psi\lambda}$. In addition, the subscripts a and σ on $(G^{\psi\lambda}_{\mu\nu})$ in (3.174) should be interchanged, the denominator of the second term on the right-hand side of (3.176) should be $48p^4$, and the left-hand side of (3.176) should be multiplied by $\frac{1}{4}$.

(q) The last line of (3.192), and the last term in the fourth line of that equation, should be removed.

2. Corrections to Ref. [5]

(a) Equations (30) and (31) should read, respectively,

$$\begin{aligned} y_i^{\alpha\beta} &= 2k\kappa \left(D_{\tilde{\phi}^i} \ln\tilde{e}\right)\left[P^{\alpha\beta,\delta\sigma}\eta^{\gamma\epsilon} + \frac{1}{4\kappa}\eta^{\gamma\alpha}\eta^{\epsilon\beta}\eta^{\delta\sigma}\right]\mathrm{Tr}\tilde{\mathcal{F}}_{\epsilon\sigma}\tilde{\mathcal{F}}_{\gamma\delta}, \\ K_a^{\alpha\beta,\epsilon} &= -4\kappa\left[P^{\alpha\beta,\delta\sigma}\eta^{\gamma\epsilon} + \frac{1}{4\kappa}\eta^{\gamma\alpha}\eta^{\epsilon\beta}\eta^{\delta\sigma}\right]\left[\left(\tilde{D}_\sigma\tilde{\mathcal{F}}_{\delta\gamma}\right)_a - [\tilde{\nabla}_\sigma, \ln\tilde{e}]\left(\tilde{\mathcal{F}}_{\delta\gamma}\right)_a\right]. \end{aligned} \tag{C12}$$

(b) The sentence before (44) should read "Also, $-2\hat{f}^i\eta^\nu_\alpha(C_\mu)^\alpha_{bi}(\hat{\mathcal{D}}^\mu)^b_a\hat{\mathcal{A}}^a_\mu = \ldots$."

(c) The signs of the $C^\alpha C_\alpha$ terms in (46) should be changed.

(d) There is a $\partial^\mu \ln\tilde{e}\Delta_{\mu\nu}\partial^\nu \ln\tilde{e}$ term missing from (53).

(e) The last sentence of the paragraph following (53) should read "...where N' is N without the Ω and C terms ...."

(f) The following corrections to (58) should be made.

Replace $\frac{3}{4}N'^{ab}_{\mu\nu}\delta_{ab}\eta^{\mu\nu}$ by $\left(N - \Omega - \frac{1}{4}N'\right)^{ab}_{\mu\nu}\delta_{ab}\eta^{\mu\nu}$.

Replace $-\frac{3}{2}\mathcal{S}^a_{\mu i}\mathcal{S}^b_{\nu j}Z^{ij}\delta_{ab}\eta^{\mu\nu}$ by $-(2\mathcal{S}^a_{\mu i}\mathcal{S}^b_{\nu j} - \frac{1}{2}\mathcal{S}'^a_{\mu i}\mathcal{S}'^b_{\nu j})Z^{ij}\delta_{ab}\eta^{\mu\nu}$.

Replace $\frac{1}{2}(N'_{\mu\nu}N'^{\nu\mu})^{ab}\delta_{ab}$ by $([N-\Omega]_{\mu\nu}[N-\Omega]^{\nu\mu} - \frac{1}{2}N'_{\mu\nu}N'^{\nu\mu})^{ab}\delta_{ab}$.

Replace $2N'_{\mu\nu}\Omega^{\mu\nu}$ by $2(N-\Omega)_{\mu\nu}\Omega^{\mu\nu}$.

(g) The text following (58) should read "...where $\mathcal{S}' = S + s$, and N' is given by N in (46) without the Ω and C^2 terms, ..."

(h) The second sentence of the Appendix should read "The space-time metric $g_{\mu\nu}$ has the flat limit $\eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$."

(i) Replace e by $\tilde{e}$ in the definition of Q, Eq. (A1).

[1] L. Dixon, V. Kaplunovsky, and J. Louis, Nucl. Phys. **B355**, 649 (1991); G.L. Cardoso and B.A. Ovrut, *ibid.* **B369**, 351 (1992); J.-P. Derendinger, S. Ferrara, C. Kounnas, and F. Zwirner, Phys. Lett. B **271**, 307 (1991); P. Binétruy, G. Girardi, R. Grimm, and M. Müller, *ibid.* **265**, 111 (1991); P. Adamietz, P. Binétruy, G. Girardi, and R. Grimm, Nucl. Phys. **B401**, 257 (1993); P. Mayr and S. Stieberger, Garching-Munich Report No. MPI-Ph/93-19, TUM-TH-153/93, 1993 (unpublished).

[2] M.K. Gaillard and T.R. Taylor, Nucl. Phys. **B381**, 577 (1992).

[3] P. Binétruy and M.K. Gaillard, Phys. Lett. B **232**, 83 (1989); Nucl. Phys. **B358**, 121 (1991).

[4] J.W. Burton, M.K. Gaillard, and V. Jain, Phys. Rev. D **41**, 3118 (1990).

[5] M.K. Gaillard and V. Jain, Phys. Rev. D **46**, 1786 (1992).

[6] P. Binétruy and M.K. Gaillard, Nucl. Phys. **B312**, 341 (1989).

[7] M. Srednicki and S. Theisen, Phys. Rev. Lett. **54**, 278 (1985).

[8] C. Chiou-Lahanas, A. Kapella-Economu, A.B. Lahanas, and X.N. Maintas, Phys. Rev. D **42**, 469 (1990); **45**, 534 (1992).

[9] G.A. Vilkoviski, Nucl. Phys. **B234**, 125 (1984); E.S. Fradkin and A.A. Tseytlin, *ibid.* **B234**, 509 (1984).

[10] M.K. Gaillard and V. Jain, Report No. LBL-34948, UCB-PTH-93/37, ITP-SB-93-89 (unpublished).

[11] S. Honerkamp, Nucl. Phys. **B36**, 130 (1973); L. Alvarez-Gaumé, D.Z. Freedman, and S. Mukhi, Ann. Phys. (N.Y.) **134**, 85 (1981); S. Mukhi, Nucl. Phys. **B264**, 640 (1986).

[12] M.K. Gaillard, Nucl. Phys. **B268**, 669 (1986).

[13] E. Cremmer, S. Ferrara, L. Girardello, and A. Van Proeyen, Nucl. Phys. **B212**, 413 (1983).

[14] P. Binétruy, G. Girardi, R. Grimm, and M. Muller, Phys. Lett. B **189**, 83 (1987); P. Binétruy, G. Girardi, and

R. Grimm, Annecy Report No. LAPP-TH-275-90, 1990 (unpublished).

[15] R. Barbieri, S. Ferrara, L. Maiani, F. Palumbo, and C.A. Savoy, Phys. Lett. **115B**, 212 (1982).

[16] I. Antoniadis, E. Gava, K.S. Narain, and T.R. Taylor, Nucl. Phys. **B407**, 706 (1993).

[17] M.T. Grisaru, Physica D **15**, 278 (1985); S.J. Gates, M.T. Grisaru, M. Rocek, and W. Siegel, *Superspace* (Benjamin-Cummings, Reading, MA, 1983), and references therein.

[18] G.K. 't Hooft and M. Veltman, Ann. Inst. Henri Poincaré **20**, 69 (1974).

[19] V. Jain, Phys. Rev. D **40**, 1213 (1989).

[20] R. Barbieri and C. Cecotti, Z. Phys. C **17**, 183 (1983); P. Binétruy and M.K. Gaillard, Nucl. Phys. **B254**, 388 (1985).

[21] S.W. Christensen and M.J. Duff, Nucl. Phys. **B154**, 301 (1979).

[22] P. Binétruy and M.K. Gaillard, Phys. Lett. B **220**, 68 (1989).

[23] E. Witten, Phys. Lett. **155B**, 151 (1985).

[24] S. Ferrara, C. Kounnas, and M. Porrati, Phys. Lett. B **181**, 263 (1986).

[25] L.J. Dixon, V.S. Kaplunovsky, and J. Louis, Nucl. Phys. **B329**, 27 (1990); S. Ferrara, D. Lüst, and S. Theisen, Phys. Lett. B **233**, 147 (1989).