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Covariant symplectic structure of two-dimensional dilaton gravity

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We study the symplectic structure on phase space of the classical solutions of dilaton gravity coupled to scalar matter. The symplectic form clearly reveals the conjugate pair of global degrees of freedom, and shows the modification of the canonical structure of matter fields due to dilaton gravity.

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I. INTRODUCTION

A theory of two-dimensional dilaton gravity coupled to scalar matter has attracted much attention since the pioneering work by Callan, Giddings, Harvey, and Strominger (CGHS) [1]. It was hoped that this toy model for studying the formation, evaporation, and back reaction of black holes could resolve the elusive issues associated with the end point of black-hole evaporation [2]. The initial expectation of CGHS turned out not to be correct, as shown by several authors [3], because there arise singularities in the solutions of semiclassical equations. There have appeared many articles which investigated the nature of semiclassical solutions including modification of the equations [4], or numerical analysis [5]. Studies in other gauges than the simplest conformal gauge [6], and path integral quantization approaches [7] have also been attempted, and yet no definitive resolution to the central issues has emerged. Thus it is still worth studying the theory with a different method of investigation.

In this paper we apply the covariant description of canonical formalism proposed by Witten [8], Zuckerman [9], and others [10] to investigate the theory of dilaton gravity coupled to scalar matter. The usual canonical approach ruins Poincaré invariance from the beginning through an explicit choice of a time coordinate, but the covariant method preserves this symmetry as well as other relevant symmetries.

Chu *et al.* applied this method for the analysis of the Wess-Zumino-Witten model on a circle [11], Navarro-Salas, Navarro, and Aldaya applied this method for the investigation of two-dimensional gravity [12], and Hwang *et al.* applied it for the case of a particle moving in the Jackiw-Teitelboim geometry [13].

The CGHS model is particularly suitable to the covariant approach because the model is classically exactly solvable; therefore, the covariant description of phase space is explicitly given.

In this paper we will follow the Crnkovic-Witten procedure [14] for the construction of a symplectic structure on the phase space of classical solutions. First, we consider dilaton gravity without a matter field, and we find that the symplectic structure most clearly reveals the conjugate pair of the global degrees of freedom: the black-hole mass parameter and an integral of gauge function. Next, we include the matter field in construction of the symplectic form, whose final result shows the effects of dilaton gravity on the canonical structure of the matter field. In the last section we invert the matrix of the symplectic form to calculate Poisson brackets, but only mention difficulties about quantizing the theory.

II. SYMPLECTIC STRUCTURE

A phase space is the manifold of solutions of the equations of motion of field theory. This definition has the advantage of manifest covariance, and was introduced by Witten [8], Zuckerman [9], and also earlier by others [10]. The symplectic structure on phase space can be given by the symplectic two-form ω expressed as the integral

$$\omega = \int_\Sigma \omega^\mu d\sigma_\mu \ , \quad (1)$$

where Σ is an initial value hypersurface and ω^μ is a symplectic current. This definition is independent of Σ if ω^μ is conserved, $\partial_\mu \omega^\mu = 0$, and the condition for the Jacobi identity for Poisson brackets to hold requires ω to be closed, $\delta\omega = 0$, which is satisfied if ω^μ itself is closed.

A canonical expression for ω^μ in a theory with fields ϕ^j, based on a Lagrangian density $\mathcal{L}$, is

$$\omega^\mu = \delta\phi^j \wedge \delta \left[\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^j)} \right] , \quad (2)$$

where δ denotes the exterior derivative on phase space. So obtained, ω_μ is automatically closed, and $\partial_\mu \omega^\mu = 0$.

In our theory the action in $(1+1)$-dimensional spacetime is given by

$$S = \frac{1}{2\pi} \int d^2x \sqrt{-g} \, \{ e^{-2\phi} [R + 4(\nabla\phi)^2 + 4\lambda^2] - \tfrac{1}{2} (\nabla f)^2 \} \ , \quad (3)$$

where $g_{\mu\nu}$, ϕ, and f are the metric, dilaton, and matter field, respectively, and λ^2 is a cosmological constant. In order to obtain the field equations and the symplectic current we take the variation of the action as

$$\delta S=\frac{1}{2\pi}\int d^2x\sqrt{-g}\,((\Box f)\delta f+\{\Phi[R+4(\nabla\phi)^2+4\lambda^2]+4(\Box\Phi)\}\delta\phi$$
$$+\{(\Box\Phi-2\lambda^2\Phi)g_{\alpha\beta}-\nabla_\alpha\nabla_\beta\Phi+4\Phi[\partial_\alpha\phi\,\partial_\beta\phi-\tfrac{1}{2}g_{\alpha\beta}(\nabla\phi)^2]+[\tfrac{1}{4}(\nabla f)^2g_{\alpha\beta}-\tfrac{1}{2}\partial_\alpha f\,\partial_\beta f]\}\delta g^{\alpha\beta})$$
$$+\frac{1}{2\pi}\int d^2x\,\partial_\mu\sqrt{-g}\,\{(-g^{\mu\alpha}\partial_\alpha f)\delta f-(4g^{\mu\alpha}\partial_\alpha\Phi)\delta\phi-\Phi[\nabla_\sigma\delta g^{\sigma\mu}+2g^{\mu\sigma}\partial_\sigma(\delta(\ln\sqrt{-g}\,))]$$
$$+(\partial_\sigma\Phi)[\delta g^{\sigma\mu}+2g^{\sigma\mu}\delta(\ln(\sqrt{-g}\,))]\}\,, \tag{4}$$

where $\Phi\equiv e^{-2\phi}$, $\Box\equiv\nabla_\mu\nabla^\mu$, and ∇_ν is a covariant derivative.

The phase space and its symplectic structure can be conveniently analyzed in the conformal gauge:

$$g_{+-}=-\tfrac{1}{2}e^{2\rho}\,,\quad g_{++}=g_{--}=0\,, \tag{5}$$

where $x^\pm=t\pm x$. The equations of motion are

$$e^{-2\rho}\partial_+\partial_-f=0\,, \tag{6}$$

$$e^{-2\phi}e^{-2\rho}(-4\partial_+\partial_-\phi+4\partial_+\phi\partial_-\phi+2\partial_+\partial_-\rho+\lambda^2e^{2\rho})=0\,, \tag{7}$$

$$T_{++}=e^{-2\phi}(4\partial_+\rho\,\partial_+\phi-2\partial_+^2\phi)+\tfrac{1}{2}\partial_+f\partial_+f=0\,, \tag{8}$$

$$T_{--}=e^{-2\phi}(4\partial_-\rho\partial_-\phi-2\partial_-^2\phi)+\tfrac{1}{2}\partial_-f\partial_-f=0\,, \tag{9}$$

$$T_{+-}=e^{-2\phi}(2\partial_+\partial_-\phi-4\partial_+\phi\partial_-\phi-\lambda^2e^{2\rho})=0\,. \tag{10}$$

The general solution of these equations is given by CGHS [1] as

$$f(x)=f_+(x^+)+f_-(x^-)\,, \tag{11}$$

$$2\rho-2\phi\equiv G=G_+(x^+)+G_-(x^-)\,, \tag{12}$$

$$\Phi\equiv e^{-2\phi}=\frac{M}{\lambda}-\lambda^2\int^{x^+}e^{G_+}\int^{x^-}e^{G_-}-\frac{1}{2}\int^{x^+}e^{G_+}\int^{x^+}e^{-G_+}(\partial_+f_+)^2-\frac{1}{2}\int^{x^-}e^{G_-}\int^{x^-}e^{-G_-}(\partial_-f_-)^2\,, \tag{13}$$

where M is an integration constant. The conformal subgroup of diffeomorphisms, which is not fixed by the conformal gauge, might be fixed by a suitable choice of G, for example, $G=0$. In the case $f=0$ and $M=0$ the solution represents the linear dilaton vacuum, while the solution with $M\neq0$ and $f=0$ represents a black-hole solution where M corresponds to the black-hole mass. In the situation where an f shock wave travels in the x^- direction with magnitude a described by the stress tensor

$$\tfrac{1}{2}\partial_+f\,\partial_+f=a\delta(x^+-x_0^+)\,, \tag{14}$$

the solution, in the gauge $G=0$, represents a formation of a black hole [1].

The symplectic current in the conformal gauge can be read from (4) as

$$\omega^\pm=\delta f\wedge\delta(\partial_\mp f)+2e^{-2\phi}[\delta(\partial_\mp G)\wedge\delta\phi+\delta(\partial_\mp\phi)\wedge\delta G+2(\partial_\mp\phi)\delta G\wedge\delta\phi]$$
$$=\delta f\wedge\delta(\partial_\mp f)-[\delta(\partial_\mp G)\wedge\delta\Phi-\delta G\wedge\delta(\partial_\mp\Phi)]\,. \tag{15}$$

Before proceeding further, we note that the symplectic currents of the dilaton gravity and the matter fields are completely separated, and those of the dilaton gravity are expressed in terms of Φ and G [$\equiv2(\rho-\phi)$]. When pushing down $\omega^\pm$ on phase space, since G is not a dynamical field but only a subconformal gauge function, we find that dilaton gravity does not have a local physical degree of freedom. It is well known from previous works that the dilatonic gravity action is equivalent to a topological gauge theory [15]; therefore, it is expected that the symplectic current depends on a pure gauge function, and physical degrees of freedom are of global origin.

The covariant symplectic structure $\omega=\int_C\omega^\mu d\sigma_\mu$ is independent of the choice of the initial value hypersurface C. So we take

$$\omega=\int\omega^0dx=\int dx(\delta f\wedge\delta\dot f+\delta\Phi\wedge\delta\dot G-\delta\dot\Phi\wedge\delta G)\,, \tag{16}$$

where $\dot f\equiv\partial_0f=(\partial_++\partial_-)f$, etc.

Inserting the general solution (11)–(13) into the symplectic two-form (16), we can obtain the desired symplectic structure. First, in the case $f=0$, we have

$$(\partial_++\partial_-)\Phi=-\lambda^2e^{G_+(x^+)}\int^{x^-}e^{G_-(x^-)}-\lambda^2\left[\int^{x^+}e^{G_+(x^+)}\right]e^{G_-(x^-)}\,, \tag{17}$$

$$\delta\Phi=\frac{\delta M}{\lambda}-\lambda^2\left[\int^{x^+}e^{G_+(x'_+)}\delta G_+(x'_+)\right]\left[\int^{x^-}e^{G_-(x'_-)}\right]-\lambda^2\left[\int^{x^+}e^{G_+(x'_+)}\right]\left[\int^{x^-}e^{G_-(x'_-)}\delta G_-(x'_-)\right]\,, \tag{18}$$

and the symplectic current of dilaton gravity $\omega^0_{\rm DG}$ is

$$\omega_{DG}^0=-\lambda^2\left[\left(\int^{x^+}e^{G_+(x'^+)}\delta G_+(x'^+)\right)\left(\int^{x^-}e^{G_-(x'^-)}\right)+\left(\int^{x^+}e^{G_+(x'^+)}\right)\left(\int^{x^-}e^{G_-(x'^-)}\delta G_-(x'^-)\right)\right]\wedge\delta\dot{G}$$
$$+\lambda^2\left\{\left[\left(\int^{x^-}e^{G_-(x'^-)}\right)e^{G_+(x^+)}\delta G_+(x^+)+\left(\int^{x^+}e^{G_+(x'^+)}\delta G_+(x'^+)\right)e^{G_-(x^-)}\right]\right.$$
$$\left.+\left[e^{G_+(x^+)}\int^{x^-}e^{G_-(x'^-)}\delta G_-(x'^-)+\left(\int^{x^+}e^{G_+(x'^+)}\right)e^{G_-(x^-)}\delta G_-(x^-)\right]\right\}\wedge\delta G+\frac{1}{\lambda}\delta M\wedge\delta\dot{G}\ . \tag{19}$$

Since G is a subconformal gauge function it depends upon continuous coordinate transformations, and can be divided into topologically distinctive classes. Here we will limit only to the simplest case, namely, $G_+=G_-=0$ and its continuously transformed functions. So δG is also limited to this class of functions, and Eq. (19) is simplified in this case, by letting $G_+=G_-=0$ with an appropriate choice of coordinates, as

$$\omega_{DG}^0=-\lambda^2\left[x^-\int^{x^+}dx'^+\delta G_+(x'^+)+x^+\int^{x^-}dx'^-\delta G_-(x'^-)\right]\wedge\delta\dot{G}$$
$$+\lambda^2\left[x^-\delta G_+(x^+)+x^+\delta G_-(x^-)+\int^{x^+}dx'^+\delta G_+(x'^+)+\int^{x^-}dx'^-\delta G_-(x'^-)\right]\wedge\delta G+\frac{1}{\lambda}\delta M\wedge\delta\dot{G}\ . \tag{20}$$

Now for the evaluation of the symplectic two-form ω_{DG}, we make use of the manifest covariance of the formalism, i.e., the freedom of choice of the integral curve, and take $t=0$, namely, $x^\pm=\pm x$. Then we have

$$\omega_{DG}=\int dx\,\lambda^2\left[-x(\delta G_+(x)+\delta G_-(-x))-\int^x dx'\delta G_+(x')+\int^{-x}dx'\delta G_-(x')\right]\wedge\delta(G_+(x)-G_-(-x))$$
$$+\int dx\,\lambda^2\left[-x(\delta G_+(x)-\delta G_-(-x))+\int^x dx'\delta G_+(x')+\int^{-x}dx'\delta G_-(x')\right]\wedge\delta(G_+(x)+G_-(-x))$$
$$+\int dx\,\frac{1}{\lambda}\delta M\wedge\delta\dot{G}\ , \tag{21}$$

where, in deriving the first line, we used

$$\delta\dot{G}(t=0)=\delta\left[\frac{d}{dx^+}G_+(x^+)\bigg|_{x^+=x}+\frac{d}{dx^-}G_-(x^-)\bigg|_{x^-=-x}\right]$$
$$=\delta\frac{d}{dx}[G_+(x)-G_-(-x)]\ ,$$

and integration by parts assuming vanishing total derivative terms. The result immediately reduces to

$$\omega_{DG}=\int dx\left[2\lambda^2\left\{\int^{-x}dx'\delta G_-(x')\wedge\delta G_+(x)+\int^x dx'\delta G_+(x')\wedge\delta G_-(-x)\right\}+\frac{\delta M}{\lambda}\wedge\delta\dot{G}\right]. \tag{22}$$

The first integral in the curly brackets can be reexpressed, after changing the order of double integration, and exchanging the integration variables x and x', as

$$\int dx\int^x dx'\delta G_-(-x)\wedge\delta G_+(x')\ , \tag{23}$$

which cancels the second term in the curly brackets. Therefore, we are left with the last term only, and it is

$$\omega_{DG}=\frac{1}{\lambda}\delta M\wedge\left[\int dx\,\dot{G}\right]. \tag{24}$$

This final result clearly exhibits that there is only one degree of freedom of a global nature: the apparent local function $\dot{G}$ does not enter in the final formula but only its integration over the whole range. The black-hole mass parameter M appears as the only dynamical degree, and it is in agreement with Miković's analysis of the dilaton gravity in a unitary gauge [16]. Our study, however, explicitly shows the conjugate variable of M, and its relation to the gauge function. It is perhaps worth mentioning that the conjugate pair depends upon the topology of the base manifold. For instance, in the case of cylindrical geometry the monodromy group variable appears instead of the gauge function G. This problem was studied in the case of the Wess-Zumino-Witten model [11], two-dimensional induced gravity [12], and Jackiw-Teitelboim gravity [13].

Having obtained the symplectic structure of dilaton gravity without a matter field, we proceed to the case where $f\neq 0$. With the matter field the symplectic form of dilaton gravity is modified. If we denote this modification by $\Delta\omega$, we see that

$$\Delta\omega=\Delta\omega_{DG}=\int dx\,\Delta\omega_{DG}^0$$
$$=\int dx\,\Delta(\delta\Phi)\wedge\delta\dot{G}-\Delta(\delta\dot{\Phi})\wedge G\ , \tag{25}$$

where

$$\Delta(\delta\Phi)=-\frac{1}{2}\int^{x^+}dx'_+e^{G_+(x'_+)}\int^{x'_+}dx''_+e^{-G_+(x''_+)}[(\delta G_+(x'_+)-\delta G_+(x''_+))(\partial_+f_+(x''_+))^2+\delta(\partial_+f_+(x''_+))^2]$$
$$-\frac{1}{2}\int^{x^-}dx'_-e^{G_-(x'_-)}\int^{x'_-}dx''_-e^{-G_-(x''_-)}[(\delta G_-(x'_-)-\delta G_-(x''_-))(\partial_-f_-(x''_-))^2+\delta(\partial_-f_-(x''_-))^2]\ , \tag{26}$$

$$\Delta(\partial_++\partial_-)\Phi=-\frac{1}{2}e^{G_+(x^+)}\int^{x^+}dx'_+e^{-G_+(x'_+)}(\partial_+f_+(x'_+))^2-\frac{1}{2}e^{G_-(x^-)}\int^{x^-}dx'_-e^{-G_-(x'_-)}(\partial_-f_-(x'_-))^2 \tag{27}$$

and

$$\Delta(\delta\dot\Phi)=-\frac{1}{2}e^{G_+(x^+)}\int^{x^+}dx'_+e^{-G_+(x'_+)}[(\delta G_+(x^+)-\delta G_+(x'_+))(\partial_+f_+(x'_+))^2+\delta(\partial_+f_+(x'_+))^2]$$
$$-\frac{1}{2}e^{G_-(x^-)}\int^{x^-}dx'_-e^{-G_-(x'_-)}[(\delta G_-(x^-)-\delta G_-(x'_-))(\partial_-f_-(x'_-))^2+\delta(\partial_-f_-(x'_-))^2]\ . \tag{28}$$

As before we let $G_+=G_-=0$ by a suitable coordinate transformation, and take the integral curve $t=0$. Then $x^\pm=\pm x$, and we have

$$\Delta\omega^0_{\rm DG}=-\frac{1}{2}\int^x dy\int^y dz\left[[\delta G_+(y)-\delta G_+(z)]\left[\frac{d}{dz}f_+(z)\right]^2+\delta\left[\frac{d}{dz}f_+(z)\right]^2\right]\wedge\delta\frac{d}{dx}(G_+(x)-G_-(-x))$$
$$-\frac{1}{2}\int^{-x}dy\int^y dz\left[[\delta G_-(y)-\delta G_-(z)]\left[\frac{d}{dz}f_-(z)\right]^2+\delta\left[\frac{d}{dz}f_-(z)\right]^2\right]\wedge\delta\frac{d}{dx}(G_+(x)-G_-(-x))$$
$$+\frac{1}{2}\int^x dy\left[[\delta G_+(x)-\delta G_+(y)]\left[\frac{d}{dy}f_+(y)\right]^2+\delta\left[\frac{d}{dy}f_+(y)\right]^2\right]\wedge\delta(G_+(x)+G_-(-x))$$
$$+\frac{1}{2}\int^{-x}dy\left[[\delta G_-(-x)-\delta G_-(y)]\left[\frac{d}{dy}f_-(y)\right]^2+\delta\left[\frac{d}{dy}f_-(y)\right]^2\right]\wedge\delta(G_+(x)+G_-(-x))\ . \tag{29}$$

After integration, and ignoring the total derivative terms, we get

$$\Delta\omega=\int dx\left\{\int^x dy\left[\delta\left[\frac{d}{dy}f_+(y)\right]^2-\left[\frac{d}{dy}f_+(y)\right]^2\delta G_+(y)\right]\right\}\wedge\delta G_+(x)$$
$$+\int dx\left\{\int^{-x}dy\left[\delta\left[\frac{d}{dy}f_-(y)\right]^2-\left[\frac{d}{dy}f_-(y)\right]^2\delta G_-(y)\right]\right\}\wedge\delta G_-(-x)\ . \tag{30}$$

Here we notice that the left-moving and right-moving sectors are separated, and the matter field mixes with the local gauge function. The dilaton gravity coupled with a matter field is no longer a pure topological gauge theory. This contrasts with the gauge invariant matter coupling in the topological gauge formalism of dilaton gravity [15], where one may expect that only global degrees of freedom appear in the symplectic structure. The second terms in the curly brackets show the weights in the symplectic structures of $\delta G_\pm$'s, and after changing the order of integration we can rewrite them as

$$-\int dx\left[\frac{d}{dx}f_+(x)\right]^2\delta G_+(x)\wedge\int^x dy\,\delta G_+(y)$$
$$-\int dx\left[\frac{d}{dx}f_-(x)\right]^2\delta G_-(x)\wedge\int^x dy\,\delta G_-(-y)\ . \tag{31}$$

The total symplectic two-form is

$$\omega=\int dx\left\{\left[\delta f_+(x)\wedge\delta\left[\frac{d}{dx}f_+(x)\right]-\delta f_-(x)\wedge\delta\frac{d}{dx}f_-(-x)\right]\right.$$
$$+\left[\int^x dy\,\delta\left[\frac{d}{dy}f_+(y)\right]^2\wedge\delta G_+(x)+\int^{-x}dy\,\delta\left[\frac{d}{dy}f_-(y)\right]^2\wedge\delta G_-(x)\right]$$
$$\left.+\left[\int^x dy\,\delta G_+(y)\wedge\delta G_+(x)\left[\frac{d}{dx}f_+(x)\right]^2+\int^x dy\,\delta G_-(-y)\wedge\delta G_-(x)\left[\frac{d}{dx}f_-(x)\right]^2\right]\right\}$$
$$+\frac{\delta M}{\lambda}\wedge\delta\left[\int dx\,\dot G\right]\ . \tag{32}$$

In the first set of curly brackets we used

$$\dot{f} \equiv (\partial_+ + \partial_-) f = \frac{d}{dx}[f_+(x) - f_-(-x)] ,$$

and the crossed term $\delta f_+ \wedge \delta f_-$ cancels out. The left- and right-moving sectors are completely separated except for the global degrees of freedom. The cosmological constant appears only through the ratio M/λ in the last term; hence, it seems to have no other role but providing a scale of mass. This concludes our evaluation of the symplectic structure.

III. DISCUSSION

Having obtained the symplectic structure the next step is to calculate the Poisson brackets. For this we must invert the form ω: if we write

$$\omega = \tfrac{1}{2}\omega_{ij}(X)\delta X^i \delta X^j ,$$

where X^i are coordinates on the phase-space manifold, and the Poisson brackets are

$$\{F,K\} = \omega^{ij}(X)\frac{\partial F}{\partial X^i}\frac{\partial K}{\partial X^j} , \tag{33}$$

where $\omega_{ij}(X)\omega^{jk}(X) = \delta_i^k$. Since the left-moving, the right-moving, and the global degrees of freedom are mutually separated, the inversion can be performed in each sector. We consider, for simplicity, only one case, and the others are entirely similar. First we notice that

$$\omega_+ \equiv \int dx\, \delta f_+(x) \wedge \delta f'_+(x) + \int dx \int^x dy [\delta f'^2(y) - f'^2(y)\delta G_+(y)] \wedge \delta G_+(x) \tag{34}$$

can be rewritten

$$\omega_+ = \int dx\, \delta f_+(x) \wedge \delta f'_+(x) + \int dx \int dy \left[\frac{\theta(x-y) f'^2_+(y) - \theta(y-x) f'^2_+(x)}{2} \right] \times [\delta(G_+ - F_+)(x) \wedge \delta(G_+ - F_+)(y) - \delta F_+(x) \wedge \delta F_+(y)] , \tag{35}$$

where

$$f'_+(x) \equiv \frac{d}{dx} f_+(x) ,$$

and

$$\dot{f}_+(x) \equiv \exp[F_+(x)] .$$

By defining

$$K_+(x) \equiv G_+(x) - F_+(x) \tag{36}$$

we have

$$\omega_+ = \int dx\, \delta f_+ \wedge \delta f'_+ - \frac{1}{2}\int dx\, dy \left[\theta(x-y)\frac{f'_+(y)}{f'_+(x)} - \theta(y-x)\frac{f'_+(x)}{f'_+(y)} \right] \delta f'_+(x) \wedge \delta f'_+(y) + \int \int dx\, dy\, \theta(x-y) f'^2_+(y) \delta K_+(x) \wedge \delta K_+(y) . \tag{37}$$

This division into two orthogonal blocks (f_+, f'_+) and $K_+(x)$ made the inversion simplified: the inversion in the (f_+, f'_+) block is immediate, and the inverse of $\theta(x-y) f'^2_+(y)$ is

$$\frac{1}{f'^2_+(z)}\delta(z-x)\frac{d}{dx} .$$

It is noteworthy that the conjugate pairs in flat spacetime of the matter field (f_+, f'_+) are no longer canonical pairs, but some combinations of them are.

For quantization of the model one proceeds with the replacement of the Poisson brackets by commutation relations of operators on a Hilbert space. But in our problem we have to take care of the conformal anomaly and Hawking radiation. At present there is no known straightforward way of dealing with these in the formalism [17], so we leave it for future investigation. For the topologically distinctive classes of the gauge function G other than $G_+ = G_- = 0$, the symplectic structure may be quite different from the one which we presented in this article. Another point that requires more careful analysis is the neglect of the total derivative terms which we assumed in deriving (21) and (30).

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