Extended inflation with a curvature-coupled inflaton

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We examine extended inflation models enhanced by the addition of a coupling between the inflaton field and the space-time curvature. We examine two types of model, where the underlying inflaton potential takes on second-order and first-order forms, respectively. One aim is to provide models which satisfy the solar system constraints on the Brans-Dicke parameter ω. This constraint has proven very problematic in previous extended inflation models, and we find circumstances where it can be successfully evaded, though the constraint must be carefully assessed in our model and can be much stronger than the usual $\omega > 500$. In the simplest versions of the model, one may avoid the need to introduce a mass for the Brans-Dicke field in order to ensure that it takes on the correct value at the present epoch, as seems to be required in hyperextended inflation. We also briefly discuss aspects of the formation of topological defects in the inflaton field itself.

I. INTRODUCTION

Extended inflation [1,2] has proven to be an extremely interesting addition to the collection of inflationary universe models [3,4], as it reintroduces the possibility that inflation might proceed via a first-order phase transition, ending with bubble nucleation, rather than a second-order phase transition involving the continuous evolution of the inflaton field into the minimum of its potential. Such a transition is allowed to complete via the incorporation of extensions to Einstein's gravity, which alter the space-time dynamics in the presence of a metastable vacuum state. Such a theory offers a richer phenomenology than traditional "chaotic" inflationary models [5,4], in that the earliest bubbles to nucleate may be dramatically stretched by the subsequent inflation, providing an extra source of density perturbations over and above those caused by quantum fluctuations of scalar fields as present in all inflationary models.

Despite this, extended inflation is not without its problems; indeed, the problems are closely linked to this bubble production. In original versions [6,7], the "big bubble problem" simply noted that in the Brans-Dicke version of the theory, one could only have acceptably small microwave background perturbations from the bubbles provided the Brans-Dicke parameter ω took on a value below around 25, in conflict with present-day limits from solar system time-delay experiments of $\omega > 500$ [8]. More detailed calculations [9] can tighten this number yet further. Nevertheless, the bubble constraint can easily be evaded by further extensions of the gravitational sector [10,7,11,12] such as a potential for the Brans-Dicke field; that the bubble constraint applies during inflation and the timing experiment at the present epoch leaves plenty of room for theoretical maneuvering.

More recently, the big bubble problem has been extended and reexpressed in a much more restrictive way [13]. This arises through the recognition that in order to avoid unacceptably large fluctuations from bubbles, the modifications to gravity must act to break the scale invariance of the bubble distribution. This breaking of the scale invariance inevitably leads to a breaking of scale invariance in the density perturbation spectrum *from quantum fluctuations* [14], which is taken as requiring the form needed to explain large scale structure data [15]. Recent data, particularly that from the Cosmic Background Explorer (COBE) satellite [16], has imposed strong constraints on the extent to which the density perturbation spectrum can deviate from scale invariance. The crucial difference in the big bubble problem expressed in this way is that we now have two constraints acting in opposite directions *at the same cosmological time*. Further, they appear strong enough to rule out a wide range of extended inflation models [13], including all those known in which inflation is brought to an end by a first-order transition. We are at present aware of only two "extended inflation" models in the literature which safely survive these constraints. One is the Steinhardt-Accetta hyperextended inflation model [17–19], in which inflation is brought to an end via some complicated gravitational dynamics *before* any substantial bubble nucleation has taken place — the phase transition then completes in the postinflationary universe. The second is the "plausible double inflation" model [12], in which the first-order phase transition is followed by a second, slow-rolling, phase of inflation which serves to at least partly erase the bubble perturbations (for many parameters this slow-roll phase lasts long enough, more than 70 e-foldings, to completely erase the memory of the first-order inflation). Note that in neither of these models is inflation ended by the first-order transition.

In this paper, we examine a very simple extension to the basic extended inflation model which has not been considered in this context before. That is to include a coupling of the inflaton field to the scalar curvature R. This is a particularly natural term to consider adding to the theory as it does not involve the introduction of any new fields, and further it has been argued [20] that in

such theories terms of this form are generated by quantum corrections even if they are not present in the bare (classical) action. Such a term acts so as to favor the inflaton χ being trapped in a metastable $\chi = 0$ state, even if the potential for the inflaton possesses a local maximum, rather than a minimum, at the origin. In Einstein gravity such a term is known to be problematical, because if the field is trapped by the curvature term in such a state the expansion rate tends toward de Sitter in which R is a constant. Such an inflationary state, at least in the classical theory, persists forever.

Extended inflation offers a crucial difference, because even if the field is trapped in a metastable state, the solutions governing the expansion do not tend to a time translation invariant state. For instance, in a Brans-Dicke theory the scalar curvature falls off as t^2 while inflation proceeds. Consequently, one can arrange a scenario in which the curvature coupling term initially plays a critical role in aiding inflation, but later becomes insignificant allowing inflation to end. It is to this possibility that we devote this paper.

Curvature coupling has arisen in several contexts in the inflation literature. Our model has connections with all of the following.

(1) Our motivation actually came from a model by Yokoyama [21] aiming to combine cosmic strings with inflation; he used a curvature coupling term for the string field in order to keep it in its symmetric phase. Inflation was provided by a slow-rolling field, which naturally gives a decreasing R which eventually frees the string field.

(2) Curvature coupling has been added to chaotic inflation scenarios [22]. In fact, it is subject there to a very strong constraint that the coupling constant ξ be less than 0.002; otherwise the scenario is ruined.

(3) A large negative curvature coupling has been utilized by Salopek, Bond, and Bardeen [23] in their "variable Planck mass" model (see also the earlier work of Spokoiny [24]). As we shall see, their scenario has several similarities with that discussed here.

(4) Two coupled scalar fields were used by Berkin, Maeda, and Yokoyama in their "soft inflation" scenario [25], and the results obtained were similar to those of Salopek, Bond, and Bardeen [23]. The form of their action allows one field to be interpreted as the Brans-Dicke field via conformal transformation; however, the fact that the fields in the original action were coupled means that the second field in the transformed action is coupled to the Brans-Dicke field through the conformal transformation.

Our aim is to explore the possibilities introduced by this curvature coupling. The most dramatic new possibility is that inflation can proceed as if driven by a false vacuum even if the underlying inflaton potential is of second-order form possessing no metastable state, because of the ability of the curvature term to stabilize the inflaton at the origin. We shall concentrate most of our attention on this possibility. In the final section we shall widen our attention to discuss first-order potentials which already possess metastable states even without the curvature coupling; this section is closer in spirit to usual extended inflation scenarios, and the introduction of the curvature coupling reopens the possibility of ending inflation via bubble nucleation in extended gravity theories, while evading any constraints from excessively large bubbles.

II. THE THEORY

A. Equations of motion

We shall concentrate our entire discussion on the Jordan-Brans-Dicke theory [26], enhanced by a curvature coupling of the inflaton, although presumably many of these ideas go through in more generally extended models. The action is given by

$$S = \int d^4x \sqrt{-g}\left[-\Phi R + \omega \frac{\partial^\mu \Phi \partial_\mu \Phi}{\Phi} + \frac{1}{2}\partial^\mu \chi \partial_\mu \chi + V(\chi) \right. $$
$$\left. + \frac{1}{2}\xi R \chi^2 \right], \tag{1}$$

where Φ is the Brans-Dicke scalar field, ω the Brans-Dicke constant, χ the inflaton field, R the scalar curvature, and ξ the curvature coupling.[1] We shall not at this stage make a specific choice for the inflaton potential $V(\chi)$; indeed, we shall not even at this stage state whether it is of first-order form (metastable vacuum at the origin) or second order (local maximum at the origin). Apart from the last term, this is exactly the action used in the original extended inflation model.

We shall work with the action in this form. However, before proceeding it is worth looking at the action with the Brans-Dicke field redefined to have the dimensions and kinetic term of a canonical scalar field, by defining a new field $\phi^2 = 8\omega\Phi$ (be careful to distinguish lower and upper cases). The redefined action becomes

$$S_{\text{redef}} = \int d^4x \sqrt{-g}[\tfrac{1}{2}\zeta\phi^2 R + \tfrac{1}{2}\partial^\mu\phi\partial_\mu\phi + \tfrac{1}{2}\partial^\mu\chi\partial_\mu\chi$$
$$+ V(\chi) + \tfrac{1}{2}\xi R\chi^2], \tag{2}$$

where $\zeta = -1/4\omega$ confirming the Brans-Dicke field as conformally coupled for $\omega = -3/2$. We see that the two scalar fields χ and ϕ appear almost symmetrically in the action, a symmetry which would be further enhanced by incorporating a potential for ϕ. Note further that the action in this form has no pure Einstein-Hilbert term. The different roles of the two scalar fields, one as inflaton and one as gravitational scalar, are brought about by the different choices for the potentials (mass scales of the fields) and by the opposing signs of their curvature couplings.

Returning to the theory as defined by Eq. (1), the field equations obtained from S by variation with respect to the Brans-Dicke field, the inflaton field and the metric are

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[1]One must be wary of the different conventions used in the literature. With our conventions, conformal coupling occurs for $\xi = +1/6$.

$$\partial^\mu \partial_\mu \Phi = \frac{\partial^\mu \Phi \partial_\mu \Phi}{2\Phi} - \frac{R\Phi}{2\omega}, \tag{3}$$

$$\partial^\mu \partial_\mu \chi = \frac{dV(\chi)}{d\chi} + \xi R\chi, \tag{4}$$

$$
\begin{aligned}
\left(2\Phi - \xi\chi^2\right)\left(R_{\alpha\beta} - \tfrac{1}{2}Rg_{\alpha\beta}\right) =\ & \frac{2\omega}{\Phi}\left(\partial_\alpha \Phi \partial_\beta \Phi - \tfrac{1}{2}g_{\alpha\beta}\partial^\mu \Phi \partial_\mu \Phi\right) - V(\chi)g_{\alpha\beta} \\
& + \left(\partial_\alpha \chi \partial_\beta \chi - \tfrac{1}{2}g_{\alpha\beta}\partial^\mu \chi \partial_\mu \chi\right) + 2\left(\partial_\alpha \partial_\beta \Phi - g_{\alpha\beta}\partial^\mu \partial_\mu \Phi\right) \\
& - 2\xi\left[\chi\partial_\alpha \partial_\beta \chi + \partial_\alpha \chi \partial_\beta \chi - g_{\alpha\beta}\left(\chi\partial^\mu \partial_\mu \chi + \partial^\mu \chi \partial_\mu \chi\right)\right].
\end{aligned}
\tag{5}
$$

We shall consider these equations in a spatially flat Robertson-Walker space-time with line element $ds^2 = -dt^2 + a^2(t)d\Omega_3^2$, where $a(t)$ is of course the scale factor. In accordance with the assumption of isotropy, we require that our fields have no spatial dependence. Writing the Hubble parameter as $H = \dot{a}/a$, the equations of motion become

$$\ddot{\Phi} + 3H\dot{\Phi} = \frac{\dot{\Phi}^2}{2\Phi} + \frac{R\Phi}{2\omega}, \tag{6}$$

$$\ddot{\chi} + 3H\dot{\chi} + \xi R\chi = -\frac{dV(\chi)}{d\chi}, \tag{7}$$

$$H^2 = \frac{1}{3\left(2\Phi - \xi\chi^2\right)}\left[\omega\frac{\dot{\Phi}^2}{\Phi} + \frac{\dot{\chi}^2}{2} + V(\chi) - 6H\dot{\Phi} + 6\xi H\chi\dot{\chi}\right]. \tag{8}$$

The standard equations of motion for Einstein gravity with a curvature coupled scalar (with a sufficiently small ξ) can be recovered by setting $\Phi = m_{\rm Pl}^2/16\pi$, where $m_{\rm Pl}$ is the present-day Planck mass, and the effects of the curvature coupling term can be removed from these equations by taking their limit as ξ tends to zero. The scalar curvature R is given by

$$R = \frac{-\omega}{(2\omega + 3)\Phi - \xi\omega\chi^2(1 - 6\xi)}\left[(2\omega + 3)\frac{\dot{\Phi}^2}{\Phi} + (1 - 6\xi)\dot{\chi}^2 - 4V(\chi) + 6\xi\chi\frac{dV(\chi)}{d\chi}\right]. \tag{9}$$

B. Singularities in the equations

As in the Einstein gravity case, Eqs. (8) and (9) contain nontrivial singularities for particular values of the fields. Equation (8) contains a singularity when

$$\chi = \chi_1 = \pm\sqrt{\frac{2\Phi}{\xi}}, \tag{10}$$

and Eq. (9) contains a singularity when

$$\chi = \chi_2 = \pm\sqrt{\frac{(2\omega + 3)\Phi}{(1 - 6\xi)\xi\omega}}. \tag{11}$$

In the large ω limit (which we shall be considering throughout) we have $\chi_1 \simeq \sqrt{1 - 6\xi}\,\chi_2$. For $\xi \leq 1/6$ the equation shows us that the singularity in H occurs at a smaller value of the field χ than the singularity in R. As we shall be considering a scenario in which the inflaton field evolves from zero to larger values, only the H singularity need be considered for $\xi \leq 1/6$. It can also be seen from the above equations that for $\xi \geq 1/6$ that the singularity in R does not exist and we need only consider

the singularity in H. This shows that as long as we can avoid the singularity in H, the singularity in R is either also avoided or does not exist. Note that the singularity in the H equation involves the effective Planck mass becoming zero, presumably on route to negative values, which is certainly something we wish to avoid.

In practice we have shown numerically that the equations are well behaved for the entire evolution of the system. That this should in fact be the case is clear in the conformally transformed frame where the potential for the system rises rapidly as the fields evolve toward those values where the above singularities arise. The details of this calculation are given in Ref. [27].

C. Other restrictions on the fields

In addition to the restrictions imposed on the fields in order to avoid the above singularities, we get further restrictions on the fields due to the requirement that H and R be both positive and real. The former is required to have an expanding universe; the latter is not required for dynamical consistency but is a necessary ingredient for the scenario we shall outline.

Consider first the requirement that H be positive. Equation (8) expresses H entirely as a function of the fields of the theory. It is easily shown that

$$\left(2\Phi - \xi\chi^2\right) H = \xi\chi\dot{\chi} - \dot{\Phi}$$

$$\pm\frac{1}{3}\left[9\left(\dot{\Phi} - \xi\chi\dot{\chi}\right)^2 + 3\left(2\Phi - \xi\chi^2\right)\right.$$

$$\left.\times\left(\omega\frac{\dot{\Phi}^2}{\Phi} + \frac{\dot{\chi}^2}{2} + V(\chi)\right)\right]^{1/2}. \quad (12)$$

The requirement that H be positive obviously leads us to the use of the positive root in our calculations and we also get a restriction on the field values due to the requirement that the argument of the root in Eq. (12) be greater than zero. However, this is easily seen to be satisfied as long as $\chi \leq \chi_1$ (assuming only that the Brans-Dicke field is positive), and hence provides no additional restrictions.

The model that we are considering requires us to keep R positive, at least in the initial stages of the evolution of the system. Looking at Eq. (9) it can be seen that if we take ω positive (which we always do) then the prefactor to the square brackets will always be negative if $\chi \leq \chi_2$. As this condition will always be satisfied (as we require our system to evolve in such a way as to avoid the singularities in the equations) the requirement that R be positive becomes

$$(2\omega + 3)\frac{\dot{\Phi}^2}{\Phi} + (1 - 6\xi)\dot{\chi}^2 - 4V(\chi) + 6\xi\chi\frac{dV(\chi)}{d\chi} \leq 0. \quad (13)$$

In general it is difficult to say whether or not this condition is satisfied. However, in the scenarios outlined below it can be shown that in what we shall refer to as the trapped phase of the system this inequality becomes a restriction on the Brans-Dicke constant $\omega \geq -1/2$. It will also be seen that in the case of the second-order potential that, for the values of the parameters which we have chosen to meet the various constraints on the system, the system evolves in such a way as to ensure that this condition will always be satisfied. Numerical simulation of the system's evolution has also shown that for the parameters chosen the inequality Eq. (13) is always satisfied.

III. CURVATURE-COUPLED EXTENDED INFLATION

A. Initial conditions

We shall consider a scenario in which the inflaton field is initially trapped, with the aid of its curvature coupling, at $\chi = 0$. As we shall see, it depends on parameters whether or not the trapped phase is actually relevant for details such as density perturbation production. For definiteness, let us assume for the time being that the underlying potential $V(\chi)$ is of the simplest form for a second-order phase transition, that is

$$V(\chi) = \frac{\lambda}{4}\left(\chi^2 - \chi_0^2\right)^2, \quad (14)$$

where λ and χ_0 are constants.[2] Many results will depend only on the energy of the metastable state, and where that is the case we shall denote it by M^4. For this potential $M^4 = \lambda\chi_0^4/4$.

Ordinarily this potential would not admit metastable states, as $\chi = 0$ is a local maximum of the potential. The introduction of the curvature coupling changes this. We shall often refer loosely to an "effective potential"

$$V_{\text{eff}}(\chi) = V(\chi) + \tfrac{1}{2}\xi R\chi^2, \quad (15)$$

though we stress that as R depends on all the fields of the theory this terminology is not strictly accurate. However, the equations of motion show that it applies provided R is slowly varying, and it is very useful for building a picture of the possible dynamics. By analogy to the temperature-corrected effective potential, provided R is sufficiently large the field can be trapped at $\chi = 0$. For the potential above, one requires

$$R > R_c = \frac{\lambda\chi_0^2}{\xi}. \quad (16)$$

As long as R remains greater than R_c, $\chi = 0$ remains the appropriate stable solution to the classical equations. As noted in the Introduction, in general relativity the corresponding solution for R is that it remains constant, and so classically the solution pertains forever.

We pause briefly to comment on how the field may have become trapped in the first place. There is an important difference from a temperature correction in that R depends on χ. Indeed, it is the potential energy of the χ field that provides the dominant contribution to R. Ordinarily there is another perfectly good minimum of the energy functional where χ sits close to its classical minimum χ_0, and R is very small with only the Brans-Dicke kinetic contribution. Indeed, presumably it is possible, even in general relativity, for the field to tunnel via a form of gravitational instanton into this second minimum. This highlights the dangers of thinking of Eq. (15) as an effective potential, as this possibility would be very hard to see. Although tunneling by gravitational instanton provides some interesting possibilities, they are beyond the scope of this paper and we shall assume from here on that the probability of such tunneling is negligibly small.

Despite having set aside tunneling, one must still address why one should not expect the field to originally settle in the true vacuum state. This is easily resolved — it seems likely that the field can become trapped in the metastable $\chi = 0$ state by thermal effects, exactly

as envisaged in any first-order inflation model. Certainly one expects the gravitational term to assist, rather than interfere, with this process. Having deposited the field close to the $\chi = 0$ state, the thermal effects can then fall off as inflationary supercooling takes place, leaving the curvature term to hold the field in the metastable state. (Later we shall also consider the possibility that the bare potential itself has a metastable state.)

B. Approximations to inflationary evolution

Before embarking on a study of the exact solutions in various regimes, we shall here derive an approximate relationship between the scale factor and the Brans-Dicke field during inflation which shall prove very pertinent later. This result is geometric in nature, and does not depend on the details of the fields driving inflation. Suppose that inflation is proceeding with $a(t) \propto t^p$ for some high exponent p. As R has the geometrical definition

$$R = 6 \left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^2 \right] , \tag{17}$$

one finds

$$R = \left(12 - \frac{6}{p} \right) H^2 \simeq 12 H^2 . \tag{18}$$

That is, as long as inflation is proceeding sufficiently rapidly the scalar curvature is given simply by $12H^2$. Taking this to be the case, and substituting this into the equation for Φ, dropping the $\ddot{\Phi}$ term by use of a slow-roll approximation,[3] yields an algebraic relation between $\dot{\Phi}/\Phi$ and H:

$$6H \frac{\dot{\Phi}}{\Phi} - \frac{\dot{\Phi}^2}{\Phi^2} - \frac{12}{\omega} H^2 = 0 , \tag{19}$$

the appropriate solution to which is

$$H \simeq \frac{\omega}{2} \frac{\dot{\Phi}}{\Phi} . \tag{20}$$

Integrating yields

$$a(t) \propto \Phi^{\omega/2}(t) . \tag{21}$$

Although approximate, this result is useful because it applies regardless of whether inflation is driven by a trapped or rolling field in a Brans-Dicke theory. Further, we have found numerically that it provides an excellent approximation, giving an analytic link between the number of e-foldings of inflation and the growth of the Brans-Dicke field.

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[3]One might suggest that instead one should define a canonical scalar field such as ϕ used in Eq. (2) and drop the $\ddot{\phi}$ term; this alters the coefficient of the second term in Eq. (19) but does not change the approximate solution Eq. (20).

C. The trapped phase

While in the trapped phase, the solution is extremely simple, because the equations reduce directly to the standard extended inflation ones [29,1]. Along with $\chi(t) = 0$, one has

$$\Phi(t) = \Phi_0 \left(1 + Bt \right)^2 , \tag{22}$$

$$a(t) = a_0 \left(1 + Bt \right)^{\omega + 1/2} , \tag{23}$$

where $B^2 = M^4/6\alpha^2\Phi_0$ and $\alpha^2 = (2\omega + 3)(6\omega + 5)/12$. This is not the most general solution, but it is known to be a late-time attractor. It almost satisfies the approximate relation Eq. (21), with a small correction accounting for the avoidance of a slow-roll approximation. One can readily calculate the behavior of the scalar curvature to be

$$R(t) = \frac{6\omega (2\omega + 1) B^2 \Phi_0}{\Phi(t)} = \frac{6\omega (2\omega + 1)}{(2\omega + 3)(6\omega + 5)} \frac{2M^4}{\Phi(t)}, \tag{24}$$

where the prefactor on the right-hand side is very close to unity. As advertised, the scalar curvature falls as t^2 while in the trapped phase. It is thus inevitably destined to pass below R_c, bringing the trapped phase to an end. Regardless of initial conditions, this occurs at the same value of the Brans-Dicke field — for the second-order potential this is at

$$\Phi_c = \frac{6\omega (2\omega + 1)}{(2\omega + 3)(6\omega + 5)} \frac{\xi \chi_0^2}{2} . \tag{25}$$

Sufficient inflation is arranged by choosing a small enough initial value of Φ. However, one must be careful to avoid breaching the classical regime. Comparing the action with the Einstein-Hilbert one, the effective Planck mass at a given epoch during the trapped phase is $m_{\mathrm{Pl}}^{\mathrm{eff}} = \sqrt{16\pi\Phi}$, requiring $\Phi_0 \gg M^2/16\pi$.

D. The rolling phase

With this second-order potential, once $R < R_c$ it becomes favorable for the field to roll down the potential toward the minimum at $\chi = \chi_0$. It will be displaced from the local maximum of the potential by quantum fluctuations. When the inflaton field begins to evolve, the simple solutions of the trapped phase no longer apply and an analytic solution to the equations of motion is no longer possible. A very important feature of this rolling is that the Planck mass begins to *decrease*, as the curvature coupling of the χ field makes its influence felt. As we shall see, this is very significant.

In general it is possible for a significant amount of inflation to occur during this phase. The amount which occurs will depend on all the parameters of the theory, and we can identify two cases with distinct physics. The first is that the number of e-foldings in the rolling phase is substantially less than 60. In such a case, the field was still in the trapped phase when scales corre-

sponding to the present observable Universe crossed the Hubble radius during inflation, and particularly significantly this means that the density perturbations leading to large scale structure formation were generated during the trapped phase. The second case corresponds to having more than 60 e-foldings in the rolling phase. In that case, none of the physics pertaining to the presently observable Universe is linked directly to the physics of the trapped phase. Nevertheless, even in this case the prior existence of the trapped phase is assumed, leading to a precise form of initial conditions for the much more complex rolling phase.

Once the field reaches its minimum, it will oscillate and give a conventional reheating scenario. There is unlikely to be much difference here from conventional treatments, and in any case the advent of at least the possibility of electroweak baryogenesis [30] has effectively removed any stringent reheating constraint anyway. What is much more significant in this phase is the behavior of the Brans-Dicke field, as we shall shortly discuss.

In the limit of small ξ (i.e., where the inflaton is decoupled from the curvature) it has been shown by Linde [31] that the relationship between the scale factor and the Brans-Dicke field Φ given by Eq. (21) holds even in the rolling phase. The analytic arguments of Sec. III B indicate that it should be a good approximation even at large ξ and we have verified this numerically. This has some very important consequences for this theory which are discussed below.

IV. PARAMETER CONSTRAINTS

Our model is rich in parameters. Even ignoring initial conditions (which it is assumed are rendered irrelevant by the trapped phase inflationary attractor), one has ω, ξ, and at least two parameters describing the inflaton potential. However, there are substantial constraints that must be obeyed, and in other cases there are at least guideline values. In the latter category is the value of the symmetry breaking scale for the inflaton potential, χ_0. Primarily we shall consider χ_0 to be a typical grand unified theory (GUT) scale parameter in the range $10^{-5} m_{\rm Pl}$ to $10^{-3} m_{\rm Pl}$, though in complete generality its range of plausible values is much wider.

Let us progress through the constraints, assuming the second-order potential of Eq. (14).

A. Classical behavior and sufficient inflation

We have already remarked above that classicality demands $\Phi_0 > M^2/16\pi$. When we couple this to the requirement that there be a trapped phase, $\Phi_0 < \Phi_c$, this requires $\sqrt{\lambda} < 8\pi\xi$. This can be further strengthened by the requirement of sufficient inflation, the usual requirement being a minimum of 70 e-foldings to ensure the solution of the usual cosmological problems. The number N of e-foldings required in the trapped phase depends on the number available in the rolling phase; once this has been calculated the trapped phase solutions yield

$\Phi_0 < \exp\left[-4N/(2\omega + 1)\right]\Phi_c$ tightening the constraint on λ somewhat.

B. Normalization of the present-day Planck mass

This is the most crucial constraint on the scenario. One must reproduce the present-day value of the Planck mass, which with our conventions requires that the coefficient of R be $-m_{\rm Pl}^2/16\pi$. By the present the χ field will have fallen into its true vacuum state, so the requirement is

$$\Phi_p - \xi\chi_0^2/2 = m_{\rm Pl}^2/16\pi, \tag{26}$$

where Φ_p is the present-day value. We see a crucial aspect of the scenario here; with this choice of potential, were Φ to retain its critical value then the effective Planck mass would be canceled almost exactly between the two fields. Hence the present-day Planck mass is entirely given by the growth in Φ since the critical point. It is well known that for large ω the fractional growth in Φ during conventional radiation or matter domination is small [29,6] if Φ_c is a free field. Hence without a potential the only phase during which Φ may grow significantly beyond Φ_c is during the rolling phase. A precise calculation of this amount of growth requires numerical study, but the analytic relationship Eq. (21) provides an excellent indication.

Let us parametrize this growth, whether its source be dynamical or a potential, via a quantity k, which in general depends on all the parameters in the action, by writing

$$\Phi_p = k\Phi_c. \tag{27}$$

Then the normalization condition for this potential requires

$$(k - 1)\Phi_c = m_{\rm Pl}^2/16\pi, \tag{28}$$

which for the second-order potential implies that one must have

$$k = 1 + \frac{(2\omega + 3)(6\omega + 5)m_{\rm Pl}^2}{6\omega(2\omega + 1)8\pi\xi\chi_0^2} = 1 + f(\omega, \xi, \chi_0), \tag{29}$$

if the Brans-Dicke field is to be correctly normalized. In general, the evolution of Φ will lead to a k value that does not satisfy this constraint, symptomizing a failure to match the present-day Planck mass. The parameters must be selected in order that this condition is met.

As an excellent guide, Eq. (21) gives an expression for k in terms of the number of e-foldings N that occur during the rolling phase

$$N = \frac{\omega}{2}\ln k. \tag{30}$$

If the fractional growth of the field after the trapped phase is small, then the critical value of Φ is larger than would be required to give the present-day Planck mass because of the counteracting effect of the χ field; k will be less than 2 in Eq. (28). That is, the effective Planck

mass *increases* up to the critical point, and thereafter *decreases* to its present-day value.

We shall find that the near cancellation of the two contributions is an obstacle to model building. Our discussion will follow three possibilities, of roughly decreasing attractiveness.

(1) The present-day Planck mass is determined by the dynamical growth of Φ after passing the critical point.

(2) The present-day Planck mass is determined by the introduction of a potential for Φ as utilized in hyperextended inflation [19]. This potential will have a minimum at $\Phi = m_{\rm Pl}^2/16\pi + \xi\chi_0^2/2$, which is generically at a larger value of Φ than the critical value. It must be acknowledged though that such a potential involves very serious fine-tuning if it is not to interfere with the inflationary dynamics [32].

(3) The cancellation between the two scalar field terms is a consequence of the specific potential chosen, as it is caused by a precise relation between the curvature at the top of the bare potential $V(\chi)$ and the vacuum expectation of χ in the true vacuum. For more general potentials such a cancellation need not occur, and the factor $(k-1)$ would be unlikely to be much less than unity. However, the potential as given is the most general renormalizable potential with a $\chi \to -\chi$ symmetry and zero potential energy in the true vacuum state, and one would need strong motivation to go to a more general case.

C. Large enough ω

In a pure Brans-Dicke theory, the present-day limit on ω is that it should exceed 500 [8], though there are no constraints if the Brans-Dicke field is anchored by a potential. Our model is more complex, due to the competing contributions to the present gravitational "constant," and so one must be careful in assessing the constraint.

In the literature [33], constraints are quoted via post-Newtonian parameters derived from a gravitational action written in the form

$$S_{\rm grav} = \int d^4x\sqrt{-g}\left[-\tilde{\Phi}R + \tilde{\omega}(\tilde{\Phi})\frac{\partial^\mu\tilde{\Phi}\partial_\mu\tilde{\Phi}}{\tilde{\Phi}}\right], \quad (31)$$

which is that of a general scalar-tensor theory. At present it is fair to assume that the χ field is fixed in its vacuum, and so the relevant transformation is

$$\tilde{\Phi} = \Phi - \xi\chi_0^2/2, \quad (32)$$

yielding

$$\tilde{\omega}(\tilde{\Phi}) = \omega\frac{\tilde{\Phi}}{\tilde{\Phi} + \xi\chi_0^2/2}. \quad (33)$$

The post-Newtonian parameters depend on $\tilde{\omega}$ and $d\tilde{\omega}/d\tilde{\Phi}$, but for this $\tilde{\omega}$ one can ignore the latter (which gives a $1/\tilde{\omega}$ correction) and take the constraint as $\tilde{\omega} > 500$ [33]. Using the k parametrization of the above subsection then yields

$$\tilde{\omega} = \frac{(k-1)\,\omega}{k}. \quad (34)$$

Any system we propose must yield a value of $\tilde{\omega}$ greater than 500. As long as the cancellation between the two contributions to the Planck mass is not very precise, this gives the standard result. But, as we have commented, the fractional growth of Φ from the critical point can be very small, for some parameters only a few percent. In that case, the solar system constraint is much stronger than expected. The physical reason for this is clear; the larger the Φ field's value, the more dramatic the effects of its variation and the larger ω has to be to damp them down.

Let us finally remind ourselves that in the case where there is also a potential for Φ, there is no ω constraint as the potential anchors the field, preventing the (spatial) variation which gives rise to the deviations from the general relativistic predictions in our solar system.

D. Density perturbations

The strongest argument in favor of inflation is its ability to produce adiabatic density perturbations, and it is reasonable to require that these be of the appropriate size to match observations. We take this constraint as requiring that the COBE $10°$ microwave anisotropy is reproduced. The perturbations on interesting scales are produced around 60 e-foldings from the end of inflation, and there are two possible situations that may arise.

(1) *The χ field is in the trapped phase 60 e-foldings from the end of inflation.* In this case the calculation is just that of the standard extended inflation scenario [14]. The standard technique [23] for calculating density perturbations in scalar-tensor theories is the conformal transformation, by which the gravitational term is brought into the Einstein-Hilbert form, followed by field redefinitions to render the Brans-Dicke kinetic term into canonical form. In the trapped phase we can assume that the inflaton degrees of freedom are negligible, and concentrate only on the Brans-Dicke field. Utilizing a conformal factor so that $\tilde{g}_{\mu\nu} = (16\pi\Phi/m_{\rm Pl}^2)g_{\mu\nu}$, this yields a transformed action

$$S_{\rm con} = \int d^4x\sqrt{-\tilde{g}}\left[-\frac{m_{\rm Pl}^2\tilde{R}}{16\pi} + \frac{1}{2}\partial_\mu\psi\partial^\mu\psi + \exp\left(-\frac{2\psi}{\psi_0}\right)M^4\right], \quad (35)$$

where the field ψ is defined as

$$\psi = \psi_0\ln\left(\frac{16\pi\Phi}{m_{\rm Pl}^2}\right), \quad (36)$$

$$\psi_0^2 = \frac{(2\omega + 3)\,m_{\rm Pl}^2}{16\pi}. \quad (37)$$

One can then apply the standard formula for calculating the perturbation amplitude. The final step is to assume that at late times the conformally transformed frame and the untransformed frame coincide to high accuracy, so that the result applies in the original frame.

Although this appears largely motivated by a laziness regarding transforming back, the known accurate constancy of the gravitational coupling over the last few Hubble times effectively guarantees it. The density perturbation δ_H (effectively the density contrast at horizon crossing and formally defined as in [15]) is given by

$$\delta_H = \sqrt{\frac{512\pi}{75}} \frac{V^{3/2}(\psi)}{m_{\rm Pl}^3 [dV(\psi)/d\psi]} = \frac{1}{20\pi} \left[\frac{(2\omega+3)}{6}\right]^{1/2}$$
$$\times \left(\frac{M}{m_{\rm Pl}}\right)^2 \frac{m_{\rm Pl}^2}{\Phi}, \quad (38)$$

where the right-hand side is evaluated at horizon crossing $\kappa = aH$, where κ is the comoving wave number. The COBE result [16] requires[4] that δ_H evaluated 60 e-foldings from the end of inflation equals 1.7×10^{-5} [15]. The equation for δ_H can be evaluated explicitly 60 e-foldings from the end of inflation if we make the approximation that Φ at the end of inflation is equal to Φ_c, the value of Φ at the critical point. One can then use the trapped phase solutions to evaluate Φ 60 e-foldings from the end of inflation. This actually gives us a slight overestimate of the size of δ_H, but for the cases we discuss the correction (which appears in the exponential term below) is at most a few percent. This gives us a value for δ_H of

$$\delta_H = \frac{F(\omega)}{20\sqrt{3}\,\pi} \sqrt{\frac{\omega\lambda}{\xi^2}}, \quad (39)$$

where $F(\omega)$ is given by

$$F(\omega) = \left(\frac{2\omega+3}{2\omega}\right)^{\frac{1}{2}} \frac{(6\omega+5)(2\omega+3)}{6\omega(2\omega+1)} \exp\left(\frac{240}{2\omega+1}\right),$$
$$(40)$$

and is close to unity for large ω. If $N_{\rm roll}$ e-foldings occur during the rolling phase, the term in the exponential is modified to $(240-4N_{\rm roll})/(2\omega+1)$, but for typical parameters (primarily large ω) this is a tiny correction, saving one the trouble of the numerical calculation of $N_{\rm roll}$.

(2) *The χ field is in the rolling phase 60 e-foldings from the end of inflation.* This is a much trickier situation. Because both scalar fields are evolving, and both couple to the curvature tensor, the simple conformal transformation above cannot be used. We do not intend to investigate this situation here due to this complexity, though unfortunately this will restrict our ability to make precise statements in this case. A general formalism for dealing numerically with multiple nonminimally coupled fields is provided elsewhere [27], and would be applicable in more general circumstances, including the small ξ limit of the theory as investigated by Linde [28].

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[4]This value is appropriate to nearly flat spectra with negligible gravitational waves, which is exactly the situation here as in the transformed frame the model corresponds to power-law inflation with a very high exponent.

E. Putting it all together

With a free Brans-Dicke field, it proves impossible to satisfy all the constraints if one demands that large scale structure perturbations were generated during the trapped phase. This only requires the solar system constraint and the approximate relation between N and k; one has

$$N \simeq \frac{\omega}{2} \ln k > 250\frac{k}{k-1} \ln k. \quad (41)$$

By definition $k > 1$, which implies that the k-dependent factor always exceeds unity and hence the number of e-foldings in the rolling phase always greatly exceeds 60 when the solar system bounds are satisfied (note that this has not even required the correct Planck mass normalization). The trapped phase can therefore only find its role in providing unique initial conditions for the rolling phase.

By consequence, the scenario is substantially complicated in that the density perturbation calculation fits into the second case, where analytic progress is not possible. Nevertheless, there is no reason to suppose that a satisfactory level of density perturbations cannot be achieved, as the parameter λ is not fixed by any other criterion and can therefore be selected to fit the perturbation constraint. This is because we find numerically, but have been unable to confirm analytically, that the amount of inflation and the growth of Φ are independent of λ, as is the case in general relativity. Consequently the details of the dynamical normalization are unaffected by adjusting λ to ensure the correct level of perturbations. A detailed investigation of the perturbation constraint must however be reserved for future work.

Setting aside the perturbation constraint, it is clear that the correct present-day Planck mass can be achieved, as Eq. (29) amounts to a single constraint on the ξ-χ_0-ω parameter space. This constraint yields a two-dimensional surface in the parameter space, which must be calculated numerically. For every point on that surface there is then associated a value of the parameter $\bar{\omega}$ which may or may not satisfy the solar system timing constraint; this too can only be determined by use of numerical calculations. We have shown numerically that it is indeed possible to find parameters satisfying all the constraints, though the parameter space is too extensive for us to have investigated its limits. For example, the correct present-day value of the Planck mass is obtained if we choose the parameters to be $\chi_0 = 10^{-3}$, $\xi = 7 \times 10^5$, and $\omega = 1000$. This particular choice of parameters does not however result in a model which satisfies the constraint on the parameter $\bar{\omega}$; it gives a value of $\bar{\omega} = 53$.

Working from this example, it is possible to obtain a picture of the two-dimensional surface in the ξ-χ_0-ω parameter space. First, further working models can be obtained by changing the values of the parameters ξ and χ_0 but keeping the product $\xi\chi_0^2$ constant, i.e., $\chi_0 = 10^{-5}$, $\xi = 7 \times 10^9$, and $\omega = 1000$ produces a model with exactly the same values for the Planck mass and $\bar{\omega}$.

Second, it is possible to obtain further models which produce the correct value of the Planck mass, by chang-

ing ξ and χ_0 in such a way that the product $\xi \chi_0^2$ does not remain constant, but then we must also use a different value for ω. For example, $\chi_0 = 10^{-3}$, $\xi = 7 \times 10^6$, and $\omega = 10^5$. In this case $\tilde{\omega} = 546$, satisfying the constraint on this parameter. Of course in this case the system is in the rolling phase 60 e-foldings from the end of inflation, as the calculation above proves it must be. Again, equivalent models, with the same value for ω and $\tilde{\omega}$, can be obtained by varying ξ and χ_0 but keeping their product constant.

If the use of a free Brans-Dicke field is abandoned, then models become much easier to construct, as the potential invalidates both the ω constraint and the need to normalize the Brans-Dicke field dynamically. It is assumed that such fine-tuning has been introduced [32] as to make the Brans-Dicke potential flat enough during inflation to have negligible affect on the dynamics.

By allowing $\tilde{\omega}$ to be less than 500, we can obtain working models with a wide range of values of the parameters ω, χ_0, and ξ. In particular, it becomes possible, through a sufficiently low choice of ω, that the inflaton is still in the trapped phase 60 e-foldings from the end of inflation, as was the case for the first example quoted above. Then the simpler density perturbation calculation applies and can be used directly to fix the value of λ; the possible range of values for λ is also wide. For the first example set of parameters quoted above, λ is of order 10^3.

V. CURVATURE COUPLING AND FIRST-ORDER INFLATION

In the sense that the relevant evolution is completely classical, we have up until now been discussing what is essentially a slow-rolling inflation model, with the novelty of the inflaton field being trapped in a false vacuum state for a finite duration by the curvature coupling. In this section we consider a case closer to the heart of the original extended inflation model, by considering an underlying inflaton potential $V(\chi)$ which itself exhibits a metastable vacuum state in which the field is trapped. Such a situation may arise, for example, via a Coleman-Weinberg-type potential [34]. For simplicity we model this potential via a quartic polynomial

$$V(\chi) = M^4 + \beta \chi^2 + \gamma \chi^3 + \delta \chi^4, \tag{42}$$

which will allow us to utilize tunneling results found by Adams [35]. An appropriate choice of the parameters gives a metastable vacuum at the origin and a true vacuum state on the positive χ side. Although this potential does not have the symmetry about $\chi = 0$ that would ensure full consistency with our incorporation of the curvature term, we expect that the effect of the curvature coupling term would be the same for a wide range of first-order potentials. This particular potential has been chosen as a simple illustration of the effect of the curvature coupling.

Ordinarily in extended inflation this theory suffers from the big bubble problem, because the efficiency of nucleation is only a modest function of time during in-

flation. Nucleation efficiency is normally expressed via the dimensionless parameter describing the nucleation per hubble volume per hubble time

$$\epsilon = \Gamma/H^4, \tag{43}$$

where Γ is the nucleation per unit volume per unit time. For an unchanging potential Γ is constant. The criteria for the phase transition to complete is that ϵ be of order one, which is brought about in extended inflation by the time variation of H. However, ϵ grows slowly by comparison with the scale factor, so bubble nucleation can still be sufficiently probable in the earlier stages (say 50 e-foldings from the end) to allow large bubbles to be generated.

Our aim is to use the curvature coupling to suppress this early nucleation. One can see the motivation from the "effective potential." When R is sufficiently large, it is the $\chi = 0$ state which is the minimum, and there can be no tunneling, barring the possibility of gravitational instantons as discussed earlier.[5] Only as R drops does the "effective potential" develop the second minimum. Once it falls below the $\chi = 0$ minimum, nucleation then becomes favorable, increasingly so as R falls. Ideally, the underlying potential should possess a very rapid nucleation rate ϵ, so that once the effect of the curvature coupling term is diminished the phase transition rapidly completes. The desired overall effect then is for the curvature coupling to inject a time dependence into Γ, which will act in concert with the time dependence in H to make ϵ a more rapidly varying function than would otherwise be the case and thus suppress the formation of large bubbles. In the worst case, where Γ is approximately constant over the last 60 or so e-foldings, we will reproduce the standard extended inflation bubble spectrum. Any time dependence in Γ helps.

At the point at which the minima become degenerate it can be shown that the value of the Brans-Dicke field is

$$\Phi_{\text{deg}} = \frac{24\omega \, (2\omega + 1)}{(6\omega + 5) \, (2\omega + 3)} \frac{\xi \delta M^4}{\gamma^2 - 4\beta\delta}. \tag{44}$$

Φ enters this calculation through the trapped phase solution for the scalar curvature.

We restrict the potential in such a way as to ensure that it is always positive, and denote the zero of the potential by $\chi = \chi_0$. With these restrictions, we find that for generic parameters the critical value is similar to the second-order case with $\Phi_{\text{deg}} \simeq \Phi_c$. Because we are dealing with more general potentials than in the last section, the cancellation between the contributions to the present-day Planck mass from the critical Φ and the vacuum value of χ is not exact, so a k factor as defined earlier need not be extremely close to unity. In this case therefore the solar system constraint on ω for a free Φ field is close to the usual one. The question of normalizing the Brans-Dicke field hinges on three possibilities.

(1) Φ_{deg} is around the value needed to give the correct

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[5] Of course, further work is required to decide whether or not this possibility really can be consistently ignored.

value of the Planck mass today. We will assume here that the value of ξ is such that the subsequent growth in the field Φ does give us the correct value of the Planck mass today.

(2) A mechanism adjusts Φ after the phase transition completes.

(3) If Φ_{deg} is extremely small one could suppose that bubble nucleation takes place so slowly that it has time to grow to the value needed to produce the correct value of the Planck mass at the point at which the phase transition completes.

In practice however, the third possibility is of no utility, as by implication so much inflation would have to occur while Φ grew that the epoch during which the curvature term was important would be inflated completely away. It may also give rise to problems due to the negative χ contribution to the Planck mass being greater than the positive Φ contribution in those bubbles that nucleate early. We shall therefore assume that the first of the above possibilities is realized.

In a typical extended inflation scenario the requirement that the parameter ϵ be of order one before there is enough time for "big bubbles" to grow is found to impose a strong limit of the Brans-Dicke parameter $\omega \leq 20$ [9], which is in disagreement with the limit imposed upon it by solar system timing experiments. In order to examine the rate of growth of the bubble nucleation parameter ϵ in our model, we have used the tunneling calculations given by Adams [35] for a general quartic potential.

First we can fix parameters using the density perturbation expression Eq. (38), which remains valid, M being as in the second-order case the false vacuum energy density. As before, large ω allows us to use Φ_{deg} in this expression, and assuming field normalization according to case 1 above we get

$$\frac{M^4}{\xi^2 \chi_0^4} = N_1, \qquad (45)$$

where the number N_1 is defined to be

$$N_1 = \frac{300 \pi^2 \delta_H^2}{\left(\omega + \frac{3}{2}\right)\left[\exp\left(\frac{60}{2\omega+1}\right) - 1\right]^2} \simeq \frac{\pi^2}{3} \, \omega \, \delta_H^2, \qquad (46)$$

with the last approximate equality holding at large ω.

We make the stringent assumption that we shall only allow 45 e-foldings between the potential being degenerate and ϵ being of order unity. The value 45 is chosen because bubbles would have to nucleate before this in order to be stretched to observationally awkward sizes. If the constraints we derive seem very strong, it is worth recalling that a much less stringent condition would also reduce the bubble spectrum on large scales, and further investigation (hopefully including a discussion of gravitational instantons) is required before one can be definitive on the merits of our proposal.

We shall be making use here of results from a paper by Adams [35]. He provides a numerically derived fitting function for the Euclidean four action, expressed in terms of the numerical coefficients of the quartic potential, which is precisely the result we need. Using the usual equation for the decay probability per unit volume

per unit time, which is proportional to the exponential of the Euclidean four action, and our analytic solution for the Hubble parameter in the trapped phase, it can be shown that the condition for the parameter ϵ, as defined above, to be of order one within 45 e-foldings of the minima becoming degenerate is

$$N_2 \left(\frac{\chi_0^4}{M^4}\right) \leq \ln\left[\left(\frac{3}{8\pi}\right)^2 \frac{m_{\text{Pl}}^4}{M^4}\right]. \qquad (47)$$

The numerical factor N_2 on the left of this equation depends on the choice of ω, the number of e-foldings between degeneracy of the minima and $\epsilon = 1$, and the geometry of the bare potential. If the geometry of the bare potential is chosen such that the parameter ϵ would be of order one at an early stage during inflation were the curvature coupling not present (that is, so that the bare ϵ at the end of inflation would be well in excess of unity), then the dependence of N_2 on the geometry of the bare potential in the above equation is small. N_2 changes significantly when we change either ω or the number of allowed e-foldings between degeneracy of the minima and $\epsilon = 1$. Assuming 45 e-foldings, then in the cases $\omega = 10000$, $\omega = 1000$, $\omega = 500$, $\omega = 50$, and $\omega = 20$ this numerical factor is found to be $N_2 \sim 10^5$, $N_2 = 400$, $N_2 = 80$, $N_2 = 0.9$, and $N_2 = 0.07$, respectively.

Assuming the value of the present-day Planck mass is obtained as in case 1 above then it is easily shown that this implies a condition on ξ as

$$\xi = \frac{m_{\text{Pl}}^2}{8\pi \chi_0^2 \left[\exp\left(\frac{180}{2\omega+1}\right) - 1\right]} \equiv \frac{N_3 m_{\text{Pl}}^2}{\chi_0^2}. \qquad (48)$$

Using Eqs. (45), (47), and (48) the condition for the phase transition to complete within 45 e-foldings of the tunneling rate becoming nonzero, with the correct value of the Planck mass and reproducing the correct level of density perturbations, is easily shown to be

$$\frac{N_2 \chi_0^4}{N_1 N_3^2 m_{\text{Pl}}^4} \leq \ln\left[\left(\frac{3}{8\pi}\right)^2 \frac{1}{N_1 N_3^2}\right]. \qquad (49)$$

Given that we have fixed the geometry of the bare potential and have chosen to allow only 45 e-foldings between degeneracy of the minima and $\epsilon = 1$ then the above inequality basically determines the maximum possible value of χ_0 for any given ω. For each ω the numbers N_1 and N_3 are easily evaluated and values of the more complicated number N_2 are given above for some interesting values of ω.

What one finds when this inequality is evaluated for the various values of ω given above is that for all ω in agreement with the constraint $\omega > 500$, all χ_0 values in what we specified to be our preferred range satisfy the inequality. As ω is reduced the upper end of this range is excluded by the inequality.

This appears to be a very interesting result as it implies that the scenario outlined above allows one to overcome the big bubble problem, even for large values of ω which are in agreement with observational constraints; unfortunately, as in the second-order case the parameters of

the theory are large, taking on their smallest values at the high end of the preferred χ_0 range. For example, for $\omega = 1000$ and using $\chi_0 = 10^{-3}$ (which is allowed by the inequality) then ξ is of order 10^5 and the quantity $N_1 \xi^2$, which is the closest equivalent to the coupling λ in the second-order case, is also of order 10^5. The size of these quantities increases with increasing ω (at constant χ_0) and decreases with increasing χ_0 (at constant ω).

We would like further to point out that in this case introducing a mechanism to normalize Φ (and hence the Planck mass) after inflation makes the constraints harder to satisfy. However, we remind the reader that a detailed analysis of the bubble constraint would not require such a stringent condition on the evolution of ϵ, and that any time dependence in Γ must necessarily aid the evasion of the big bubble problem. Only a small suppression is necessary to enable one to evade the problems associated with the required flatness of the perturbation spectrum, though should the model still possess $\omega < 500$ during inflation one would not be able to dispense with a mechanism such as a mass for Φ to avoid violation of the solar system constraints.

In both this and the second-order case, if we demand that the inflationary perturbations were contributing only part the COBE signal (the remainder coming say from topological defects) then the couplings could be kept small for a wider range of χ_0 values.

The results above certainly deserve further detailed examination.

VI. DISCUSSION AND CONCLUSIONS

We have investigated models based on extended inflation, with the additional feature that the inflaton field is coupled to the space-time curvature. This extension is a particularly natural one, as such a term would be generated by quantum corrections even if absent in the bare (classical) action.

Several new physical features arise, including the following.

(1) Even with a second-order potential, it becomes possible to have a period of inflation driven as if by a trapped vacuum phase. Unlike in Einstein gravity, such a phase ends naturally via an instability induced by the decreasing spatial curvature.

(2) It seems likely that working models exist for ω chosen large enough to satisfy the solar system timing constraints with such second-order models, though a detailed analysis of the rolling phase is essential to confirm this. Introducing a potential for the Brans-Dicke field introduces yet more possibilities at the cost of fine-tuning.

(3) With an underlying first-order potential, it is possible to construct a model where inflation ends by bubble nucleation, in a theory with a modified gravitational sector. This can even be achieved with values of ω satisfying the solar system bounds without introduction of a potential for the Brans-Dicke field. Successful completion of the phase transition is brought about by the inflaton's curvature coupling introducing a time variation into the nucleation rate Γ, a mechanism which bears some similarity to models where modifications of the particle sector

alone allow inflation to end via bubble nucleation [36].

We note also that, in common with all extended inflation models, one expects the production of topological defects at (or close to) the end of inflation [37]. This is because the direction of symmetry breaking in the inflaton field is only decided at the end of inflation, when the field becomes unstable to rolling or tunneling from the origin. (Such an effect is also possible with two field models of inflation using field couplings and Einstein gravity [38].) The role of topological defects can be very important in providing extra restrictions on the parameters, and indeed our choice of range for χ_0 is more or less motivated by this criterion. All types of defects are problematical if the characteristic scale χ_0/m_{Pl} is too large.

For the potential as we have been writing it thus far, it would be domain walls which would form, which would of course be disastrous. All of global strings, monopoles, textures, and nontopological textures exhibit scaling solutions, and are allowed provided that the energy scale is sufficiently low, less than around $\chi_0 = 10^{-3} m_{Pl}$ [39]. In that case all we have said goes through as before with χ replaced by $|\chi|$, the angular degrees playing no role until the instability sets in. One could also imagine that the inflaton symmetry might be gauged, in which case gauge strings, which also scale, would not cause any problems if the characteristic scale is below about the same value [40]. Gauge monopoles are rather more interesting, as they are the only defect whose existence is inevitable if the symmetry is connected to a grand unified theory. Further, gauge monopoles do not scale, and the constraints on their formation are much tighter than the other cases, leading to a much lower limit of $\chi_0 < 10^{-8} m_{Pl}$ or even less [4].

However, even if the values of χ_0 above are violated, it is possible for consistency with observations to be attained if enough inflation occurs after the defects form, during the rolling phase. For defects which possess scaling solutions, one needs to expand the correlation length practically to the present horizon size, and so one really needs to erase all observational impact of the trapped phase. For gauge monopoles, however, one can get away with much less, as one needs only sufficiently dilute their overabundance. The standard estimates [4] suggest around 10^{25} times too high a number density at formation, which can thus be remedied by an increase in the correlation length due to inflation of around 10^8, corresponding to about 20 e-foldings. Models with a free Brans-Dicke field are always very safe, but those with a potential or based on a first-order potential are capable of only diluting monopoles to just below observable limits.

In standard inflationary models, the defect constraint is usually weak, as with standard inflation it is nearly generic for the inflaton scale governed by large scale structure constraints to be lower than the defect scale required to have a marked effect on structure formation and microwave anisotropies [41]. In this case the restrictions are stronger, because the Planck mass during the trapped phase is higher than at present, and so the inflationary scale can be higher.

Of course, according to one's taste, if the symmetries are of a type thought useful for structure formation (pri-

marily gauge strings or global texture), then one could try and use them, giving three possibilities.

(1) Use the inflaton perturbations for everything, and ensure the defect scale is too low for any noticeable effects.

(2) Use the defects for everything, ensuring that the inflaton perturbations are too small, to have a noticeable effect. As stated above, this is rather tricky, but perhaps not impossible in models with a small λ to suppress the vacuum energy with respect to the expectation value of the true vacuum.

(3) Use them both in combination. This is a rather ugly possibility, but irritatingly it is probably the one best suited to the present data. In inflation models, the COBE result is generally regarded as being a bit higher than one would like based on prejudice regarding galaxy formation [15]. Meanwhile, defects are recognized as providing larger microwave anisotropies for a given size of density perturbation [39,42], but probably to a greater extent than one would like. In combination, the defects could be used to soak up some of the excess microwave anisotropies, without having too much of an effect on the density perturbations.

We have investigated the possibilities introduced by the addition of the inflaton's curvature coupling. One can use the curvature coupling to temporarily stabilize the false vacuum even with an underlying second-order potential. This permits extended inflation to end by classical field evolution while satisfying the solar-system constraints on ω, though we have not been able to provide a definitive density perturbation calculation here as the relevant inflation occurs in the rolling phase rather than the trapped one. With an underlying first-order potential, it is possible to evade the big bubble problem which has plagued previous models, and while the overall theory lacks the simplicity of models such as chaotic inflation, it does have the advantage of being based on what appears to be fairly plausible particle physics, and provides a potentially rich phenomenology with topological defects being formed at the end of inflation. The most likely versions would however rely on the inflaton field perturbation to provide structure formation, in which case the generic prediction of a very flat spectrum and negligible gravitational wave production may be testable in the near future [43].

However, there is much scope for further work providing an accurate delineation of the parameter space of the model, including the development of a better understanding of density perturbations induced by the rolling phase. From our preliminary numerical studies, the main worry is an unusual one for inflationary models — the coupling constants appear to be generically rather large (in many cases greater than unity) instead of unpopularly small. This effect has already been noted in the "variable Planck mass" model [23], which has some loose connections with the scenario we have discussed here, and it would be interesting to discover whether or not this effect is generic.

ACKNOWLEDGMENTS

[1] D. La and P. J. Steinhardt, Phys. Rev. Lett. **62**, 376 (1989).

[2] E. W. Kolb, Phys. Scr. **T36**, 199 (1991).

[3] A. H. Guth, Phys. Rev. D **23**, 347 (1981).

[4] E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, Redwood City, CA, 1990).

[5] A. D. Linde, Phys. Lett. **129B**, 177 (1983); *Particle Physics and Cosmology* (Harwood Academic, Switzerland, 1990).

[6] E. Weinberg, Phys. Rev. D **40**, 3950 (1989).

[7] D. La, P. J. Steinhardt, and E. Bertschinger, Phys. Lett. B **231**, 231 (1989).

[8] R. D. Reasenberg et al., Astrophys. J. Lett. **234**, L219 (1979).

[9] A. R. Liddle and D. Wands, Mon. Not. R. Astron. Soc. **253**, 637 (1991); Phys. Lett. B **276**, 18 (1992).

[10] F. S. Accetta and J. J. Trester, Phys. Rev. D **39**, 2854 (1989).

[11] R. Holman, E. W. Kolb, and Y. Wang, Phys. Rev. Lett. **65**, 17 (1990).

[12] R. Holman, E. W. Kolb, S. Vadas, and Y. Wang, Phys. Rev. D **43**, 3833 (1991); Phys. Lett. B **269**, 252 (1991).

[13] A. R. Liddle and D. H. Lyth, Phys. Lett. B **291**, 391 (1992); Ann. N.Y. Acad. Sci. **688**, 647 (1993).

[14] E. W. Kolb, D. S. Salopek, and M. S. Turner, Phys. Rev. D **42**, 3925 (1990); A. H. Guth and B. Jain, *ibid.* **45**, 426 (1992); D. H. Lyth and E. D. Stewart, Phys. Lett. B **274**, 168 (1992).

[15] A. R. Liddle and D. H. Lyth, Phys. Rep. **231**, 1 (1993).

[16] G. F. Smoot et al., Astrophys. J. Lett. **396**, L1 (1992).

[17] P. J. Steinhardt and F. S. Accetta, Phys. Rev. Lett. **64**, 2740 (1990).

[18] A. R. Liddle and D. Wands, Phys. Rev. D **45**, 2665 (1992).

[19] R. Crittenden and P. J. Steinhardt, Phys. Lett. B **293**, 39 (1992).

[20] A. D. Linde, Phys. Lett. **114B**, 431 (1982).

[21] J. Yokoyama, Phys. Lett. B **212**, 273 (1988); Phys. Rev. Lett. **63**, 712 (1989).

[22] T. Futamase and K. Maeda, Phys. Rev. D **39**, 399 (1989); T. Futamase, T. Rothman, and R. Matzner, *ibid.* **39**, 405 (1989).

[23] D. S. Salopek, J. R. Bond, and J. M. Bardeen, Phys. Rev. D **40**, 1743 (1989).

[24] B. L. Spokoiny, Phys. Lett. **147B**, 39 (1984).

[25] A. L. Berkin, K. Maeda, and J. Yokoyama, Phys. Rev. Lett. **65**, 141 (1990); A. L. Berkin and K. Maeda, Phys. Rev. D **44**, 1691 (1991).

[26] C. Brans and R. H. Dicke, Phys. Rev. **124**, 925 (1961).

[27] A. M. Laycock (unpublished).

[28] A. D. Linde, Phys. Rev. D **49**, 748 (1994).

[29] H. Nariai, Prog. Theor. Phys. **42**, 544 (1969).

[30] A. D. Dolgov, Phys. Rep. **222**, 309 (1992).

[31] A. D. Linde, Phys. Lett. B **238**, 160 (1990).

[32] F. C. Adams, K. Freese, and A. H. Guth, Phys. Rev. D **43**, 965 (1991).

[33] C. M. Will, *Theory and Experiment in Gravitational Physics* (Cambridge University Press, Cambridge, England, 1981).

[34] S. Coleman and E. J. Weinberg, Phys. Rev. D **7**, 1888 (1973).

[35] F. C. Adams, Phys. Rev. D **48**, 2800 (1993).

[36] A. D. Linde, Phys. Lett. B **249**, 18 (1990); F. C. Adams and K. Freese, Phys. Rev. D **43**, 353 (1991).

[37] E. J. Copeland, E. W. Kolb, and A. R. Liddle, Phys. Rev. D **42**, 2911 (1990).

[38] A. D. Linde, Phys. Lett. B **259**, 38 (1991).

[39] U.-L. Pen, D. N. Spergel, and N. Turok, Phys. Rev. D **49**, 692 (1994).

[40] D. P. Bennett, A. Stebbins, and F. R. Bouchet, Astrophys. J. Lett. **399**, L5 (1992).

[41] H. M. Hodges and J. Primack, Phys. Rev. D **43**, 3155 (1990).

[42] A. Stebbins, Ann. N.Y. Acad. Sci. **688**, 824 (1993).

[43] E. J. Copeland, E. W. Kolb, A. R. Liddle, and J. E. Lidsey, Phys. Rev. Lett. **71**, 219 (1993); R. Crittenden, J. R. Bond, R. L. Davis, G. Efstathiou, and P. J. Steinhardt, *ibid.* **71**, 324 (1993); E. J. Copeland, E. W. Kolb, A. R. Liddle, and J. E. Lidsey, Phys. Rev. D **48**, 2529 (1993).