Choice of filters for the detection of gravitational waves from coalescing binaries. II. Detection in colored noise

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Coalescing systems of compact binary stars are one of the most important sources for the future laser interferometric gravitational wave detectors. The signal from such a source will, in general, be completely swamped out by the photon-counting noise in the interferometer. However, since the wave form can be modeled quite accurately, it is possible to filter the signal out of the noise by the well known technique of matched filtering. The filtering procedure involves correlating the detector output with a copy of the expected signal called a matched filter or a template. When the signal parameters are unknown, as in the case of the coalescing binary signal, it is necessary to correlate the output through a number of filters each with a different set of values for the parameters. The ranges in which the values of the parameters lie are determined from astrophysical considerations and the set of filters must together span the entire ranges of the parameters. In this paper, we show how a choice of filters can be made so as not to miss any signal of amplitude larger than a certain minimum value, called the minimal strength. The number of filters and the spacing between filters in the parameter space are obtained for different values of the minimal strength of the signal. We also present an approximate analytical formula which relates the spacing between filters to the minimal strength. We discuss the problem of detection and false dismissal probabilities for a given data output and how a given set of filters determines these probabilities.

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I. INTRODUCTION

The detection of gravitational waves has been an outstanding problem in experimental physics for over three decades now. Starting from the pioneering experiments of Weber using a bar detector [1], there has been a lot of effort in building detectors of higher sensitivity (see [2] for a 1987 review of the bar detector program). In recent years, several groups around the globe have been planning to build long base line laser interferometric gravitational wave detectors [3–6], prototypes of which already exist in Germany, Great Britain, and the USA. Because of their inherent broadband nature, interferometric detectors can be used to detect wideband gravitational waves emitted during the inspiral of compact binary systems during the final stages of their evolution. The rate of such coalescences is estimated to be about three per year out to a distance of 100–200 Mpc [7].

Because of their extragalactic origin the amplitude of these signals is, in general, expected to be too low for them to be seen in the time series. However, since these sources are well modeled, the nature of the gravitational wave emitted during the inspiral is accurately known. With an accurate knowledge of the wave form it is possible to extract the signal out of the detector noise by the use of a data analysis technique called matching filtering or Weiner filtering. This technique consists of correlating the output of a detector with a filter which, in the Fourier domain, is nothing but the Fourier transform of the expected signal weighted by the noise power spectral density. When the parameters of the filter exactly match those of the signal we have what is called a matched filter. It is well known in the theory of hypothesis testing that of all the linear filters a matched filter performs the best in extracting a given signal buried in noisy data. The performance of other filters will degrade, depending on the degree of mismatch of the values of their parameters in relation to those of the actual signal present in the detector output. An experimenter would not know beforehand what the parameters of the signal are and hence will not be in a position to use a matched filter. It is, therefore, mandatory to filter the detector output with a set of filters, each corresponding to a fixed set of values of the parameters. The hope is that when an arbitrary coalescing binary wave form is present in the detector output, its parameters would be close enough to that of at least one of the filters in the set to enable its detection. Clearly, if the number of filters in the set is very large then the chance of detection of an arbitrary signal will be higher. However, having a large number of filters would mean a heavy load on computing and data analysis. This is an unfavorable aspect, especially because gravitational wave detectors have formidable data output rates and filtering

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each piece of data, with a large number of filters, might be unreasonably time consuming. On the other hand, having too low a number of filters means a lower chance of detection, which would defeat the very purpose of the whole exercise. Clearly, an optimal solution is in order. In fact matched filtering is akin to drawing a net through the sea of parameter space. If the net mesh is very fine then many signals will be captured but it will be very hard to draw the net through the sea quickly, while if the net is too coarse then the sea can be dragged quickly but no signal will be caught. In this paper we address the question of how to make a net optimally. More precisely, we show how the number of filters and the distance between them is related to the ranges of parameters. The specific objectives of this paper are (a) to estimate the number of filters needed to filter out signals from archetypal binaries located up to a distance of 500 Mpc, (b) to show how the spacing between filters in the parameter space depends on the noise characteristics of an interferometer, (c) to relate the number of filters to the probability of detecting a signal of a given amplitude, and (d) to obtain an analytical relation for the spacing of filters in the parameter space in terms of the amplitude of detectable signals.

As mentioned earlier, we can hope to have a reasonably large event rate if a gravitational wave detector is able to detect signals out to a distance of ~ 500 Mpc. Interferometric detectors aim at achieving very high sensitivities to match the low amplitude signals of extragalactic origin. An important factor that determines the sensitivity of an interferometric detector is the effective laser power available in its arms. The sensitivity is enhanced by a special arrangement of the optical components so that maximum usage of the input laser light is made. The first one of such techniques to be invented [8] and implemented [9] is called *standard recycling* or *power recycling.* The detector noise in the frequency range $\sim 100-2000$ Hz is chiefly contributed by the photon-shot noise. It has been shown that when Fabry-Pérot cavities are used in the arms of the detector, and in particular when standard recyling is employed, the noise power spectral density is frequency dependent [2,10–12]; that is to say that the noise in above range of frequency is *colored.* Normally, the detector noise is expected to be described by a Gaussian density distribution function. However, the analysis of data from detectors at Glasgow and Garching shows that the noise distribution is not strictly Gaussian [13,14]. This means that there are a number of other, as yet ununderstood, sources of noise present in the current prototypes. In this paper we take into account the frequency dependence of the noise power spectral density but assume that the detector noise is described by a Gaussian density distribution. It is well known that even when the detector noise is colored, as is the case when power recycling is implemented, the method of maximum likelihood ratio and the associated matched filtering technique [15] can be employed to detect known signals buried in a noisy data. However, the characteristics of the set of filters used in detecting signals with arbitrary parameter values will depend on the noise power spectral density. This is because, as we remarked earlier, an optimal filter is weighted by the power spectrum of noise. In an earlier paper (henceforth referred to as paper I) [16] we have worked out an algorithm to make a choice of filters by assuming that the detector noise has a flat power spectral density. In this paper we drop this assumption and extend the algorithm for the case of an interferometric detector operating with power recycling.

In post-linear gravity (i.e., with the post-Newtonian corrections to the wave form and binary evolution ignored) and when a single interferometric detector is used the coalescing binary wave form is characterized by three parameters, apart from the amplitude of the signal. These are (i) a certain combination of the masses of the component stars, (ii) the time of arrival of the signal and (iii) the phase of the signal at the time of arrival. In order to apply the technique of matched filtering for its detection it is necessary to construct a set of filters, variously known as a *bank of filters* or a *lattice of filters,* corresponding to different values of the three parameters, in their relevant range. The amplitude of the signal at the detector is also a signal parameter, but, as we shall see, for a fixed amplitude the lattice of filters is uniquely determined. Moreover, the amplitude of the signal has negligible covariance with the other parameters since the detectors are broadband. Hence the signal amplitude need not be considered in computing errors in the estimation of parameters: For a given amplitude the error in the estimation of parameters is fixed. Therefore, in our discussions concerning setting up a lattice of filters, we will normalize the amplitude of the filter and consider the other three quantities listed above as the parameters of the filter.

The paper is organized as follows. In Sec. II we briefly review the efforts in understanding the nature of the gravitational wave from coalescing binary systems, known as a chirp wave form. Among other things, we also discuss in Sec. II, the assumptions made about the noise characteristics, the Fourier transform of the chirp, matched filters for colored noise and their normalization, spectral density of noise in detectors that work with power recycling, and correlation of a chirp signal with a filter that differs from it in all the parameter values. Since the output of a detector is a random variable, a signal is said to be present only when the detector output crosses a preset amplitude, called the threshold. The threshold is set so that the noise generated amplitudes are most unlikely to exceed this value in a given duration of time. In Sec. III, we first discuss how to estimate the threshold for the filtered output for a given event rate of the signals. We then present an algorithm for the construction of a bank of filters to detect signals buried in colored noise. These results are then applied to the case of power recycling. The random nature of the detector output has one other implication. It is impossible to definitively assert that a signal is present in the data train even when the detector output exceeds the threshold value. When the output of a detector is of a certain amplitude there is only a definite probability that the output actually contains a signal. This is known as the detection probability. In Sec. IV we discuss the detection probabilities for filtered signals and whether or not a choice of a detection probability changes the nature of the set of filters obtained in Sec. III. In Sec. V we summarize our results.

II. CHIRP WAVE FORM AND OPTIMAL FILTERS

With the aid of the point mass approximation Peter and Mathews [17] predicted the flux of gravitational waves from a system of binary stars and showed that the system radiates energy in the form of gravitational waves at an average rate that increases as the inverse fifth power of the distance between the two stars. The gravitational waves also carry away the angular momentum of the system. The energy and angular momentum are carried away in such a balance that elliptical orbits become circularized on a time scale smaller than the time scale for the two stars to coalesce. In particular, circular orbits remain circular. As the two stars approach each other there will be an increase in the orbital frequency and a consequent enhancement in the amplitude and frequency of the waves. Eventually, the two stars coalesce emitting a burst of gravitational waves with a very characteristic wave form. The nature of such a wave form was worked out by Clark and Eardley [18] in the quadrupole approximation using Newtonian orbits for the point mass stars. More recently, Kochanek [19] and Bildsten and Cutler [20] have made a more realistic analysis of the coalescence of a binary system. The analysis of the nature of the waves emitted by the system at the very last stages is, as yet, an unsolved problem in general relativity. In the past couple of years a lot of effort has gone into the understanding of the nature of the late time signals using the post-Newtonian and the post-Minkowskian formalisms [21–28]. Several groups have tried to simulate the evolution of the system and estimate the associated emission of gravitational waves by numerical methods using fast computers [29]. However, the final word on the nature of the signal during the very last stages of the evolution has not yet been said.

In spite of a lack in our understanding of the wave form there is already an extensive discussion, in the literature, of the application of the matched filtering technique to the detection of the Newtonian wave form [2,30,31] and the first order post-Newtonian correction to it [12,32]. Such an effort is to be expected in view of the worldwide proposals for building large scale interferometric detectors. These investigations give us an idea of the order-of-magnitude estimate of the average signal-to-noise ratio to be expected for such systems.

A. The chirp wave form

In the transverse traceless (TT) gauge the gravitational wave emitted by a coalescing binary system is described in terms of the two polarizations usually denoted by $h_+(t)$ and $h_\times(t)$. The noise-free reponse of the detector is a linear combination of the two polarization amplitudes with coefficients depending on the orientation of the detector relative to the direction of propagation of the wave [2, 33–35]. In paper I it has been shown that the effect of an arbitrary relative orientation of the detector and the plane of the orbit of the binary is *only to alter the amplitude and the phase of the signal* at the site of the detector without affecting its time dependence. Therefore, for our purpose of constructing the matched filter, it is enough to consider, say, the + polarization which will be the noise-free response of the detector when the detector is optimally oriented and the plane of the orbit coincides with that of the sky. The wave form from such a system of total mass M and reduced mass μ located at a distance r is given by

$$h(t)\equiv h_+(t)=N_h a(t)^{-1}\cos\left[2\pi\int_{t_a}^{t} f(t')dt'+\Phi\right] . \tag{2.1}$$

The quantities appearing in the above equation are defined as follows.

t_a and Φ are, respectively, the time of arrival and the phase of the signal when the instantaneous gravitational wave frequency of the signal reaches some fiducial frequency, say f_a.

$\mathcal{M}=(\mu^3M^2)^{1/5}$ is called the chirp mass; the Newtonian wave form depends only on this parameter instead of the two individual masses of the stars.

ξ is the time taken for the two stars to theoretically coalesce starting from a time when the instantaneous frequency is f_a:

$$\xi=3.00\left[\frac{\mathcal{M}}{M_\odot}\right]^{-5/3}\left[\frac{f_a}{100\ \text{Hz}}\right]^{-8/3}\text{sec} . \tag{2.2}$$

The coalescence time ξ serves as a parameter to characterize the wave instead of the chirp mass $\mathcal{M}$.

$a(t)$ is the time-dependent normalized distance between the stars [normalized to $a(t_a)=1$]:

$$a(t)=\left[1-\frac{t-t_a}{\xi}\right]^{1/4} . \tag{2.3}$$

$f(t)$ is the instantaneous gravitational wave frequency given by

$$f(t)=f_a a(t)^{-3/2} . \tag{2.4}$$

The constant N_h is given by

$$N_h=2.56\times10^{-23}\left[\frac{\xi}{3\ \text{sec}}\right]^{-1}\left[\frac{r}{100\ \text{Mpc}}\right]^{-1}\times\left[\frac{f_a}{100\ \text{Hz}}\right]^{-2} . \tag{2.5}$$

The wave form (2.1) is derived in the quadrupole approximation assuming that the system consists of two point masses in orbit about each other. It is now realized that the secular corrections to the wave form arising from the post-Newtonian terms are far more important than as was thought previously [28,36]. When we have a better understanding of the wave form the work described in this paper needs to be readdressed. The time dependence of the amplitude and the frequency of the signal are given by Eqs. (2.3) and (2.4), respectively. They both diverge in the limit of $t\rightarrow t_a+\xi$. However, much before that the physical assumptions made in deriving them break down. Near coalescence, the post-Newtonian and other higher order corrections became important so that the wave form (2.1) is valid for orbital frequencies $\lesssim 400$ Hz or equivalently for gravitational frequencies $\lesssim 800$ Hz [32]. Nevertheless, in this paper, we work with the above wave form. When the full form of the signal is known it is straightforward to extend the present analysis.

B. Modeling the noise

There is as yet no viable model of the noise in the entire bandwidth of the interferometer. In the frequency range of 10–100 Hz to a few kHz the detector noise is chiefly contributed by the photon counting noise. If we restrict ourselves to this range of frequencies then we can make the following assumptions about the character of the noise. (A detailed discussion of the noise in the present prototypes can be found in [31].)

(i) At each instant of time t, the noise $n(t)$ is a random variable with zero mean; i.e., $\langle n(t)\rangle=0$. Here the angular brackets denote the ensemble average.

(ii) The noise is stationary. This means that it can be described by the *one-sided* power spectral density $S_h(f)$ defined by the equation

$$\langle \tilde{n}(f)\tilde{n}^*(f')\rangle = S_h(f)\delta(f-f') , \tag{2.6}$$

where $\tilde{n}(f)=\int_{-\infty}^{\infty}n(t)\exp(-2\pi i f t)dt$ is the Fourier transform of the noise.

(iii) The seismic vibrations cause the noise in an interferometric detector to steeply rise below a certain frequency f_a. We thus assume that $S_h(f)=\infty$ for $f\le f_a$. This is equivalent to introducing a lower frequency cutoff f_a in the frequency response of the detector. In the present prototype detectors f_a is between 200 and 400 Hz. In future interferometric detectors it is expected to be around 40 Hz initially and in advanced detectors it will be lowered to about 10 Hz by using special seismic isolation techniques [3,37]. We choose $f_a=100$ Hz to compute specific physical quantities but we retain f_a as a free parameter in much of our discussion.

(iv) Finally, we assume that the noise can be described by a Gaussian distribution function. In what follows we use assumptions (i)–(iii) while (iv) will be used in Sec. III A in estimating the threshold for filtered signals.

C. Fourier transform of the chirp

In specifying a matched filter, in addition to the above modeling of the noise we need to know the Fourier transform of the signal. In the stationary phase approximation, the positive frequency components of the Fourier transform of Eq. (2.1) are given by [16,38]

$$\tilde{h}(f)=\int_{-\infty}^{\infty}h(t)\exp(-2\pi i f t)dt = N_h\sqrt{\xi}\tilde{H}(f) , \tag{2.7a}$$

where

$$\tilde{H}(f)=\left[\frac{2}{3f_a}\right]^{1/2}\left[\frac{f}{f_a}\right]^{-7/6}\exp[i\psi(f)] ,$$

$$\psi(f)=-2\pi f t_a+2\pi f_a\xi\alpha(f)+\Phi+\frac{\pi}{4} , \tag{2.7b}$$

$$\alpha(f)=\frac{1}{5}\left[8-3\left[\frac{f}{f_a}\right]^{-5/3}-5\frac{f}{f_a}\right] .$$

Since $h(t)$ is real, the negative frequency components can be found by using the relation $\tilde{h}(-f)=\tilde{h}^*(f)$, where here and below a $*$ denotes the operation of complex conjugation. The quantity $\tilde{H}(f)$ is chosen to have unit normalization, i.e.,

$$2\int_{f_a}^{\infty}|\tilde{H}(f)|^2df=1 . \tag{2.8}$$

The lower limit in the integral is taken to be f_a and not zero since, as discussed earlier, the detector response has a lower frequency cutoff. With the aid of (2.7) we see that

$$2\int_{f_a}^{\infty}|\tilde{h}(f)|^2df=N_h^2\xi . \tag{2.9}$$

We shall see below that in setting up a lattice of filters to detect a signal $h(t)$ it is both sufficient and convenient to deal with the normalized Fourier transform of the wave form $\tilde{H}(f)$ instead of $\tilde{h}(f)$. Notice that the inverse Fourier transform of the function $\tilde{H}(f)$ contains all the time dependence of the wave form (2.1) differing from it only in the constant part (2.5) of the full amplitude.

D. Matched filters and their normalization

We are now ready to define a matched filter along the lines of [31]. Consider an output data stream $o(t)$ of a detector. This output consists of two components: the noise $n(t)$ and the signal $s(t)$,

$$o(t)=n(t)+s(t-t_a) , \tag{2.10}$$

where we have assumed that the noise is simply additive and that the arrival time of the signal is t_a: $s(t-t_a)=0$, for $t<t_a$. If the coalescence time of the signal is ξ then $s(t-t_a)=0$, for $t>t_a+\xi$. Let $q(t)$ denote a linear filter. The correlation of the output of the detector with the filter $q(t)$ with a time shift Δt of the filter is defined as

$$C(\Delta t)\equiv\int_{-\infty}^{\infty}o(t)q(t+\Delta t)dt = \int_{-\infty}^{\infty}\tilde{o}(f)\tilde{q}^*(f)e^{2\pi i f\Delta t}df , \tag{2.11}$$

where $\tilde{o}(f)$ and $\tilde{q}(f)$ are the Fourier transforms of $o(t)$ and $q(t)$, respectively. The second equality in the above equation follows by transforming the first integral to the Fourier domain and using the property of the Fourier transform for real functions: $\tilde{q}^*(f)=\tilde{q}(-f)$. The correlation $C(\Delta t)$ is a random variable and it is the statistic used in determining whether or not a given signal is present in the data stream. $C(\Delta t)$ is sometimes referred to as the filtered output. Notice that if the time-series noise $n(t)$ is Gaussian then it follows that $C(\Delta t)$ too is a Gaussian random variable since it has been obtained by a linear operation on the Gaussian random variables $n(t)$ (see, e.g., Helstrom [15]). The ensemble average of the filtered output is given by

$$\langle C(\Delta t)\rangle=\int_{-\infty}^{\infty}\langle\tilde{o}(f)\rangle\tilde{q}^*(f)e^{2\pi i f(\Delta t+t_a)}df .$$

As a consequence of the assumption (i) above, the ensemble average of the filtered output is independent of the noise and is equal to the correlation of the filter with the signal. For this reason it is called the *filtered signal* and we denote it by $\mathcal{S}(t)$:

$$\mathcal{S}(t)\equiv\langle C(\Delta t)\rangle=\int_{-\infty}^{\infty}\tilde{s}(f)\tilde{q}^*(f)e^{2\pi if(\Delta t+t_a)}df\ , \qquad (2.12)$$

where $\tilde{s}(f)$ is the Fourier transform of the signal $s(t)$. Consider now the variance of the correlation. The signal being just a constant does not contribute to the variance of C. The only contribution to the variance comes from the noise. The variance $V(C)\equiv\langle C^2\rangle-\langle C\rangle^2$ is given by

$$V(C)=\int_{-\infty}^{\infty}df\int_{-\infty}^{\infty}df'\tilde{q}^*(f)\tilde{q}(f')\times e^{2\pi i(f-f')\Delta t}\langle\tilde{n}(f)\tilde{n}^*(f')\rangle\ . \qquad (2.13)$$

The square root of this variance is the *noise of the filtered output* or *correlation noise* and we shall denote it by $\mathcal{N}$. With the aid of Eq. (2.6) we have

$$\mathcal{N}^2\equiv V(c)=\int_{-\infty}^{\infty}|\tilde{q}(f)|^2S_h(f)df\ . \qquad (2.14)$$

From (2.12) and (2.14) we see that the *signal-to-noise ratio*, for the filtered output, is

$$\frac{\mathcal{S}(\Delta t)}{\mathcal{N}}=\frac{\int_{-\infty}^{\infty}\tilde{s}(f)\tilde{q}^*(f)e^{2\pi if(\Delta t+t_a)}df}{\left[\int_{-\infty}^{\infty}S_h(f)|\tilde{q}(f)|^2df\right]^{1/2}}\ , \qquad (2.15)$$

It is well known in the theory of hypothesis testing that when the noise is stationary, among all linear filters the filter that produces the maximum signal-to-noise ratio for any time shift Δt is given by ([15, 16, 38–40])

$$\tilde{q}(f)=N_f\frac{\tilde{s}(f)}{S_h(f)}\ , \qquad (2.16)$$

where N_f is an arbitrary constant. Such a filter is called a *matched filter*. Notice that the signal-to-noise ratio (2.15) is independent of the choice of N_f: Two filters that only differ from each other by a constant factor produce the same signal-to-noise ratio. We use this fact to our advantage and *choose N_f so that the variance of the correlation is unity*. Since $S_h(f)=\infty$ for $f\le f_a$, this choice of normalization implies that

$$2\int_{f_a}^{\infty}|\tilde{q}(f)|^2S_h(f)df=1\ . \qquad (2.17)$$

This equation completely fixes the normalization constant N_f of the filter. There are several advantages in normalizing the filter in this way. Notice that the above normalization of noise, together with the assumptions (i)–(iii) above concerning the noise characteristics of a detector, imply that C is a standard normal variate with mean zero and standard deviation equal to unity. Moreover, with this choice of normalization the optimal signal-to-noise-ratio, that is the signal-to-noise ratio obtained by using an optimal filter, is just the correlation of the signal with the filter. We shall soon see other advantages of this choice of normalization.

The time Δt which appears in the definition of the signal-to-noise ratio has a special meaning. We noted earlier that Δt is the relative time shift between the filter and the output $o(t)$. When an optimal filter is used, the signal-to-noise ratio will peak at a time shift $\Delta t=-t_a$ and for other values of the time shift the signal-to-noise ratio will be lower. It is in this sense that the signal-to-noise ratio is to be thought of as a function of time.

We now set out to obtain the normalization constant N_f for chirp wave forms. In what follows we take $\tilde{s}(f)$ to be the normalized Fourier transform of the signal $\tilde{H}(f)$ given by (2.7b). The normalization constant N_f is obtained from (2.17) by substituting for $\tilde{q}(f)$ the expression for the optimal filter (2.16). Thus,

$$N_f=\frac{\sqrt{S_0}}{J} \qquad (2.18)$$

where we have introduced a constant S_0 of dimensions Hz^{-1} for future convenience and the dimensionless quantity J is given by

$$J^2=2S_0\int_{f_a}^{\infty}\frac{|\tilde{H}(f)|^2}{S_h(f)}df=\frac{4}{3}S_0f_a^{4/3}\int_{f_a}^{\infty}\frac{df}{f^{7/3}S_h(f)}\ . \qquad (2.19)$$

For white noise, the power spectral density is a constant. If this constant is equal to S_0 we have $J=1$ as a consequence of (2.8).

Standard recycling. When the noise is colored we need to know the noise power spectral density in order to compute the normalization. The power spectral density of noise in detectors that employ Fabry-Pérot cavities in their arms and use standard recycling to enhance the intensity of light in their cavities has been shown to be of the form [2]

$$S_h(f)=s_0\left[1+\frac{f^2}{f_k^2}\right]\ , \qquad (2.20)$$

where $s_0=\mathrm{const}\times f_k$ and f_k are constants depending on the parameters of the detector. The quantity f_k is the so-called *knee frequency* which the experimenters can set by an appropriate choice of mirror reflectivities. We observe that the power spectral density of noise in standard recycling is roughly a constant for frequencies below the knee frequency but rises quadratically above it. It therefore becomes increasingly harder to extract the signal at higher frequencies. This means that hardly any improvement in the signal-to-noise ratio can be achieved by increasing the upper frequency cutoff beyond a certain value determined by f_k. Such an analysis is made in Ref. [41] and here we merely mention the result that for the case under consideration, viz., standard recycling in Fabry-Pérot cavities, it is sufficient to sample coalescing binary filters up to about 400 Hz: By the time the signal reaches this frequency more than 98% of the signal power is extracted.

If we identify the constant s_0 appearing in Eq. (2.20) with the constant S_0 defined earlier and make a change of variables to $x=f/f_a$ and $\gamma=f_k/f_a$, then the quantity J is given by

$$J^2(\gamma)=\frac{4}{3}\gamma^2\int_1^{\infty}\frac{dx}{x^{7/3}(x^2+\gamma^2)}\ . \qquad (2.21)$$

For coalescing binary signals, the maximum signal-to-noise ratio (or equivalently J) can be obtained by choosing f_k to be [2]

$$f_k=1.44f_a\ . \qquad (2.22)$$

For this value of f_k (i.e., $\gamma=1.44$) a numerical integration of Eq. (2.21) gives $J\simeq 0.62$.

E. Strength of a signal

We now define the strength of a signal along the lines of paper I. Consider a coalescing binary signal $h(t)$ and a matched filter $q(t)$. The correlation of the signal with the filter is defined as usual by

$$C(\Delta t)=\int_{-\infty}^{\infty} h(t)q(t+\Delta t)dt \ .$$

$C(\Delta t)$ now stands for the correlation of the filter with a pure signal and not with the detector output which is contaminated by noise. Therefore, it is a deterministic quantity and not a random variable. Now, a signal is said to be of strength S if the maximum of the correlation function $C(\Delta t)$ has the value S:

$$S\equiv \max_{\Delta t} C(\Delta t) \ .$$

Using Eqs. (2.16), (2.7), and (2.18) in the above equation we obtain

$$S=N_h\left[\frac{\xi}{S_0}\right]^{1/2} J \ . \qquad (2.23a)$$

Substituting for N_h from Eq. (2.5) we obtain the strength of the signal in terms of r, ξ, and f_a:

$$S(r,\xi,f_a)=44.5\left[\frac{\xi}{3\ \text{sec}}\right]^{-1/2}\left[\frac{f_a}{100\ \text{Hz}}\right]^{-2} \times\left[\frac{r}{100\ \text{Mpc}}\right]^{-1}\left[\frac{S_0}{10^{-48}\ \text{Hz}^{-1}}\right]^{-1/2} J \ . \qquad (2.23b)$$

There are several points to be noted. In our definition of the strength of a signal we have incorporated the sensitivity of a detector through the quantity of J. Hence, the strength of the signal as it appears here is what is *seen* in the filtered output. Secondly, the choice of our normalization of the filters also means that the signal-to-noise ratio is just equal to the strength of the signal S. Finally, it may appear that the standard recycling noise is lowering the signal strength as compared to the case when the detector noise is white, since $J<1$ in the former case and $J=1$ in the latter. However, recall that we have chosen the constant power spectral density of white noise (S_0) to be the minimum level of noise in standard recycling which occurs at $f=f_a$. This was done purely for the purpose of illustration and in reality this is not the case. What the experimenters achieve by implementing standard recycling is to lower the value of S_0. Hence, the signal strength will, indeed, turn out to be larger in the case of standard recycling if we use realistic values of S_0 for the two cases. However, in this paper we shall continue to use the same value of S_0 for both the cases.

F. The correlation function for colored noise

The statistic used in deciding the presence or absence of a signal is the correlation function of the output of the detector with a filter. It is therefore essential to have an understanding of the behavior of this correlation function in setting up an algorithm to construct a set of filters for the detection of arbitrary signals. In this section we consider the noise-free correlation function of a chirp wave form with a normalized filter. The effect of noise will be discussed in Sec. IV.

Consider the chirp $h(t,\xi,\Phi)$ and a filter $q(t,\xi+\Delta\xi,\Phi+\Delta\Phi)$ whose coalescence time and phase differ from that of the signal by $\Delta\xi$ and $\Delta\Phi$, respectively. The correlation function of these two wave forms is given by

$$C(\Delta t,\xi,\Delta\xi,\Phi,\Delta\Phi)=\int_{-\infty}^{\infty} h(t,\xi,\Phi) \times q(t+\Delta t,\xi+\Delta\xi,\Phi+\Delta\Phi)dt \ . \qquad (2.24)$$

Note that as in Sec. II E above, C is a deterministic function. Going over to the Fourier domain and using the stationary phase approximation for the Fourier transform of $h(t)$, Eq. (2.7), we have

$$C(\Delta t,\Delta\xi,\Delta\Phi) =N_c\int_{f_a}^{\infty}\frac{\cos[2\pi f\Delta t+2\pi\alpha(f)f_a\Delta\xi+\Delta\Phi]}{f^{7/3}S_h(f)}df \ , \qquad (2.25)$$

where $N_c=\frac{4}{3}f_a^{4/3}\xi^{1/2}N_hN_f$. As in the white noise case treated in paper I here also the correlation function depends only on the *differences* Δt, $\Delta\xi$, and $\Delta\Phi$ in the parameters of the two wave forms. Hence, we have dropped the dependence of C on ξ and ϕ and have expressed it as a function of just the three variables Δt, $\Delta\xi$, and $\Delta\Phi$.

We note the following properties of C.

(i) The maximum value of C is $C(0,0,0)=N_h\sqrt{\xi/S_0}J$. [Cf. Eq. (2.23a).]

(ii) Reflection symmetry about the maximum,

$$C(\Delta t,\Delta\xi,\Delta\Phi)=C(-\Delta t,-\Delta\xi,-\Delta\Phi) \ , \qquad (2.26)$$

as is obvious from (2.25). For our purposes it is important to note that the maximum, over Δt and $\Delta\Phi$, of the correlation function, for a fixed $\Delta\xi$, depends only on its modulus.

(iii) Finally, let us note that

$$C(\Delta t,\Delta\xi,\Delta\Phi)=C(\Delta t,\Delta\xi,0)\cos\Delta\Phi +C(\Delta t,\Delta\xi,\pi/2)\sin\Delta\Phi \ , \qquad (2.27)$$

which states that the correlation of a signal with a filter of arbitrary phase can be expressed as a linear combination of its correlation with two filters: one with the phase equal to 0 and another with the phase equal to $\pi/2$.

There is a word of caution about the statements made about the properties of the correlation function. The expression for the Fourier transform of the chirp is derived in the stationary phase approximation and it is this expression which has enabled us to show the simple dependence of the correlation function on its parameters.

Therefore these properties are also approximate and as discussed in detail in paper I they hold good only for values of coalescence time more than about 0.3 sec. It is important to remember this while generating a set of filters for low values of the coalescence time which correspond to high values of the chirp mass.

III. CHOICE OF FILTERS

In this section we first discuss the nature of the probability density function of the correlation noise when the time series noise is colored. Using a certain *false alarm probability* we obtain a *threshold* for filtered signals. We then introduce the idea of *minimal strength*. The minimal strength will then be related to the spacing of filters in the parameter space. Towards the end of this section we give an approximate analytical relation for the spacing between filters as a function of the minimal strength. The details of the calculation are given in the Appendix. This relation will be shown to hold good for values of minimal strength close to the threshold.

A. Threshold and minimal strength

In the absence of the signal, the correlation function $C(\Delta t)$ is a random variable and is in general a function of the parameters of the filter; for each filter we have a different random variable. However, since the filters are all normalized [cf. Eq. (2.17)] and no signal is present, *all these random variables are identically distributed* and all of them are described by the same probability density function. In Sec. II D we have shown that, for a stationary, Gaussian time-series noise, the noise in the filtered output is also Gaussian with mean zero. A great simplification brought about by the choice of our normalization is that the correlation noise has unit variance. Had we not normalized the filters in accordance with (2.17) the variance would depend on the filter parameters as can be seen from Eqs. (2.14), (2.16), and (2.18). The probability density function for the correlation noise, with our choice of normalization, is therefore given by

$$p(C)=\frac{1}{\sqrt{2\pi}}e^{-C^2/2}\ . \tag{3.1}$$

This enables us to find the threshold for filtered data. Because of the random nature of noise there is a definite probability, called a *false alarm*, that the noise amplitude crosses a preset level, called the *threshold*, even when no signal is present in the data stream. Because of the same reason, there is also a definite probability, called a *false dismissal*, that when a signal of a given strength *is* present in the data stream the output amplitude is below the threshold. Similarly, the probability that the output crosses the threshold when a signal is present in the detector is called a *true alarm*, and the probability that a decision in favor of the absence of a signal is made when the output has no signal in it is called a *true dismissal*. In detection problems normally the threshold is set by considering some combination of the false alarm and the false dismissal probabilities depending on the *risks* involved in making a false decision.

Here the aim almost certainly is to avoid a false alarm. Hence the threshold must be set high enough so that the false alarm probability is very small. The threshold η is set by the requirement that the number of times the statistic C exceeds the threshold in a given length of data, purely due to noise, is much smaller than the expected number of true events. Following paper I, we consider a data segment for a one year period, and allow for just one false alarm in this period; i.e., the expected number of times C can cross η just due to noise fluctuations, in a year's time, is one.

Let p_0 be the probability that one sample of the output consisting only of noise cross the threshold η. For a sampling rate of a few kHz there are thus $N\sim 10^{10}$ samples in a year's data. The probability that at least one of them registers as a signal is then given by $1-(1-p_0)^N\sim Np_0$ for p_0 sufficiently small. Assuming that there are m filters we obtain m filtered outputs. We take m to be ~ 1000. Now for the sake of simplicity we also assume that the filtered outputs are uncorrelated. This is certainly not the case and a detailed analysis of this aspect needs to be performed. However, if we take signals with larger strengths which allow coarser filter spacing, then the covariances between the filtered outputs will be relatively smaller and we may not be unjustified in neglecting them. Here we ignore the covariances and do a rough analysis to get an idea of the threshold. Since each filtered output is a normal deviate with zero mean and unit variance we get the following relation for p_0:

$$1-p_0=\left[\left[\frac{1}{2\pi}\right]^{1/2}\int_{-\eta}^{\eta}dx\, e^{-x^2/2}\right]^m \tag{3.2a}$$

which for $\eta \gg 1$ yields

$$1-p_0\sim 1-\left[\frac{2}{\pi}\right]^{1/2}\frac{e^{-\eta^2/2}}{\eta}m\ ,\quad \eta\gg 1\ . \tag{3.2b}$$

For $N\sim 10^{10}$, $m\sim 1000$ we get $\eta\simeq 7.5$. If C exceeds this threshold value of η, we say that the signal is detected; otherwise it is not.

The effect of the covariances between filtered outputs can be gauged qualitatively. Consider the extreme case when the m filtered outputs are perfectly correlated (this will almost certainly not be the case here); then effectively we have just one independent filtered output and the threshold can be obtained by putting $m=1$ in the above equation, which in this case would give $\eta\sim 6.6$. However, here we have an intermediate case, so that effectively we have less than m independent filtered outputs and hence the values of the threshold will be reduced from the one corresponding to *m independent* outputs. In our case, the threshold value is insensitive to these modifications and will be marginally reduced from the value quoted above, for which all filtered outputs were taken to be independent.

Clearly, we can only filter the data through a finite number of filters, with each filter corresponding to a distinct set of values of the parameters. In other words, we can only have a discrete lattice of filters. The discrete nature of the lattice dictates that the minimal strength of detectable signals should be somewhat larger than the detector threshold. The reason for this is the following:

A signal present in the output of a detector will have a definite set of values of the parameters. For such a signal, the signal-to-noise ratio will be the largest when it is filtered using a template that matches all its parameters. In a discrete lattice of filters it is very unlikely that there will be a filter that exactly matches all the parameters of the signal. In that case for a given signal, the maximum signal-to-noise ratio is obtained with a filter that has the least mismatch with the actual parameters of the signal. This will, in general, be less than what one gets using a perfectly matched filter; the reduction in signal-to-noise on the average is less, if one uses a larger number of filters. Thus, in order to detect a signal with unknown parameters we need a large number of closely spaced filters in the parameter space.

Consider signals of such a strength that they produce a correlation just above the threshold when they are filtered with perfectly matched templates. It is clear from the foregoing discussion that it is unlikely that such signals will be detected using a *discrete* lattice of filters: On the average such signals will produce a correlation lower than the threshold as they will not find a perfectly matched filter in the lattice. We shall therefore consider constructing a lattice of filters for signals of a certain *minimal strength* $S_{\min}$ slightly larger than the threshold:

$$S_{\min} \equiv \kappa\eta \ , \tag{3.3}$$

where $\kappa > 1$; i.e., the minimum strength of the signal is κ times the threshold. This means that with a right filter (a filter whose parameters exactly match those of the signal) the correlation of a signal whose strength is larger than the minimal strength will be at least $\kappa\eta$. The aim now is to construct a bank of filters so that every signal of strength greater than or equal to the minimal strength $S_{\min}$ will be detected. It is to be expected that the parameter κ we have introduced here determines the bank of filters.

B. Bank of filters

We now consider the problem of constructing a bank of filters so that, given an arbitrary chirp wave form of strength larger than the minimal strength, at least one filter in the bank obtains a signal-to-noise ratio larger than the threshold. We follow the same procedure as in paper I but the results obtained here are quantitatively different since the power spectral density $S_h(f)$ is frequency dependent. For the sake of completeness we give a brief description of the procedure.

As in the white-noise case of paper I, here too, a two-dimensional basis exists for the signal parameter Φ. A chirp filter of arbitrary phase can be expressed as a linear combination of two filters: one with phase equal to 0 and another with phase equal to $\pi/2$. Consequently, the correlation of a given data set with a filter of arbitrary phase can be expanded in terms of the correlation of the same data set with the two basis filters [cf. Eq. (2.27)]. Moreover, the phase of the filter which maximizes the correlation can be found analytically once the correlations are calculated with the basis set: the maximum correlation (with respect to the phase) is just the square root of the sum of squares of the correlations obtained from the orthogonal basis set (see paper I for details).

In constructing a lattice of filters for the chirp mass, or equivalently for the coalescence time, we make use of the symmetry of the correlation function, namely, that it depends only on the difference, and not on the absolute values, of the coalescence times of the signal and filter. We fix the phase to be zero, i.e., $\Phi=0$. The set of filters for $\Phi=\pi/2$ are going to consist of the same set of chirp masses as the set of filters for $\Phi=0$.

We start with the chirp mass $\mathcal{M}_1$ at the minimum of the range of chirp masses which the filter bank has to span. This corresponds to a coalescence time, say, ξ_1. A typical value of $\mathcal{M}_1$ could be $0.5M_\odot$ which for $f_a=100$ Hz corresponds to $\xi_1 \simeq 9.54$ sec. This gives us the first filter in the set, namely, $q(t,\xi_1,0)$.

The next filter $q(t,\xi_2,0)$ is obtained as follows: We consider the set of all signals, $h_{\min}(t,\xi,\Phi)$, of minimal strength with $\xi \le \xi_1$ and $0 \le \Phi \le 2\pi$. By the definition of the minimal strength we have the relation

$$\int_{t_a}^{t_a+\xi} h_{\min}(t,\xi,\Phi)q(t,\xi,\Phi)dt=\kappa\eta \ . \tag{3.4}$$

Consider the signal $h_{\min}(t,\xi_1,0)$. The correlation of $h_{\min}(t,\xi_1,0)$ with the first filter is $\kappa\eta$. If we reduce the coalescence time of the signal slightly, say to $\xi=\xi_1-\Delta\xi$ where $\Delta\xi>0$, then the correlation will drop below $\kappa\eta$. We now solve the following equation for $\Delta\xi$:

$$\max_{\Delta t,\Delta\Phi} \int_{t_a}^{t_a+\xi} h_{\min}(t+\Delta t,\xi_1-\Delta\xi,\Delta\Phi)q(t,\xi_1,0)dt=\eta \ . \tag{3.5a}$$

The maximization with respect to Δt and $\Delta\Phi$ has been carried out to obtain the largest possible value of $\Delta\xi$ and leads to the coarsest possible lattice. The maximization is necessary as there is a covariance between the parameters. Thus, a signal of minimal strength with $\xi=\xi_1-\Delta\xi$, where $\Delta\xi$ is the solution of Eq. (3.5a), is just about picked up by the first filter. If $\xi<\xi_1-\Delta\xi$ for a signal of minimal strength then this signal is not picked up by the first filter. Therefore, there ought to be some other filter in the bank which should pick up this signal. We choose ξ_2 such that the maximum value of the correlation with respect to Δt and $\Delta\Phi$ of the filter $q(t,\xi_2,0)$ with a signal of minimal strength having $\xi=\xi_1-\Delta\xi$ is η. That is, ξ_2 is the solution of the equation

$$\max_{\Delta t,\Delta\Phi} \int_{t_a}^{t_a+\xi} h_{\min}(t+\Delta t,\xi_1-\Delta\xi,\Delta\Phi)q(t,\xi_2,0)dt=\eta \ . \tag{3.5b}$$

Note that the $\Delta\xi$ in the above equation is the solution of (3.5a). Equation (3.5b) can be solved using the reflection symmetry of the correlation function [cf. (2.26)]. Thus, we have the relation $\xi_2+\Delta\xi=\xi_1-\Delta\xi$, or

$$\xi_2=\xi_1-2\Delta\xi \ . \tag{3.6}$$

The process is then repeated until the upper end of the range of the chirp mass is reached. Thus, the $(k+1)$th filter has the coalescence time

$$\xi_{k+1}=\xi_1-2k\Delta\xi \ . \tag{3.7}$$

We observe that the filters in the ξ dimension are spaced with a constant spacing of $2\Delta\xi$. By employing iterative methods to Eqs. (3.5a) and (3.5b) one can numerically solve for $\Delta\xi$. Such numerical computations corroborate our analytical result (3.7) except at the higher end of the range of the chirp mass, where the stationary phase approximation breaks down.

How many filters do we need? If we take the upper limit to the chirp mass as $\simeq 10M_\odot$ then the corresponding $\xi \ll \xi_1$ and hence the number of filters with $\Phi=0$ are just $\xi_1/2\Delta\xi$. The full set of filters is obtained by including the filters with phase $\pi/2$ and with the same set of ξ_k. Therefore, the total number of filters n_f is

$$n_f=\frac{\xi_1}{2\Delta\xi}\times 2=\frac{\xi_1}{\Delta\xi} \ . \tag{3.8}$$

From Eq. (2.23) we also obtain the furthest distance $r_{\max}$ up to which a coalescing binary could be detected with a bank of filters corresponding to the minimal strength $\kappa\eta$. Thus,

$$r_{\max}=630\kappa^{-1}J\left[\frac{\eta}{7}\right]^{-1}\left[\frac{\xi}{3\ \text{sec}}\right]^{-1/2} \times\left[\frac{f_a}{100\ \text{Hz}}\right]^{-2}\left[\frac{S_0}{10^{-48}\ \text{Hz}^{-1}}\right]^{-1/2}\ \text{Mpc} \ . \tag{3.9}$$

In the limit of infinite number of filters, $\kappa=1$, $r_{\max}$ tends to a maximal limit r_0 which is determined only by the threshold.

C. Bank of filters for the standard recycling case

In this section we present the numerical analysis for setting up a bank of filters. Specifically, we state a formula relating the number of filters to the spacing between them for different values of the minimal strength. We assume that the noise power spectral density is that of a detector in standard recycling configuration. We take the knee frequency f_k to be 144 Hz. We obtain an approximate analytical relation for $\Delta\xi$, which is half the spacing between the consecutive filters in the bank. The details of the calculation are given in the Appendix.

Using the algorithm developed in Sec. III it is straightforward to find the spacing between the filters given a certain value of κ. Equation (3.5a) is first solved numerically for $\Delta\xi$. This gives half the spacing between the first two filters. Having determined the spacing between the first two filters in this manner, one can construct the rest of the filters in the lattice with the aid of Eq. (3.7). This procedure is accurate enough for most of the filters in the lattice. However, since the properties of the correlation function for low values of coalescence time ($\xi \lesssim 0.3$ sec, the exact value depending on the value of κ) may not strictly hold good, it is necessary to adopt the iterative method, discussed in detail in paper I, for such values. Following such a procedure the bank of filters has been obtained for $\kappa^{-1}=0.8$ and 0.9. The filters labeled by their chirp mass have been presented in Tables I(a) and I(b), respectively. We observe that the quantity $\Delta\xi$ is more or less constant for a given value of κ and grows

TABLE I. Banks of filters labeled by chirp masses for two different values of κ^{-1}: (a) $\kappa^{-1}=0.8$, (b) $\kappa^{-1}=0.9$. The distance between filters in the parameter space is a constant except for high values of the chirp mass.

$\mathcal{M}$ ($M_\odot$)	$\Delta\xi$ (ms)
(a)	
$\mathcal{M}\le 1.530$	18.6
$1.542\le\mathcal{M}\le 2.375$	18.7
$2.413\le\mathcal{M}\le 2.796$	18.8
2.856	19.0
2.920	18.9
2.988	19.0
3.060	19.0
3.138	19.2
3.221	19.2
3.311	19.2
3.408	19.0
3.512	19.2
3.626	19.1
3.750	19.2
3.886	19.2
4.036	19.0
4.202	19.2
4.388	19.2
4.598	19.2
4.839	19.2
5.116	19.5
5.444	19.5
5.835	19.8
6.319	19.8
6.928	19.9
7.729	19.2
8.790	18.6
10.298	18.5
(b)	
$\mathcal{M}\le 2.004$	10.1
$2.016\le\mathcal{M}\le 3.897$	10.2
$3.976\le\mathcal{M}\le 4.929$	10.0
5.075	9.7
5.229	9.9
5.399	10.0
5.588	10.1
5.797	10.1
6.028	10.1
6.285	10.0
6.571	9.7
6.885	10.0
7.250	9.9
7.670	9.8
8.156	10.0
8.750	10.5
9.509	11.0
10.529	11.3
11.957	11.3
14.044	10.3

larger for lower values of κ^{-1}. For $\kappa^{-1}=0.9$ and 0.8, $\Delta\xi \simeq 0.010$ and 0.019 sec, respectively. The number of filters n_f is related to $\Delta\xi$ and ξ_1 [or equivalently $\mathcal{M}_1$ as given in Eq. (3.8)]. Table II displays for various values of κ^{-1} (column 1), the distance $\Delta\xi$ for white noise (column 2) and standard recycling noise (column 3) and the number of filters n_f in the standard recycling case for $\mathcal{M}_1=0.25$ (column 4) and $0.5M_\odot$ (column 5). Note that the distance between filters is roughly doubled in the standard recycling case as compared to white noise and consequently the number of filters is halved. For example, for $\kappa^{-1}=0.8$ and $\mathcal{M}_1=0.5M_\odot$, the number of filters is 513 as compared to 1150 in the case of white noise.

This can be understood in the following way: The noise rises very quickly once f increases beyond f_k, so that when $f \gtrsim 400$ Hz, the signal is basically drowned in the noise. This implies that the Fourier transform of the matched filter, $\tilde{q}(f)$, decreases rapidly in amplitude as f increases beyond f_k so that the signal is effectively cutoff beyond about 400 Hz. Thus, most of the contribution to the signal-to-noise ratio comes from $f \lesssim 400$ Hz. Now, the time rate of change of the frequency, $df(t)/dt$, of the chirp wave form increases with time and the chirp mass. This means that if a signal is cut off prematurely, it is harder to determine the chirp mass, as the rapid acceleration in the frequency, in the final stages, is not detected. Consequently, the correlation function near the peak becomes flatter. Figure 1 depicts the correlation functions for the white noise and the standard recycling noise. It is clear from the figure that a wider spacing of filters results in the standard recycling case.

This has the following important implication: In the case of standard recycling the computing time is considerably less than in the case of white noise for a similar value of κ. This is because (i) lesser number of filters are required to span the same range of chirp mass (by a factor of about 2) and (ii) the template can be cutoff at about 400 Hz, so that a lower sampling rate ~ 1 kHz is adequate as compared to the 2 or 2.5 kHz rate required for the white noise.

The saving in computation time is not due to any speciality of the algorithm that we have developed. Even in the case of white noise we could have chopped off the template at 400 Hz and got similar results. But then that would have meant a drop in the signal-to-noise ratio. However, in the case of standard recycling, the sensitivity of a detector is enhanced in a relatively narrow band of frequencies at the lower end of the detector bandwidth at the cost of a greater noise at higher frequencies. Therefore, the signal power at higher frequencies cannot be extracted. Thus, in the standard recycling case we can consider a frequency limited template without an appreciable loss of signal-to-noise ratio. Another advantage of standard recycling is that, for a given computability, a lower

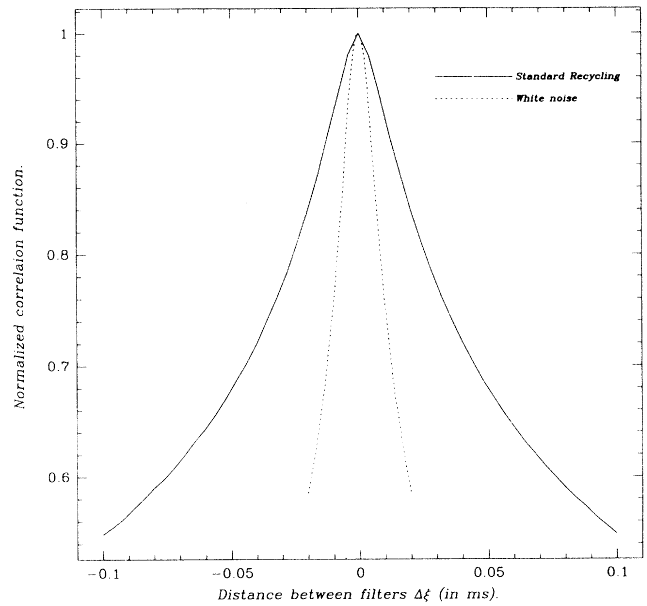

FIG. 1. Plots of the correlation functions for standard recycling noise and white noise with their maxima normalized to unity. The correlation function corresponding to standard recycling noise drops slower than that corresponding to the white noise counterpart. This behavior implies that in the standard recycling case the filters are more coarsely spaced.

TABLE II. Distance between consecutive filters for power spectral density corresponding to white noise (column 2) and noise in interferometers with standard recycling (column 3) for different values of the parameter κ (column 1). The spacing between filters in the latter case is smaller since the template in that case filters only the lower frequency part of the signal where it is harder to distinguish between two chirp wave forms of different chirp mass values. Also quoted are the number of filters required in the case of standard recycling for two different ranges of the chirp masses: $\mathcal{M} \in [0.25, 20]M_\odot$ (column 4) and $\mathcal{M} \in [0.5, 20]M_\odot$ (column 5).

κ^{-1}	$\Delta\xi$ (ms) White noise	$\Delta\xi$ (ms) SR noise	n_f $\mathcal{M}_1=0.25M_\odot$	n_f $\mathcal{M}_1=0.50M_\odot$
0.95	3.4	6.2	4914	1548
0.90	4.9	10.1	2994	943
0.85	6.4	14.1	2152	678
0.80	8.2	18.6	1631	513
0.75	10.2	24.0	1261	397
0.70	12.2	30.4	995	313
0.65	15.4	38.2	793	249

value of κ, or equivalently a lower minimal strength, can be chosen than in the white noise case.

We now derive an approximate analytical formula for $\Delta\xi$ as a function of κ. This is achieved by Taylor expanding the correlation function about the peak and taking a "slice" at $C=\kappa^{-1}C(0,0,0)$. The intersection is an ellipsoid in the parameter space, from which the spacing between filters can be obtained. The details of the computation are given in the Appendix. Denoting the analytical value of the distance between filters by $\Delta\xi_{an}$, we have

$$\Delta\xi_{an}=\frac{1}{2\pi f_a}\left[\frac{\Gamma_{11}\Gamma_{33}-\Gamma_{13}^2}{\Gamma_{11}\Gamma_{22}\Gamma_{33}-(\Gamma_{11}\Gamma_{23}^2+\Gamma_{22}\Gamma_{13}^2+\Gamma_{33}\Gamma_{12}^2)+2\Gamma_{12}\Gamma_{23}\Gamma_{13}}\right]^{1/2}\sqrt{2(1-\kappa^{-1})}\ , \tag{3.10}$$

where the Γ matrix is

$$\Gamma=A\begin{bmatrix}\int_1^\infty \frac{x^2}{S(x)}dx & \int_1^\infty \frac{xa(x)}{S(x)}dx & \int_1^\infty \frac{x}{S(x)}dx\\ * & \int_1^\infty \frac{a^2(x)}{S(x)}dx & \int_1^\infty \frac{a(x)}{S(x)}dx\\ * & * & \int_1^\infty \frac{dx}{S(x)}\end{bmatrix}, \tag{3.11}$$

and

$$S(x)=x^{7/3}(x^2+\gamma^2)\ ,\quad a(x)=\tfrac{8}{5}-\tfrac{3}{5}x^{-5/3}-x\ ,$$

$$\text{and } A=\left[\int_1^\infty \frac{dx}{S(x)}\right]^{-1}. \tag{3.12}$$

The Γ matrix is symmetric and the "stars" in the matrix denote elements obtained by symmetry. When the noise power spectral density is flat [i.e., $S_h(f)$=const] most of the integrals in the above formula diverge. However, for a realistic detector the frequency response is band limited and therefore we can replace the limits in the above integrals with that corresponding to the bandwidth of the detector. For the standard recycling case, with $\gamma=1.44$, we get

$$\Gamma=\begin{bmatrix}3.07 & -1.02 & 1.58\\ * & 0.54 & -0.34\\ * & * & 1.00\end{bmatrix} \tag{3.13a}$$

and for the white noise (with lower frequency cutoff at 100 Hz and upper frequency cutoff at 2.5 kHz) we get

$$\Gamma=\begin{bmatrix}15.30 & -11.37 & 2.70\\ * & 9.36 & -1.34\\ * & * & 1.00\end{bmatrix}. \tag{3.13b}$$

The values quoted are obtained by numerical integration of the integrals in (3.11).

The filter spacings obtained in this way are given in Tables III(a) and III(b), for the white noise and standard recycling noise, respectively. For values of κ^{-1} (column 1) close to 1 there is indeed a good agreement between the numerical (column 2) and the analytical (column 3) results since the quadratic approximation is expected to be adequate.

We comment that the matrix appearing in Eq. (3.11) is the so-called Fischer information matrix and its inverse,

$$\gamma_{ij}=[\Gamma^{-1}]_{ij}\ , \tag{3.14}$$

is the expected covariance of errors of the various parameters of the signal, namely, t_a,ξ,Φ, or more accurately, the scaled dimensionless parameters p_1,p_2,p_3, defined in the Appendix. The square root of the diagonal elements of γ, namely, $(\gamma_{ii})^{1/2}$, represent the expected errors in the parameters p_i. A detailed discussion of this is being published elsewhere [41] (also see Refs. [38,40]).

TABLE III. Distance between consecutive filters in a particular lattice, specified by κ^{-1} (column 1), found by numerical methods (column 2) using Eqs. (3.5a) and (3.5b) and by analytical formula (3.10) (column 3), for (a) white noise and (b) colored noise. In evaluating the integrals numerically, the lower frequency cutoff is taken to be 100 Hz and the upper frequency cutoff is chosen to be 2.5 kHz. The analytical method becomes less accurate as κ^{-1} decreases.

κ^{-1}	$\Delta\xi$ (ms) Numerical	$\Delta\xi$ (ms) Analytical
	(a)	
0.95	3.4	3.7
0.90	4.9	5.2
0.85	6.4	6.3
0.80	8.2	7.3
0.75	10.2	8.1
	(b)	
0.99	2.49	2.15
0.98	3.54	3.04
0.97	4.47	3.73
0.96	5.35	4.30
0.95	6.19	4.81

IV. DETECTION PROBABILITIES

The foregoing analysis needs some modification if we are to apply it to a given output data train. The filtered output $C(\Delta t)$ with a given filter, ξ and Φ fixed, is a random variable for each value of Δt as seen from Eq. (2.11). However, in an actual data analysis problem we have to consider the fact that we have only one random output $C(\Delta t)$ to contend with and the decision whether the signal is present or absent has to be made based on this output. The analysis in the previous sections is valid only when we consider the expectation value $\langle C(\Delta t)\rangle$ of $C(\Delta t)$.

Let a signal of strength greater than the minimal strength be present in the detector output. Although in a given situation $\langle C(\Delta t)\rangle$ may exceed the threshold for some filter, there is no guarantee that $C(\Delta t)$ will also exceed the threshold, as there is noise present in the output. It may so happen that for a particular time shift Δt when $\langle C(\Delta t)\rangle$ exceeds the threshold, a sufficiently large negative noise component "pulls" $C(\Delta t)$ below the threshold level, leaving the signal undetected. The reverse may also take place: a correlation whose expectation value is below the threshold can get "pushed up" above the threshold due to a positive noise component. There is also the added problem that when the arrival time of the signal is t_a the statistic $C(\Delta t)$ may not peak at a time shift $\Delta t=-t_a$. In other words, the maximum, over the time shifts Δt, of $C(\Delta t)$ will, in general, be different from the maximum of $\langle C(\Delta t)\rangle$.

In this section we give a lower bound on the minimal strength of the signal that it be detected with a certain probability, called the detection probability, typically 95%. We find that we need to modify the previous results to some extent. We also justify below that considering just the maximum of the correlation function is sufficient to decide a detection. This is done in Sec. IV A. Section IV B deals with detection probabilities and the modified thresholds.

A. Covariance of the correlation at different time shifts

Consider a signal $h(t,\xi,\Phi)$ such that ξ is closest to the filter with $\xi=\xi_i$, i.e., $\xi=\xi_i-\Delta\xi_0$ where $0<|\Delta\xi_0|<\Delta\xi$. Let us consider the case $\Delta\xi_0>0$. The argument for $\Delta\xi_0<0$ is analogous. The correlation of $h(t,\xi,\Phi)$ with the filter with $\xi=\xi_i$ is given by

$$C(\Delta t,\Delta\xi_0,\Delta\Phi)=h(t,\xi_i-\Delta\xi_0,\Phi)\circ q(t+\Delta t,\xi_i,\Phi+\Delta\Phi)\ , \tag{4.1}$$

where the $\circ$ denotes the operation of correlation. Note that to obtain the filter for a general Φ a suitable linear combination of the filters for $\Phi=0$ and $\Phi=\pi/2$ has to be taken. Let us denote the values of Δt and $\Delta\Phi$ at which C attains a maximum by Δt_m and $\Delta\Phi_m$, respectively, and consider the function $C(\Delta t,\Delta\xi_0,\Delta\Phi_m)$ for different values of Δt. In Eq. (2.25) we fix $\Delta\xi=\Delta\xi_0$ and $\Delta\Phi=\Delta\Phi_m$ and allow Δt to vary. Our aim is to find out the time scale in which the correlation drops to zero. If $S_h(f)$ is basically flat near f_a (as in the case of white noise or standard recycling), the steep wall at f_a due to the seismic noise, and the rapid falloff in the power of the signal ($\propto f^{-7/3}$), produces a Dirac-Δ-like function with a peak at $f\gtrsim f_a$. From Eq. (2.25) we see that the correlation function C is approximately proportional to $\sim\cos(2\pi f_a\delta t)$ where $\Delta t=\Delta t_m+\delta t$. This leads to the correlation function dropping to zero for $\delta t\sim\pm 1/4f_a$. For $f_a=100$ Hz, $\delta t\sim\pm 2.5$ msec. This agrees with the numerical contour plots obtained for the correlation function by Schutz [42].

We argue that this is the same time scale over which the correlation at different instants is correlated. (The time instant in these discussions corresponds to the time shift Δt of the filter relative to the output of the detector.) It is not too hard to show that the covariance of the correlation between the instants Δt and $\Delta t+\delta t$ is given by

$$\begin{aligned}\langle C(\Delta t_m,\Delta\xi_0,\Delta\Phi_m)C(\Delta t_m+\delta t,\Delta\xi_0,\Delta\Phi_m)\rangle\\ -\langle C(\Delta t_m,\Delta\xi_0,\Delta\Phi_m)\rangle\langle C(\Delta t_m+\delta t,\Delta\xi_0,\Delta\Phi)\rangle\\ =B\int_{f_a}^{\infty}\frac{\cos(2\pi f\delta t)}{f^{7/3}S_h(f)}df\ ,\end{aligned} \tag{4.2}$$

where B is a constant. This equation again shows by the foregoing argument that the covariance $\propto\cos(2\pi f_a\delta t)$ and goes to zero over the time scale $1/4f_a$. The quantity δt is also called the decorrelation time [31] and basically gives a time scale over which the correlations computed at instants differing by a time interval greater than δt are uncorrelated.

The above considerations show that over the time scale when $\langle C(\Delta t)\rangle$ is appreciable it is correlated to $C(\Delta t_m)$ and hence it is not unjustified if we base our conclusions on the statistic $C(\Delta t_m)$ to decide the presence or absence of a signal.

B. Detection probabilities

Since the C is a random variable the probability that a decision in favor of detection will be made, when a signal is actually present, is *not* unity. Since the noise is assumed to be Gaussian, the C at the time shift Δt_m is also a Gaussian variate with mean $\tilde\eta=\langle C(\Delta t_m,\xi_i-\Delta\xi_0,\Delta\Phi_m)\rangle$ and variance unity.

We give below a rough estimate of the detection probabilities. The relevant statistic here is the maximum of the filtered outputs and this in general not a normal deviate. As in Sec. III A here too we derive the results for the case when the filter spacings are fairly large ($\kappa^{-1}\sim 0.8$ or even less) and the correlation of the signal with, say, the ith filter, denoted by C_i in short, is much larger than its correlation with the neighboring filters. Ignoring covariances between the filtered outputs we argue that the maximum of the filter outputs has roughly the same distribution as C_i and hence is approximately a normal deviate.

Let us consider C_{i+1}. We examine the case when $\Delta\xi_0\ll\Delta\xi$ and $\kappa^{-1}\sim 0.8$. Then the mean of C_{i+1} denoted by $\langle C_{i+1}\rangle$ will be less than $\kappa^{-1}\tilde\eta$ (more like 0.6 $\tilde\eta$ for $\kappa^{-1}\sim 0.8$). For $\tilde\eta\sim 7.5$ this number is about 4.5, i.e., three times the standard deviation away from the mean of C_i. Therefore, C_{i+1} will have very little effect on the distribution of the maximum of C_i. The effect of other filters on the maximum is even less and we ignore it in our analysis. Therefore, in this case the distribution of the maximum of C_i over i is approximately the same as that of C_i itself. The detection probability Q_d is given by

$$Q_d\simeq\frac{1}{\sqrt{2\pi}}\int_{\eta}^{\infty}\exp\left[-\frac{(C-\tilde\eta)^2}{2}\right]dC\ . \tag{4.3}$$

For example if $\tilde\eta$ is just the threshold level η then by Eq. (4.3), $Q_d=0.5$. This implies that there is an equal chance that the fluctuations due to the noise may either bring C down or push it up above the threshold. It is desirable to have the detection probability as high as possible. For example, if $Q_d = 0.975$, then $\tilde{\eta} \simeq 9.5$. Therefore, the effect of the noise for a given data output is to increase the threshold level higher than the basic threshold level η if we impose the condition of high detection probability.

When $\Delta\xi_0$ is chosen to have a larger value $\sim \Delta\xi$, near about the filter spacing, the above approximation breaks down since the means of C_i and C_{i+1} are more or less equal. Thus, the maximum of C_i and C_{i+1} is no more a normal deviate. However, when $\Delta\xi_0 \sim \Delta\xi$, i.e., when the signal lies exactly between the two filters, both C_i and C_{i+1} are distributed identically with mean $\tilde{\eta}$ and variance unity. If we ignore the covariances, the maximum of C_i over i, denoted by $C_{\max}$, has the probability density

$$p(C_{\max}) = \frac{d}{dC} I(C)^2 \bigg|_{C = C_{\max}} , \tag{4.4}$$

where

$$I(C) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{C} \exp\{-[(x-\tilde{\eta})^2/2]\} dx \ .$$

(This formula may be easily derived by differentiating the joint distribution function of C_i and C_{i+1}.) The mean of $C_{\max}$ is greater than $\tilde{\eta}$ and Q_d will be greater than the previous case of $\Delta\xi_0 \ll \Delta\xi$ for the same value of $\tilde{\eta}$. But a larger signal strength, larger approximately by a factor κ, will be needed to produce the same mean $\tilde{\eta}$ than in the previous case since the signal and filter parameters are mismatched.

What happens to the bank of filters? We have seen that the above considerations amount to shifting the threshold level from η to $\tilde{\eta}$, because if $\langle C \rangle$ is at least $\tilde{\eta}$ then C will exceed η with probability at least equal to Q_d. We can ensure the detection probability to be greater than a given Q_0 if we take the minimal strength of signals to be $S_{\min} = \kappa\tilde{\eta}$, where $\tilde{\eta}$ is obtained by solving Eq. (4.3) in which Q_d is replaced by Q_0. Signals of this minimal strength will be detected with a probability greater than Q_d, if a bank of filters corresponding to κ is used. This minimal strength is probably a slight overestimate in the light of Eq. (4.4) and a slightly lower value of $S_{\min}$ should be adequate to give a detection probability greater than Q_0.

V. CONCLUSIONS

In this paper, the analysis of setting up a bank of filters to extract arbitrary Newtonian chirp wave forms buried in noisy data is investigated. Earlier analysis carried out for the case of white noise has been extended to the case of colored noise.

The detector noise is assumed to be stationary, characterized by the Gaussian normal distribution. The well-known technique of matched filtering is employed to filter out a signal buried in noisy data. The idea of normalized filters is introduced which greatly simplifies the problem of setting up a lattice of filters. A general formalism is given for making a choice of filters, with each filter having a different set of values for its parameters, so as to detect arbitrary chirp signals, of strength greater than a certain minimal strength, buried in arbitrary colored noise. It is then applied to the specific case of noise found in detectors that employ standard recycling. Banks of filters are obtained for different values of the minimal strength for this case. It is found that the spacing between filters is more than twice, as compared to the case when the detector noise is white, implying that only half the number of filters are required to span a given range of parameters of the chirp wave form as compared to the latter. This is basically due to the fact that the noise power spectral density in standard recycling is larger at higher frequencies and consequently the signal power at those frequencies is effectively cut off. Thus, the filter can be chosen to have an upper frequency cutoff of 400 Hz, and a lower sampling rate is adequate for the data train.

Further, an approximate analytic formula is obtained for the spacing between filters in terms of the minimal strength. This is achieved by expanding the correlation function, of a filter and a signal, about its maximum and demanding that this maximum be equal to the minimal strength. In the Appendix an elegant geometrical construction is given for deriving this formula. Strictly speaking, this formula exists for the standard recycling case, and not for the idealistic white noise case, where the correlation function is not sufficiently smooth at the maximum. However, for the bandwidth limited white noise the analytic formula does exist but the results depend upon the bandwidth.

Finally, we have discussed the case of a single data output and detection probabilities. For a detection probability of 0.975 it is shown that the threshold must be raised by about two sigma. However, the same bank of filters can be used for this modified threshold.

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APPENDIX: ANALYTICAL RELATION BETWEEN MINIMAL STRENGTH AND SPACING BETWEEN FILTERS

In Sec. III C a numerical method of determining the lattice spacing of filters was described which is inevitable for large values of $\Delta\xi$. However, for values of κ close to unity, i.e., when the spacing between the filters is small, an analytical relation can be found. This analytical relation has been compared with numerical results and good agreement is obtained for $\kappa \gtrsim 1$. The relation is basically derived by Taylor expanding the cross-correlation function about its maximum up to the second order and equating the result to the threshold level.

Let us write the correlation of two chirp wave forms that differ slightly in their parameter values as [cf. Eq. (2.25)]

$$C(\Delta t,\Delta\xi,\Delta\Phi) = \tilde{A}\int_{f_a}^{\infty}\frac{\cos[2\pi f\Delta t+2\pi\alpha(f)f_a\Delta\xi+\Delta\Phi]}{f^{7/3}S_h(f)}df\ , \quad (A1)$$

where $\tilde{A}$ is an overall normalization constant. It is convenient to use dimensionless quantities and to this end we set

$$x=f/f_a,\ \ \gamma=f_k/f_a,\ \ a(x)=\tfrac{1}{5}(8-3x^{-5/3}-5x)\ , \quad (A2)$$

$$p_1=2\pi f_a t,\ \ p_2=2\pi f_a\xi,\ \ p_3=\Phi\ . \quad (A3)$$

In terms of these new variables the correlation function takes the form

$$C(\Delta p_i)=A\int_1^{\infty}\frac{\cos[x\Delta p_1+a(x)\Delta p_2+\Delta p_3]}{S(x)}dx\ , \quad (A4)$$

where $S(x)$ and A have already been defined in Eq. (3.12). The normalization can be chosen arbitrarily as long as one chooses the threshold level accordingly. We choose the maximum of the C to be 1 and this occurs at $\Delta p_i=0$. Thus,

$$C(\Delta p_i=0)=1\ . \quad (A5)$$

Since the maximum of C is just κ times the threshold level, the threshold level is κ^{-1}. This problem therefore reduces to finding the maximum values of Δp_2 with the condition

$$C(\Delta p_i)=\kappa^{-1}\ , \quad (A6)$$

the other parameters Δp_1 and Δp_3 being otherwise free. To this end we Taylor expand C up to the second order about its maximum:

$$C(\Delta p_i)=C(\Delta p_i=0)-\tfrac{1}{2}\Gamma_{ij}\Delta p_i\Delta p_j\ , \quad (A7)$$

where

$$\Gamma_{ij}=-\left[\frac{\partial^2 C}{\partial\Delta p_i\partial\Delta p_j}\right]_{\Delta p_i=0}\ . \quad (A8)$$

In the above equation summation convention has been used: i.e., repeated indices are summed over. We observe that Γ_{ij} is a positive definite and symmetric matrix, since C has a maximum at the origin.

The above equations have been obtained in the noise-free case for which the quantities involved are ordinary functions. However, in the realistic case when noise exists the output of the detector is a random variable. Consequently, quantities such as the cross correlation C are also random variables. In particular, Γ_{ij} is a random tensor. But now the corresponding quantities will be the expectation values of these random variables which are expected to match with those in the foregoing noise-free treatment. We observe that the expectation value of Γ_{ij} is then just the *Fischer information matrix* [19].

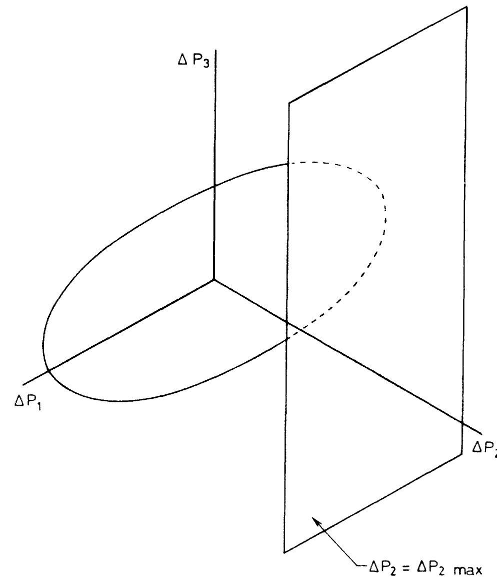

FIG. 2. Schematic diagram showing the ellipsoid of Eq. (A9) in the Appendix. The plane $\Delta p_2=\Delta p_{2\,\max}$ touches this ellipsoid. The distance of this plane from the origin gives the lattice spacing in terms of p_2.

Using (A5) and (A6) in (A7) we have

$$f(\Delta p_i)\equiv\Gamma_{ij}\Delta p_i\Delta p_j=2(1-\kappa^{-1})\ . \quad (A9)$$

Geometrically, this is an equation of an ellipsoid in $(\Delta p_1,\Delta p_2,\Delta p_3)$ space. Further, in the four-dimensional space spanned by $(\Delta p_i, C)$, the Γ_{ij} can be interpreted as curvatures of the cross-correlation hypersurface. The problem then is to find the maximum value of Δp_2 with the constraint described by Eq. (A9). Geometrically, this amounts to the following construction: One may imagine a $\Delta p_2=$const plane which is tangent to the ellipsoid. The distance of this plane from the $\Delta p_2=0$ plane is the required $\Delta p_{2\,\max}$ (see Fig. 2). Since the normal to this tangent plane must be parallel to the Δp_2 axis, we have

$$\frac{\partial f}{\partial\Delta p_1}=\frac{\partial f}{\partial\Delta p_3}=0\ . \quad (A10)$$

Equations (A10) written out explicitly in terms of the Fischer information matrix are

$$\Gamma_{1i}\Delta p_i=0\ , \quad (A11a)$$

$$\Gamma_{3i}\Delta p_i=0\ . \quad (A11b)$$

We now eliminate Δp_1 and Δp_3 from (A9) and (A11a) and (A11b). This yields the result

$$\Delta p_{2\max}=\left[2(1-\kappa^{-1})\frac{\Gamma_{11}\Gamma_{33}-\Gamma_{13}^2}{\Gamma_{11}\Gamma_{22}\Gamma_{33}-(\Gamma_{11}\Gamma_{23}^2+\Gamma_{22}\Gamma_{13}^2+\Gamma_{33}\Gamma_{12}^2)+2\Gamma_{12}\Gamma_{23}\Gamma_{13}}\right]^{1/2}, \quad (A12)$$

where the Γ matrix is

$$\Gamma = A \begin{vmatrix} \int_1^\infty \frac{x^2}{S(x)} dx & \int_1^\infty \frac{x a(x)}{S(x)} dx & \int_1^\infty \frac{x}{S(x)} dx \\ * & \int_1^\infty \frac{a^2(x)}{S(x)} dx & \int_1^\infty \frac{a(x)}{S(x)} dx \\ * & * & \int_1^\infty \frac{dx}{S(x)} \end{vmatrix} . \tag{A13}$$

The "stars" denote matrix elements obtained by symmetry. This formula, as it stands, cannot be applied to white noise since many of the Γ_{ij} are infinite in this idealistic case. However, the upper limit of the integrals will be finite for a realistic detector. For instance, if the sensitivity of a detector is band limited in a range of frequencies, say 100 Hz to 2 kHz, then the limits in the above integrals are from 1 to 20.

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