Weak decays of D mesons to pseudoscalar-tensor final states

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(Received 7 May 1993)

In this Brief Report we study the pseudoscalar-tensor meson weak decays of D mesons in a nonrelativistic quark model using the factorization scheme. Branching ratios for the Cabibbo-angle-favored decays are calculated.

PACS number(s): 13.25.Ft, 14.40.Cs, 14.40.Lb

The spectator model using the factorization ansatz [1–7] has achieved remarkable success in explaining most exclusive two-body D-meson decays. It involves an expansion of transition amplitudes in terms of invariant form factors which provide essential information on the structure of mesons and other rare phenomena such as CP violation. Though there exists no exact dynamical calculation for these form factors, many phenomenological models have been proposed [1,7].

In this Brief Report, we calculate branching ratios for Cabibbo-angle-favored $D\to PT$ decays in the nonrelativistic quark model [7]. Since the combined mass of the final state particles lies close to the D-meson mass, the decays involving p-wave mesons are expected to be suppressed [8]. Recently, axial-meson emitting decay modes $D\to\bar{K}a_1$ have been measured [9], which apparently seem to be in conflict with theoretical predictions [10]. However, for $D\to PT$ modes, only the upper limits of two decays $D^0\to K^-a_2^+$ and $D^+\to\bar{K}^0a_2^+$ are available [9,11]. In the present work, we find that $D\to\bar{K}a_2$ decays are suppressed and suggest that the better candidates to look for in experiment are $D\to\pi\bar{K}_2^*$ decays. These decays are enhanced in comparison with $D\to\bar{K}a_2$ by two orders of magnitude.

TABLE I. Amplitudes for the Cabibbo-angle-favored $D\to PT$ decays. $F^{D\to T}=[k+(m_D^2-m_T^2)b_+ + m_P^2 b_-]$ and $f-f'$ mixing angle $\phi_T=\theta_T$ (physical) $-\theta$(ideal).

Serial No.	Decay	Amplitude $\left[\frac{G_F}{\sqrt{2}}\cos^2\theta_C\right]$
1.	$D^0\to K^-a_2^+$	0
2.	$D^0\to\bar{K}^0a_2^0$	$(1/\sqrt{2})a_2f_KF^{D\to a_2}$
3.	$D^0\to\bar{K}^0f_2$	$(1/\sqrt{2})a_2f_K\cos\phi_TF^{D\to f_2}$
4.	$D^0\to\pi^+K_2^{*-}$	$a_1f_\pi F^{D\to K_2^*}$
5.	$D^0\to\pi^0\bar{K}_2^{*0}$	0
6.	$D^+\to\bar{K}^0a_2^+$	$a_2f_KF^{D\to a_2}$
7.	$D^+\to\pi^+\bar{K}_2^{*0}$	$a_1f_\pi F^{D\to K_2^*}$
8.	$D_s^+\to\pi^+f_2$	$-a_1f_\pi\sin\phi_TF^{D_s\to f_2}$
9.	$D_s^+\to\pi^+f_2'$	$-a_1f_\pi\cos\phi_TF^{D_s\to f_2'}$
10.	$D_s^+\to\eta a_2^+$	0
11.	$D_s^+\to K^+\bar{K}_2^{*0}$	0
12.	$D_s^+\to\bar{K}^0K_2^{*+}$	$a_2f_KF^{D_s\to K_2^*}$

The effective weak Hamiltonian for the charm $\to$hadronic decays in the Cabibbo-angle-favored [1,10] mode is given by

$$H_W(\Delta C=\Delta S=-1)=\frac{G_F}{\sqrt{2}}\cos^2\theta_C[a_1(\bar{u}d)_H(\bar{s}c)_H + a_2(\bar{u}c)_H(\bar{s}d)_H] , \quad (1)$$

where a_1 and a_2 are the QCD coefficients. We take $a_1=1.2$ and $a_2=-0.5$, as guided by $D\to PP,PV,VV$ data [1,6,10]. The notation $(\bar{q}q)$ is shorthand for a color singlet combination $\bar{q}\gamma_\mu(1-\gamma_5)q$ and a subscript H indicates that the parentheses should be treated as an effective hadronic field.

For the spectator process, the factorization hypothesis expresses the decay amplitudes as the product of the matrix element of weak currents as

$$A(D\to PT)=\langle P|J^\mu|0\rangle\langle T|J_\mu|D\rangle+\langle T|J^\mu|0\rangle\langle P|J_\mu|D\rangle . \quad (2)$$

However, tensor mesons cannot be extracted from the vacuum, since

$$\langle T(q_\mu)|J_\mu|0\rangle=0 \quad (3)$$

due to the fact that

$$q^\mu\epsilon_{\mu\nu}=0 , \quad (4)$$

where $\epsilon_{\mu\nu}$ is polarization tensor of the tensor mesons. For the remaining matrix elements, we use the definitions [7]

TABLE II. β parameters for s- and p-wave mesons [7].

Quark content:	$u\bar{d}$	$u\bar{s}$	$u\bar{c}$	$u\bar{b}$
β_S (GeV)	0.31	0.34	0.39	0.41
β_P (GeV)	0.27	0.30	0.34	

$$\langle P(k_\mu)|J_\mu|0\rangle = -if_P k_\mu \; ;$$
$$\langle T(P_T)|J_\mu|D(P_D)\rangle = ih\epsilon_{\mu\nu\lambda\rho}\epsilon^{*\nu\alpha}P_{D\alpha}(P_D+P_T)^\lambda(P_D-P_T)^\rho + k\epsilon^*_{\mu\nu}P_D^\nu + b_+(\epsilon^*_{\alpha\beta}P_D^\alpha P_D^\beta)(P_D+P_T)_\mu + b_-(\epsilon^*_{\alpha\beta}P_D^\alpha P_D^\beta)(P_D-P_T)_\mu \; . \tag{5}$$

Finally the decay amplitude becomes

$$A(D\to PT) = -if_P\epsilon^*_{\mu\nu}P_D^\mu P_D^\nu F^{D\to T} \tag{6}$$

up to the scale factor $(G_F/\sqrt{2})\cos^2\theta_C$, where

$$F^{D\to T} = [k + (m_D^2 - m_T^2)b_+ + m_P^2 b_-] \tag{7}$$

is the combination of the form factors appearing in (5). The contribution arising from the last term in (7) is small [7], and is neglected in the present analysis. The decay amplitudes using (6) are listed in Table I.

The decay rate is given by

$$\Gamma(D\to PT) = [p_c^5/(12\pi m_T^2)]|A(D\to PT)|^2 \tag{8}$$

where p_c is the magnitude of three-momentum of the final state particles in the rest frame of D meson and is given by

$$p_c = \{[(M_D^2+m_P^2-m_T^2)/2M_D]^2 - m_P^2\}^{1/2} \tag{9}$$

where M_D, m_P, and m_T are the masses of the D meson, pseudoscalar meson, and tensor meson, respectively.

To calculate form factors k, b_+ appearing in $F^{D\to T}$, we use the nonrelativistic quark model [7], which yields

$$k = \sqrt{2}F_5\frac{m_d}{\beta_D} \; , \tag{10a}$$

$$b_+ = -F_5\frac{m_d}{2\sqrt{2}\tilde{m}_T m_c\beta_D}\left[1 - \frac{m_d m_c\beta_T^2}{2\mu_+\tilde{m}_D\beta_{DT}^2} + \frac{m_d m_c\beta_T^2}{4\mu_-\tilde{m}_D\beta_{DT}^2}\left[1 - \frac{m_d\beta_T^2}{2\tilde{m}_D\beta_{DT}^2}\right]\right] , \tag{10b}$$

$$F_5 = \left[\frac{\tilde{m}_T}{\tilde{m}_D}\right]^{1/2}\left[\frac{\beta_D\beta_T}{\beta_{DT}^2}\right]^{5/2}\exp\left\{\left[-\frac{m_d^2}{4\tilde{m}_T\tilde{m}_D}\right]\frac{(t_m-t)}{\beta_{DT}^2}\right\} , \tag{10c}$$

where $\tilde{m}$ represents the mock mass of corresponding meson, and

$$\beta_{DT}^2 = \tfrac{1}{2}(\beta_D^2+\beta_T^2) \; , \quad t_m = (m_D - m_T)^2 \; ,$$
$$t = (P_D - P_T)^2 = m_P^2 \; ,$$
$$\mu_\pm = [m_q^{-1} \pm m_c^{-1}]^{-1} \; .$$

The subscript q depends upon the quark currents $\bar{q}\gamma_\mu c$ and $\bar{q}\gamma_\mu\gamma_5 c$ in the weak Hamiltonian. With $m_u = m_d = 0.33$ GeV, $m_s = 0.51$ GeV, $m_c = 1.60$ GeV, and β's are given in Table II. We calculate the form factors k, b_+, and $F^{D\to T}$ at maximum momentum transfer t_m for various possible decays and list them in Table III. For the decay $D_s^+\to\pi^+f_2'$, m_d in the above equation (10) should be taken as m_s.

Gluon self-coupling in QCD suggests that, in addition to the conventional $q\bar{q}$ meson states, tensor mesons may have components from glueballs, and or hybrid ($q\bar{q}g$) states. However, experimentally [9,12] the tensor meson nonet behaves very well with respect to SU(3) flavor symmetry and the quark model. The two $1\,^3P_2\; q\bar{q}$ states are very likely the well-known $f_2(1.270)$ and $f_2'(1.525)$, although the observation by Breakstone [13] of $f_2(1.270)$ production by gluon fusion could indicate that it has a glueball component [9]. In the present analysis, we take the well-established tensor meson nonet as a pure $q\bar{q}$ state. The mixing angle between the two isoscalars f_2 and f_2' is $\theta_T = 28°$ (quadratic mass) [9]. We use $f_\pi = 0.132$ MeV, $f_K = 0.169$ MeV [1,9], and $F^{D\to T}$ at real momentum transfer t. Further β_S for $s\bar{c}$ is assumed to be equal to β_S for $u\bar{c}$ as it has small dependence on quark flavor [7]. The branching ratios calculated for $D\to PT$ decays are listed in Table IV. Some salient features are given below.

(1) The decays $D_s^+\to\pi^+a_2^0$, $D_s^+\to\pi^0a_2^+$ are forbidden by isospin invariance.

(2) The decay $D_s^+\to\pi^+f_2$ is forbidden in the limit of ideal mixing for $f_2 - f_2'$ states. The value $B(D_s^+\to\pi^+f_2) = 1.5\times10^{-4}\%$ arises through the physical mixing angle.

(3) The decays $D^0\to K^-a_2^+$, $D^0\to\pi^0\bar{K}_2^{*0}$, $D_s^+\to\eta a_2^+$, $D_s^+\to\eta' a_2^+$, and $D_s^+\to K^+\bar{K}_2^{*0}$ are forbidden due to the condition given in Eq. (4). Out of these, $D^0\to K^-a_2^+$ and $D^0\to\pi^0\bar{K}_2^{*0}$ may arise through the possible elastic final

TABLE III. Form factors at maximum momentum transfer ($t = t_m$).

Transition	k	b_+ (GeV^{-2})	$F^{D\to T}$
$D\to a_2$	0.593	−0.109	0.403
$D\to f_2$	0.593	−0.109	0.391
$D\to K_2^*$	0.693	−0.106	0.541
$D_s\to f_2$	0.593	−0.109	0.348
$D_s\to f_2'$	1.181	−0.124	0.989
$D_s\to K_2^*$	0.693	−0.099	0.513

TABLE IV. Branching ratios (%) for $D \to PT$ decays. (f_2-f_2' mixing angle $\theta_T=28°$).

Serial No.	Decay	Momentum (p_c) (GeV)	Theory	Experiment
1.	$D^0 \to K^- a_2^+$	0.197	0	<0.6 [9] ≤ 0.2 [11]
2.	$D^0 \to \bar{K}^0 a_2^0$	0.190	8.4×10^{-6}	
3.	$D^0 \to \bar{K}^0 f_2$	0.263	4.1×10^{-5}	
4.	$D^0 \to \pi^+ K_2^{*-}$	0.367	2.4×10^{-3}	
5.	$D^0 \to \pi^0 \bar{K}_2^{*0}$	0.363	0	
6.	$D^+ \to \bar{K}^0 a_2^+$	0.199	5.4×10^{-5}	<0.8 [9] ≤ 0.3 [11]
7.	$D^+ \to \pi^+ \bar{K}_2^{*0}$	0.365	5.8×10^{-3}	
8.	$D_s^+ \to \pi^+ f_2$	0.559	1.5×10^{-4}	
9.	$D_s^+ \to \pi^+ f_2'$	0.374	8.0×10^{-3}	
10.	$D_s^+ \to \eta a_2^+$	0.288	0	
11.	$D_s^+ \to K^+ \bar{K}_2^{*0}$	0.179	0	
12.	$D_s^+ \to \bar{K}^0 K_2^{*+}$	0.186	2.2×10^{-5}	

state interaction (FSI) effects.

Since D-meson masses lie in the resonance region, rescattering effects of outgoing mesons may be important. FSI's induced by strong interactions may cause mixing of the decay channels having the same quantum numbers. Weak decays $D \to PP, PV, VV$ definitely indicate the need of FSI's [1,4–6]. Therefore, one may expect these to be significant for $D \to PT$ decays also. At the isospin level the elastic FSI's introduce appropriate phase factors in the different isospin channels. Obviously, the decays involving a single isospin channel remain unaffected. FSI modified amplitudes for $D \to \bar{K} a_2$ are

$$\begin{aligned} A(D^0 \to K^- a_2^+) &= A^{\bar{K}a_2} \exp(i\delta_{1/2}^{\bar{K}a_2}) \left[1 + \frac{r^{\bar{K}a_2}}{2} \exp(-i\delta^{\bar{K}a_2}) \right], \\ A(D^0 \to \bar{K}^0 a_2^0) &= -\frac{1}{\sqrt{2}} A^{\bar{K}a_2} \exp(i\delta_{1/2}^{\bar{K}a_2}) [1 - r^{\bar{K}a_2} \exp(-i\delta^{\bar{K}a_2})], \\ A(D^+ \to \bar{K}^0 a_2^+) &= A_{3/2}^{\bar{K}a_2} \exp(i\delta_{3/2}^{\bar{K}a_2}), \end{aligned} \tag{11}$$

where $A^{\bar{K}a_2} = \frac{2}{3} A_{1/2}^{\bar{K}a_2}$, $r^{\bar{K}a_2} = A_{3/2}^{\bar{K}a_2} / A_{1/2}^{\bar{K}a_2}$, and the phase angle

$$\delta^{\bar{K}a_2} = (\delta_{1/2}^{\bar{K}a_2} - \delta_{3/2}^{\bar{K}a_2}) .$$

Leaving aside the scale factor, the isospin reduced amplitudes can be expressed as

$$-\tfrac{1}{2} A_{1/2}^{\bar{K}a_2} = A_{3/2}^{\bar{K}a_2} = a_2 f_K F^{D\to a_2} . \tag{12}$$

This yields

$$r^{\bar{K}a_2} = -2 , \quad A^{\bar{K}a_2} = 11.267\times 10^{-3}\ \text{GeV} . \tag{13}$$

A similar analysis for $\pi \bar{K}_2^*$ decay modes gives

$$r^{\pi \bar{K}_2^*} = 1 , \quad A^{\pi \bar{K}_2^*} = 55.862\times 10^{-3}\ \text{GeV} . \tag{14}$$

Then, the branching ratios (%) are modified to

$$\begin{aligned} B(D^0 \to K^- a_2^+) &= 4.54\times 10^{-6}(1-\cos\delta^{\bar{K}a_2}) , \\ B(D^0 \to \bar{K}^0 a_2^0) &= 0.93\times 10^{-6}(5+4\cos\delta^{\bar{K}a_2}) , \\ B(D^+ \to \bar{K}^0 a_2^+) &= 54.10\times 10^{-6} , \\ B(D^0 \to \pi^+ K_2^{*-}) &= 0.27\times 10^{-3}(5+4\cos\delta^{\pi \bar{K}_2^*}) , \\ B(D^0 \to \pi^0 \bar{K}_2^{*0}) &= 0.99\times 10^{-3}(1-\cos\delta^{\pi \bar{K}_2^*}) , \\ B(D^+ \to \pi^+ \bar{K}_2^{*0}) &= 5.83\times 10^{-3} . \end{aligned} \tag{15}$$

Using $-180° \le (\delta^{\bar{K}a_2}) \le 180°$, we plot the branching ratios for decays $D^0 \to K^- a_2^+$ and $D^0 \to \bar{K}^0 a_2^0$ in Fig. 1. Similarly, we plot the branching ratios for the decays $D^0 \to \pi^+ K_2^{*-}$ and $D^0 \to \pi^0 \bar{K}_2^{*0}$ in Fig. 2. Thus, the maximum branching ratios for the decays $D^0 \to K^- a_2^+$ and $D^0 \to \pi^0 \bar{K}_2^{*0}$ that can be reached are $9.09\times 10^{-6}\%$ and $1.98\times 10^{-3}\%$, respectively.

We have calculated the branching ratios of $D \to PT$ decays with and without the FSI. Form factors appearing

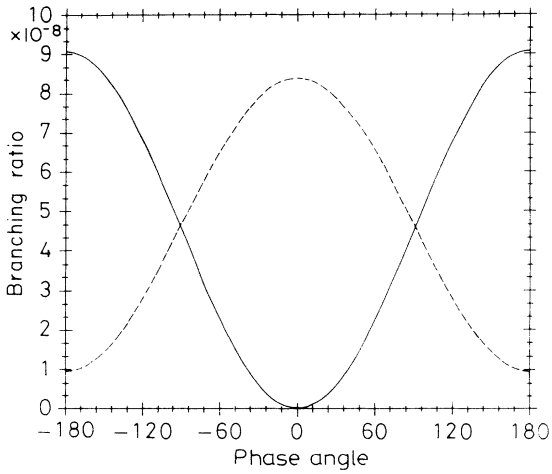

FIG. 1. Branching ratio vs phase difference $\delta^{\bar{K}a_2}$. Solid line for $D^0 \to K^- a_2^+$; dashed line for $D^0 \to \bar{K}^0 a_2^0$.

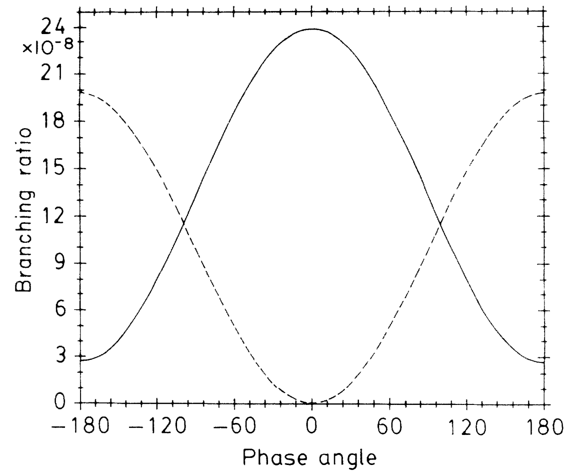

FIG. 2. Branching ratio vs phase difference $\delta^{\pi\bar{K}_2^*}$. Solid line for $D^0 \to \pi^+ K_2^{*-}$; dashed line for $D^0 \to \pi^0 \bar{K}_2^{*0}$.

in the decay matrix elements of weak currents are calculated in the nonrelativistic quark model [7]. This technique has worked well in explaining $D \to PP/PV/VV$ decays. The calculated branching ratios are found to be relatively small, but possibly in reach of current experiments. These decays, being d wave, are generally suppressed due to the kinematical factors. In this work we have employed the factorization scheme. Inclusion of nonfactorizable contributions may modify the decay rates. However, these contributions are expected to be small as is the case with $D \to PP/PV/VV$ decay modes.

Though many decay channels (PT) are available for D mesons, the dominant decays are $D^+ \to \pi^+ \bar{K}_2^{*0}$, $D^0 \to \pi^+ K_2^{*-}$, and $D_s^+ \to \pi^+ f_2'$. So far, the experimental efforts have been made for $D \to \bar{K}a_2$ decays, which are found to be suppressed in comparison to $D \to \pi\bar{K}_2^*$ decays. In fact, the dominant cause of the suppression of $D \to \bar{K}a_2$ mode is the lesser phase space available. Notice that $[p_c(D \to \bar{K}a_2)/p_c(D \to \pi\bar{K}_2^*)]^5 \approx 0.045$. QCD corrections suppresses $D \to \bar{K}a_2$ mode further by a factor $(a_2/a_1)^2 \approx 0.17$.

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