Quenched heavy-light decay constants

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(Received 24 August 1993)

We present results for heavy-light decay constants, using both propagating quarks and the static approximation, in $O(a)$-improved, quenched lattice QCD. At $\beta=6.2$ on a $24^3\times 48$ lattice we find $f_D=185^{+4}_{-3}(\text{stat})\,^{+42}_{-7}(\text{syst})$ MeV, $f_B=160^{+6}_{-6}\,^{+53}_{-19}$ MeV, $f_{D_s}/f_D=1.18^{+2}_{-2}$, and $f_{B_s}/f_B=1.22^{+4}_{-3}$, in good agreement with earlier studies. From the static theory we obtain $f_B^{\text{stat}}=253^{+16}_{-15}\,^{+105}_{-15}$ MeV. We also present results from a simulation at $\beta=6.0$ on a $16^3\times 48$ lattice, which are consistent with those at $\beta=6.2$. In order to study the effects of improvement, we present a direct comparison of the results using both the Wilson and the improved action at $\beta=6.0$.

PACS number(s): 12.38.Gc, 14.40.Lb, 14.40.Nd

I. INTRODUCTION

The leptonic decay constants f_P of pseudoscalar mesons composed of a heavy and a light quark play an important role in weak-interaction phenomenology. In particular, f_B, or more strictly $f_B\sqrt{B_B}$ (where B_B, the B parameter of B^0-$\bar{B}^0$ mixing, is expected to be close to one), is one of the principal unknown quantities needed for the determination of the *CP*-violating phase in the standard model, as well as other properties of weak decays. Lattice QCD offers the opportunity for a nonperturbative computation of the operator matrix elements, which are necessary for the determination of the decay constants and B parameters.

During the last few years there have been several lattice computations of the decay constants of "heavy-light" pseudoscalar (and vector) mesons. The results for the decay constant of the D meson, obtained using the Wilson action for the quarks, are in the region of 200 MeV (using a normalization for which $f_\pi=132$ MeV). For example, in his 1989 review Sharpe [1] quoted

$$f_D \simeq 180\pm 25(\text{stat})\pm 30(\text{syst})\ \text{MeV}$$

as his summary of the lattice results. More recent simulations with Wilson fermions also give results in this range [2–5]. The experimental bound is $f_D<290$ MeV [6].

In the heavy-quark limit the scaling law for the decay constant of a heavy-light pseudoscalar meson is $f_P\sqrt{M_P}\sim$const (up to mild logarithmic corrections). Lattice simulations using heavy-quark masses in the charm region indicate that there are large corrections to this scaling law (of order 40% at the charm quark mass, decreasing to about 15% at the mass of the bottom quark) [2–4]. The value of the decay constant of the B meson deduced from these simulations is in the region of 180 MeV. The conclusion that there are violations of the scaling law is supported by the large value for $f_P\sqrt{M_P}$ deduced from simulations obtained using a static (i.e., infinitely massive) heavy quark [5,7–12].

The important results and conclusions quoted above were obtained from simulations in which the mass of the heavy quark is large in lattice units (up to about a half). One may therefore worry that discretization errors significantly contaminate the results. In this paper we present the results for decay constants of heavy-light mesons computed using the $O(a)$-improved lattice action proposed by Sheikholeslami and Wohlert [13], with which the discretization errors in operator matrix elements (and hence in the computed values of the decay constants) can be reduced from $O(m_Q a)$ to $O(\alpha_s m_Q a)$, where m_Q is the mass of the heavy quark [14]. This formal reduction in discretization errors provides an important check on the stability of results and conclusions obtained with Wilson fermions.

The results presented in this paper were obtained from two simulations of quenched QCD, using the Sheikholeslami-Wohlert (SW) or "clover" fermion action for the quarks (see Sec. I A below). Our main results come from a simulation on a $24^3\times 48$ lattice at $\beta=6.2$, for which 60 gauge field configurations were generated. Details of this simulation and the determination of the

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values of the Wilson hopping parameter corresponding to the chiral limit $\kappa_{\rm crit}$ and to the mass of the strange quark have been presented in Ref. [15]. The heavy quarks have masses in the region of the charm quark mass and we study the behavior of the decay constants with the mass of the heavy quark. Interpolating to the mass of the charm quark itself, and extrapolating the results to the mass of the b quark, we find that our best results for the decay constants of the B and D mesons are

$$f_D = 185^{+4}_{-3}({\rm stat})^{+42}_{-7}({\rm syst})\ {\rm MeV}\ , \tag{1}$$

$$f_B = 160^{+6+53}_{-6-19}\ {\rm MeV}\ , \tag{2}$$

$$f_{D_s}/f_D = 1.18^{+2}_{-2}\ , \tag{3}$$

$$f_{B_s}/f_B = 1.22^{+4}_{-3}\ . \tag{4}$$

The details of this calculation and a complete set of results are presented in Sec. II. The second simulation is on a $16^3\times48$ lattice at $\beta=6.0$, using 36 configurations. The results, which are consistent with those mentioned above, are presented in Sec. III. In order to study the effects of improvement on the calculation of heavy-light decay constants, we have repeated the computation for both the Wilson and SW actions using a subset of 16 of these configurations. The results and a discussion are presented in Sec. III B. We have also computed f_B in the static approximation [in which contributions of $O(1/m_b)$ are neglected]. A discussion of the calculation and of the results is presented in Sec. IV. The result from the simulation at $\beta=6.2$, on 20 of the 60 configurations, is

$$f_B^{\rm stat} = 253^{+16}_{-15}({\rm stat})^{+105}_{-14}({\rm syst})\ {\rm MeV}\ , \tag{5}$$

and the result at $\beta=6.0$ on all 36 configurations is

$$f_B^{\rm stat} = 286^{+8}_{-10}{}^{+67}_{-42}\ {\rm MeV}\ . \tag{6}$$

Finally, Sec. V contains our conclusions.

A. Improved action and operators

The SW action is

$$S_F^{\rm SW} = S_F^W - i\frac{\kappa}{2}\sum_{x,\mu,\nu}\bar q(x)F_{\mu\nu}(x)\sigma_{\mu\nu}q(x)\ , \tag{7}$$

where S_F^W is the Wilson action:

$$S_F^W = \sum_x \Big\{ \bar q(x)q(x) - \kappa\sum_\mu [\bar q(x)(1-\gamma_\mu)U_\mu(x)q(x+\hat\mu) + \bar q(x+\hat\mu)(1+\gamma_\mu)U_\mu^\dagger(x)q(x)] \Big\}\ . \tag{8}$$

The decay constants of heavy-light pseudoscalar and vector mesons are computed using lattice axial-vector and vector currents as interpolating operators. In order to obtain $O(a)$-improved matrix elements we use "rotated" operators [14]:

$$\bar Q(x)(1+\tfrac{1}{2}\gamma\cdot\overleftarrow{D})\Gamma(1-\tfrac{1}{2}\gamma\cdot\overrightarrow{D})q(x)\ , \tag{9}$$

where Γ is one of the Dirac matrices (either $\gamma^\mu\gamma^5$ or γ^μ), and Q and q represent the fields of the heavy and light quark, respectively.

In the static effective theory, in which the heavy propagator is expressed in terms of the link variables [16], it is sufficient to rotate the light-quark fields only [17], i.e., to use the operators

$$\bar Q(x)\Gamma(1-\tfrac{1}{2}\gamma\cdot\overrightarrow{D})q(x)\ , \tag{10}$$

in order to eliminate the $O(a)$-discretization errors.

B. Renormalization constants Z_V and Z_A

In order to determine the physical values of the decay constants from those obtained in lattice simulations using the interpolating operators in Eq. (9), it is necessary to know the corresponding renormalization constants. These are defined by requiring that $Z_A A_\mu^{\rm latt}$ and $Z_V V_\mu^{\rm latt}$ are the correctly normalized currents, where the superscript "latt" denotes that the operator is a lattice current. These renormalization constants have been calculated at one-loop order in perturbation theory for the SW action with rotated operators [18]:

$$Z_V = 1-0.10g^2\ , \tag{11}$$

$$Z_A = 1-0.02g^2\ . \tag{12}$$

In this paper they are evaluated using the "boosted" coupling suggested in Ref. [19]; specifically, we use $g^2=6/(\beta u_0^4)$, where u_0 is a measure of the average link variable, for which we take $u_0=(8\kappa_{\rm crit})^{-1}$. It has been suggested [19,20] that the use of such an effective coupling, rather than the bare lattice coupling, resums some of the large higher-order corrections and, in particular, some of the tadpole diagrams. Using the measured values of $\kappa_{\rm crit}$ from our simulations, we obtain $Z_A\simeq0.97$ (0.96) and $Z_V\simeq0.83$ (0.82) for the simulation at $\beta=6.2$ (6.0).

In a recent nonperturbative determination of these renormalization constants, obtained by requiring that the correctly normalized currents obey the continuum Ward Identities, it was found that $Z_V=0.824(2)$ and $Z_A=1.09(3)$ [21]. These results were obtained from a simulation at $\beta=6.0$ for one value of the quark mass. It remains to be checked that the results are independent of the quark mass and insensitive to small variations in β. For this reason, we use the perturbative values, given above, throughout the paper. We note, however, that the nonperturbative value of Z_A may be larger by about 15%. In Ref. [15] we obtained $f_\pi/m_\rho=0.138^{+6}_{-9}$, using the perturbative value of Z_A. A larger value of Z_A, such as $Z_A=1.09$, would bring this result closer to the physical value of 0.172. However, we also observed that f_K/f_π, which does not require Z_A, was in very good agreement with the experimental value and, therefore, we quote values for the ratios f_D/f_π and f_B/f_π in the following sections. The normalization of the axial-vector current in the static effective theory is discussed in Sec. IV.

C. Error estimation

Statistical errors are obtained from a bootstrap procedure [22]. This involves the creation of 1000 bootstrap samples from the original set of N configurations by randomly selecting N configurations per sample (with replacement). Correlators are fitted for each bootstrap sample by minimizing χ^2. During the fits, correlations among different time slices are taken into account, whereas correlations among different values of the quark mass are neglected. The latter correlations are preserved by using the same sequence of bootstrap samples at each quark mass. When extrapolating our results to the chiral limit and physical meson masses, the correlation matrix for the fitted quantities is estimated from the full bootstrap ensemble. All quoted statistical errors are obtained from the central 68% of the corresponding bootstrap distributions [23].

We attempt to quantify the systematic error arising from the uncertainty in the value of the lattice spacing, a, determined from properties of light hadrons, and from the string tension [15]. The differences between results obtained using our central value for a^{-1} (GeV) and our upper and lower estimates are quoted as systematic uncertainties in the final estimates for decay constants in physical units. Hereafter, where we quote two errors, the first is statistical and the second is systematic.

D. Extended interpolating operators

In order to isolate the ground state in correlation functions effectively, it is useful to use extended (or "smeared") interpolating operators for the mesons. In particular, in the static theory it has been found to be essential to use smeared operators in order to obtain any signal for the ground state [24]. In this study we use gauge-invariant Jacobi smearing on the heavy-quark field (described in detail in Ref. [25]), in which the smeared field, $Q^S(\mathbf{x},t)$ is defined by

$$Q^S(\mathbf{x},t)\equiv\sum_{\mathbf{x}'}J(x,x')Q(\mathbf{x}',t)\ , \tag{13}$$

where

$$J(x,x')=\sum_{n=0}^{N}\kappa_S^n\Delta^n(x,x') \tag{14}$$

and

$$\Delta(x,x')=\sum_{i=1}^{3}[\delta_{\mathbf{x}',\mathbf{x}-\hat{i}}U_i^\dagger(\mathbf{x}-\hat{i},t)+\delta_{\mathbf{x}',\mathbf{x}+\hat{i}}U_i(\mathbf{x},t)]\ . \tag{15}$$

Wuppertal smearing [26], which uses the operator $(1-\kappa_S\Delta)^{-1}$ as the kernel of the smearing, corresponds to $N=\infty$, provided that κ_S is sufficiently small to guarantee convergence. Following the discussion in Ref. [25], we choose $\kappa_S=0.25$ and use the parameter N to control the smearing radius, defined by

$$r^2\equiv\frac{\sum_{\mathbf{x}}|\mathbf{x}|^2|J(x,0)|^2}{\sum_{\mathbf{x}}|J(x,0)|^2}\ . \tag{16}$$

The values of N and r used in each of the calculations below will be quoted in the corresponding sections.

II. DECAY CONSTANTS FROM THE SIMULATION AT $\beta=6.2$

In this section we present the results obtained for the decay constants of heavy-light mesons from our simulation on 60 configurations of a $24^3\times48$ lattice at $\beta=6.2$, using the SW action in the quenched approximation. The computations are performed for four different values of the mass of the heavy quark, corresponding to $\kappa_h=0.121$, 0.125, 0.129, and 0.133, and for three values of the mass of the light quark, corresponding to $\kappa_l=0.141\,44$, 0.142 26, and 0.142 62. The mass of the charm quark corresponds approximately to $\kappa_h=0.129$. The value of the hopping parameter corresponding to the mass of the strange quark is $\kappa_s=0.1419^{+1}_{-1}$ and the critical value is $\kappa_{\rm crit}=0.143\,15^{+2}_{-2}$ [15].

The decay constants are determined by computing two-point correlation functions of the form

$$C^{QR}_{J_1J_2}(t)=\sum_{\mathbf{x}}\langle 0|J_1^Q(x)J_2^{\dagger R}(0)|0\rangle\ , \tag{17}$$

where J_1 and $J_2^\dagger$ are interpolating operators which can annihilate or create the pseudoscalar or vector meson being studied. The labels Q and R denote whether a local (L) or smeared (S) interpolating operator is being used. In this simulation we use Jacobi smearing with $N=75$, corresponding to a smearing radius of $r=5.2$. The decay constants are obtained from the matrix elements of the local operators, which are determined by computing both the C^{SS} and C^{LS} correlation functions.

In order to determine the decay constant, it is necessary to know the value of the lattice spacing in physical units. This can be done by relating the lattice measurements of some dimensionful quantity to its physical value, e.g., the mass of a light hadron or f_π. Among the other frequently used choices are the string tension $\sqrt{K}$ and the $1P-1S$ mass splitting in charmonium. Using m_ρ to set the scale in our study of light hadrons [15] we found $a^{-1}(m_\rho)=2.7(1)$ GeV, and a mass spectrum in physical units which was close to experimental values. Furthermore, our determination of the string tension [23] gave $a^{-1}=2.73(5)$ GeV. Encouraged by the consistency of these results, we use

$$a^{-1}=2.7\ \text{GeV}\ . \tag{18}$$

However, the study described in Ref. [15] showed that the measurement of the pion decay constant gave a higher value for the scale, i.e, $a^{-1}(f_\pi)=3.4^{+2}_{-1}$ GeV, using the perturbative value for Z_A. In order to get an estimate of the systematic uncertainties in the final numbers, we evaluate all our result using the central value of $a^{-1}(f_\pi)$ as well, and quote the difference as the upper systematic error on decay constants. The lower systematic error is obtained from the uncertainty of -0.1 GeV in $a^{-1}(m_\rho)$.

In an attempt to reduce the systematic errors associated with the value of the renormalization constant of the axial-vector current, we also compute $f_{D,B}/f_\pi$, and determine $f_{D,B}$ by using the physical value of f_π.

A. Decay constants of pseudoscalar mesons

In order to determine the pseudoscalar decay constants, we start by fitting the two-point correlation function

$$C_{PP}^{SS}(t) \equiv \sum_{\mathbf{x}} \langle 0 | P^S(\mathbf{x},t) P^{\dagger S}(0) | 0 \rangle \to \frac{Z_{PS}^2}{2M_P} \exp(-M_P L_t/2) \cosh[M_P(L_t/2-t)] \,, \tag{19}$$

where P is the pseudoscalar density, $Z_{PS} = \langle 0 | P^S(0) | P \rangle$, and L_t is the temporal extent of the lattice. This correlation function gives the best determination of the masses of the heavy-light pseudoscalars. Symmetrizing in Euclidean time, the fitting range was chosen to be $13 \le t \le 22$ for all three values of the light-quark mass. Good plateaus in the effective mass were observed and stable fits obtained. The values of the masses of the pseudoscalar and vector mesons for the twelve κ_h-κ_l combinations are presented in Table I. The values obtained by linear extrapolation to the chiral limit for the light quark are also tabulated. At large values of t, the ratio of correlation functions,

$$\frac{C_{AP}^{LS}(t)}{C_{PP}^{SS}(t)} \to \frac{\langle 0 | A_4^L(0) | P \rangle}{\langle 0 | P^S(0) | P \rangle} \tanh[M_P(L_t/2-t)] \,, \tag{20}$$

is used to extract the pseudoscalar decay constant, where A_4 is the temporal component of the axial-vector current. The ratio is fitted in the range $15 \le t \le 22$ with the pseudoscalar mass M_P (in each bootstrap sample) constrained to its value extracted from fits to Eq. (19). In Fig. 1 we plot the ratio of correlators together with the fit to Eq. (20) as a function of t. Using the value of Z_{PS} obtained from the fits of Eq. (19), the matrix elements of the local axial-vector current are obtained. Although there are other ways of determining these matrix elements, we find that the ratios in Eq. (20) give the most precise determination.

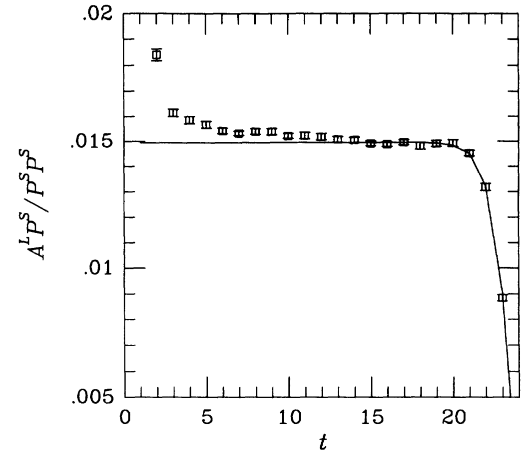

FIG. 1. The ratio of correlators defined in Eq. (20) plotted versus t for $\kappa_h = 0.129$ and $\kappa_l = 0.142\,62$. The curve represents the fit using time slices 15–22.

In Table II we present the results for the decay constants (in lattice units) of the pseudoscalar mesons for the

TABLE I. Masses (in lattice units) of the pseudoscalar and vector mesons for the twelve κ_h-κ_l combinations at $\beta = 6.2$ on a $24^3 \times 48$ lattice. Also presented are the values obtained by linear extrapolation to the chiral limit ($\kappa_l \to \kappa_{crit} = 0.143\,15$).

κ_h	κ_l	M_P	M_V
0.121	0.141 44	0.924^{+3}_{-1}	0.944^{+4}_{-2}
	0.142 26	0.900^{+3}_{-2}	0.920^{+4}_{-3}
	0.142 62	0.890^{+4}_{-3}	0.909^{+5}_{-4}
	κ_{crit}	0.875^{+4}_{-3}	0.894^{+6}_{-4}
0.125	0.141 44	0.822^{+3}_{-1}	0.847^{+4}_{-2}
	0.142 26	0.799^{+3}_{-2}	0.823^{+4}_{-3}
	0.142 62	0.789^{+4}_{-2}	0.811^{+5}_{-4}
	κ_{crit}	0.773^{+5}_{-2}	0.797^{+6}_{-4}
0.129	0.141 44	0.715^{+3}_{-1}	0.745^{+4}_{-2}
	0.142 26	0.691^{+3}_{-2}	0.721^{+4}_{-3}
	0.142 62	0.681^{+4}_{-2}	0.711^{+5}_{-4}
	κ_{crit}	0.665^{+5}_{-2}	0.695^{+6}_{-4}
0.133	0.141 44	0.599^{+3}_{-1}	0.637^{+3}_{-2}
	0.142 26	0.574^{+3}_{-2}	0.613^{+4}_{-3}
	0.142 62	0.564^{+4}_{-2}	0.603^{+5}_{-4}
	κ_{crit}	0.546^{+4}_{-2}	0.588^{+6}_{-4}

TABLE II. The decay constants (in lattice units) of the pseudoscalar and vector mesons. Also shown are the results for the combination $f_P\sqrt{M_p}$ which in the heavy-quark limit is independent of the heavy-quark mass (up to mild logarithmic corrections).

κ_h	κ_l	f_P/Z_A	$f_P\sqrt{M_P}/Z_A$	$1/(f_V Z_V)$
0.121	0.141 44	0.086^{+1}_{-1}	0.083^{+2}_{-1}	0.124^{+2}_{-3}
	0.142 26	0.079^{+1}_{-1}	0.075^{+2}_{-1}	0.116^{+3}_{-3}
	0.142 62	0.076^{+2}_{-2}	0.071^{+2}_{-2}	0.111^{+3}_{-3}
	κ_{crit}	0.071^{+2}_{-1}	0.066^{+2}_{-1}	0.105^{+4}_{-4}
0.125	0.141 44	0.086^{+1}_{-1}	0.078^{+1}_{-1}	0.141^{+3}_{-3}
	0.142 26	0.079^{+1}_{-1}	0.070^{+1}_{-1}	0.132^{+3}_{-3}
	0.142 62	0.076^{+2}_{-2}	0.067^{+2}_{-2}	0.128^{+4}_{-4}
	κ_{crit}	0.071^{+2}_{-1}	0.062^{+2}_{-1}	0.123^{+4}_{-4}
0.129	0.141 44	0.085^{+1}_{-1}	0.071^{+1}_{-1}	0.163^{+3}_{-3}
	0.142 26	0.078^{+1}_{-1}	0.065^{+1}_{-1}	0.155^{+3}_{-3}
	0.142 62	0.075^{+2}_{-1}	0.062^{+1}_{-1}	0.151^{+4}_{-4}
	κ_{crit}	0.071^{+2}_{-1}	0.057^{+2}_{-1}	0.146^{+5}_{-5}
0.133	0.141 44	0.082^{+1}_{-1}	0.063^{+1}_{-1}	0.193^{+3}_{-4}
	0.142 26	0.076^{+1}_{-1}	0.057^{+1}_{-1}	0.186^{+3}_{-4}
	0.142 62	0.073^{+1}_{-1}	0.055^{+1}_{-1}	0.183^{+5}_{-5}
	κ_{crit}	0.069^{+1}_{-1}	0.051^{+1}_{-1}	0.179^{+5}_{-5}

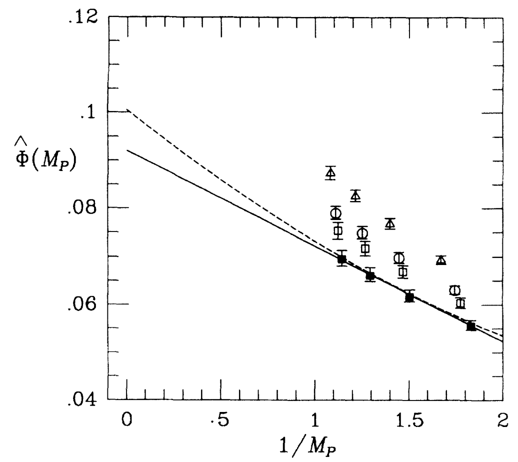

FIG. 2. The data for $\hat{\Phi}(M_P)$ plotted against the inverse meson mass. The open symbols denote points with $\kappa_l < \kappa_{\rm crit}$, whereas full symbols denote those extrapolated to $\kappa_{\rm crit}$. The solid line represents the linear fit to the chirally extrapolated points using the three heaviest meson masses, whereas the dashed curve results from a quadratic fit to all four.

twelve κ_h-κ_l combinations, as well as the values obtained by linearly extrapolating the results to the chiral limit. We also tabulate the results for the quantity $f_P\sqrt{M_P}$, which, in the heavy-quark limit, is independent of the mass of the heavy quark (except for a mild logarithmic dependence).

We start the discussion of our results with the behavior of the pseudoscalar decay constants as a function of the mass of the meson, with all dimensionful quantities given in lattice units. In the heavy-quark limit, the quantity $f_P\sqrt{M_P}$ scales like

$$f_P\sqrt{M_P}=\text{const}\times[\alpha_s(M_P)]^{-2/\beta_0}\ , \quad M_P\to\infty\ . \tag{21}$$

In order to detect possible deviations from this scaling law we plot in Fig. 2 the quantity[1]

$$\hat{\Phi}(M_P)\equiv[\alpha_s(M_P)/\alpha_s(M_B)]^{2/\beta_0}Z_A^{-1}f_P\sqrt{M_P} \tag{22}$$

as a function of $1/M_P$. We approximate $\alpha_s(M)$ by

$$\alpha_s(M)=\frac{2\pi}{\beta_0\ln(M/\Lambda_{\rm QCD})}\ , \tag{23}$$

where we take $\Lambda_{\rm QCD}=200$ MeV, and $\beta_0=11-\frac{2}{3}n_f$, with $n_f=0$ in the quenched approximation. From the figure we see that $\hat{\Phi}(M_P)$ increases as the mass of the heavy quark is increased (in agreement with the behavior found using the Wilson action for the quarks [2–4]). In order to quantify this behavior, we fit $\hat{\Phi}(M_P)$ to either a linear or quadratic function of $1/M_P$:

$$\hat{\Phi}(M_P)=A\left[1-\frac{B}{M_P}\right] \tag{24}$$

or

$$\hat{\Phi}(M_P)=C\left[1-\frac{D}{M_P}+\frac{E}{M_P^2}\right]\ . \tag{25}$$

We have performed these fits twice, once using the values of $f_P\sqrt{M_P}$ for all four values of κ_h, and once using those for only the smallest three κ_h's (i.e., for the heaviest three heavy-quark masses). The results of the fits are given in Table III. We find that the nonscaling corrections are $\sim 30\%$ for f_D and $\sim 10\%$ for f_B, in agreement with previous results obtained using Wilson fermions [2–4]. From the quadratic fit to the data at all four heavy-meson masses we find, in physical units,

$$\begin{aligned} Ca^{-3/2}&=0.45^{+2}_{-2}{}^{+19}_{-3}\ \text{GeV}^{3/2}\ ,\\ Da^{-1}&=0.84^{+11}_{-8}{}^{+22}_{-3}\ \text{GeV}\ ,\\ Ea^{-2}&=0.28^{+7}_{-9}{}^{+16}_{-2}\ \text{GeV}^2\ . \end{aligned} \tag{26}$$

The second error in Eq. (26) corresponds solely to the uncertainty in the scale. It should be mentioned that ignoring the residual logarithmic dependence of $f_P\sqrt{M_P}$ on M_P makes the slope more pronounced. However, it is clear from Fig. 2 and Table III that the logarithmic corrections to the scaling law can by no means account for the observed slope in $f_P\sqrt{M_p}$.

We use the parameters of the fits in Table III to make our predictions for the values of the decay constants f_D and f_B. The results corresponding to the four fits are presented in Table IV. From this table it is clear that there is a further systematic uncertainty in f_B of about 11 MeV from extrapolating using either linear or quadratic fits. In contrast with this, since we interpolate to m_D, the results for f_D are very stable. It should be emphasized that choosing a different value for $\Lambda_{\rm QCD}$ (e.g., $\Lambda_{\rm QCD}=250$ MeV), or for the anomalous dimension (e.g., by taking $n_f=4$) changes the results by only about 1 MeV.

Taking the results from the quadratic fit using all four κ_h values we find

$$f_D=185^{+4}_{-3}{}^{+42}_{-7}\ \text{MeV}\ , \tag{27}$$

$$f_B=160^{+6}_{-6}{}^{+53}_{-19}\ \text{MeV}\ , \tag{28}$$

where we have included the uncertainty of 11 MeV from the extrapolations in the systematic error quoted for f_B. We take the results presented in Eqs. (27) and (28) as our best estimates of the decay constants of the D and B mesons.

In Ref. [3] it was found useful to use the pion decay constant f_π to set the scale in the computations of the decay constants of heavy-light mesons. By calculating f_D/f_π and f_B/f_π it may be expected that some of the systematic errors cancel, since, in particular, the ratios are independent of Z_A. Our results for the decay constants obtained in this way are, as expected, close to the upper systematic error margins in Table IV. We find

$$(f_D/f_\pi)\times 132\ \text{MeV}=232^{+12}_{-5}\ \text{MeV}\ , \tag{29}$$

$$(f_B/f_\pi)\times 132\ \text{MeV}=201^{+12}_{-8}\ \text{MeV}\ . \tag{30}$$

Finally, in this subsection, we present our results for

[1]The normalization factor $\alpha_s(M_B)^{-2/\beta_0}$ is convenient when comparing these results with those obtained in the static theory.

TABLE III. Values of the parameters of the linear and quadratic fits to the behavior of the pseudoscalar decay constants with the mass of the mesons (as defined in the text).

	Linear fit		Quadratic fit		
	A	B	C	D	E
4 κ_h's	0.089^{+3}_{-2}	0.199^{+5}_{-7}	0.101^{+5}_{-5}	0.31^{+4}_{-3}	0.038^{+9}_{-13}
3 κ_h's	0.092^{+3}_{-3}	0.216^{+9}_{-10}	0.102^{+6}_{-8}	0.33^{+10}_{-5}	0.045^{+18}_{-39}

f_{D_s} and f_{B_s}. These are obtained by interpolating the measured values of the decay constants given in Table II to $\kappa_l=\kappa_s=0.1419^{+1}_{-1}$ [15]. The extrapolations in the heavy-quark masses are done as above. We find

$$f_{D_s}/f_D=1.18^{+2}_{-2}\,, \tag{31}$$

$$f_{B_s}/f_B=1.22^{+4}_{-3}\,. \tag{32}$$

In physical units we obtain

$$f_{D_s}=212^{+4+46}_{-4-7}\ \text{MeV}\,, \tag{33}$$

$$f_{B_s}=194^{+6+62}_{-5-9}\ \text{MeV}\,. \tag{34}$$

Recently, the first measurement of f_{D_s} has been made by the WA75 Collaboration [27], who found

$$f_{D_s}=(232\pm45\pm20\pm48)\ \text{MeV}\,.$$

Our result is in good agreement with the measured value, and also with previous lattice calculations using Wilson fermions [3,5].

B. Decay constants of vector mesons

In this subsection we present our results for the decay constants of heavy-light vector mesons. These are defined by

$$\langle 0|V_\mu|V\rangle\equiv\epsilon_\mu\frac{M_V^2}{f_V}=Z_V\langle 0|V_\mu^L(0)|V\rangle\,, \tag{35}$$

where $|V\rangle$ represents a state containing a vector meson V, with mass M_V, polarization vector ϵ_μ, and decay constant f_V. V_μ^L denotes the local lattice vector current, defined in Eq. (9), with $\Gamma=\gamma_\mu$, which has to be multiplied by the renormalization constant Z_V. The vector mass M_V is extracted from fits to the correlator

$$C_{VV}^{SS}\equiv\sum_{j=1}^{3}\sum_{\mathbf{x}}\langle 0|V_j^S(\mathbf{x},t)V_j^S(0)|0\rangle$$
$$\to-\frac{3Z_{V^S}^2}{2M_V}\exp(-M_VL_t/2)\cosh[M_V(L_t/2-t)]\,, \tag{36}$$

where V_j^S is the jth spatial component of the smeared vector operator and Z_{V^S} is defined through

$$\langle 0|V_j^S(0)|V\rangle=\epsilon_jZ_{V^S}\,. \tag{37}$$

Fitting time slices $14\le t\le 23$, symmetrized, we obtain the vector meson masses shown in Table I. In order to extract the matrix element of the local vector current we fit the ratio

$$\frac{C_{VV}^{LS}(t)}{C_{VV}^{SS}(t)}\to-\frac{\sum_{j=1}^{3}\langle 0|V_j^L(0)|V\rangle\epsilon_j^*}{3Z_{V^S}} \tag{38}$$

to a constant in the fitting interval $15\le t\le 23$.

The results for $1/f_V$ for the twelve κ_h-κ_l combinations are presented in Table II, together with those obtained after extrapolation to the chiral limit. The chirally extrapolated values for $f_V^{-1}Z_V^{-1}$ are now interpolated to the D^* mass using a quadratic fit to the data at all four values of κ_h, giving

$$f_{D^*}^{-1}=0.110^{+5+36}_{-5-5}\,. \tag{39}$$

This is slightly below, but still compatible with, earlier studies (e.g., Ref. [3]) when the systematic error is taken into account. This result remains unaltered if a linear fit is used instead of a quadratic one.

C. A test of the heavy-quark symmetry

In the heavy-quark limit, the decay constants of heavy-light pseudoscalar and vector mesons are related by [28]

$$U(M)\equiv\frac{f_Vf_P}{M}=\left[1+\frac{8}{3}\frac{\alpha_s(M)}{4\pi}+O\left[\frac{1}{M}\right]\right]\,, \tag{40}$$

where we take the heavy mass scale M to be the spin-averaged meson mass, $M=(M_P+3M_V)/4$.

In order to test the predicted behavior of $U(M)$, we take the chirally extrapolated values for both the pseudoscalar and vector decay constants, and fit

$$\tilde{U}(M)\equiv U(M)\Big/\left[1+\frac{8}{3}\frac{\alpha_s(M)}{4\pi}\right] \tag{41}$$

TABLE IV. Values of the decay constants f_B and f_D in MeV, corresponding to the linear and quadratic fits.

	Linear fit		Quadratic fit	
	f_D	f_B	f_D	f_B
4 κ_h's	185^{+4+45}_{-3-7}	149^{+5+52}_{-3-7}	185^{+4+42}_{-3-7}	160^{+6+53}_{-6-8}
3 κ_h's	186^{+4+41}_{-3-7}	154^{+5+53}_{-4-8}	185^{+4+42}_{-3-7}	160^{+7+54}_{-7-7}

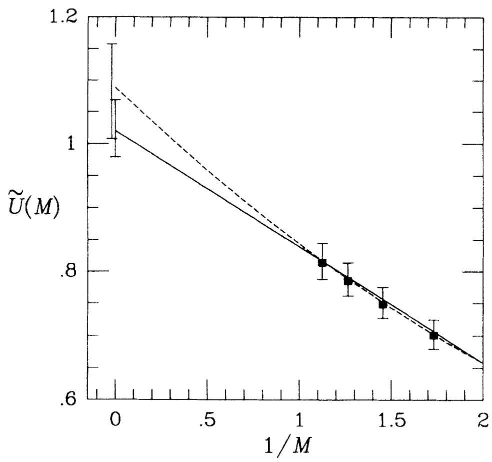

FIG. 3. The quantity $\tilde{U}(M)$ plotted against the inverse spin-averaged mass. Linear and quadratic fits are represented by the solid and dashed curves, respectively. Also shown are the statistical errors of the extrapolation to the infinite mass limit.

to either a linear or quadratic function of $1/M$. The data together with the fits are shown in Fig. 3, and we display our results in Table V. The perturbative values of Z_A and Z_V are used.

The fact that $\tilde{U}(\infty)$ is around one in Table V provides support for our parametrizations, in Eqs. (24) and (25), of the nonscaling behavior of the decay constants for finite heavy-quark masses.

III. DECAY CONSTANTS FROM THE SIMULATION AT $\beta=6.0$

In this section we describe the results of a computation of the decay constants using the SW fermion action at $\beta=6.0$ on a $16^3\times48$ lattice. These results were obtained using 36 configurations, with light-quark masses corresponding to $\kappa_l=0.1432$, 0.1440, and 0.1445. The corresponding light-light pseudoscalar and vector meson masses, and pseudoscalar decay constants, all in lattice units, are presented in Table VI. The values of the hopping parameter corresponding to the chiral limit and the strange quark mass are $\kappa_{\rm crit}=0.145\,56^{+6}_{-6}$ and $\kappa_s=0.1437^{+4}_{-5}$, respectively. Using the mass of the ρ meson to determine the value of the lattice spacing, we find $a^{-1}=2.0^{+3}_{-2}$ GeV, while using f_π we find $a^{-1}=2.1^{+2}_{-1}$ GeV. These two results are compatible, and below we will use the value

$$a^{-1}=2.0^{+3}_{-2}\ \text{GeV} \tag{42}$$

to convert the results from lattice to physical units.

TABLE V. The quantity $\tilde{U}(M)$ obtained from linear and quadratic fits.

M	Linear fit	Quadratic fit
∞	1.02^{+5}_{-4}	1.09^{+7}_{-8}
$(M_B+3M_B^*)/4$	0.93^{+4}_{-3}	0.96^{+4}_{-5}
$(M_D+3M_D^*)/4$	0.77^{+2}_{-2}	0.77^{+2}_{-2}

TABLE VI. Masses of light-light pseudoscalar and vector mesons, and the pseudoscalar decay constants at $\beta=6.0$.

κ_l	m_π	m_ρ	f_π/Z_A
0.1432	0.386^{+4}_{-4}	0.51^{+2}_{-1}	0.088^{+2}_{-3}
0.1440	0.311^{+6}_{-5}	0.47^{+3}_{-2}	0.080^{+2}_{-4}
0.1445	0.257^{+5}_{-6}	0.43^{+6}_{-3}	0.075^{+2}_{-5}
$\kappa_{\rm crit}=0.145\,56^{+6}_{-6}$		0.38^{+5}_{-4}	0.065^{+2}_{-6}

We have computed the heavy-light correlation functions as series in κ_h (the hopping-parameter expansion [29]), thus enabling us to obtain the decay constants at any value of the mass of the heavy quark, without explicitly computing the heavy-quark propagators. The decay constants are obtained by fitting to Eqs. (19) and (20), over the range $12\le t\le 18$ for both fits. We employ the Jacobi smearing algorithm with $N=50$, corresponding to a smearing radius of $r=4.2$.

In an attempt to improve our understanding of the discretization errors, we have also computed the decay constants for the Wilson action at one value of the light-quark mass, using a subset of 16 of the 36 configurations. The comparison of the results for the two actions is presented in Sec. III B.

A. Pseudoscalar decay constants

In Fig. 4 we plot the chirally extrapolated values of $\hat{\Phi}(M_P)$ as a function of $1/M_P$ for 11 values of the heavy-quark mass. We fit the points corresponding to the five lightest meson masses [for which $m_Q=1/2(1/\kappa_h-1/\kappa_{\rm crit})<0.7$, as was the case at $\beta=6.2$] to Eq. (25), and this is shown as the solid curve in the figure. For the coefficients of the fit we find

$$C=0.18^{+3}_{-3}\,,\quad D=0.45^{+13}_{-5}\,,\quad E=0.08^{+4}_{-9}\,. \tag{43}$$

In physical units we obtain

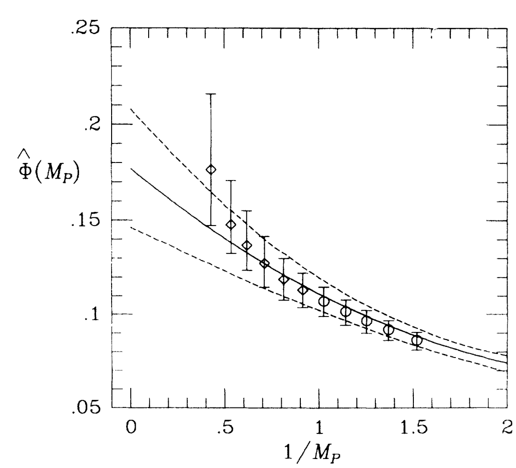

FIG. 4. The chirally extrapolated data for $\hat{\Phi}(M_P)$ at $\beta=6.0$ plotted against the inverse meson mass. The solid curve represents a quadratic fit to the points denoted by circles. Points represented by diamonds are not included in the fit. The dashed curves are the 68% confidence bounds on the fit.

$$Ca^{-3/2}=0.50^{+9+12}_{-9-7}\ \mathrm{GeV}^{3/2}\ ,$$
$$Da^{-1}=0.91^{+21+14}_{-26-9}\ \mathrm{GeV}\ , \quad (44)$$
$$Ea^{-2}=0.32^{+16+10}_{-36-6}\ \mathrm{GeV}^2$$

in good agreement with the results at $\beta=6.2$, quoted in Eq. (26). It should be noted that the inclusion of all 11 data points makes no significant difference to the fit.

The values for the decay constants in physical units are

$$f_D=199^{+14+27}_{-15-19}\ \mathrm{MeV}\ , \quad (45)$$
$$f_B=176^{+25+33}_{-24-15}\ \mathrm{MeV}\ , \quad (46)$$
$$f_{D_s}/f_D=1.13^{+6}_{-7}\ , \quad (47)$$
$$f_{B_s}/f_B=1.17^{+12}_{-12}\ . \quad (48)$$

All these numbers are in good agreement with the corresponding results from the simulation at $\beta=6.2$. Finally, for the ratios f_D/f_π and f_B/f_π we obtain

$$(f_D/f_\pi)\times 132\ \mathrm{MeV}=211^{+26}_{-11}\ \mathrm{MeV}\ , \quad (49)$$
$$(f_B/f_\pi)\times 132\ \mathrm{MeV}=186^{+35}_{-21}\ \mathrm{MeV}\ . \quad (50)$$

B. Comparison of results using Wilson and SW actions

For 16 of the configurations, we have computed the decay constants for the Wilson fermion action, again using the hopping-parameter expansion. We compute light-quark propagators at a single value of the hopping parameter, $\kappa_l^W=0.155$, corresponding to a pseudoscalar-meson mass of 0.30^{+1}_{-1}. This was chosen to match the SW pseudoscalar-meson mass of 0.31^{+2}_{-1} obtained at $\kappa_l^{SW}=0.144$ on the same set of configurations.

In Fig. 5 we plot $f_P\sqrt{M_P}$ as a function of $1/M_P$. Using the conventional normalization of $\sqrt{2\kappa}$ for the quark

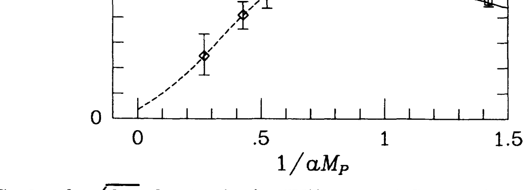

FIG. 5. $f_P\sqrt{M_P}$ for both the Wilson and SW actions. Diamonds denote points obtained with the Wilson action in the conventional normalization, $\sqrt{2\kappa}$, whereas squares denote points normalized by $\sqrt{1-6\tilde{\kappa}}$. Results using the SW action are represented by circles. The solid curves are quadratic fits in $1/M_P$ to $f_P\sqrt{M_P}$ for the Wilson action, with fields normalized by $\sqrt{1-6\tilde{\kappa}}$, and for the SW action. The dashed curve is to guide the eye. Note that the renormalization factor, Z_A, is not included in the plot.

fields, we see a clear divergence between the results for the two actions for $m_Q>0.7$; the Wilson results turn over and decrease. However, we note that uncorrelated χ^2 fits of the Wilson points, at the lightest few meson masses, to Eqs. (24) and (25) would yield coefficients of the $1/M_P$ term broadly consistent with previous Wilson analyses.

The figure also shows the results obtained with the Wilson action, but using the normalization $\sqrt{1-6\tilde{\kappa}}$ for the quark fields [5,19,30,31], where $\tilde{\kappa}=u_0\kappa$ and $u_0=1/(8\kappa_{\mathrm{crit}})$. It has been suggested that this normalization may absorb some of the discretization errors [19,31], and indeed the corresponding results agree remarkably with those obtained using the SW action. This agreement provides considerable motivation for a theoretical study to investigate whether there is any formal connection between the ansatz above and the improvement program initiated by Symanzik [32].

IV. f_B IN THE STATIC LIMIT

An alternative and complementary approach to heavy-quark physics using lattice QCD was proposed by Eichten [16]. This technique is based on an expansion of the heavy-quark propagator in inverse powers of the quark mass. In practice, one keeps just the leading term, given by (at zero velocity)

$$S_Q(\mathbf{x},t;\mathbf{0},0)=\left[\theta(t)e^{-m_Qt}\frac{1+\gamma^4}{2}+\theta(-t)e^{m_Qt}\frac{1-\gamma^4}{2}\right]\times\delta^{(3)}(\mathbf{x})\mathcal{P}_0(t,0)\ , \quad (51)$$

where $\mathcal{P}_0(t,0)$ is the product of links from $(\mathbf{0},t)$ to the origin, for example, for $t>0$:

$$\mathcal{P}_0(t,0)=U_4^\dagger(\mathbf{0},t-1)U_4^\dagger(\mathbf{0},t-2)\cdots U_4^\dagger(\mathbf{0},0)\ . \quad (52)$$

At sufficiently large times

$$\sum_{\mathbf{x}}\langle A_4(\mathbf{x},t)A_4^\dagger(0)\rangle\rightarrow\frac{f_P^2M_P}{2}e^{-M_Pt}\ , \quad (53)$$

where A_μ is the improved axial-vector current of Eq. (10) with $\Gamma=\gamma_\mu\gamma_5$. Since the only dependence on m_Q in Eq. (53) arises through the exponential factor in Eq. (51), we deduce the scaling law that $f_P\sqrt{M_P}$ is independent of the heavy-quark mass. Matching the result from the heavy-quark effective theory with that in the full theory introduces the logarithmic corrections in Eq. (21). The full scaling law is of the form

$$f_P\sqrt{M_P}=\mathrm{const}\times\{[\alpha_s(M_P)]^{-2/\beta_0}[1+O(\alpha_s)]+O(1/M_p)\}\ . \quad (54)$$

The objective of lattice computations is to determine the constant. We refer to the value of f_B obtained using Eq. (54), but dropping the $O(1/M_B)$ corrections, as f_B^{stat}. We compute the two correlation functions, C^{SS} and C^{LS}, defined by[2]

[2]In principle, the behavior of the correlation functions in Eqs. (55) and (56) is given by a cosh [as in Eq. (19)]; however, the contribution of the backward-propagating meson is negligible in the time intervals we will be considering.

$$C^{SS}(t)=\sum_{\mathbf{x}}\langle 0|\,A_4^S(\mathbf{x},t)A_4^{\dagger S}(\mathbf{0},0)|0\rangle \to (Z^S)^2 e^{-\Delta E t}\ , \quad (55)$$

$$C^{LS}(t)=\sum_{\mathbf{x}}\langle 0|\,A_4^L(\mathbf{x},t)A_4^{\dagger S}(\mathbf{0},0)|0\rangle \to Z^L Z^S e^{-\Delta E t}\ , \quad (56)$$

where ΔE is the (unphysical) difference between the mass of the meson and the bare mass of the heavy quark. The matrix element of the local operator A_4^L is obtained from the two correlation functions C^{SS} and C^{LS} as follows. By fitting $C^{SS}(t)$ to the functional form given in Eq. (55) we obtain Z_S (and ΔE). At sufficiently large times the ratio

$$C^{LS}(t)/C^{SS}(t) \to Z^L/Z^S\ ,$$

so that Z^L can be determined.

A. Results at $\beta=6.2$

We now report on a computation of $f_B^{\rm stat}$ at $\beta=6.2$. The results presented here were obtained using a subset of 20 of the 60 configurations discussed in Sec. II, at the three values of the light-quark mass. The values of $f_B^{\rm stat}$ and $f_{B_s}^{\rm stat}$ were determined by extrapolating the results to $\kappa_{\rm crit}$ and κ_s, respectively.

In view of the difficulty in isolating the ground state in correlation functions using the static effective theory, we have compared results obtained with different numbers of iterations of the Jacobi smearing algorithm [34]. For N less than about 80 the plateaus do not start until at least $t=7$. In this paper we present our results obtained with $N=110$ and 140, corresponding to $r=5.9$ and 6.4, respectively, where plateaus begin as early as $t=4$ and hence statistical errors are smaller.

In Fig. 6(a) we show the effective masses obtained from $C^{SS}(t)$, and in Fig. 6(b) the ratio $C^{LS}(t)/C^{SS}(t)$, both at $\kappa_l=0.142\,26$ and $N=140$. Excellent plateaus are obtained, giving us confidence that the ground state has indeed been isolated. In Table VII we present the results for ΔE, $(Z^S)^2$, Z^L/Z^S, and Z^L at all three values of κ_l, from fits over the range $5\le t\le 11$, without symmetrization in Euclidean time. ΔE is obtained from the fit to $C^{SS}(t)$ for $N=140$; consistent values are obtained for $N=110$.

Extrapolating the results for Z^L in Table VII to the chiral limit and to the mass of the strange quark we find

$$Z^L=0.124^{+8}_{-7} \quad \text{at } \kappa_l=\kappa_{\rm crit}\ , \quad (57)$$

$$Z^L=0.140^{+7}_{-6} \quad \text{at } \kappa_l=\kappa_s \quad (58)$$

when obtained using smeared interpolating operators with $N=110$ and

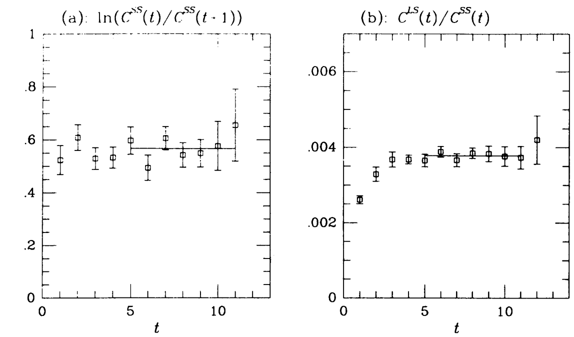

FIG. 6. (a) The effective mass obtained from $C^{SS}(t)$ and (b) the ratio $C^{LS}(t)/C^{SS}(t)$ at $\beta=6.2$, $\kappa_l=0.142\,26$, and $N=140$. The solid lines represent fits from $5\le t\le 11$.

$$Z^L=0.117^{+7}_{-7} \quad \text{at } \kappa_l=\kappa_{\rm crit}\ , \quad (59)$$

$$Z^L=0.134^{+7}_{-6} \quad \text{at } \kappa_l=\kappa_s \quad (60)$$

when using interpolating operators with $N=140$.

When matching the static lattice theory to the full theory at a scale m_b, the factor required is [36]

$$Z_A^{\rm stat}=\mathcal{Z}_A\left[1+\frac{\alpha_s(a^{-1})}{3\pi}\left[\frac{3}{2}\ln a^2 m_b^2-2\right]\right]\ . \quad (61)$$

$\mathcal{Z}_A$, relating the axial-vector current in the static lattice theory to the static continuum one, has been calculated in perturbation theory for the SW action [17,35]:

$$\mathcal{Z}_A=1-0.127g^2\simeq 0.79\ . \quad (62)$$

The value of 0.79 on the right-hand side of Eq. (62) was estimated using the boosted coupling at $\beta=6.2$. For the remaining factor in Eq. (61), we take $m_b=5$ GeV, α_s given by Eq. (23) with $n_f=0$, and $\Lambda_{\rm QCD}=200$ MeV, yielding a number close to one (note that this is insensitive to small changes in m_b). Thus $Z_A^{\rm stat}=0.79$ also. We find

$$f_B^{\rm stat}=266^{+17+110}_{-15-14}\left[\frac{Z_A^{\rm stat}}{0.79}\right]\ \text{MeV}\ , \quad (63)$$

$$f_{B_s}^{\rm stat}=300^{+14+125}_{-13-16}\left[\frac{Z_A^{\rm stat}}{0.79}\right]\ \text{MeV}\ , \quad (64)$$

when using the interpolating operators with $N=110$, and

$$f_B^{\rm stat}=253^{+16+105}_{-15-14}\left[\frac{Z_A^{\rm stat}}{0.79}\right]\ \text{MeV}\ , \quad (65)$$

TABLE VII. Values of ΔE, $(Z^S)^2$, Z^L/Z^S, and Z^L at the three value of κ_l. ΔE is obtained from the fit to $C^{SS}(t)$.

κ_l	ΔE	$(Z^2)^2$		Z^L/Z^S		Z^L	
	$N=140$	$N=110$	$N=140$	$N=110$	$N=140$	$N=110$	$N=140$
0.141 44	0.59^{+1}_{-1}	141^{+12}_{-11}	130^{+10}_{-11}	0.0125^{+3}_{-3}	0.0039^{+1}_{-1}	0.149^{+7}_{-7}	0.142^{+7}_{-6}
0.142 26	0.57^{+1}_{-1}	127^{+12}_{-11}	119^{+10}_{-11}	0.0121^{+3}_{-3}	0.0038^{+1}_{-1}	0.137^{+7}_{-6}	0.130^{+7}_{-6}
0.142 62	0.56^{+2}_{-1}	119^{+12}_{-11}	112^{+9}_{-11}	0.0120^{+3}_{-3}	0.0037^{+1}_{-1}	0.131^{+7}_{-6}	0.125^{+7}_{-7}

$$f_{B_s}^{\rm stat}=287^{+14+119}_{-13-15}\left[\frac{Z_A^{\rm stat}}{0.79}\right]\ {\rm MeV} \tag{66}$$

when using those with $N=140$. The systematic errors quoted arise from the uncertainty in the scale. We take the results in Eqs. (65) and (66) as our best values, and these give for the ratio

$$\frac{f_{B_s}^{\rm stat}}{f_B^{\rm stat}}=1.14^{+4}_{-3}\ . \tag{67}$$

B. Results at $\beta=6.0$

We have performed a similar analysis on the 36 configurations at $\beta=6.0$, using Jacobi smearing with $N=50$ and 150, corresponding to $r=4.2$ and 6.2, respectively. The results obtained using the two smearing radii are consistent, and our best results are those at $N=50$ for which $Z_L=0.211^{+6}_{-7}$, comparable to $Z_L=0.22(1)$ for the SW action obtained in Ref. [10], yielding

$$f_B^{\rm stat}=286^{+8}_{-10}{}^{+67}_{-42}\left[\frac{Z_A^{\rm stat}}{0.78}\right]\ {\rm MeV}\ , \tag{68}$$

$$f_{B_s}^{\rm stat}=323^{+14+75}_{-14-47}\left[\frac{Z_A^{\rm stat}}{0.78}\right]\ {\rm MeV}\ , \tag{69}$$

$$\frac{f_{B_s}^{\rm stat}}{f_B^{\rm stat}}=1.13^{+4}_{-3}\ . \tag{70}$$

These results at $\beta=6.0$ are consistent with those at $\beta=6.2$ presented in Eqs. (65)–(67).

The results plotted in Fig. 6(b) for the ratio $C^{LS}(t)/C^{SS}(t)$ appear to have considerably smaller errors than in some recent studies [8,10], in spite of our limited statistics. We attribute this to the fact that we use the C^{LS} correlation function in which the smearing is performed at the source, rather than the C^{SL} correlation function in which the smearing is performed at the sink.[3] Of course, with sufficiently many configurations, the results are independent of this choice. However, in the C^{LS} correlation function, the heavy-quark propagator is sampled at many spatial points, whereas in the C^{SL} correlation function only the heavy-quark propagator at $\mathbf{x}=0$ contributes. Thus the statistical errors are considerably reduced using the C^{LS} correlation function [7,12].

To check this, we have computed the ratios $C^{LS}(t)/C^{SS}(t)$ and $C^{SL}(t)/C^{SS}(t)$ at $\beta=6.0$, with $\kappa_l=0.1432$. The results are plotted in Fig. 7, and indeed confirm that there is an enormous improvement in precision when the correlation function C^{LS} is used. We believe that this, rather than the different method of smearing, is the reason for the relatively large statistical error in Ref. [10].

[3]In both Refs. [8,10] it was in fact the C^{SL}, and not the C^{LS}, correlation function which was computed [37].

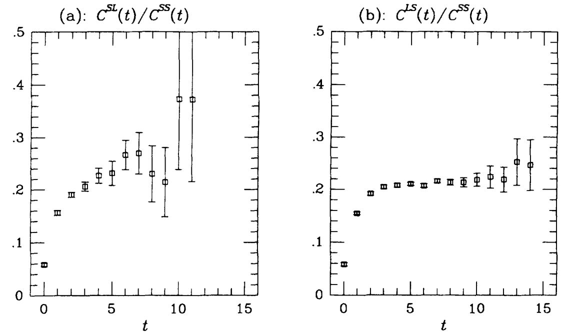

FIG. 7. (a) The ratio $C^{SL}(t)/C^{SS}(t)$ and (b) $C^{LS}(t)/C^{SS}(t)$ plotted against t for $\beta=6.0$, $\kappa_l=0.1432$, and $N=50$ iterations used in the Jacobi smearing algorithm.

C. Discussion

We begin with a comparison of the static and propagating results. Because we do not yet have static results for the full set of configurations at $\beta=6.2$, we focus on a comparison at $\beta=6.0$. In Fig. 8 we plot our results for the scaling quantity

$$f_P\sqrt{M_P}[\alpha_s(M_P)/\alpha_s(M_B)]^{6/33}$$

from the simulation at $\beta=6.0$ as a function of $1/M_P$ (in lattice units), together with our result for $f_B^{\rm stat}\sqrt{M_B}$. The quadratic fit which we used to obtain our estimate for f_B in Sec. III A gives an intercept at $1/M_P=0$ which is about 25% and two standard deviations below the static result; a similar discrepancy is observed at $\beta=6.2$. There are a number of possible reasons for this, e.g., uncertainties in the renormalization constants (which are

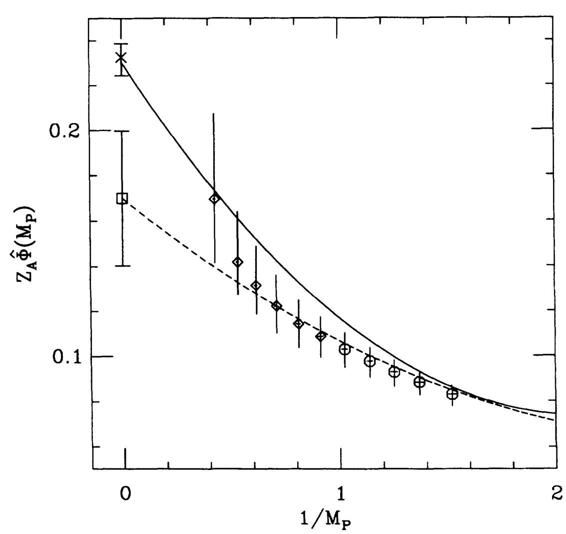

FIG. 8. $Z_A\hat{\Phi}(M_P)$ at $\beta=6.0$ from the simulation with propagating quarks (open symbols) and the static theory (cross). The dashed curve is the fit to the open circles, with the parameters of Eq. (43); the square is the intercept at $1/M_P=0$. The solid curve is the fit with the static point included.

TABLE VIII. Compilation of lattice results for f_B^{stat}.

Ref.	Action	β	a^{-1} (GeV)	Z_A^{stat}	f_B^{stat} (MeV)
[11]	Wilson	5.9	1.75	0.79	319 ± 11
[8]	Wilson	6.0	2.0 ± 0.2	0.8	$310\pm25\pm50$
[9]	Wilson	6.0	2.2 ± 0.2	0.8	$366\pm22\pm55$
[10]	Wilson	6.0	$2.11\pm0.05\pm0.10$	0.8	$350\pm40\pm30$
[10]	SW	6.0	2.05 ± 0.06	0.89	370 ± 40
This work	SW	6.0	2.0^{+3}_{-2}	0.78	$286^{+8}_{-10}{}^{+67}_{-42}$
This work	SW	6.2	2.7^{+7}_{-1}	0.79	$253^{+16}_{-15}{}^{+105}_{-14}$
[5]	Wilson	6.3	$3.21\pm0.09\pm0.17$	0.69	$235\pm20\pm21$
[12]	Wilson	5.74, 6.0, 626	1.12, 1.88, 278	0.71(8)	$230\pm22\pm26$

different for the static and propagating quarks), residual discretization errors in the simulation of the propagating quarks, and uncertainties in the various extrapolations.

A better way of determining the consistency of the static and propagating results is to include the static result in the quadratic fit. Such a fit using the full correlation matrix at $\beta=6.0$ gives a χ^2/N_{DF} of 1.5. This is still acceptable, and provides further evidence that using rotated operators with the SW action gives a sensible normalization for propagating heavy-quark fields. From this fit we obtain

$$f_B=220^{+6}_{-7}{}^{+40}_{-27}\ \text{MeV}\ ,$$

which is 44 MeV higher than that obtained from the propagating points alone.

In Table VIII we present the results for f_B^{stat} obtained by other groups, together with our values. Although at $\beta=6.0$ the values of Z^L found by all groups and for both actions are in broad agreement, the different treatment of systematics leads to the spread of results in f_B^{stat}. It has been suggested that f_B^{stat} decreases as $a\to0$ [12]. However, the agreement of the results obtained with the Wilson and SW actions at $\beta=6.0$, together with consistency between our results at $\beta=6.0$ and 6.2, suggests that the discretization errors are small.

V. CONCLUSIONS

In this paper we have carried out an extensive study of the decay constants of heavy-light mesons using the SW action for the quarks. The use of the SW action confirms the large, negative $O(1/M_P)$ corrections to the scaling law $f_P\sqrt{M_P}\sim\text{const}$ at the mass of the charm quark and the significant corrections at the mass of the b quark, previously observed with the Wilson action. However, from our comparison of results for the Wilson and SW actions at $\beta=6.0$, we observe clear evidence that the $\sqrt{2\kappa}$ normalization of the Wilson quark fields fails for large quark mass. This failure is presumably due to large $O(m_Q a)$ effects. It has been suggested that such effects may largely be absorbed by the use of a different normalization [5]. We find that such a normalization yields results in agreement with those obtained using the SW action with rotated operators and the $\sqrt{2\kappa}$ normalization for the quark fields.

Our best estimates of f_D and f_B are

$$f_D=185^{+4}_{-3}(\text{stat})^{+42}_{-7}(\text{syst})\ \text{MeV}\ , \quad (71)$$

$$f_B=160^{+6}_{-6}{}^{+53}_{-19}\ \text{MeV}\ , \quad (72)$$

obtained using propagating quarks at $\beta=6.2$. Our analysis at $\beta=6.0$ yields entirely consistent results. The latter analysis also suggests that including the static result in the fits is likely to increase the value of f_B by around 40 MeV.

The most urgent extension of this work is to determine the B parameter for B^0-$\bar{B}^0$ mixing, since it is the combination $f_B\sqrt{B_B}$ which is directly relevant for phenomenological studies of the mixing and of *CP* violation. A recent simulation with Wilson fermions found $B_B=1.16\pm0.07$ [3], and it is important to confirm this result with the improved action.

ACKNOWLEDGMENTS

We are grateful to G. Martinelli for many helpful discussions. C.T.S. and A.D.S. acknowledge the support of the Science and Engineering Research Council. This work was supported by SERC Grant Nos. GR/32779, GR/H 49191, GR/H 01069, GR/H 53624, the University of Edinburgh and Meiko Limited. We are grateful to Mike Brown of Edinburgh University Computing Service, and to Arthur Trew of EPCC, for provision and maintenance of service on the Meiko *i*860 Computing Surface and the Thinking Machines CM-200. We wish to thank Brian Murdoch for access to a Digital 7640 ACP ("Alpha") system placed at the University of Edinburgh Computing Service by Digital Equipment Corporation for field test.

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