Heavy quarkonium potential model and the $^1P_1$ state of charmonium

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A theoretical explanation of the observed splittings among the P states of charmonium is given with the use of a nonsingular potential model for heavy quarkonia. We also show that the recently observed mass difference between the center of gravity of the ^3P_J states and the $^1P_1$ state of $c\bar{c}$ does not provide a direct test of the color hyperfine interaction in heavy quarkonia. Our theoretical value for the mass of the $^1P_1$ state is in agreement with the experimental result, and its $E1$ transition width is 341.8 keV. The mass of the η_c' state is predicted to be 3622.3 MeV.

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I. INTRODUCTION

A quantum-chromodynamic potential model was proposed by us [1] in 1982, which not only yielded results for the $c\bar{c}$ and $b\bar{b}$ energy levels and their spin splittings in good agreement with the existing experimental data but its predictions were also confirmed by later experiments at the Cornell Electron Storage Ring [2]. An essential feature of our model was the inclusion of the one-loop radiative corrections to the quark-antiquark potential, which had been derived by us in an earlier investigation [3]. Subsequently, the model was improved by using relativisitic kinematics [4] and a nonsingular form of the quarkonium potential [5]. As shown by us, in addition to the energy levels of $c\bar{c}$ and $b\bar{b}$, our model also yields results in good agreement with the experimental data for the leptonic and $E1$ transition widths. It was further shown by Zhang, Sebastian, and Grotch [6] that the $M1$ transition widths for $c\bar{c}$ and $b\bar{b}$ obtained from our model are in better agreement with the experimental data than those predicted using other potential models.

Recently the mass of the $^1P_1$ state of charmonium has been determined by the E760 Collaboration [7] in $p\bar{p}$ annihilations at Fermilab, and the splitting between the center of gravity of the ^3P_J states and the $^1P_1$ state, denoted as ΔM_P, is found to be approximately -0.9 MeV. This experimental result has created much interest since it provides a new test for the potential models for heavy quarkonia.

If the spin-dependent forces in the quarkonium potential could be treated perturbatively, the ΔM_P splitting would arise solely from the spin-spin (color hyperfine) interaction. However, the spin-dependent forces are known to be quite large and, as observed by Lichtenberg and Potting [8], the contributions of the spin-orbit and tensor interactions to ΔM_P cannot be ignored in a nonperturbative treatment. We shall analyze this complex situation with the use of our model which avoids the use of an illegitimate perturbative treatment, and provide an explanation for the observed splittings of the charmonium P states.

Several authors [9–11] have recently shown that a theoretical value for ΔM_P in close agreement with the experimental value can be readily obtained from the spin-spin interaction terms in the quarkonium potential. However, since they have employed an illegitimate perturbative treatment, the significance of this simple interpretation remains an open question.

Only a quarkonium model which is in good overall agreement with the experimental data can be taken seriously. Our model for heavy quarkonia satisfies this requirement.

II. $c\bar{c}$ SPECTRUM

Our model is based on the Hamiltonian

$$H = H_0 + V_p + V_c, \tag{1}$$

where

$$H_0 = 2(m^2 + \mathbf{p}^2)^{1/2} \tag{2}$$

is the relativistic kinetic energy term, and V_p and V_c are nonsingular quasistatic perturbative and confining potentials, which are given in the Appendix. We found a trial wave function introduced by Jacobs, Olsson, and Suchyta [12] particularly suitable for obtaining the quarkonium energy levels and wave functions.

Our results for the energy-level splittings as well as the individual energy levels of $c\bar{c}$ are given in Tables I and II. For experimental data we have generally relied on the Particle Data Group [13], but for the η_c state we have used the new result announced by the E760 Collaboration

TABLE I. $c\bar{c}$ energy level splittings in MeV. Theoretical splittings and parameters correspond to the scalar-exchange and the scalar-vector-exchange forms of the confining potential. The experimental value of the ψ'-η'_C splitting is not used for the determination of the $c\bar{c}$ parameters because of the uncertainty regarding the η'_c mass.

	Scalar	Scalar-vector	Expt
$\psi' - J/\psi$	587.7	588.9	589.07±0.13
$J/\psi - \eta_c$	105.1	109.0	109.03±3.1
$\psi' - \eta'_c$	60.5	63.5	
$\chi_{c.o.g} - J/\psi$	430.5	428.6	428.35±1
$\chi_{c2} - \chi_{c1}$	28.6	44.6	45.64±0.18
$\chi_{c1} - \chi_{c0}$	80.0	95.8	95.43±1
$\chi_{c.o.g} - h_c$	−5.8	−0.9	−0.93±0.19 ± 0.2
m_c (GeV)	2.375	2.208	
μ (GeV)	3.329	2.580	
α_s	0.295	0.313	
A (GeV^2)	0.183	0.181	
B		0.245	

[14]. The two sets of theoretical results in these tables correspond to the scalar-exchange and the scalar-vector-exchange forms of the confining potential, given by

$$V_c = V_S, \tag{3a}$$

and

$$V_c = (1 - B)V_S + BV_V, \tag{3b}$$

respectively. The results obtained with the scalar-exchange confining potential are unsatisfactory, while the scalar-vector-exchange results are in surprisingly close agreement with the experimental data, including the observed mass of the $^1P_1$ state and the ΔM_P splitting. The scalar-vector mixing parameter B is found to be about $\frac{1}{4}$.

In Table III we display the contributions to ΔM_P from the various types of terms in the Hamiltonian (1) with the confining potential (3b). The table shows comparable contributions to ΔM_P from several sources, which brings out the complexity of this splitting when spin-dependent potential terms are included in the unperturbed Hamiltonian. The ΔM_P splitting, therefore, does not provide a direct test of the spin-spin interaction in heavy quarkonia.

TABLE II. $c\bar{c}$ energy levels in MeV corresponding to the scalar-exchange and the scalar-vector-exchange forms of the confining potential.

	Scalar	Scalar-vector	Expt
$1\,^3S_1\,(J/\psi)$	3096.9	3096.9	3096.93±0.09
$1\,^1S_0\,(\eta_c)$	2991.8	2987.9	2987.9±3.1
$2\,^3S_1\,(\psi')$	3684.6	3685.8	3686.0±0.1
$2\,^1S_0\,(\eta'_c)$	3624.1	3622.3	
$1\,^3P_2\,(\chi_{c2})$	3549.0	3556.0	3556.17±0.13
$1\,^3P_1\,(\chi_{c1})$	3520.4	3511.3	3510.53±0.12
$1\,^3P_0\,(\chi_{c0})$	3440.4	3415.5	3415.1±1
$1\,^1P_1\,(h_c)$	3533.2	3526.3	3526.2±0.15 ± 0.2

TABLE III. Contributions to the $\chi_{c.o.g.}$-$^1P_1$ splitting in MeV from the various types of terms in the $c\bar{c}$ Hamiltonian. The spin-independent, spin-spin, spin-orbit, and tensor potential terms are denoted as V_{SI}, V_{SS}, V_{LS}, and V_T, respectively.

Hamiltonian term	$\chi_{c.o.g.}$-$^1P_1$
H_0	19.4
V_{SI}	−9.6
V_{SS}	5.2
V_{LS}	−13.7
V_T	−2.2
Total	−0.9

In Tables IV and V we give the results for the leptonic and $E1$ transition widths corresponding to the scalar-vector-exchange confining potential by using the formulas

$$\Gamma_{ee}(^3S_1 \to e^+e^-) = \frac{16\pi\alpha^2 e_Q^2}{M^2(Q\bar{Q})}\,|\Psi(0)|^2 \left(1 - \frac{16\alpha_s}{3\pi}\right), \tag{4}$$

and

$$\begin{aligned} \Gamma_{E1}(^3S_1 \to {}^3P_J) &= \frac{4}{9}\frac{2J+1}{3}\alpha e_Q^2 k_J^3 |r_{fi}|^2, \\ \Gamma_{E1}(^3P_J \to {}^3S_1) &= \tfrac{4}{9}\alpha e_Q^2 k_J^3 |r_{fi}|^2, \\ \Gamma_{E1}(^1P_1 \to {}^1S_0) &= \tfrac{4}{9}\alpha e_Q^2 k_J^3 |r_{fi}|^2. \end{aligned} \tag{5}$$

The photon energies for the $E1$ transition widths have been obtained from the energy difference of the initial and the final $c\bar{c}$ states by taking into account the recoil correction. Our results are in good agreement with the available experimental data [13], and our prediction for the $E1$ transition width of $1^1P_1 \to 1^1S_0$ is 341.8 keV.

III. CONCLUSION

We conclude with explanatory remarks concerning some features of our quarkonium potential.

A. Renormalization scheme

We have used the Gupta-Radford (GR) renormalization scheme [15] for the one-loop radiative corrections to the quarkonium potential rather than the modified minimal-subtraction ($\overline{\text{MS}}$) scheme. The GR scheme is a simplified momentum-space subtraction scheme, and the parameter μ can be interpreted as representing the momentum scale of the physical process. This scheme also has the desirable feature that it satisfies the decoupling theorem [16]. On the other hand, in the $\overline{\text{MS}}$ scheme μ appears as a mathematical parameter, and in this scheme

TABLE IV. $c\bar{c}$ leptonic widths in keV.

State	Γ_{ee} (theory)	Γ_{ee} (expt)
$1\,^3S_1$	6.68	5.36±0.29
$2\,^3S_1$	3.25	2.14±0.21

TABLE V. $E1$ transition widths for $c\bar{c}$ in keV. The matrix elements $|r_{fi}|$ for these transitions are given in GeV^{-1}.

Transition	J	$\|r_{fi}\|$	Γ_{E1} (theory)	Γ_{E1} (expt)
$2\,^3S_1 \to 1\,^3P_J$	2	2.19	24.2	21.7±3.3
	1	1.96	28.1	24.2±3.6
	0	1.47	18.3	25.9±3.9
$1\,^3P_J \to 1\,^3S_1$	2	1.60	293.5	270.0±33
	1	1.62	225.2	240.0±41
	0	1.62	105.2	92.4±42
$1\,^1P_1 \to 1\,^1S_0$		1.39	341.8	

decoupling-theorem-violating terms are simply ignored.

The one-loop radiative corrections in the GR scheme can be converted into those in the $\overline{\text{MS}}$ scheme by means of the relation [15]

$$\alpha_s = \bar{\alpha_s}\left[1 + \frac{\bar{\alpha_s}}{4\pi}\left(\frac{49}{3} - \frac{10}{9}n_l + \frac{2}{3}\sum_{n_h}\ln\frac{m^2}{\mu^2}\right)\right], \tag{6}$$

where $\bar{\alpha_s}$ refers to the $\overline{\text{MS}}$ scheme, and n_l and n_h are the numbers of light and heavy quark flavors. If we drop the decoupling-theorem-violating terms that appear in the $\overline{\text{MS}}$ scheme, we can put $n_l = n_f$ and $n_h = 0$, and (6) reduces to

$$\alpha_s = \bar{\alpha_s}\left[1 + \frac{\bar{\alpha_s}}{4\pi}\left(\frac{49}{3} - \frac{10}{9}n_f\right)\right]. \tag{7}$$

B. Quasistatic potential

In an earlier investigation [4], we arrived at the surprising conclusion that while the quasistatic form of the quarkonium potential yields results in good agreement with the experimental data, this is not the case for the momentum-dependent form. This conclusion has also been confirmed by the recent investigations of Gara *et al.* [17] and Lucha *et al.* [18].

It appears to us that the success of the quasistatic potential is related to the phenomenon of quark confinement. Since a rigorous treatment of quark confinement does not exist at this time, we shall only offer a plausible argument. It was argued earlier [19] with the use of a renormalization-group-improved quantum-chromodynamic treatment that quark confinement can be understood as a consequence of the fact that quarks and antiquarks are unable to exchange low-momentum gluons. Moreover, since, for the quark-antiquark scattering in the center-of-mass system,

$$\mathbf{p}^2 = \tfrac{1}{4}\mathbf{k}^2 + \tfrac{1}{4}\mathbf{s}^2, \tag{8}$$

where

$$\mathbf{k} = \mathbf{p}' - \mathbf{p}, \qquad \mathbf{s} = \mathbf{p}' + \mathbf{p}, \tag{9}$$

it follows that if $\mathbf{k}^2$ is allowed to take only large values, $\mathbf{s}^2$ can be treated as small. This may be regarded as a justification for the quasistatic approximation in which terms of second and higher orders in $\mathbf{s}$ are ignored.

Our quarkonium perturbative and confining potentials are not only quasistatic but also nonsingular. In the momentum space, these potentials are obtained by first expanding in powers of $\mathbf{p}^2/(m^2 + \mathbf{p}^2)$, and then approximating $\mathbf{p}^2$ as $\frac{1}{4}\mathbf{k}^2$. The perturbative potential in powers of $\mathbf{p}^2/(m^2 + \mathbf{p}^2)$ includes, among others, terms of the form

$$f(\mathbf{p}^2) = \frac{a + b\,\mathbf{S}_1\cdot\mathbf{S}_2}{m^2 + \mathbf{p}^2}, \tag{10}$$

which becomes, in the quasistatic approximation,

$$f(\mathbf{k}^2) = \frac{a + b\,\mathbf{S}_1\cdot\mathbf{S}_2}{m^2 + \frac{1}{4}\mathbf{k}^2}. \tag{11}$$

It has been observed by Grotch, Sebastian, and Zhang [11] that while the contribution of $f(\mathbf{p}^2)$ vanishes for the P states due to the vanishing of the wave function at the origin, $f(\mathbf{k}^2)$ yields a small but nonvanishing contribution for these states. Consequently, for P and higher angular-momentum states it would be more accurate to drop terms of the form (10) than to convert them into the approximate form (11). We agree with the observation of Grotch *et al.* Accordingly, in the treatment of states with $l \neq 0$ we shall drop terms of the form (11) in the momentum-space potentials and the corresponding terms of the form

$$f(\mathbf{r}) = \frac{a + b\,\mathbf{S}_1\cdot\mathbf{S}_2}{\pi r}e^{-2mr} \tag{12}$$

in the coordinate-space potentials.

C. Confining potential

In our theoretical treatment, our aim has been to avoid phenomenology except in the choice of the long-range confining potential, which cannot be derived sufficiently accurately by any known theoretical technique. **It is indeed remarkable that the results obtained from our field-theoretical perturbative potential supplemented with an appropriate confining potential are in excellent overall agreement with the experimental data including the ΔM_P splitting. It should be noted that we have neglected the effect of coupling of the energy levels to virtual decay channels and possibly other small effects. Such effects presumably have also been taken into account in our phenomenological confining potential.**

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APPENDIX: NONSINGULAR QUARKONIUM POTENTIALS

The nonsingular quarkonium potentials can be obtained [20] by appropriate modifications of the singular potentials in the momentum space, and transforming them to the coordinate space. The nonsingular potentials obtained by this procedure are given below. Some unwanted terms for states with $l \neq 0$ have been dropped as explained in Sec. III.

1. Perturbative quantum-chromodynamic potential

The perturbative potential V_p consists of the direct potential V_p' and the annihilation potential V_p'', and, in momentum space,

$$V_p(\mathbf{k}) = V_p'(\mathbf{k}) + V_p''(\mathbf{k}), \tag{A1}$$

where

$$\begin{aligned}
V_p'(\mathbf{k}) = &-\frac{16\pi\alpha_s}{3\mathbf{k}^2}\left[1-\frac{3\alpha_s}{2\pi}-\frac{\alpha_s}{12\pi}(33-2n_f)\ln\left(\frac{\mathbf{k}^2}{\mu^2}\right)\right] \\
&+\frac{16\pi\alpha_s}{3(\mathbf{k}^2+4m^2)}\left[\delta_{l0}\left(1-\frac{3\alpha_s}{2\pi}\right)-\frac{\alpha_s}{12\pi}(33-2n_f)\ln\left(\frac{\mathbf{k}^2}{\mu^2}\right)-\frac{7\pi\alpha_s}{3}\frac{m}{|\mathbf{k}|}\right] \\
&+\frac{128\pi\alpha_s}{9}\frac{\mathbf{S}_1\cdot\mathbf{S}_2}{\mathbf{k}^2+4m^2}\left[\delta_{l0}\left(1-\frac{35\alpha_s}{12\pi}\right)-\frac{\alpha_s}{12\pi}(33-2n_f)\ln\left(\frac{\mathbf{k}^2}{\mu^2}\right)+\frac{21\alpha_s}{8\pi}\ln\left(\frac{\mathbf{k}^2}{m^2}\right)\right] \\
&-32\pi\alpha_s\frac{i\mathbf{S}\cdot(\mathbf{k}\times\mathbf{p})}{\mathbf{k}^2(\mathbf{k}^2+4m^2)}\left[1-\frac{11\alpha_s}{18\pi}-\frac{\alpha_s}{12\pi}(33-2n_f)\ln\left(\frac{\mathbf{k}^2}{\mu^2}\right)+\frac{\alpha_s}{\pi}\ln\left(\frac{\mathbf{k}^2}{m^2}\right)\right] \\
&-\frac{64\pi\alpha_s}{3}\frac{\mathbf{S}_1\cdot\mathbf{k}\;\mathbf{S}_2\cdot\mathbf{k}-\frac{1}{3}\mathbf{k}^2\mathbf{S}_1\cdot\mathbf{S}_2}{\mathbf{k}^2(\mathbf{k}^2+4m^2)}\left[1+\frac{4\alpha_s}{3\pi}-\frac{\alpha_s}{12\pi}(33-2n_f)\ln\left(\frac{\mathbf{k}^2}{\mu^2}\right)+\frac{3\alpha_s}{2\pi}\ln\left(\frac{\mathbf{k}^2}{m^2}\right)\right],
\end{aligned} \tag{A2}$$

$$V_p''(\mathbf{k}) = \delta_{l0}\frac{32\alpha_s^2}{3(\mathbf{k}^2+4m^2)}(1-\ln 2)\left(\mathbf{S}_1\cdot\mathbf{S}_2-\frac{1}{4}\right). \tag{A3}$$

In coordinate space, the potential takes the form

$$V_p(\mathbf{r}) = V_p'(\mathbf{r}) + V_p''(\mathbf{r}), \tag{A4}$$

where

$$\begin{aligned}
V_p'(\mathbf{r}) = &-\frac{4\alpha_s}{3r}\left\{1-\frac{3\alpha_s}{2\pi}+\frac{\alpha_s}{6\pi}(33-2n_f)\left[\ln(\mu r)+\gamma_E\right]\right\} \\
&+\frac{4\alpha_s}{3r}\left\{\delta_{l0}\left(1-\frac{3\alpha_s}{2\pi}\right)e^{-2mr}+\frac{\alpha_s}{6\pi}(33-2n_f)\left[\ln(\mu r)e^{-2mr}+E_+(2mr)\right]\right. \\
&\qquad\left.-\frac{7\alpha_s}{3}\left[\ln(2mr)e^{-2mr}-E_-(2mr)\right]\right\} \\
&+\frac{32\alpha_s}{9r}\mathbf{S}_1\cdot\mathbf{S}_2\left\{\delta_{l0}\left(1-\frac{35\alpha_s}{12\pi}\right)e^{-2mr}+\frac{\alpha_s}{6\pi}(33-2n_f)\left[\ln(\mu r)e^{-2mr}+E_+(2mr)\right]\right. \\
&\qquad\left.-\frac{21\alpha_s}{4\pi}\left[\ln(mr)e^{-2mr}+E_+(2mr)\right]\right\} \\
&+\frac{8\alpha_s}{r}\mathbf{L}\cdot\mathbf{S}\left\{\left(1-\frac{11\alpha_s}{18\pi}\right)f_1(2mr)+\frac{\alpha_s}{6\pi}(33-2n_f)\left[f_1(2mr)\ln(\mu r)+g_1(2mr)\right]\right. \\
&\qquad\left.-\frac{2\alpha_s}{\pi}\left[f_1(2mr)\ln(mr)+g_1(2mr)\right]\right\} \\
&+\frac{4\alpha_s}{3r}S_T\left\{\left(1+\frac{4\alpha_s}{3\pi}\right)f_2(2mr)+\frac{\alpha_s}{6\pi}(33-2n_f)\left[f_2(2mr)\ln(\mu r)+g_2(2mr)\right]\right. \\
&\qquad\left.-\frac{3\alpha_s}{\pi}\left[f_2(2mr)\ln(mr)+g_2(2mr)\right]\right\},
\end{aligned} \tag{A5}$$

$$V_p''(\mathbf{r}) = \delta_{l0}\frac{8\alpha_s^2 e^{-2mr}}{3\pi r}(1-\ln 2)\left(\mathbf{S}_1\cdot\mathbf{S}_2-\frac{1}{4}\right). \tag{A6}$$

Note that the tensor operator is defined as

$$S_T = 3\,\boldsymbol{\sigma}_1 \cdot \hat{\mathbf{r}}\,\boldsymbol{\sigma}_2 \cdot \hat{\mathbf{r}} - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2, \tag{A7}$$

the functions $E_\pm$ are expressible in terms of the exponential-integral function Ei as

$$E_\pm(x) = \tfrac{1}{2}\left[e^x \mathrm{Ei}(-x) \pm e^{-x}\mathrm{Ei}(x)\right] \mp e^{-x}\ln x, \tag{A8}$$

and

$$\begin{aligned} f_1 &= \frac{1-(1+x)e^{-x}}{x^2}, \\ f_2 &= \frac{1-\left(1+x+\frac{1}{3}x^2\right)e^{-x}}{x^2}, \\ g_1 &= \frac{\gamma_E - [E_+(x) - xE_-(x)]}{x^2}, \\ g_2 &= \frac{\gamma_E - \left[\left(1+\frac{1}{3}x^2\right)E_+(x) - xE_-(x)\right]}{x^2}. \end{aligned} \tag{A9}$$

2. Phenomenological confining potential

The scalar-exchange and the vector-exchange confining potentials in the momentum space are

$$V_S(\mathbf{k}) = -8\pi A\left[\frac{1}{\mathbf{k}^4} - \frac{2i\mathbf{S}\cdot(\mathbf{k}\times\mathbf{p})}{\mathbf{k}^4(\mathbf{k}^2+4m^2)}\right], \tag{A10}$$

and

$$\begin{aligned} V_V(\mathbf{k}) = -8\pi A\Bigg[&\frac{1}{\mathbf{k}^4} - \frac{1+\frac{8}{3}\mathbf{S}_1\cdot\mathbf{S}_2}{\mathbf{k}^2(\mathbf{k}^2+4m^2)} + \frac{6i\mathbf{S}\cdot(\mathbf{k}\times\mathbf{p})}{\mathbf{k}^4(\mathbf{k}^2+4m^2)} \\ &+4\frac{\mathbf{S}_1\cdot\mathbf{k}\,\mathbf{S}_2\cdot\mathbf{k} - \frac{1}{3}\mathbf{k}^2\mathbf{S}_1\cdot\mathbf{S}_2}{\mathbf{k}^4(\mathbf{k}^2+4m^2)}\Bigg]. \end{aligned} \tag{A11}$$

The coordinate-space potentials are given by

$$V_S(\mathbf{r}) = Ar - \frac{A}{2m^2r}\mathbf{L}\cdot\mathbf{S}\left[1-2f_1(2mr)\right], \tag{A12}$$

and

$$\begin{aligned} V_V(\mathbf{r}) = Ar &+ \frac{A}{2m^2r}\left(1+\frac{8}{3}\mathbf{S}_1\cdot\mathbf{S}_2\right)\left(1-e^{-2mr}\right) \\ &+\frac{3A}{2m^2r}\mathbf{L}\cdot\mathbf{S}\left[1-2f_1(2mr)\right] \\ &+\frac{A}{12m^2r}S_T\left[1-6f_2(2mr)\right]. \end{aligned} \tag{A13}$$

It is understood that the confining potential also contains an additive phenomenological constant C.

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