Is large lepton mixing excluded?

Alexei Yu. Smirnov
Institute for Advanced Study, Princeton, New Jersey 08540;
International Centre for Theoretical Physics, 34100 Trieste, Italy;
and Institute for Nuclear Research, 117312 Moscow, Russia

David N. Spergel
Princeton University Observatory, Princeton, New Jersey 08540

John N. Bahcall
Institute for Advanced Study, Princeton, New Jersey 08540
(Received 6 May 1993)

The original $\bar{\nu}_\mu$- (or $\bar{\nu}_\tau$-) energy spectrum from the gravitational collapse of a star has a larger average energy than the spectrum for $\bar{\nu}_e$ since the opacity of $\bar{\nu}_e$ exceeds that of $\bar{\nu}_\mu$ (or ν_τ). Flavor neutrino conversion $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$ induced by lepton mixing results in partial permutation of the original $\bar{\nu}_e$ and $\bar{\nu}_\mu$ spectra. An upper bound on the permutation factor $p \leq 0.35$ (99% C.L.) is derived using the data from SN 1987A and a range of models of the neutrino emission. The relation between the permutation factor and the vacuum mixing angle is established, which leads to the upper bound on this angle. The upper bound $\sin^2 2\theta > 0.7-0.9$ excludes the large mixing angle solutions of the solar neutrino problem: "just-so" and, partly, MSW, as well as part of the region of the ν_e-ν_μ oscillation space which could be responsible for the atmospheric muon neutrino deficit. These limits are sensitive to the predicted neutrino spectrum and can be strengthened as supernova models improve.

PACS number(s): 14.60.Pq, 14.60.Lm, 96.60.Kx, 97.60.Bw

I. INTRODUCTION

There are several hints that lepton mixing does exist and might even be much bigger than that in the quark sector. Solar neutrino data [1] can be reconciled with predictions of the standard solar model [2] by long length vacuum oscillations ("just-so" solution) [3]. The required values of neutrino mixing angle θ and mass squared difference Δm^2 are $\sin^2 2\theta = 0.85-1.0$, $\Delta m^2 = (0.8-1.1) \times 10^{-10}$ eV^2 [4]. The solar neutrino problem can be solved also by resonant flavor conversion, the Mikheyev-Smirnov-Wolfenstein (MSW) effect [5]. For the MSW solution, the data single out two regions of neutrino parameters, one of which involves large mixing angles: $\sin^2 2\theta = 0.6-0.9$, at $\Delta m^2 = (10^{-7}-10^{-5})$ eV^2 [6]. The deficit of the muon neutrinos in the atmospheric neutrino flux can be explained by ν_μ-ν_e oscillations with the parameters [7] $\sin^2 2\theta = 0.5-0.9$, $\Delta m^2 = (10^{-3}-10^{-2})$ eV^2 (see Fig. 3).

On the other hand it has been argued that mixing in the lepton sector can be "naturally" large. In particular, large lepton mixing may appear in models with a radiative generation of the neutrino masses (Zee mechanism [8], see [9] for review). In the "seesaw" mechanism some configurations of mass matrices result in large mixing angles (see, e.g., [10]); the "seesaw" enhancement of lepton mixing may take place at definite conditions (strong mass hierarchy in Majorana mass sector, or definite symmetry of the Majorana mass matrix and mass degeneration of the right-handed neutrino components [11]).

Large lepton mixing can be generated by some interactions at the Planck scale, which result in nonrenormalizable terms of the type $(\alpha_{ij}/M_{\rm Pl}) l_i^T l_j H^T H$ [12,13]. Here $l_i (i = e, \mu, \tau)$ are the lepton doublets of definite flavor, H is the Higgs doublet, and $M_{\rm Pl}$ is the Planck mass. At $\alpha \approx 1$, these terms generate the neutrino mass $m_{ij} = \langle H \rangle^2 / M_{\rm Pl} \approx 10^{-5}$ eV, which gives Δm^2 in the region of "just-so" solutions. Furthermore, it was argued in [13] that the "Planck-scale interaction" related to gravity does not respect lepton number, and moreover all coupling constants in the flavor basis have the same value $\alpha_{ij} = \alpha_0$ [13]. All of the elements in the corresponding matrix are equal. In this case, the electron neutrino mixes with only one state, namely, with the combination $(\nu_\mu - \nu_\tau)/\sqrt{2}$, and the mixing parameter is $\sin^2 2\theta = \frac{8}{9}$, i.e., precisely in the "just-so" region. Although there is no real model for the "Planck-scale interaction" the coincidence of parameters is remarkable.

In this paper we will discuss the limits on large lepton mixing that could be obtained using the observational data from the SN 1987A [14–16].

The effects of lepton mixing on the neutrino fluxes from gravitational collapses of stars have been widely discussed [17–22,5]. In particular, it was noted that large flavor mixing results significantly distorts the $\bar{\nu}_e$ spectra at the Earth through the appearance of a high-energy tail, and increases the average energy of the detected events relative to the no mixing case [17,19,20]. Comparison of the observed energy distribution to theoretical predictions [21] has led some authors to argue that $\theta > 50°$ is disfavored while others [22] suggest that 20–30% transition would improve the agreement between theory and observations. At large mixing angles, the oscillations in the matter of the Earth result in different signals in Kamiokande and IMB detectors; this could explain the more energetic spectrum seen by IMB [20]. Here, we refine the consideration of the large mixing effects to obtain statistically significant upper bounds on the mixing angle by making use of the existing data from SN 1987A.

In Sec. II we introduce the permutation factor—a single energy-independent parameter which describes any conversion of $\bar{\nu}_e$ into neutrino of other flavor. In Sec. III a statistical analysis is performed to get the upper bounds on the permutation factor. The sensitivity of the bound to the parameters assumed for the original neutrino spectra are studied. It is argued that the time integral characteristics of these spectra are well restricted as the mass of the collapsing star and the duration of the neutrino signal are known. In Sec. IV we apply the obtained bounds to the conversion stipulated by large vacuum mixing. The dynamics of the neutrino transitions in matter for the nonresonant channel is considered and the relations between the mixing angle and the permutation factor are found. In Sec. V, we discuss the obtained upper bounds on mixing angle.

II. PERMUTATION OF $\bar{\nu}_e$ AND $\bar{\nu}_\mu$ SPECTRA. PERMUTATION FACTOR

Consider the influence of transitions $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$ on the $\bar{\nu}_e$-energy spectrum. Since $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$ have, to high accuracy, the same production and detection properties, the results will be the same for transitions to $\bar{\nu}_\tau$ or to any combination of $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$. (This remark applies also for transitions into ν_μ and ν_τ.) We will comment on three-neutrino mixing latter (in Sec. IV), although many cases can be reduced to two neutrino mixing.

Let $F_0(\bar{\nu}_e)$ and $F_0(\bar{\nu}_\mu)$ be the original $\bar{\nu}_e$, and $\bar{\nu}_\mu$ spectra, and let p be the probability of a $\bar{\nu}_e \to \bar{\nu}_\mu$ transition on the way from a core of collapsing star to the detector. Since the $\bar{\nu}_e$, and $\bar{\nu}_\mu$ spectra emitted by neutrinospheres are incoherent, the $\bar{\nu}_e$ flux in the detector can be written as

$$F(\bar{\nu}_e) = (1-p)F_0(\bar{\nu}_e) + pF_0(\bar{\nu}_\mu) \ . \tag{1}$$

Obviously, there is no observable effect when the original spectra are the same: $F_0(\bar{\nu}_e) = F_0(\bar{\nu}_\mu)$.

The energy spectra of $\bar{\nu}_e$'s and $\bar{\nu}_\mu$'s that are emitted from the core of a collapsing star are different: the $\bar{\nu}_e$ spectrum has a mean energy that is 1.5–2 times smaller than that of the $\bar{\nu}_\mu$ spectrum [23–33]. This general feature follows from the fact that $\bar{\nu}_e$ interacts with matter more strongly than $\bar{\nu}_\mu$ does; neutral current scattering and charged current absorption on protons, $\bar{\nu}_e + p \to n + e^+$, are allowed for $\bar{\nu}_e$ but not for $\bar{\nu}_\mu$. Also, because of the charge current interaction, the cross section of $\bar{\nu}_e$ scattering on electrons is larger than that for $\bar{\nu}_\mu$. Therefore $\bar{\nu}_e$'s encounter a larger opacity and consequently are emitted from more external and colder layers of the star. This essentially model-independent feature plays a key role in our determination of the maximum allowed mixing angles. Another crucial point is that the cross section of the detection reaction, $\bar{\nu}_e + p \to e^+ + n$, is approximately proportional to the neutrino energy squared. Therefore even a small permutation (or admixture of a higher energy spectrum) can result in an appreciable effect.

In general, the probability p depends on the energy of the neutrino, the mass splitting, the vacuum mixing angle, and the density profiles of the supernova and the Earth. However, as we will show, for most of the interesting ranges of neutrino parameters p turns out to be practically independent of energy. This comes about from the averaging effects or/and from the loss of the coherence or from the fact that the conversion probability itself does not depend on energy in a wide region of parameters. It is the assumption $p(E)=\text{const}$ that allows us to perform an extensive statistical analysis. The cases where p depends on energy will be considered separately

If $p=1$ (complete transformation), the detected $\bar{\nu}_e$ spectrum will coincide with the original $\bar{\nu}_\mu$ spectrum: $F(\bar{\nu}_e) = F_0(\bar{\nu}_\mu)$ and, vice versa, the final $\bar{\nu}_\mu$ will coincide with original spectra of $\bar{\nu}_e$. The spectra permute and we will call the average probability p the permutation factor. If $p<1$, only partial permutation takes place and the final $\bar{\nu}_e$-energy spectrum will be a mixture of the two original $\bar{\nu}_e$ and $\bar{\nu}_\mu$ spectra.

Figure 1 depicts the expected cumulative energy spectrum of the events in Kamiokande-II and IMB detectors for different values of p. The parameters of a "conventional neutrino burst" [23–25] have been used. As many authors have concluded previously, the observed energy spectrum from SN 1987A is in reasonable agreement with that calculated without any neutrino transformations. Figure 1 shows that the $\bar{\nu}_e \to \bar{\nu}_\mu$ transition produces unobserved high energy events. We use this result to exclude large values of p.

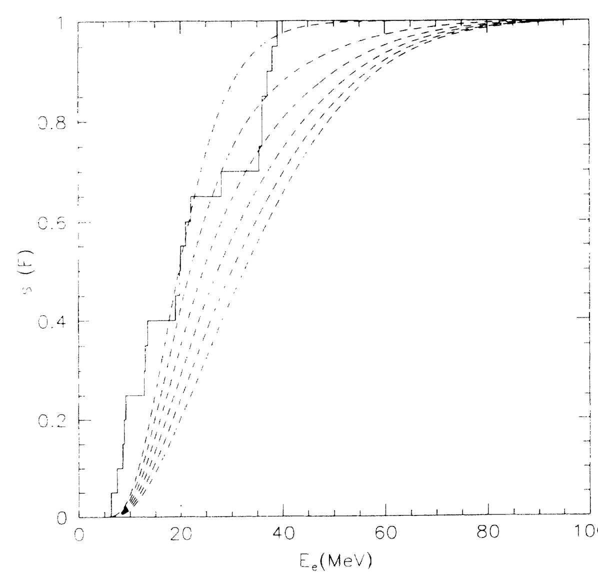

FIG. 1. The cumulative energy spectrum of the events in the Kamiokande and IMB detectors predicted for different values of permutation factor p (figures at the curves). Original spectra were taken according to the model [24]. Histogram shows the detected spectrum.

III. UPPER BOUNDS ON THE PERMUTATION FACTOR FROM SN 1987A DATA

We will compare the shapes of the predicted time-integrated energy spectra for different values of p with the observed energy distribution. The original $\bar{\nu}_e$, and $\bar{\nu}_\mu$ spectra are approximated by the modified Fermi-Dirac spectrum [26–32]:

$$\frac{dE^{\text{tot}}}{dE}=\frac{AE^3}{e^{(E/T-\eta)}+1} ,$$

where A, T, η are the fit parameters. The modification is related to the fact that the emitted spectra are superpositions of thermal fluxes (in general, Fermi-Dirac spectra with nonzero chemical potentials) from different thermalization spheres. These spectra are further modified by scattering and absorption above the thermalization spheres and by the integration over the neutrino burst time (the effective temperatures are changed during the burst). Instead of T we fix the average energy of the spectrum $\bar{E}$:

$$\bar{E}=\frac{E^{\text{tot}}}{\int \frac{dE^{\text{tot}}}{dE}\frac{dE}{E}} ,$$

where $E^{\text{tot}} \equiv \int (dE^{\text{tot}}/dE)dE$ is the total emitted energy in a given neutrino type. The distortion parameter η has the effect of a "chemical potential." The parameters of the time-integrated spectra that we use in our analysis are the average energies of the electron antineutrino $\bar{E}_e$, and the muon antineutrino $\bar{E}_\mu$, as well as the ratio of the total energies emitted in $\bar{\nu}_e$'s and $\bar{\nu}_\mu$'s: $r \equiv E^{\text{tot}}(\bar{\nu}_\mu)/E^{\text{tot}}(\bar{\nu}_e)$. The absolute value of the total energy carried away by neutrinos is eliminated by normalization; the total number of calculated events is constant and equal to the observed value $N=20$.

The cumulative energy spectra of observed and calculated neutrino events are compared by the Kholmogorov-Smirnov test, which allows us to set nonparametric upper limits on p at a definite confidence level for different values of the original spectra parameters (see Fig. 2). As is apparent from Fig. 2, the inferred upper bound depends most strongly on the average muon neutrino energy $\bar{E}_\mu$; the main difference between the calculated and the observed spectra comes from the high energy region for which the calculated events are caused by $\bar{\nu}_\mu$ converted to $\bar{\nu}_e$. If $\bar{E}_\mu < 6.5$ MeV, then the bound is $p<0.5$ and no strong limit can be obtained for the antineutrino channel (see Sec. IV). The bounds depend weakly on the total fluence emitted in $\bar{\nu}_\mu$ [Fig. 2(a)]. For example, at $\bar{E}_\mu=22$ MeV, one has $p=0.30$, 0.34, and 0.39 for $r=1.2$, 1.0, and 0.8, respectively. The bounds depend rather weakly (5% change) on $\bar{E}_e$ in the most reliable region of values 12–15 MeV [Fig. 2(b)]. At larger or smaller energies, the limits become artificially strong due to the general disagreement of the predictions and the data even without the permutation effect. The bounds are sensitive to the shape of the original spectra [Fig. 2(c)]. The more pinched the spectra (bigger η), the stronger the suppression of the number of high energy events, and, consequently, the weaker the restrictions. For fixed $\bar{E}_\mu$, the dependence of bounds on η is stronger for smaller energies $\bar{E}_\mu$. At $\bar{E}_\mu=22$ MeV, the increase of η from 0 (pure Fermi-Dirac spectra) to 3 results in the increase of the limit on p by 15%. There is a strong dependence of the inferred limits on the assumed value of the distortion parameter of the electron antineutrino spectrum η_e. A decrease of η_e results in an increase of the number of high-energy events induced by $\bar{\nu}_e$'s and therefore strengthens the limit on p. The limits on p at different confidence levels are shown in Fig. 2(e). At 95% C.L., a significant limit exists even for $\bar{E}_\mu=17-18$ MeV. At 99.9% C.L., a significant limit can be established only for $\bar{E}_\mu > 22-23$ MeV. At the representative value of energy $\bar{E}_\mu > 22-24$ MeV, the 2σ limit is 35% stronger than 3σ limit.

Since we use data from SN 1987A, the model of collapse and therefore the integral characteristics of the neutrino burst can in principle be restricted further by using information on the progenitor and the observed properties of light curve of SN 1987A. The available data suggest a mass of the iron core [26] $M_{\text{Fe}}=(1.3-1.6)M_\odot$, and, consequently, a total energy carried away by neutrinos of $E_{\text{tot}}=(2-4)\times 10^{53}$ ergs. The time interval of neutrino emission, $\Delta t \approx 13$ s, is in a good agreement with the expected value, further indicating the basic correctness of the conventional picture of neutrino transport. The observed neutrino energies versus time suggest that the average energy decreases with time, consistent with the idea that neutrinos are emitted in the cooling stage.

In Table I, the principal parameters of different models [24–32] of neutrino bursts which satisfy the above conditions are presented and the upper bounds on p are given in accordance with Fig. 2. The restrictions

$$p \le \begin{cases} 0.35, & 99\%, \\ 0.23, & 95\%, \end{cases} \tag{2}$$

can be considered as upper bounds in a representative supernova neutrino burst model.

It should be noted that in some models at $p \approx 0.10-0.15$ one obtains an even better fit of the data than at $p=0$. This value is smaller than those obtained in earlier studies [22] which is explained by higher temperatures of the original spectra used here. However, $p=0$ is also a good fit to the observations and the present data does not provide statistically significant evidence for neutrino oscillations.

Improvements in SN modeling can reduce the uncertainty in neutrino emission parameters. The difference between the $\bar{\nu}_e$, and $\bar{\nu}_\mu$ spectra is determined by the difference in interactions as well as by the structure of the star, i.e., the density, temperature, and lepton-number profiles. The latter in turn depends on the nuclear equation of state (EOS). A soft EOS results in the creation of a hot and compact protoneutron star, whereas a stiff EOS produces a colder and more expanded central object with smaller temperatures and a smaller gradient of temperature [33]. As a result, one expects smaller energies of $\bar{\nu}_\mu$ in the model with a stiff EOS. In [33], a very stiff EOS by Wolff [34] was used and the average energies $\bar{E}_e \approx 12$

MeV and $\bar{E}_\mu = 14$ MeV were obtained. This small difference in average energies probably indicates only the direction of a trend rather than a self-consistent numerical result. Indeed, the model by Mayle and Wilson [24] at $t = 0.4$ s after the bounce was used as the initial condition. This model is based on a softer EOS, so that the calculation described in [33] requires a nonphysical change of the EOS at 0.4 s. The parameters at 0.4 s were adjusted to obtain the hydrostatic configuration, whereas in the original Mayle-Wilson model at $t = 0.4$ s the star is still in the dynamical phase. In [26] no strong difference of the properties of the neutrino burst were obtained between a soft and a stiff EOS. Further, the "flux-limited diffusion method" was used in [33] to describe the neutrino transport, and the $\bar{\nu}_e$-, $\bar{\nu}_\mu$-energy distributions obtained are appreciably wider than Fermi-Dirac spectra. In particular, the calculated $\bar{\nu}_\mu$ spectrum can be approximated by a Maxwell-Boltzmann distribution ($\eta \to -\infty$). This feature [33] is in contradiction with other results obtained by the same method [26–30], as well as with results of a physically more correct method based on Monte Carlo simulations [31,32]. It is of great importance to calculate a self-consistent supernova model with the same stiff EOS [34] at all stages and to check whether

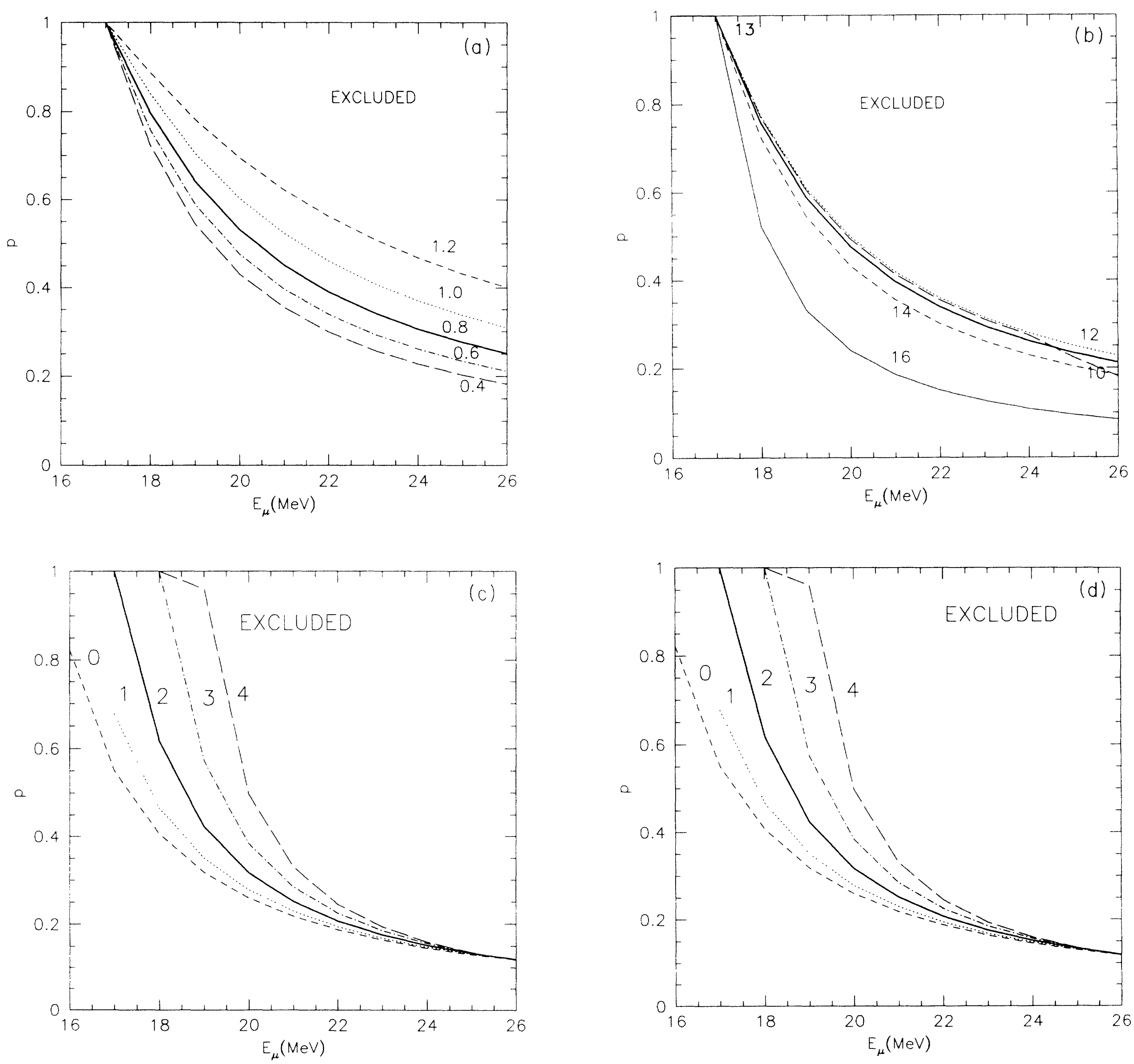

FIG. 2. Upper bounds on permutation factor from SN 1987A data as functions of the original spectra parameters. (a) The dependence of the upper bound (99% C.L.) as a function of the average muon energy $\bar{E}_\mu$ on the ratio of total energies, r, emitted in $\bar{\nu}_\mu$ and $\bar{\nu}_e$ (figures at the curves). Other parameters are fixed as follows: $\bar{E}_e = 13$ MeV, $\eta_e = \eta_\mu = 2$. (b) The dependence of the same bound as in (a) on the electron antineutrino energy, $\bar{E}_e$ (figures at the curves in MeV). Other parameters: $r = 1$, $\eta_e = \eta_\mu = 2$. (c) The dependence of the same bound as in (a) on the spectrum distortion parameter η_μ (figures at the curves). Other parameters: $\bar{E}_e = 13$ MeV, $r = 1$, $\eta_e = 2$. (d) The same dependence as in (c) for $\eta_e = 0$. (e) The upper bound on p at different confidence levels (figures at the curves).

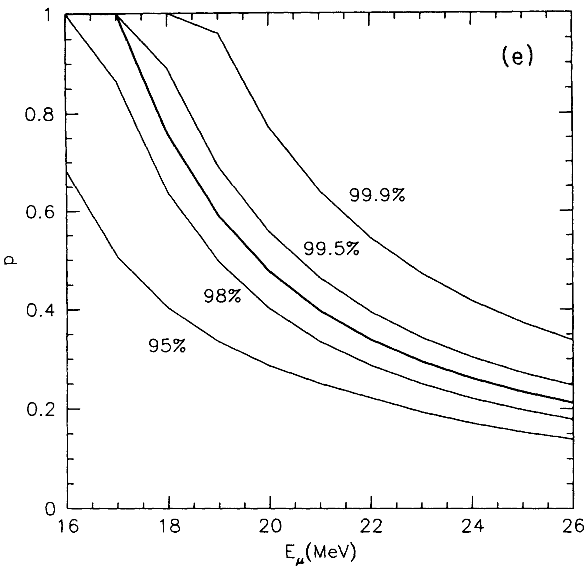

FIG. 2. (*Continued*).

such a model fits the SN 1987A data (including the neutrino luminosities and the duration of the neutrino burst).

IV. PERMUTATION FACTORS AND LEPTON MIXING

We consider in this section the propagation of neutrinos from the core of a star to detectors on Earth and determine the relations between the permutation factor p and the vacuum mixing angle θ. We assume for most of this section that the admixture to ν_e and $\bar{\nu}_e$ of the light mass component is larger than that of the heavy component. In this case, the $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$ channel is nonresonant. (Matter resonance takes place in the neutrino channel, $\nu_e \leftrightarrow \nu_\mu$, as it is implied by the MSW solution to the $\nu_\odot$ problem. We will comment on the opposite case at the end of this section.)

For the nonresonant channel $\bar{\nu}_e$-$\bar{\nu}_\mu$, the mixing angle in matter θ_m, is always smaller than the angle in vacuum:

$$\sin^2 2\theta_m = \frac{\tan^2 2\theta}{\left[\frac{\rho}{\rho_R}+1\right]^2 + \tan^2 2\theta} . \tag{3}$$

Here ρ is the density, m_N is the nucleon mass, and

$$\rho_R = \frac{m_N \Delta m^2 \cos 2\theta}{2\sqrt{2} G_F Y_e E} \tag{4}$$

is the resonant density for the *neutrino* channel (G_F is the Fermi constant, Y_e is the number of electrons per nucleon).

For values of neutrino parameters of interest, i.e., $\Delta m^2 \lesssim 10^{-2}$ eV^2, $E \gtrsim 10$ MeV, the resonant density $\rho_R \lesssim 10^4$ g/cm^3 is much smaller than the density at the neutrino production point, $\rho_0 \simeq 10^{12}$ g/cm^3. Therefore the initial mixing is strongly suppressed: $\sin^2 2\theta_m^0 \approx \tan^2 2\theta(\rho_R/\rho_0)^2$ and the initial neutrino state practically coincides with eigenstate of the instantaneous Hamiltonian of the neutrino system, $\bar{\nu}_{1m}$,: $\bar{\nu}(t=0) \equiv \bar{\nu}_e \cong \bar{\nu}_{1m}$. Further evolution of this state is determined by the adiabaticity condition [5]. (If this condition is satisfied, the transitions of the eigenstates $\bar{\nu}_{1m} \leftrightarrow \bar{\nu}_{2m}$ can be neglected.) The adiabaticity condition reads $\kappa \ll 1$, where $\kappa \equiv d\theta_m/dr/\Delta H$ is the adiabaticity parameter [5]. Here ΔH is the energy splitting between eigenvalues of the Hamiltonian, $\Delta H \equiv E(\bar{\nu}_{1m}) - E(\bar{\nu}_{2m})$. The adiabaticity parameter can be written in the form

$$\kappa = \frac{\sin^3 2\theta_m}{\sin^2 2\theta} \frac{l_\nu^2}{4\pi l_\rho l_0} , \tag{5}$$

where $l_\nu = 4\pi E/\Delta m^2$ is the oscillation length in vacuum, $l_0 = 2\pi m_N/\sqrt{2} G_F \rho Y_e$ is the refraction length, and $l_\rho \equiv \rho/(d\rho/dr)$ is the typical density scale height.

Since $\sin^3 2\theta_m \propto 1/\rho^3$ at $\rho \gg \rho_R$ and $l_0^{-1} \propto \rho$, the parameter κ is small at large densities, and the adiabaticity condition is satisfied in the early stage of neutrino propagation. When the density decreases, κ first increases as $1/\rho^2$, reaches a maximum value at $\rho_m \sim \rho_R$ ($\rho_m/\rho_R = \frac{3}{4}y - \frac{1}{4}$, where $y \equiv \sqrt{1+\frac{8}{9}\tan^2 2\theta}$), and then decreases again as ρ. For $\rho = \rho_m$ we get, from (5),

$$\kappa_m = \frac{f(\theta)}{4\pi} \frac{l_\nu}{l_\rho} , \tag{6}$$

where

$$f(\theta) = \frac{16}{9} \frac{\tan^2 2\theta}{\sin 2\theta} \frac{y - \frac{1}{3}}{[(y+1)^2 + \frac{16}{9}\tan^2 2\theta]^{3/2}} .$$

The function $f(\theta)$ increases from ≈ 0.09 at $\sin^2 2\theta = 0.3$, to ≈ 0.28 at $\sin^2 2\theta = 0.95$; ($f = 2$ at $\sin^2 2\theta = 1$). Substitut-

TABLE I. Integral characteristics of the neutrino bursts in different models and corresponding upper bounds on permutation parameters (MWS [24], Bruenn [26], Burrows [28–30], Janka [31,32]).

Model	E_e (MeV)	E_μ (MeV)	r	η_e	η_μ	p 95% C.L.	p 99% C.L.
MWS	13.8	22.3	0.9	3.8	0.6	0.27	0.42
Bruenn	13	25	0.8	2	3	0.18	0.27
Bruenn	13	25	0.8	2	2	0.17	0.26
Burrows	11.1	21	1.0	0.8	2	0.24	0.38
Janka	14	22	0.8	2	2	0.23	0.35

ing a typical value $f(\theta)=0.2$ into (6) we find

$$\kappa<\kappa_m \leq 6\times10^{-10}\left[\frac{1\ \text{eV}^2}{\Delta m^2}\right]\left[\frac{E}{10\ \text{MeV}}\right]\left[\frac{R_\odot}{l_\rho}\right]. \tag{7}$$

For $l_\rho=R_\odot$ one obtains from (7) that $\kappa_R=1$ (strong adiabaticity violation at resonance densities) at

$$\Delta m_a^2=\begin{cases}6\times10^{-10}\ \text{eV}^2 & (E=10\ \text{MeV}),\\ 3\times10^{-9}\ \text{eV}^2 & (E=50\ \text{MeV}).\end{cases} \tag{8}$$

Note that l_ρ may change from 0.1 $R_\odot$ in the region of large densities, $\rho\sim10^4$ g/cm^3, to $(3-4)R_\odot$ at small densities, $\rho\sim10^{-4}$ g/cm^3 [35].

The mass Δm_a^2 defines two extreme cases. *Adiabatic* case: $\Delta m^2 \gg \Delta m_a^2$; the adiabaticity condition is satisfied everywhere in the star. *Nonadiabatic* case: $\Delta m^2 \ll \Delta m_a^2$. Here the adiabaticity is strongly broken in the region around ρ_R, where the mixing angle varies from $\theta_m\approx0$ to $\theta_m\approx\theta$. As we will see, the dynamics of propagation in these extreme cases is simple and the results are essentially independent of the density distribution in the star. Moreover the permutation factor is practically independent of neutrino energy. Fortunately, the Δm^2 regions of interest fit these two extreme cases. The atmospheric neutrino region as well as large mixing MSW solutions are in the adiabatic domain; the "just-so" solution lies in the nonadiabatic domain.

(1) In the *adiabatic case* the neutrino state which is produced as $\bar{\nu}_e\cong\bar{\nu}_{1m}$ will everywhere practically coincide with $\bar{\nu}_{1m}$ since there are no $\bar{\nu}_{1m}\leftrightarrow\bar{\nu}_{2m}$ transitions. So the neutrino leaves the star as $\bar{\nu}_{1m}(\rho=0)$, which is the state with definite mass $\bar{\nu}_1$. No oscillations will take place on the way from the star to the Earth and the neutrino state arriving at the Earth will be $\bar{\nu}_1$. Consequently, the probability of $\bar{\nu}_e\to\bar{\nu}_\mu$ transition (permutation factor) in this case equals $p_a=|\langle\bar{\nu}_\mu|\bar{\nu}_1\rangle|^2=\sin^2\theta$ (see also [19]).

In the region of mass squared difference $\Delta m^2=(10^{-4}-10^{-7})$ eV^2, the permutation factor must be corrected for the effect of neutrino oscillations inside the Earth. Neutrino trajectories from SN 1987A to terrestrial detectors lie in the mantle of the Earth, where the density changes rather slowly. Therefore, to a good approximation, one can consider the Earth-matter effect as neutrino oscillations in matter with constant density (knowing the positions of the detectors in the moment of neutrino burst detection [14,15] and density profile of the Earth one finds $\rho_{\text{IMB}}=4.6$ g/cm^3 for IMB and $\rho_{\text{K}}=3.4$ g/cm^3 for Kamiokande-II). Neutrinos arrive at the Earth as two incoherent beams: ν_1 flux with energy spectrum $F_0(\bar{\nu}_e)$ and ν_2 with energy spectrum $F_0(\bar{\nu}_\mu)$. Considering then the ν_1-ν_2 oscillations in the matter of the Earth, one finds the permutation factor

$$p_a=\sin^2\theta-\sin2\theta_m\sin2(\theta-\theta_m)\sin^2\frac{\pi x}{l_m}, \tag{9}$$

where $\theta_m=\theta_m(\rho_i,E/\Delta m^2,\theta)$ ($i=$IMB or K) is the mixing angle in the matter of the Earth, x is the length of the neutrino trajectory inside the Earth ($x_{\text{K}}=3.9\times10^8$ cm, $x_{\text{IMB}}=8.4\times10^8$ cm for Kamiokande and IMB detectors, respectively), and

$$l_m=l_0\left[\left[1+\frac{\rho_R}{\rho}\right]^2+\left[\frac{\rho_R}{\rho}\right]^2\tan^2 2\theta\right]^{-1/2} \tag{10}$$

is the oscillation length in matter. Here $\rho_R=\rho_R(\rho_i,E/\Delta m^2,\theta)$ is defined in Eq. (4), $Y_e\approx0.5$. The first term on the right-hand side of Eq. (9) corresponds to the adiabatic result without the Earth effect. The second term is the Earth correction. Since for the nonresonant channel $\theta>\theta_m$, this second term is always negative. Therefore, the Earth-matter effect weakens the permutation and relaxes the restriction on mixing. The oscillation length is always smaller than the refraction length. Moreover, at small $E/\Delta m^2$ (big Δm^2), the oscillation length is much smaller than l_0.

The second term in (9) is an oscillating function of x as well as $E/\Delta m^2$. The amplitude of the oscillations, $\sin2\theta_m\sin2(\theta-\theta_m)$, reaches the maximal value, $\sin^2\theta$, at $\theta_m=\theta/2$. If at this point one has $x/l_m=\pi n$ (n is an integer), then the Earth effect completely compensates the effect in the star and $p_a=0$. The condition for the amplitude of the correction to be a maximum, which can be written as $\rho_R(E/\Delta m^2)=\rho\cos2\theta$, defines the Δm^2 region of strong Earth-matter effect. Taking into account that the interval of neutrino energies of interest is 10–50 MeV, we obtain that this region extends over 3 orders of magnitude around $\Delta m^2\approx10^{-5}$ eV^2: $\Delta m^2=(10^{-7}-10^{-4})$ eV^2. At $\rho_R\gg\rho\cos2\theta$ and $\rho_R\ll\rho\cos2\theta$, the matter mixing angle is, respectively, $\approx\theta$ or ≈0 and therefore the correction is negligibly small.

In the region $\Delta m^2>10^{-5}$ eV^2, the correction is a rapidly oscillating function of the neutrino energy. One can average over these oscillations, by integrating over the neutrino distribution function to yield an average $\bar{p}$, which is used in Fig. 3. Here $\theta_m=\theta_m(\Delta m^2,\bar{E},\bar{\rho})$, where $\bar{E}=(\bar{E}_e+\bar{E}_\mu)/2\approx20$ MeV and $\bar{\rho}=(\rho_{\text{K}}+\rho_{\text{IMB}})/2\approx4.0$ g/cm^3. At $\Delta m^2\leq10^{-5}$ eV^2, the approximation $p\approx$const is not valid and one must compare directly the observed distribution and the predicted one with an energy-dependent oscillation factor. In this case the Earth effect strongly depends on neutrino energy and is different for different detectors. One can use this feature to explain some difference in the energy distributions of the Kamiokande and the IMB signals [20]. Figure 4 depicts the upper bounds on $\sin^2 2\theta$ obtained with neutrino spectra from [25].

(2) *Nonadiabatic case.* If $\kappa\gg1$, neutrinos propagate nonadiabatically in the region of strong change of the mixing angle ($\rho\sim\rho_R$). The adiabaticity starts to be broken at $\rho\gg\rho_R$, where the mixing is rather small, $\theta_m\approx0$. As the first approximation, one can neglect the change of θ_m in the initial adiabatic stage counting $\theta_m=0$, and consider just the vacuum oscillations of $\bar{\nu}_e$ in the star and on the way from the star to the Earth. In this case, the permutation factor coincides with the "vacuum" permutation factor:

$$p_{\text{na}}\approx p_{\text{vac}}\equiv\tfrac{1}{2}\sin^2 2\theta. \tag{11}$$

Consider the effect of the adiabatic transformation of the neutrinos in the initial stage. Let ρ_a be the density at which $\kappa=1$ [see Eq. (5)]. Then, the neutrino flavor changes adiabatically at $\rho \gg \rho_a$; in the region $\rho \sim \rho_a$, flavor changes nonadiabatically, and at $\rho \ll \rho_a$, where $\kappa \gg 1$, one can consider just vacuum oscillations. Even if the adiabaticity is restored at $\rho \ll \rho_R$, the matter effect in this region (especially at big mixings) is negligibly small. To estimate the effect of adiabatic and nonadiabatic conversion, one can (simplifying the picture) consider the propagation before ρ_a ($\rho \geq \rho_a$) as pure adiabatic and after ρ_a ($\rho \leq \rho_a$) as strongly nonadiabatic, i.e., as oscillations in vacuum. If θ_a is the mixing angle at ρ_a: $\theta_a = \theta_m(\rho_a)$, then the neutrino state which adiabatically arrives at ρ_a can be written as $\nu_a \approx \nu_{1m} \equiv \cos\theta_a \nu_e - \sin\theta_a \nu_\mu$. Considering vacuum oscillations of ν_a in the region $\rho < \rho_a$, as well as on the way from the star to the Earth, one gets

$$p_{\rm na} = \tfrac{1}{2}[1-\cos 2(\theta-\theta_a)\cos 2\theta] \ . \qquad (12)$$

The Earth-matter effect in the nonadiabatic domain ($\Delta m^2 < 10^{-9}$ eV^2) can be neglected due to strong suppression of mixing. In the limits of very strong adiabaticity violation ($\rho_a \gg \rho_R$ and $\theta_a \cong 0$), Eq. (12) reduces to the pure vacuum oscillation result (11). In the opposite case, when the adiabaticity condition is satisfied everywhere up to zero densities ($\theta_a = \theta$), Eq. (12) reproduces the pure adiabatic permutation factor. At $\rho_a \gg \rho_R$, the condition for θ_a can be written as [see Eq. (5)]

$$\sin^2 2\theta_a \approx 4\pi \sin 2\theta \frac{l_\rho}{l_\nu} \ . \qquad (13)$$

In the region of the "just-so" solution, $\kappa_R \approx 5$; i.e., the adiabaticity condition is strongly violated. Using Eqs. (12) and (13), one finds that at $l_\rho = (1-3)R_\odot$ and $E > 20$ MeV the permutation factor decreases by (3–5)% in comparison with the vacuum value. The dependence of the correction on energy is very weak.

According to Eqs. (9)–(12), the adiabatic permutation factor is always smaller than the nonadiabatic and the vacuum (or strongly nonadiabatic) permutation factors: $p_a \leq p_{\rm na} \leq p_{\rm vac}$. Note that in the nonresonant case the nonadiabatic transition results in a stronger effect than the adiabatic transition. The adiabatic permutation factor can be used to obtain the lower limit of the permutation effect.

Using the relations in Eqs. (9) and (11) we find the upper limits on $\sin^2 2\theta$ corresponding to different upper bounds on p. In the extreme cases,

$$\sin^2 2\theta \leq \begin{cases} 4p(1-p) & \text{(adiabatic, without Earth effect)}, \\ 2p & \text{(strongly nonadiabatic)}. \end{cases} \qquad (14)$$

For the upper bounds given in Eq. (2) we get the following upper limits on mixing angle at 99% C.L.:

$$\sin^2 2\theta \leq \begin{cases} 0.9 & (\Delta m^2 \gg 10^{-9}\ \text{eV}^2), \\ 0.7 & (\Delta m^2 \ll 10^{-9}\ \text{eV}^2). \end{cases} \qquad (15)$$

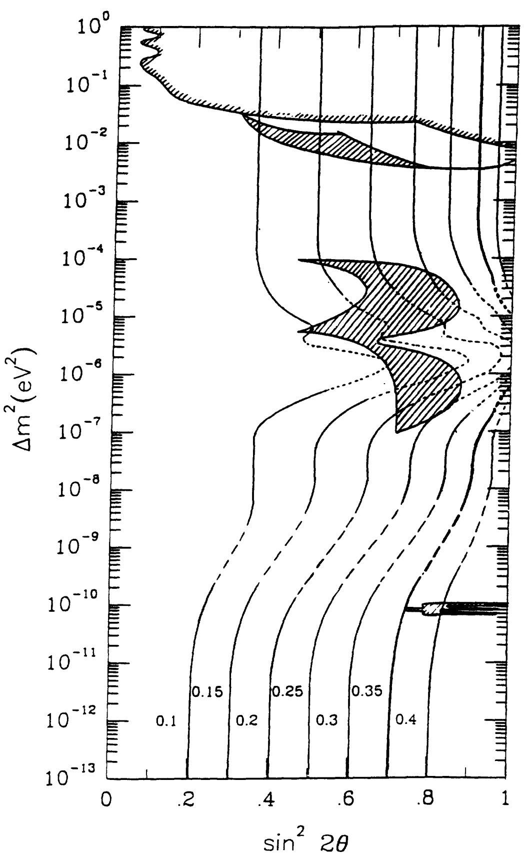

FIG. 3. The excluded regions of the neutrino parameters (to the right from the curves) for different upper bounds on permutation factor (figures at the curves). Bold lines correspond to 99% upper bound obtained from SN 87A [see Eq. (2)]. In the region $\Delta m^2 = (10^{-8}-10^{-9})$ eV^2 the restrictions (shown by dashed lines) may appreciably depend on density distribution in the star. In the region $\Delta m^2 = (10^{-5}-10^{-6})$ eV^2 the approximation $p=$const does not work due to Earth matter effect (dotted lines). Also the regions of the solar neutrino problem solutions by the MSW effect and "just-so" oscillation, as well as the region responsible for atmospheric muon neutrino deficit are shown. Shadowed curve depicts the upper bound on neutrino parameters from the reactor oscillation experiments.

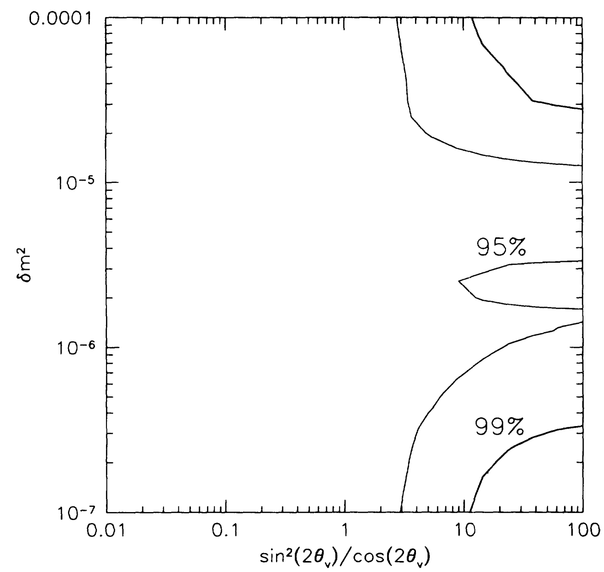

FIG. 4. The upper bounds on the mixing angle in the region of a strong Earth effect. The original spectra by MWS [25] are used.

These results are exhibited in Fig. 3.

In the case of three neutrino mixing, the permutation factor is determined by the elements of the mixing matrix U_{ei} $(i=1,2,3)$: $p_a=1-|U_{e1}|^2$ in the adiabatic limit, and $p_{na}=1-\sum_{i=1,2,3}|U_{ei}|^4$ in the strongly nonadiabatic limit.

For the resonant channel (neutrino transitions $\nu_e-\nu_\mu$, or antineutrino transitions in the case of inverse mass hierarchy), the permutation factor can be found from the result obtained above: $p_a^{\rm res}=1-p_a$. Now $p_a^{\rm res}=\cos^2\theta$ without Earth-matter effects and the Earth decreases again the transition, because now $\theta_m>\theta$ [see Eq. (9)]. In vacuum, $p_{\rm vac}^{\rm res}=p_{\rm vac}$, if there is no CP violation. For the resonant channel, the relation between different permutation factors reads $p_a>p_{\rm na}>p_{\rm vac}$.

V. DISCUSSION

(1) We have obtained an upper bound on the permutation parameter p [see Eq. (2)] using observational data on the neutrino burst from SN 1987A and the original neutrino spectra predicted by neutrino burst models that describe well the observed luminosity and the burst duration. We have derived the relation between the permutation factor and the vacuum mixing angle and have shown that this relation is practically independent of the structure of the star in physically interesting regions of neutrino parameter space. The relation allows one to set upper bounds on the lepton mixing angle (see Fig. 3). The excluded region of neutrino parameters covers the region of the "just-so" solution of the solar neutrino problem, part of the region of the large-mixing-angle MSW solution, and part of the region of $\nu_e-\nu_\mu$ oscillations which could be responsible for atmospheric muon neutrino deficit.

(2) The upper limit on p derived here can be directly applied to any transformations of $\bar\nu_e$ to $\bar\nu_\mu$, or ν_μ, or $\bar\nu_\tau$, or ν_τ which are independent of, or only weakly depend on, the neutrino energy. Spin-flavor conversion, $\bar\nu_{eR}\leftrightarrow\nu_{\mu L}$ (or ν_τ), can result in spectra permutation with p up to $\frac{1}{2}$. This maximal value could be realized if there is some region inside the star in which the interaction with the magnetic field B dominates over the vacuum and the matter effects: $\mu B \gg G_F\rho/m_N, \Delta m^2/E$, and the neutrino propagates up to this region adiabatically. The limit on p set in this paper can be converted to a limit on the product $\mu B(r)$, although this restriction depends sensitively on the structure of the star.

If neutrino mixing is induced by some flavor off-diagonal interaction with the ambient medium ("massless oscillations") [36], then both neutrino and antineutrino channels can be resonant. In this case p may be bigger than $\frac{1}{2}$ [37]. The upper bound set here on p translates into the upper bounds on the coupling constants of the new interaction [37].

(3) The upper bounds on p, and therefore on lepton mixing, depend strongly on the parameters of the original neutrino spectra (Fig. 2). The integral characteristics of the neutrino burst (such as total energies emitted in neutrinos, or the average energy of time integrated spectra) are determined in large part by the initial mass of iron core, $M_{\rm Fe}$, and are independent of most details of the model or of the explosion mechanism [25]. Since $M_{\rm Fe}$ and the duration of the neutrino burst are fixed by observations, the integral parameters of the neutrino burst can, in principle, be strongly constrained. The difference in fluxes and the average energies of neutrinos of different species are determined by the known difference in interactions of these neutrinos. Moreover, the effective temperatures of neutrino spectra enter as T^4 in the luminosity and as T^5 in the interaction rates. This means that small changes in T imply appreciable changes of other characteristics of the supernova; this circumstance is reflected in the relatively small spread of calculated model parameters (see Table I).

It is of great importance that supernova modelers refine their predictions for integral characteristics of neutrino energy spectra. One needs to find the reliable regions, as well as the allowed limits, for parameters characterizing the energy spectra by making use of all available information on SN 1987A (excluding, of course, the information on the neutrino burst).

(4) Future observations of SN 1987A (light curve, possible manifestations of remnant), when combined with improvements in the theory of neutrino transport and supernova explosion may allow one to make stronger inferences. Better statistics from a future neutrino burst would make it possible to look for the deviations from simple Fermi-Dirac spectra modified by a "chemical potential," especially the appearance of a high-energy tail. Confronting the model calculations with data on both the charged current and on the neutral current interactions [as is possible with the Sudbury Neutrino Observatory (SNO), Large Volume Detector (LVD), Super-Kamiokande] would sharpen the conclusions of the paper.

(5) We have described in this paper a method of constraining lepton mixing using data on a neutrino burst produced by gravitational collapse. Depending on the skepticism of the reader, the results obtained from SN 1987A can be considered as either a demonstration of the method or as an indication that large-angle lepton mixing is excluded.

ACKNOWLEDGMENTS

We would like to thank E. Kh. Akhmedov, A. Burrows, A. Dar, M. Fukugita, S. P. Mikheyev, D. N. Schramm, and G. Senjanovic for valuable discussions. One of us (A.S.) is grateful to H. Suzuki and H.-T. Janka for sending their data and for extensive comments. A.S. was supported in part by the National Science Foundation (Grant No. NSF PHY92-45317) and the Ambrose Monell Foundation. D.N.S. was supported in part by NSF Grant No. AST 88-58145 and by NASA Grant No. NAGW2448.

[1] R. Davis, talk given at the International Symposium on Neutrino Astrophysics, Takayama/Kamioka, Japan, 1992 (unpublished), p. 47; K. S. Hirata *et al.*, Phys. Rev. Lett. **66**, 9 (1991); Phys. Rev. D **44**, 2241 (1991); SAGE Collaboration, A. I. Abazov *et al.*, Phys. Rev. Lett. **67**, 3332 (1991); GALLEX Collaboration, P. Anselmann *et al.*, Phys. Lett. B **85**, 376 (1992).

[2] J. N. Bahcall and M. H. Pinsonneault, Rev. Mod. Phys.

64, 885 (1992); I.-J. Sackmann *et al.*, Astrophys. J. **360**, 727 (1990); R. Sienkiewicz, J. N. Bahcall, and B. Paczynski, *ibid.* **349**, 641 (1989); J. N. Bahcall and R. K. Ulrich, Rev. Mod. Phys. **60**, 297 (1988); S. Turck-Chieze *et al.*, Astrophys. J. **335**, 415 (1988); in *Neutrino '92,* Proceedings of the XVth International Conference on Neutrino Physics and Astrophysics, Granada, Spain, 1992, edited by A. Morales [Nucl. Phys. B (Proc. Suppl.) **31**, (1993)].

[3] V. N. Gribov and B. M. Pontecorvo, Phys. Lett. **28**, 493 (1967); J. N. Bahcall and S. C. Frautschi, Phys. Lett. **29B**, 623 (1969); V. Barger, R. J. N. Phillips, and K. Whisnant, Phys. Rev. D **24**, 538 (1981); S. L. Glashow and L. M. Krauss, Phys. Lett. B **190**, 199 (1987).

[4] V. Barger, R. J. N. Phillips, and K. Whisnant, Phys. Rev. D **43**, 1110 (1991); A. Acker, S. Pakvasa, and J. Pantaleone, *ibid.* **43**, 1754 (1991); P. I. Krastev and S. T. Petcov, Phys. Lett. B **285**, 85 (1992).

[5] S. P. Mikheyev and A. Yu. Smirnov, Yad. Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1985)]; Prog. Part. Nucl. Phys. **23**, 41 (1989); L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); **20**, 2634 (1979); T. K. Kuo and J. Pantaleone, Rev. Mod. Phys. **61**, 937 (1989).

[6] P. Anselmann *et al.*, Phys. Lett. B **285**, 389 (1992); S. A. Bludman, D. C. Kennedy, and P. G. Langacker, Phys. Rev. D **45**, 1810 (1992); S. A. Bludman, N. Hata, D. C. Kennedy, and P. G. Langacker, *ibid.* **47**, 2220 (1993); X. Shi and D. N. Schramm, Phys. Lett. B **283**, 305 (1992); X. Shi, D. N. Schramm, and J. N. Bahcall, Phys. Rev. Lett. **69**, 717 (1992); J. M. Gelb, W. Kwong, and S. P. Rosen, *ibid.* **69**, 1846 (1992); W. Kwong and S. P. Rosen, *ibid.* **68**, 748 (1992); A. Yu. Smirnov, ICTP Report No. IC/92/429, 1992 (unpublished); P. I. Krastev and S. T. Petcov, Phys. Lett. B **299**, 99 (1993); L. M. Krauss, E. Gates, and M. White, *ibid.* **298**, 94 (1993); Phys. Rev. D **46**, 1263 (1992); Phys. Rev. Lett. **70**, 375 (1993).

[7] K. S. Hirata *et al.*, Phys. Lett. B **280**, 146 (1992); R. Becker-Szendy *et al.*, Phys. Rev. D **46**, 3720 (1992). The atmospheric neutrino deficit can be explained also by $\nu_\mu - \nu_\tau$ oscillations, but supernova data do not give bounds on this channel.

[8] A. Zee, Phys. Lett. **93B**, 389 (1980); K. S. Baby and E. Ma, Phys. Lett. B **228**, 508 (1989); W. Grimus and H. Neufeld, Nucl. Phys. **B325**, 18 (1989).

[9] S. M. Bilenky and S. T. Petcov, Rev. Mod. Phys. **59**, 671 (1987); J. W. F. Valle, Prog. Part. Nucl. Phys. **26**, 91 (1991).

[10] C. H. Albright, Phys. Rev. D **43**, R3595 (1991); **45**, R725 (1992).

[11] A. Yu. Smirnov, in *Proceedings of the Joint International Lepton-Photon Symposium and Europhysics Conference on High Energy Physics,* Geneva, Switzerland, 1991, edited by S. Hegarty, K. Potter, and E. Quercigh (World Scientific, Singapore, 1992), Vol. 1, p. 623.

[12] R. Barbieri, J. Ellis, and M. K. Gaillard, Phys. Lett. **90B**, 249 (1980).

[13] E. Kh. Akhmedov, Z. G. Berezhiani, and G. Senjanovič, Phys. Rev. Lett. **69**, 3013 (1992).

[14] K. Hirata *et al.*, Phys. Rev. Lett. **58**, 1490 (1987).

[15] R. M. Bionta *et al.*, Phys. Rev. Lett. **58**, 1494 (1987).

[16] E. N. Alexeyev *et al.*, Phys. Lett. B **205**, 209 (1988).

[17] L. Wolfenstein, Phys. Rev. D **20**, 2634 (1979); P. Reinartz and L. Stodolsky, Z. Phys. C **27**, 507 (1985).

[18] S. P. Mikheyev and A. Yu. Smirnov, Zh. Eksp. Teor. Fiz. **91**, 7 (1986) [Sov. Phys. JETP **64**, 4 (1986)]; in *Massive Neutrinos in Astrophysics and Particle Physics,* Proceedings of the Sixth Moriond Workshop, Tignes, France, 1986, edited by O. Fackler and J. Tran Thanh Van (Editions Frontieres, Gif-sur-Yvette, 1986), p. 355; G. M. Fuller *et al.*, Astrophys. J. **322**, 795 (1987); G. M. Fuller *et al.*, *ibid.* **389**, 517 (1992).

[19] L. Wolfenstein, Phys. Lett. B **194**, 197 (1987).

[20] A. Yu. Smirnov, in *Proceedings of the Twentieth International Cosmic Ray Conference,* Moscow, USSR, 1987, edited by V. A. Kozyarivsky (Nauka, Moscow, 1987); see [5].

[21] J. A. Frieman, H. E. Haber, and K. Freese, Phys. Lett. B **200**, 115 (1988).

[22] L. M. Krauss, Nature (London) **329**, 689 (1987); A. E. Chudakov, Ya. S. Elensky, and S. P. Mikheyev, in *Cosmic Gamma Rays, Neutrinos and Related Astrophysics,* Proceedings of the NATO Advanced Study Institute, Erice, Italy, 1988, edited by M. M. Shapiro and J. P. Wefel, NATO ASI Series C: Vol. 270 (Kluwer Academic, Dordrecht, 1989), p. 131; T. K. Kuo and J. Pantaleone, Phys. Rev. D **37**, 298 (1988).

[23] V. S. Imshennik and D. K. Nadjozhin, Sov. Astrophys. Space Phys. Rev. **8**, 1 (1989).

[24] S. E. Woosley, J. R. Wilson, and R. Mayle, Astrophys. J. **302**, 19 (1986).

[25] R. Mayle, J. R. Wilson, and D. Schramm, Astrophys. J. **318**, 288 (1987); J. Wilson, in *Weak Interactions and Neutrinos,* Proceedings of the 12th International Symposium, Ginosar, Israel, 1989, edited by P. Singer and B. Gad Eilam [Nucl. Phys. B (Proc. Suppl.) **13**, 380 (1990)].

[26] S. W. Bruenn, Phys. Rev. Lett. **59**, 938 (1987).

[27] P. M. Giovanoni, P. C. Ellison, and S. W. Bruenn, Astrophys. J. **342**, 416 (1989).

[28] A. Burrows, Astrophys. J. **334**, 891 (1988).

[29] E. S. Myra and A. Burrows, Astrophys. J. **364**, 222 (1990).

[30] A. Burrows, D. Klein, and R. Gandhi, Phys. Rev. D **45**, 3361 (1992).

[31] H.-T. Janka and W. Hillebrandt, Astron. Astrophys. Suppl. **78**, 375 (1989); Astron. Astrophys. **224**, 49 (1989).

[32] H.-T. Janka, Ph.D. thesis, Max-Plank-Institut fur Astrophysik, 1991 (unpublished); in *Proceedings of the Workshop on Frontier Objects in Astrophysics and Particle Physics,* Vulcano, Italy, 1992, edited by F. Giovanneli and G. Mannocchi (Societa Italianadi Fisica, in press); MPA Report 720.

[33] H. Suzuki, in *Proceedings of International Workshop on Unstable Nuclei in Astrophysics,* edited by S. Kubono and T. Kajino (World Scientific, Singapore, 1992), p. 281; talk given at the International Symposium on Neutrino Astrophysics [1].

[34] W. Hillebrandt and R. G. Wolff, in *Nucleosynthesis—Challenges and New Developments,* edited by W. D. Arnett and J. W. Truran (University of Chicago Press, Chicago, 1985), p. 131.

[35] K. Nomoto and M. Hashimoto, Phys. Rep. **163**, 13 (1988).

[36] Wolfenstein, [5]; M. M. Guzzo, A. Masiero, and S. T. Petcov, Phys. Lett. B **260**, 154 (1991); V. Barger, R. J. N. Phillips, and K. Whisnant, Phys. Rev. D **44**, 1629 (1991).

[37] P. I. Krastev, S. Petcov, and A. Yu. Smirnov, in *Proceedings of the 4th International Symposium on Neutrino Telescopes,* Venice, Italy, 1992, edited by M. Baldo-Ceolin (University of Padua, 1992), p. 68.