PHYSICAL REVIEW D VOLUME 49, NUMBER 3 1 FEBRUARY 1994

Effective Lagrangian approach to precision measurements: Anomalous magnetic moment of the muon

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(Received 10 August 1992)

We investigate the use of effective Lagrangians to describe the effects on high-precision observables of physics beyond the standard model. Using the anomalous magnetic moment of the muon as an example, we detail the use of effective vertices in loop calculations. We then provide estimates of the sensitivity of new experiments measuring the muon's $g-2$ to the scale of physics underlying the standard model.

PACS number(s): 13.40.Em, 11.15.Bt, 11.15.Tk, 14.60.Ef

I. INTRODUCTION

One of the time-honored methods of probing physics beyond the known realm is to perform highly accurate measurements of precisely predicted observables. Rather than observing new particles directly, we infer their existence from the effects they have on known particles; interactions unexplainable within the framework of the accepted theory (which in our case is the standard model) imply new physics. This type of experiment, by its very nature, cannot unambiguously discriminate among the manifold possible models which extend the standard model to higher energies; there may be several competing models which affect measured quantities in the same way, but no observable to which only one of the models contributes.

Faced with this situation, it becomes questionable to try to understand the low energy effects of new physics on a model-by-model basis. The best approach is to follow a characterization which is sufficiently general to encompass all types of high-energy physics. Such a characterization is readily available in an effective Lagrangian approach [1,2], which has recently been advocated [3,4,5] as a model- and process-independent parametrization of deviations from the standard model.

The effective Lagrangian method should be contrasted with an approach in which a given model (or set of models) extending the standard model is chosen, and the effects on low energy observables are calculated. The specific model approach determines all corrections to the standard model in terms of a few couplings and masses. The effective Lagrangian approach parametrizes its predictions in terms of the coefficients of effective operators. Therefore we are faced with a tradeoff: the requirement of model independence increases the number of unknown parameters whose order of magnitude can at best be estimated. The results of the effective Lagrangian approach are very useful in determining, in a model-independent manner, the sensitivity of a given experiment to new physics, and can be used to isolate those observables most sensitive to possible new interactions.

The basic idea of the method is that processes below some energy Λ can be described by effective operators consisting of fields with masses below Λ. From these operators we hope to infer the existence of particles with masses above Λ. Thus, so long as we are below all new particle thresholds, any type of new physics can be parametrized by a series of effective operators involving standard-model particles. It is important to note that any given model will produce operators which respect the (exact) symmetries of the standard model, and that the best we can hope for from high precision measurements are statements regarding the coefficients of these operators.

The underlying physics is described by a high-energy Lagrangian, out of which all excitations with masses above ~ 100 GeV (which we label "heavy") are integrated out; what remains will be the standard model, plus an infinite series of effective operators. These must be gauge invariant[1] and describe the low-energy remnants of the full high-energy theory. We will denote by Λ the (large) energy scale at which the new physics first directly manifests itself.

There are two possible types of high-energy physics to consider: that which decouples from low-energy physics and that which does not. In the decoupling scenario, the masses of heavy degrees of freedom are large because a

[1]This is not a trivial result: Its derivation requires the introduction of a gauge-fixing technique which produces a manifestly gauge-invariant effective action [6]. This does not yield a gauge-independent effective action, but, although we have not seen it demonstrated for spontaneously-broken gauge theories, this can apparently be arranged [7]. This can be realized simply by choosing the Landau gauge among the class of gauges discussed in Ref. [6].

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dimensionful parameter respecting the symmetries of the theory is large. In this case, the decoupling theorem [1] tells us that at energies $E \ll \Lambda$ observable corrections to the low-energy theory are suppressed by powers of $1/\Lambda$ (times possible powers of $\ln\Lambda$). In the nondecoupling case, the masses of the heavy degrees of freedom in the theory are large because some dimensionless coupling constant is large. An example of this is a heavy fermion which gets its mass from spontaneous symmetry breaking and becomes heavy due to a relatively large Yukawa coupling.[2] In this case, the contributions due to physics above Λ need not be suppressed by powers of $1/\Lambda$; the corrections to standard-model processes are given by a chiral expansion in powers of p/Λ, where p is a typical momentum for the process at hand [8].

The application of effective-Lagrangian techniques to the case of high precision measurements contains a new complication, for the effective vertices can appear within loops and thereby produce new divergences. But the effective Lagrangian is completely renormalizable—power counting arguments similar to those involved in proofs of renormalizability show that all divergences multiply local operators. Since the effective Lagrangian includes all operators respecting the symmetries of the theory, such divergences simply renormalize the bare coupling constants.[3] Their only effect (associated with the logarithmic divergences) is to determine the renormalization-group running of the effective couplings. This is a well-known fact and has been applied in the context of the strong interactions [9].

We will develop and apply the techniques of effective Lagrangians using the anomalous magnetic moment of the muon as an example. This is an especially interesting observable because the Brookhaven Alternating Gradient Synchrotron experiment AGS 821 [10] is expected to achieve a precision greater than that required to observe the standard-model contributions. The best present measurement of a_μ comes from a series of experiments at CERN [11]:

$$a_{\text{CERN}} = 11\,659\,230(84)\times 10^{-10} \; . \tag{1.1}$$

The calculated effects on a_μ of weak interactions are [12]

$$a_{\text{weak}} \simeq 19.5\times 10^{-10} \; , \tag{1.2}$$

too small to be seen in the CERN experiment, but well within the projected accuracy of the AGS experiment, which should measure a_μ with an accuracy of 4×10^{-10}.

Several authors [13] have considered the effects on a_μ of gauge-boson anomalous magnetic dipole and electric quadrupole moments. We will reexamine this problem using the effective-Lagrangian formalism to discuss the effects of high-energy physics on these constants, and will examine the previous results from this point of view.

Regardless of whether or not the heavy excitations decouple, the contributions to a_μ produced by the underlying interactions can be classified in three types: δa_μ^1, produced by loops containing the effective operators; δa_μ^2, produced by the modification of the gauge boson eigenstates; $\delta a_\mu^{\text{direct}}$, produced by effective operators of the type $\bar{\mu}\sigma_{\mu\nu}\mu F^{\mu\nu}$. The contribution δa_μ^2 is due to the fact that the new interactions modify the quadratic part of the Lagrangian, so that a rediagonalization of these terms is required. We will consistently use this classification below.

II. DECOUPLING CASE

As previously mentioned in the decoupling scenario the low-energy effective Lagrangian can be written as a power series in $1/\Lambda$:

$$\mathcal{L}_{\text{eff}} = \sum_{n=0}^{\infty} \frac{1}{\Lambda^n} \alpha_{\mathcal{O}} \mathcal{O}^{(n+4)} \; , \tag{2.1}$$

where the operators $\mathcal{O}^{(n+4)}$ have dimension $[\text{mass}]^{(n+4)}$, are $SU(2)_L \times U(1)_Y$ gauge-invariant, and contain only standard-model fields. The constants $\alpha_{\mathcal{O}}$ (which must be renormalized) determine the strength of the contribution of $\mathcal{O}$. We shall assume that in this expansion $\mathcal{O}^{(4)}$ is equal to the standard model, which then takes the status of an effective Lagrangian valid for energies much less than Λ; this is true only in the decoupling case. As the structure of the interactions underlying the standard model are presently unknown, the coefficients $\alpha_{\mathcal{O}}$ cannot be evaluated; nonetheless, their order of magnitude can be estimated [8].

The determination of the contribution to a_μ from dimension-six operators is simplified by the constraint that these operators must be gauge-invariant. Additionally, we may use the classical equations of motion to remove some redundant operators [14]. Buchmüller and Wyler [15] have compiled a list of all possible gauge-invariant terms (assuming lepton and baryon number conservation) in an effective Lagrangian to order $1/\Lambda^2$. They find that there are no dimension-five operators, and 81 operators of dimension-six (for a single fermion family).

In determining $\delta a_\mu^{1,2}$ [see (1.3)], we will not treat all dimension-six operators, but will instead focus on those operators which give rise to anomalous three-gauge-boson- or two-gauge-boson-Higgs-boson couplings, and thereby to anomalous gauge-boson dipole and electric quadrupole moments. There are four such operators in [15]:

$$\begin{aligned}
\mathcal{O}_W &= -\epsilon_{IJK} W_\mu^{I\nu} W_\nu^{J\lambda} W_\lambda^{K\mu} \; , \\
\mathcal{O}_{WB} &= (\phi^\dagger \tau^I \phi) W_{\mu\nu}^I B^{\mu\nu} \; , \\
\mathcal{O}_{\phi W} &= \frac{1}{2}(\phi^\dagger\phi) W_{\mu\nu}^I W^{I\mu\nu} \; , \\
\mathcal{O}_{\phi B} &= \frac{1}{2}(\phi^\dagger\phi) B_{\mu\nu} B^{\mu\nu} \; .
\end{aligned} \tag{2.2}$$

$\mathcal{O}_W$ and $\mathcal{O}_{WB}$ contribute to Fig. 1(a) (there are three additional graphs where the W bosons are replaced by the corresponding would-be Goldstone bosons). $\mathcal{O}_{WB}$, $\mathcal{O}_{\phi W}$, and $\mathcal{O}_{\phi B}$ contribute to Figs. 1(b) and 1(c). In each figure,

[2] The coupling may still be within the perturbative regime.

[3] Note that this does not require that the underlying theory be renormalizable.

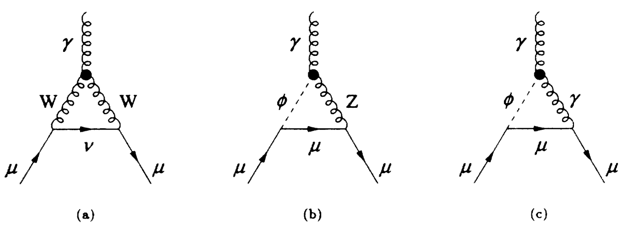

FIG. 1. Contribution to a_μ from the triple-boson vertex. The heavy dot denotes a $\mathcal{O}_{WB}$ or $\mathcal{O}_W$ vertex. There are three additional graphs where the W bosons are replaced by the corresponding would-be Goldstone bosons.

a heavy dot denotes one of these effective operators.

We must also consider the two operators which can give a direct contribution to δa_μ:

$$\mathcal{O}_{\mu W} = -(\bar{L}\sigma^{\mu\nu}\tau^I \mu_R)\phi W^I_{\mu\nu} + \text{H.c.} \,,$$
$$\mathcal{O}_{\mu B} = -(\bar{L}\sigma^{\mu\nu}\mu_R)\phi B_{\mu\nu} + \text{H.c.} \,, \tag{2.3}$$

where $L = \binom{\nu_\mu}{\mu}$. Our notation is

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g\epsilon_{\text{abc}} W^b_\mu W^c_\nu$$

for the $SU(2)_L$, $B_{\mu\nu}$ for the $U(1)_Y$ field strength, and

$$D_\mu = \partial_\mu + (i/2) g \tau^a W^a_\mu + i g' Y B_\mu$$

(Y being the hypercharge) for the covariant derivative. ϕ denotes the standard-model scalar doublet; its vacuum expectation value is

$$\langle \phi \rangle = \begin{bmatrix} 0 \\ v/\sqrt{2} \end{bmatrix} .$$

Our effective low-energy bare Lagrangian will therefore be

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2}(\alpha_W \mathcal{O}_W + \alpha_{WB}\mathcal{O}_{WB} + \alpha_{\phi W}\mathcal{O}_{\phi W} + \alpha_{\phi B}\mathcal{O}_{\phi B} + \alpha_{\mu W}\mathcal{O}_{\mu W} + \alpha_{\mu B}\mathcal{O}_{\mu B}) \,. \tag{2.4}$$

The above parametrization in terms of the $\alpha_{\mathcal{O}}$ is not standard in the literature, where the description of $\mathcal{L}_{\text{eff}}$ in terms of parameters frequently called κ and λ are used [16]. We will see that α_W is proportional to λ, and α_{WB} to $(\kappa - 1)$.

When we replace ϕ by $v/\sqrt{2}$, the effects of $\mathcal{O}_{\phi W}$ and $\mathcal{O}_{\phi B}$ on a_μ are not observable. They can be absorbed into a wave-function renormalization of B and W, together with a rescaling of the constants g and g', and will not be considered further. These two operators therefore only contribute to a_μ through Figs. 1(b) and 1(c).

As mentioned previously, we do not know the full theory, and so we cannot compute the six α's, but we can estimate their sizes. It can be shown [17] that, because of the $SU(2)\times U(1)$ symmetry of the high-energy theory, the six operators must come from some loop in the full theory with heavy internal lines. This implies that any W or B must be accompanied by the corresponding coupling g or g'. Moreover, the fact that these operators are generated by loop graphs implies that the corresponding α must contain a factor $\sim 1/16\pi^2$. In this fashion, we obtain (for the constants at scale Λ)

$$\alpha_W \sim \frac{g^2}{16\pi^2}, \quad \alpha_{WB} \sim \frac{gg'}{16\pi^2}, \quad \alpha_{\phi W} \sim \frac{g^2}{16\pi^2} \,,$$
$$\alpha_{\phi B} \sim \frac{g'^3}{16\pi^2}, \quad \alpha_{\mu W} \sim \frac{g}{16\pi^2}, \quad \alpha_{\mu B} \sim \frac{g'}{16\pi^2} \,. \tag{2.5}$$

While we cannot expect these expressions to be numerically precise, we can use them as useful order of magnitude estimates. One should, however, be aware of the possibility that there may be several such contributions or resonant effects that may enhance these estimates, perhaps by as much as an order of magnitude; on the other hand, the presence of small couplings in the underlying theory can suppress these values below the ones in (2.5). Moreover, there may even be additional suppression factors of $1/16\pi^2$, since certain couplings may arise only beyond the one-loop level in the underlying theory, e.g., the contribution of a fourth generation to $\alpha_{\mu W}$ and $\alpha_{\mu B}$ sets in at the two-loop level. Thus, the inferences to be drawn on the sensitivity to the scale Λ must be interpreted in this context and not immediately identified with the threshold for new particle production.

It might be argued that the coefficients $\alpha_{\mu W,B}$ should have a factor of m_μ/v to allow for a natural mass generation for the muon. In fact, this is not necessarily the case. Supersymmetric models can substitute for m_μ/v a factor of $m_{\tilde{Z}}/v$, for example.[4] Also without this factor are models constructed so as to allow a relatively large magnetic moment for the neutrinos while keeping their mass within experimental bounds [18]. However, models which do not contain the m_μ/v suppression are likely to render the muon mass unnaturally light. We shall present results both including and disregarding this small factor.

When ϕ is replaced by its vacuum expectation value, then

$$\mathcal{O}_{WB} = -\frac{1}{2}\frac{v^2}{\Lambda^2} W^3_{\mu\nu} B^{\mu\nu} \,, \tag{2.6}$$

which contributes to the quadratic part of the Lagrangian. This necessitates a rediagonalization of the boson fields (see [15]):

$$W^3_\mu = s_W \left[1 - s_W c_W \frac{v^2}{\Lambda^2}\alpha_{WB}\right] A_\mu + \left[c_W + s_W^3 \frac{v^2}{\Lambda^2}\alpha_{WB}\right] Z_\mu \,,$$
$$B_\mu = c_W \left[1 - s_W c_W \frac{v^2}{\Lambda^2}\alpha_{WB}\right] A_\mu - \left[s_W + c_W^3 \frac{v^2}{\Lambda^2}\alpha_{WB}\right] Z_\mu \,, \tag{2.7}$$

[4]This is not required, though: many supergravity-inspired supersymmetry (SUSY) models avoid this.

where s_W and c_W are the sine and cosine of the tree-level weak-mixing angle in the standard model. This leads to certain modifications of the standard-model parameters which we reproduce for completeness [15]:

$$e \to e^* = e\left[1 - s_W c_W \alpha_{WB}\frac{v^2}{\Lambda^2}\right],$$
$$M_Z \to M_Z^* = M_Z\left[1 + s_W c_W \alpha_{WB}\frac{v^2}{\Lambda^2}\right]. \tag{2.8}$$

$\mathcal{O}_{WB}$ does not affect v, M_W, or G_F. We will use e^*, M_Z^*, and G_F^* as our input parameters; for example, it is M_Z^* that is measured to be 91.2 GeV, not M_Z.

The standard-model electroweak one-loop contribution to a_μ is [12]

$$a_\mu^{\text{SM}} = \frac{G_F^* m_\mu^2}{6\sqrt{2}\pi^2}\left[4\left[\frac{M_W^*}{M_Z^*}\right]^4 - 6\left[\frac{M_W^*}{M_Z^*}\right]^2 + 1\right] + \frac{G_F^* m_\mu^2}{8\sqrt{2}\pi^2}\left[\frac{10}{3}\right] + \frac{e^{*2}}{8\pi^2}. \tag{2.9}$$

The change in the values of M_Z and the $Z_{\mu\bar{\mu}}$ couplings results in a change in a_μ; the anomalous magnetic moment calculated with the shifted fields (2.7) minus the standard-model result (2.9) is equal to δa_μ^2.

The inclusion of effective vertices in loop graphs requires a certain amount of care. It is assumed from the beginning, that all momenta in (2.4) lie below Λ, and this is violated by the loop momentum in Fig. 1. To deal with this problem, note that when the graphs in Fig. 1 are differentiated once with respect to an external momentum, they become ultraviolet convergent, and so the momenta entering the heavy loop can be effectively assumed to be small compared to Λ, then the use of (2.4) is justified. Integrating with respect to the above external momentum produces an undetermined integration constant times a "direct" operator $\mathcal{O}_{\mu W}$ or $\mathcal{O}_{\mu B}$. We expect that for scales $\mu = \Lambda$ this term will give a contribution of the same order of magnitude as δa_μ^1. In this manner, using the above expressions (and calculating in the Feynman gauge with dimensional regularization) we obtain, from (2.4),

$$\begin{aligned}
\delta a_\mu^{\text{direct}} &= \frac{4\sqrt{2}M_W m_\mu}{g^2 s_W \Lambda^2}(s_W\alpha_{\mu W} - c_W\alpha_{\mu B}),\\
\delta a_\mu^{1\text{a}} &= -\frac{3g}{16\pi^2}\frac{m_\mu^2}{\Lambda^2}\alpha_W + \frac{1}{8\pi^2}\frac{g}{g'}\frac{m_\mu^2}{\Lambda^2}\alpha_{WB}\left[\frac{1}{\epsilon} - \gamma + \frac{3}{2} - \ln\frac{M_W^2}{4\pi\mu^2}\right],\\
\delta a_\mu^{1\text{b}} &= \frac{4s_W^2 - 1}{16\pi^2}\frac{m_\mu^2}{\Lambda^2}\left[\frac{1}{\epsilon} - \gamma + \frac{3}{2} + \frac{m_h^2}{M_Z^2 - m_h^2}\ln\frac{m_h^2}{4\pi\mu^2} - \frac{M_Z^2}{M_Z^2 - m_h^2}\ln\frac{M_Z^2}{4\pi\mu^2}\right]\left[\alpha_{\phi W} - \alpha_{\phi B} + \frac{s_W^2 - c_W^2}{c_W s_W}\alpha_{WB}\right],\\
\delta a_\mu^{1\text{c}} &= -\frac{m_\mu^2}{4\pi^2\Lambda^2}\left[\frac{1}{\epsilon} - \gamma + \frac{3}{2} - \ln\frac{m_h^2}{4\pi\mu^2}\right](c_W^2\alpha_{\phi B} + s_W^2\alpha_{\phi W} - 2s_W c_W\alpha_{WB}),\\
\delta a_\mu^2 &= \frac{m_\mu^2}{6\pi^2\Lambda^2}\alpha_{WB}s_W c_W(1 - 4s_W^2),
\end{aligned} \tag{2.10}$$

where m_h is the mass of the Higgs boson, μ is the renormalization scale, γ is Euler's constant, and the dimension of space-time is $4-2\epsilon$. We have broken δa_μ^1 into three parts, one from each diagram in Fig. 1. To this order in Λ, the starred and unstarred parameters in (2.10) are equal. As mentioned above, the divergences are unobservable; the infinite contributions from the graphs in Fig. 1 are canceled by counterterms of the form of $\alpha_{\mu B}$ and $\alpha_{\mu W}$.

The complete expression for the new physics contribution to a_μ is given by the sum of the five contributions in (2.10). We will assume that the renormalization of the dimension-six operators has been carried out using the modified minimal subtraction scheme ($\overline{\text{MS}}$) [19] and choose $\mu^2 = \Lambda^2$, so that we may use the estimates in (2.5). If we renormalize at a different scale, for example $\mu^2 = M_W^2$, then we must run our estimates for $\alpha_{\mu W}$ and $\alpha_{\mu B}$ from scale Λ to scale M_W. This running is determined by the part of δa_μ^1 proportional to α_{WB}, and the difference in α's at scale Λ and scale M_W will be proportional to $\ln M_W/\Lambda$ such that $\delta a_\mu^{\text{direct}} + \delta a_\mu^1$ is unchanged. It is only in this sense that the logarithmic divergences in (2.10) have any effect. (The couplings e, g, and g' at scale Λ will differ from those at scale M_W by terms involving $\ln M_W/\Lambda$, but these differences are suppressed by higher powers of standard-model coupling constants and can be ignored to this order.)

We find numerically that

$$\begin{aligned}
\delta a_\mu^1 + \delta a_\mu^2 \simeq & -1\times 10^{-10}\frac{\alpha_W}{\Lambda^2}\\
& + 3\times 10^{-9}\left[1 + \frac{1}{6}\ln\Lambda^2\right]\frac{\alpha_{WB}}{\Lambda^2}\\
& - 4\times 10^{-10}\left[1 + \frac{1}{5}\ln\Lambda^2\right]\frac{\alpha_{\phi W}}{\Lambda^2}\\
& - 1\times 10^{-9}\left[1 + \frac{1}{5}\ln\Lambda^2\right]\frac{\alpha_{\phi B}}{\Lambda^2},
\end{aligned} \tag{2.11}$$

where Λ is to be expressed in TeV. Then, taking the estimates for the couplings given in (2.5), we obtain

$$|\delta a_\mu^1+\delta a_\mu^2|\simeq 10^{-12}|[4\pm1\pm0.9\pm0.3] + [0.8\pm0.2\pm0.2]\ln\Lambda^2|\frac{1}{\Lambda^2}\ , \qquad (2.12)$$

where all masses are measured in TeV, and the $\pm$ refers to the relative signs of the various α's. m_h has been set to 150 GeV (the dependence on m_h is slight). This equation shows that the CERN and Brookhaven experiments are both completely insensitive to Λ above the Z mass for our estimates of α_W and α_{WB}. To reach a bound which is at all interesting, say $\Lambda=200$ GeV, would require the α's to be about ten times larger than expected. To reach $\Lambda=1$ TeV would require the α's to be about 100 times larger than expected—an unlikely occurrence in our opinion.

These discouraging results do not apply to $\delta a_\mu^{\text{direct}}$ which (assuming no strong cancellation between the two terms) takes the value

$$\delta a_\mu^{\text{direct}} \sim \frac{9\times10^{-7}}{\Lambda^2} \qquad (2.13)$$

(Λ in TeV), which is about six orders of magnitude larger than the contributions in (2.12). This is because (2.12) contains a factor $m_\mu/(16\pi^2 v)\sim10^{-6}$ due to the loop integration and the helicity change of the muon. Equation (2.13) allows a sensitivity limit of $\Lambda\lesssim50$ TeV for the Brookhaven experiment; effects from scales beyond this value will not be observed. In fact, the CERN experiment already implies a bound $\Lambda\gtrsim10$ TeV.

If the previously mentioned factor of m_μ/v is included in $\alpha_{\mu W,B}$, the above estimate decreases to $\delta a_\mu^{\text{direct}} \sim 4\times10^{-10}/\Lambda^2$. The sensitivity is accordingly diminished to $\Lambda\lesssim1$ TeV for the Brookhaven experiment, while the CERN data implies only that $\Lambda\gtrsim0.2$ TeV.

The suggestion here is that, if Λ is of the order of the weak scale, these effects may be as large as the standard model electroweak contributions. SUSY models[5] exemplify that possibility. Nevertheless, these limits, as we mentioned previously, must be cautiously interpreted, since, in other models, small coupling constants or resonances in the underlying theory can alter these limits on Λ by an order of magnitude or more. It is important to note, however, that even if an effect is seen in the Brookhaven experiment, it will not be produced by any modification of the "anomalous" three-gauge-boson couplings: $\delta a_\mu^1+\delta a_\mu^2$ would produce a measurable effect only for scales for a few GeV, corresponding to a region already probed by the CERN e^+e^- collider LEP, Fermilab, and many other previous experiments [20], which found no evidence of new physics.

III. NONDECOUPLING CASE

We now turn our attention to the possibility that physics above the scale Λ does not decouple from the low-energy physics. In this case, the appropriate expansion of the effective Lagrangian is in powers of momentum. We will assume here that the particle spectrum is the same as the standard model's with the exception of the Higgs boson, so that the effective Lagrangian can be written as a gauged chiral model [9,21,22]. In this model, we expect that $\Lambda\sim4\pi v$ [21], and that, for energies small compared to Λ, the first terms in the expansion will provide a good approximation [8].

Specifically, if

$$U=\exp[2i\pi^a\tau^a/v] \qquad (3.1)$$

then lowest order kinetic terms in the effective Lagrangian are [5]

$$\mathcal{L}_{\text{kin}}=\frac{v^2}{4}\,\text{tr}\{\mathbf{D}_\mu U^\dagger \mathbf{D}_\mu U\}-\frac12\,\text{tr}\{\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}\} -\frac12\,\text{tr}\{\mathbf{B}_{\mu\nu}\mathbf{B}^{\mu\nu}\}\ , \qquad (3.2)$$

where we have adopted the matrix notation $\mathbf{W}_\mu=W_\mu^I\tau^I/2$, $\mathbf{B}_\mu=B_\mu\tau_3/2$, $\mathbf{D}_\mu U=\partial_\mu U+ig\mathbf{W}_\mu U-ig'U\mathbf{B}_\mu$, and τ^I denote the Pauli matrices.

There are six new $SU(2)_L\times U(1)_Y$ operators which are of chiral dimension four or lower and contain quadratic or trilinear gauge vertices [22]. The only term of chiral dimension two is

$$\mathcal{L}_1'=\frac{v^2}{4}\beta_1'(\,\text{tr}[\tau^3U^\dagger\mathbf{D}_\mu U])^2\ , \qquad (3.3)$$

and the five of order [mass]^4 are

$$\begin{aligned}
\mathcal{L}_1&=gg'\beta_1\,\text{tr}[U\mathbf{B}_{\mu\nu}U^\dagger\mathbf{W}^{\mu\nu}]\ ,\\
\mathcal{L}_2&=-2ig'\beta_2\,\text{tr}[\mathbf{B}_{\mu\nu}\mathbf{D}^\mu U^\dagger\mathbf{D}^\nu U]\ ,\\
\mathcal{L}_3&=-2ig\beta_3\,\text{tr}[\mathbf{W}_{\mu\nu}\mathbf{D}^\mu U\mathbf{D}^\nu U^\dagger]\ ,\\
\mathcal{L}_8&=\frac14 g^2\beta_8(\,\text{tr}[U\tau^3U^\dagger\mathbf{W}_{\mu\nu}])^2\ ,\\
\mathcal{L}_9&=-ig\beta_9\,\text{tr}[U\tau^3U^\dagger\mathbf{W}_{\mu\nu}]\,\text{tr}[\tau^3\mathbf{D}^\mu U^\dagger\mathbf{D}^\nu U]\ .
\end{aligned} \qquad (3.4)$$

The numbering system, prefactors, and signs are adapted from those of Ref. [22]. β_1 corresponds to L_{10} of Ref. [9], β_2 and β_3 to L_9 of that same reference. β_1' is denoted Δ_ρ in [5].

As in the decoupling case, except for $\mathcal{L}_1'$, the naive order of magnitude estimate of each β is $v^2/\Lambda^2\simeq1/16\pi^2$ [8]. The β's may be larger than expected, for example, in technicolor theories, where they are enhanced by the numbers of generations and technicolors [5], or if enhanced by a low-lying resonance. β_1', though *a priori* of order 1, violates the approximate $SU(2)_R$, and can be limited by measurements of $\rho=(M_W/M_Z\cos\theta_W)^2$ to $\beta_1'\lesssim1\%$ [23], which, coincidentally, is of the same magnitude as[6] $1/16\pi^2$.

Just as in the last section, we must also include the operators (2.3) which give a direct tree-level correction to a_μ: namely,

[5]These are summarized by Kinoshita and Marciano in Ref. [13].

[6]Once the top mass is known, the error on ρ will be reduced to a few tenths of a percent.

$$\mathcal{L}_{\text{direct}}=\frac{1}{v}\bar{\psi}_R\sigma_{\mu\nu}\mathbf{m}\psi_L[g\beta_{\mu W}W^{\mu\nu}+g'\beta_{\mu B}B^{\mu\nu}]\ ,$$

$$\mathbf{m}=\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}.$$

The β's in the direct terms are expected to be of order $1/4\pi$. We will consider separately the cases where $\beta_{\mu W,B}$ are decreased by a factor m_μ/Λ. Our complete effective Lagrangian is then given by expressions (3.2) through (3.5).

We look first at the direct terms. A quick calculation shows them to be

$$\delta a_\mu^{\text{direct}}=\frac{gm_\mu}{\sqrt{2}M_W}(\beta_{\mu W}-\beta_{\mu B})\ . \tag{3.6}$$

As in the decoupling case, there will be contributions to δa_μ from three-boson vertices and from two-boson vertices; $\mathcal{L}_1'$, $\mathcal{L}_1$, and $\mathcal{L}_8$ have bilinear terms, and all but $\mathcal{L}_1'$ have trilinear terms. We first consider the trilinear terms. To this order in the expansion of $\mathcal{L}_{\text{eff}}$, there is no contribution such as α_W since $\mathcal{O}_W$ has chiral dimension six. We can see by comparison that the contributions from β_1, β_2, β_3, β_8, and β_9 are all proportional to the α_{WB} term of the decoupling Lagrangian, so that we need only make the replacement

$$\alpha_{WB}\frac{M_W^2}{\Lambda^2}\to\frac{1}{4}g^3g'[-2\beta_1+\beta_2+\beta_3-\beta_8+\beta_9]\ , \tag{3.7}$$

and, therefore (using the $\overline{\text{MS}}$ and taking v as the renormalization scale),

$$\delta a_\mu^1=\frac{g^4}{8\pi^2}\frac{m_\mu^2}{M_W^2}[-2\beta_1+\beta_2+\beta_3-\beta_8+\beta_9]\times\left[\ln\frac{v^2}{M_W^2}+\frac{3}{2}\right]. \tag{3.8}$$

The ultraviolet divergence is, as in the decoupling case, non-observable and has been canceled by the appropriate counterterms in $\mathcal{L}_{\mu W}+\mathcal{L}_{\mu B}$.

Next, we must consider the effect of the terms quadratic in gauge bosons on δa_μ. The quadratic part of the gauge-boson Lagrangian is

$$\mathcal{L}_{W,B}^2=-\frac{1}{4}W_{\mu\nu}^IW^{I\mu\nu}+\frac{g^2}{4}\beta_8W_{\mu\nu}^3W^{3\mu\nu}-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}+\frac{1}{2}gg'\beta_1W_{\mu\nu}^3B^{\mu\nu}+\frac{v^2}{8}(g^2W_\mu^IW^{I\mu}+g'^2B_\mu B^\mu-2gg'W_\mu^3B^\mu)$$
$$-\beta_1'\frac{v^2}{4}(gW_\mu^3-g'B_\mu)(gW^{3\mu}-g'B^\mu)\ . \tag{3.9}$$

This requires the rediagonalization

$$\begin{aligned}W_\mu^3&=s_W\left[1+\frac{1}{2}g^2s_W^2(2\beta_1+\beta_8)\right]A_\mu\\&\quad+c_W\left[1-g'^2s_W^2\beta_1+\frac{1}{2}g^2(1+s_W^2)\beta_8\right]Z_\mu\ ,\\B_\mu&=c_W\left[1+\frac{1}{2}g^2s_W^2(2\beta_1+\beta_8)\right]A_\mu\\&\quad-s_W\left[1-\frac{1}{2}g^2c_W^2(2\beta_1+\beta_8)\right]Z_\mu\ .\end{aligned} \tag{3.10}$$

These expressions lead to the redefinitions

$$\begin{aligned}e\to e^*&=e\left[1+\frac{1}{2}g^2s_W^2(2\beta_1+\beta_8)\right]\ ,\\M_Z\to M_Z^*&=M_Z\left[1-\beta_1'-\frac{1}{2}g^2(2s_W^2\beta_1-c_W^2\beta_8)\right]\ ;\end{aligned} \tag{3.11}$$

while G_F and M_W remain unchanged.

As in the previous case, the above expressions produce a new term in the standard-model contribution to a_μ due to the modification in M_Z and the $Z_{\bar{\mu}\mu}$ couplings, which in turn modify a_μ. A straightforward calculation gives

$$\delta a_\mu^2=\frac{G_Fm_\mu^2}{3\sqrt{2}\pi^2}[(1-4c_W^4)\beta_1'-g^2(1-4s_W^2)(s_W^2\beta_1-c_W^2\beta_8)]\ . \tag{3.12}$$

The total change in a_μ is again given by the sum $\delta a_\mu^{\text{direct}}+\delta a_\mu^1+\delta a_\mu^2$, where these quantities are given in (3.6), (3.8), and (3.12), respectively. Numerically, $|\delta a_\mu^1+\delta a_\mu^2|\lesssim 6\times 10^{-10}$ depending on the relative signs of the β_i. The direct contribution is restricted to $|\delta a_\mu^{\text{direct}}|\lesssim 10^{-4}$ if no factor of m_μ/Λ appears in $\beta_{\mu B,W}$, or $|\delta a_\mu^{\text{direct}}|\lesssim 3\times 10^{-9}$ if this factor is present.

We see then that the situation here is marginally different from the decoupling case: if the β_i all conspire to suppress the direct contribution and enhance (by a factor of 2 or so) $\delta a_\mu^1+\delta a_\mu^2$, then the Brookhaven experiment may be sensitive to the effects of an anomalous triple-gauge-boson vertex. On the other hand, if we assume no significant cancellations between the β_i, and no unexpected enhancement of these coefficients, then the main contribution to δa_μ comes from $\delta a_\mu^{\text{direct}}$, just as in the decoupling case, while the other contributions are unobservable. In this case, current CERN data imply a very strong suppression of the direct contributions, which can be interpreted naturally as evidence for the factor m_μ/Λ in $\beta_{\mu B,W}$; when this is included, the contribution from these terms lies below the sensitivity of the existing data (but well inside that of the Brookhaven experiment).

We can conclude that, if the nondecoupling is realized in nature, the "direct" contributions must be suppressed by a m_μ/Λ factor and that the Brookhaven experiment will either observe them or set interesting limits, implying that these contributions are further diminished (for example, by arising only at two loops).

The discussion above includes the α_{WB} (or, in the conventional notation,[7] $\kappa-1$) piece of the triple-gauge-boson vertex. The α_W (or λ, in the conventional notation) part is unchanged from the decoupling case and is therefore probably unobservable.

IV. COMPARISON TO OTHER RESULTS

Several authors [13,22] have considered the effects of high-energy physics on the anomalous moments of the W. In this section, we compare these results with ours in the decoupling scenario.

In our notation, the effects considered in [13,22] are described by the replacements

$$\alpha_{WB}=(\kappa-1)gg'\Lambda^2/(4M_W^2)\ ,\qquad \alpha_W=-\lambda g\Lambda^2/(6M_W^2)\ ; \tag{4.1}$$

all other α_O are ignored. The constants λ and κ are related to the magnetic dipole and electric quadrupole moments of the W by

$$\mu_W=\frac{e}{2M_W}(1+\kappa+\lambda),\quad Q_W=-\frac{e}{M_W^2}(\kappa-\lambda)\ . \tag{4.2}$$

Note that our previous arguments imply that, taking the most benign case where $\Lambda=v$, the natural scale for these constants is $|\kappa-1|\sim 3\times10^{-3}$ and $|\lambda|\sim 2\times 10^{-3}$. With this proviso, the results obtained in [13,22] coincide with δa_μ^1 in (2.10).

V. CONCLUSIONS

We have described the formalism of loop calculations for effective-Lagrangian models, using a_μ as an example. The philosophy of our approach differs markedly from the one used in several other publications [13], and this translates into different conclusions. Firstly, we note that any divergence obtained in using an effective Lagrangian is unobservable, since it will always be canceled by appropriate counterterms appearing in other operators in $\mathcal{L}_{\rm eff}$. The only remnant of these divergences concerns the logarithmic ones which specify the renormalization group flow of the couplings due to the light fields. It is only in this sense that the logarithms in (2.10) are observable. Stronger divergences (quadratic or quartic) are completely unobservable; this argument contrasts with several other opinions [24]. Related to this issue is the constraint of gauge invariance; this allowed us to use a gauge-preserving regularization where divergences higher than logarithmic are automatically (and consistently) disregarded.

Secondly, it is important to note that the magnitudes of the dimensionful coefficients reflect assumptions regarding the scale at which new physics will become apparent. For example, if we require $\lambda\sim1$, this implies a scale of ~ 10 GeV, which is obviously irrelevant; similar results hold if $|\kappa-1|\sim 1$. It is important to note that the scale Λ in the logarithms and the one multiplying the prefactors must be the same (required by consistency), and the observability limits cannot ignore this fact.

These conclusions apply independently of the nature of the physics beyond the standard model, whether confining or weak. For example, in the approach advocated in this paper, the statement that $\lambda\sim1$ in a composite model is untenable.

Finally, we remark on the estimates we used for the couplings. In the decoupling case, we have assumed that each gauge boson is associated with a coupling-constant g or g' and that, since all the operators have dimension larger than 4, they represent the low-energy limit of a series of loop diagrams; this relies to a certain extent on perturbation theory. But, in the case where the underlying physics does not lie in the perturbative regime, we can borrow the arguments used in the chiral approach to the strong interactions [8], which lead to similar results. The nondecoupling case, again, closely parallels QCD, and we use the corresponding arguments. If the factors of $1/16\pi^2$ in our estimates are ignored, the magnitude of the contributions increases about two orders of magnitude and the conclusions are markedly altered. Since we have no reason to suppose such an anomalously large value for the α_O, or the β_i, we have not considered this possibility.

Effective Lagrangians can be used to calculate observables in a loop expansion. Among other things, these loop contributions modify the direct terms, contributing to the β functions for $\alpha_{\mu W}$ and $\alpha_{\mu B}$. Whether these "direct" terms are larger than suggested by operator mixing is model dependent, as we discussed following Eq. (2.5). Taken at face value, in the decoupling case, the BNL experiment will push the limits on Λ from their present value of 200 GeV up to about 1 TeV. However, it may well be that the relevant threshold associated with new physics is much lower than Λ, depending on the nature of the underlying theory. In the nondecoupling case, the Brookhaven experiment should again be sensitive to the underlying physics whose effects should be apparent at scales of order v. In the decoupling case, certain models ([18], SUSY) illustrate the possibility that the direct couplings are not suppressed by a m_μ/Λ factor (an unlikely possibility for the nondecoupling scenario in view of the CERN data), in which case the sensitivity of the Brookhaven experiment is increased to $\Lambda\sim 50$ TeV. For both situations, however, the measurements of a_μ will not be sensitive (or at best marginally so) to anomalous triple-gauge-boson vertices.

After this manuscript was completed, we became aware of Ref. [25], in which many of the points of principle discussed above are also addressed.

ACKNOWLEDGMENTS

Portions of this work were supported by the Department of Energy, and by the National Science Foundation (Grant No. PHY89-0435). J.W. was supported in part by the TNRLC under Grant No. FCFY9211.

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