Subleading Isgur-Wise form factor $\xi_3(v \cdot v')$ and its implications for the decays $\bar{B} \to D^{(*)}\ell\bar{\nu}$

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(Received 24 May 1993)

We calculate, in the framework of QCD sum rules and to next-to-leading order in perturbation theory, the universal function $\xi_3(v \cdot v')$ which appears at order $1/m_Q$ in the heavy quark expansion of meson weak decay form factors. We find that radiative corrections of order α_s are very important. Over the kinematic range accessible in semileptonic decays, $\xi_3(v \cdot v')$ is proportional to the leading-order Isgur-Wise function $\xi(v \cdot v')$ to very good accuracy. Taking into account all sources of uncertainty we estimate $\xi_3/\xi = (0.6 \pm 0.2)$. This reduces the theoretical uncertainty in the extraction of $|V_{cb}|$ from $\bar{B} \to D\ell\bar{\nu}$ transitions. A measurement of the form factor ratio A_2/A_1 in $\bar{B} \to D^*\ell\bar{\nu}$ decays can be used to test our prediction.

PACS number(s): 12.39.Hq, 11.55.Hx, 13.20.He

I. INTRODUCTION

In the limit where the charm and bottom quarks are considered infinitely heavy, their strong interactions with light quarks and gluons acquire additional symmetries [1–4]. In particular, the weak decay form factors describing the semileptonic transitions $\bar{B} \to D\ell\bar{\nu}$ and $\bar{B} \to D^*\ell\bar{\nu}$ become related to a single universal function. The normalization of this so-called Isgur-Wise form factor is known in the zero recoil limit, where the initial and final mesons have the same velocity. This allows a model-independent determination of the Cabibbo-Kobayashi-Maskawa matrix element $|V_{cb}|$ [3, 5], up to corrections arising from the fact that m_c and m_b are, after all, not infinitely heavy. In this work, we investigate a particular type of such corrections, which is important to the extraction of $|V_{cb}|$ from $\bar{B} \to D\ell\bar{\nu}$ transitions.

Heavy quark effective theory (HQET) provides a convenient framework to analyze the weak decays of hadrons containing a heavy quark [6–12]. It provides a systematic expansion of hadronic matrix elements in powers of $1/m_Q$. The coefficients in this expansion are universal functions of the velocities v and v' of the initial and final hadrons ($v^2 = v'^2 = 1$, $v \cdot v' \geq 1$), but they do not depend on the flavor or spin of the heavy quarks. These form factors originate from long-distance hadronic dynamics, and so they can only be investigated using some nonperturbative approach to QCD. One such method is provided by QCD sum rules [13], which have recently been widely used to calculate hadronic matrix elements in HQET [14–22]. At leading order in the heavy quark expansion, a single Isgur-Wise function $\xi(v \cdot v', \mu)$ is required to parametrize the current-induced transitions $M(v) \to M'(v')$, where M and M' are pseudoscalar or vector mesons containing a single heavy quark [4]. This is conveniently expressed by the compact trace formula [10.23]

$$\langle M'(v')| \bar{h}' \Gamma h |M(v)\rangle$$

$$= -\xi(v \cdot v', \mu) \operatorname{Tr}\{ \overline{\mathcal{M}}'(v') \Gamma \mathcal{M}(v) \}, \quad (1)$$

where Γ is an arbitrary Dirac matrix, and h and h' denote the velocity-dependent effective fields in HQET which represent heavy quarks Q and Q' moving at the hadron's velocities v and v'. The heavy mesons are represented by covariant tensor wave functions

$$\mathcal{M}(v) = \sqrt{m_M}\, \frac{(1 + \not{v})}{2} \begin{cases} -\gamma_5 & , \quad \text{pseudoscalar meson,} \\ \not{\epsilon} & , \quad \text{vector meson,} \end{cases}$$

$$(2)$$

which have the correct transformation properties under Lorentz boosts and heavy quark spin rotations. Here m_M denotes the physical meson mass, and ϵ is the polarization vector of the vector meson. Current conservation implies that the Isgur-Wise function is normalized at zero recoil: $\xi(1, \mu) = 1$. Except at this point, the universal form factor depends on a subtraction scale μ, since the velocity-changing currents in the effective theory have to be renormalized. The μ dependence of the Isgur-Wise function cancels against that of the Wilson coefficients which appear in the matching of currents of the full theory onto currents of the effective theory [10, 24].

At order $1/m_Q$, matrix elements receive contributions from higher-dimensional operators in the effective Lagrangian and in the effective currents [11]. The former give rise to three new universal functions, usually denoted by $\chi_i(v \cdot v', \mu)$ for $i = 1, 2, 3$. In the framework of QCD sum rules, we have investigated these form factors in Refs. [19, 21]. Here we shall focus on the second type of corrections, which come from operators of dimension 4 in the short-distance expansion of the weak currents in HQET. In the case of the vector or axial vector currents, there are 14 independent operators of

this type. Their Wilson coefficients have recently been calculated to next-to-leading order in renormalization-group-improved perturbation theory [25]. On dimensional grounds, any dimension-4 current operator must contain a covariant derivative acting on one of the heavy quark fields. Thus, these operators have the generic form $\bar{h}' \Gamma i D_\alpha h$ or $(i D_\alpha \bar{h}') \Gamma h$, where Γ is again an arbitrary Dirac matrix. In analogy to (1), the corresponding matrix elements can be parametrized by a tensor form factor ξ_α defined by [11]

$$\langle M'(v')| \bar{h}' \Gamma i D_\alpha h |M(v)\rangle$$

$$= -\bar{\Lambda} \operatorname{Tr}\left\{ \xi_\alpha(v, v', \mu) \overline{\mathcal{M}'}(v') \Gamma \mathcal{M}(v) \right\}, \quad (3)$$

where $\bar{\Lambda} = m_M - m_Q = m_{M'} - m_{Q'}$ denotes the mass difference between a heavy meson and the heavy quark that it contains. This parameter sets the canonical scale for power corrections in HQET. Matrix elements with a derivative acting on $\bar{h}'$ are related to (3) by Dirac conjugation. The most general decomposition of the form factor can be written as

$$\xi_\alpha(v, v', \mu) = \xi_+(v \cdot v', \mu) (v + v')_\alpha$$
$$+ \xi_-(v \cdot v', \mu) (v - v')_\alpha - \xi_3(v \cdot v', \mu) \gamma_\alpha .$$
$$(4)$$

Because of T invariance of the strong interactions, the coefficient functions $\xi_i(v \cdot v', \mu)$ are real. Furthermore, the equation of motion of HQET can be used to derive the constraints [11]

$$\xi_-(v \cdot v', \mu) = \frac{1}{2} \xi(v \cdot v', \mu) ,$$

$$\xi_+(v \cdot v', \mu) = \frac{1}{2} \frac{v \cdot v' - 1}{v \cdot v' + 1} \xi(v \cdot v', \mu)$$
$$- \frac{1}{v \cdot v' + 1} \xi_3(v \cdot v', \mu) . \quad (5)$$

These relations show that only one of the coefficient functions, say, $\xi_3(v \cdot v', \mu)$, is independent. They also suggest a close relation between the subleading functions $\xi_i(v \cdot v', \mu)$ and the leading-order Isgur-Wise function $\xi(v \cdot v', \mu)$. The origin of this relation is the invariance of the effective theory under reparametrizations of the heavy quark momentum [25, 26]. In fact, one can show that the μ dependence of the functions $\xi_i(v \cdot v', \mu)$ is the same as the μ dependence of the Isgur-Wise function. This leads us to introduce a new, renormalization-group-invariant function $\eta(v \cdot v')$ by

$$\eta(v \cdot v') \equiv \frac{\xi_3(v \cdot v', \mu)}{\xi(v \cdot v', \mu)} . \quad (6)$$

We expect that $\eta(v \cdot v')$ is a slowly varying function of order unity. By means of (5) and (6), meson matrix elements of the dimension-4 operators in the short-distance expansion of the currents can be parametrized in terms of the product $\bar{\Lambda} \xi(v \cdot v', \mu)$ and a single new function $\eta(v \cdot v')$, which is independent of the renormalization scale.

Knowledge of the function $\eta(v \cdot v')$ becomes important when one wants to extract $|V_{cb}|$ from a measurement of the differential decay rate $d\Gamma/d(v \cdot v')$ for $\bar{B} \to D \ell \bar{\nu}$ transitions near zero recoil. Because of the known normalization of the Isgur-Wise function at $v \cdot v' = 1$, hadronic uncertainties affect such a measurement only at the level of power corrections of order $1/m_c$ and $1/m_b$. Note that whereas the $\bar{B} \to D^* \ell \bar{\nu}$ decay rate is protected against first-order power corrections by Luke's theorem [11], the rate for $\bar{B} \to D \ell \bar{\nu}$ is not, due to its helicity suppression at zero recoil [27]. However, one can show that the symmetry-breaking corrections are parametrically suppressed by the "Voloshin-Shifman factor" [3]

$$S = \left(\frac{m_B - m_D}{m_B + m_D}\right)^2 \approx 0.23 . \quad (7)$$

For this reason one may hope that the theoretical uncertainty in extracting $|V_{cb}|$ from these transitions is not much worse than in the case of $\bar{B} \to D^* \ell \bar{\nu}$ decays [19]. An extrapolation of the spectrum to zero recoil gives

$$\lim_{v \cdot v' \to 1} \frac{1}{\left[(v \cdot v')^2 - 1\right]^{3/2}} \frac{d\Gamma(\bar{B} \to D \ell \bar{\nu})}{d(v \cdot v')}$$

$$= \frac{G_F^2 |V_{cb}|^2}{48\pi^3} (m_B + m_D)^2 m_D^3 \eta_V^2 \left[1 + S \cdot K\right]^2 , \quad (8)$$

where

$$K = \delta_1 + \left(\frac{\bar{\Lambda}}{2m_c} + \frac{\bar{\Lambda}}{2m_b}\right)\left[(1 + \delta_1) - 2(1 + \delta_2) \eta(1)\right] . \quad (9)$$

Here η_V and δ_i are perturbative QCD corrections arising from finite renormalizations of the currents in the intermediate region $m_b > \mu > m_c$. Numerically one finds $\eta_V \approx 1.03$, $\delta_1 \approx 11\%$, and $\delta_2 \approx 9\%$ [24, 25]. The only unknown quantity (except the heavy quark masses) in this equation is the value of the function $\eta(v \cdot v')$ at zero recoil.

It has been pointed out in Ref. [19] that certain ratios of the hadronic form factors describing $\bar{B} \to D^* \ell \bar{\nu}$ transitions are very sensitive to symmetry-breaking corrections to the heavy quark limit. Their measurement can be used to test model calculations of the subleading universal form factors. Consider the quantity

$$R_2 = \left[1 - \frac{q^2}{(m_B + m_{D^*})^2}\right] \frac{A_2(q^2)}{A_1(q^2)} , \quad (10)$$

where $A_1(q^2)$ and $A_2(q^2)$ are axial vector form factors in the notation of Ref. [28]. Introducing the variable $v \cdot v'$ instead of q^2, and performing a $1/m_Q$ expansion of the hadronic form factors, one obtains [19]

$$R_2 = 1 - \frac{\bar{\Lambda}}{v \cdot v' + 1}\left(\frac{1}{m_c} + \frac{3}{m_b}\right) \eta(v \cdot v') + \cdots . \quad (11)$$

The ellipsis represents a small "hyperfine correction" proportional to the function $\chi_2(v \cdot v', \mu)$, and higher-order power corrections of order $1/m_Q^2$. The perturbative corrections to R_2 turn out to be completely negligible. Un-

less the function $\eta(v \cdot v')$ was highly suppressed, the dominant symmetry-breaking correction to R_2 is the one shown in (11). A measurement of this ratio would therefore provide valuable information about the form factor $\eta(v \cdot v')$.

In Ref. [19], the function $\xi_3(v \cdot v', \mu)$ has been analyzed using the QCD sum rule approach and adopting the standard approximations, in which one neglects radiative corrections. Such an approach leads to the parameter-free prediction that $\xi_3(1, \mu) = \eta(1) = 1/3$. In the context of QCD sum rules, corrections to this simple result can only come from radiative corrections or higher-dimensional condensates. It was recently found that such corrections can be quite significant, however. For the subleading Isgur-Wise functions $\chi_2(v \cdot v', \mu)$ and $\chi_3(v \cdot v', \mu)$, the terms of order α_s even give the dominant contributions to the sum rules [21]. In the case of $\xi_3(v \cdot v', \mu)$ this may be expected to be even more so, as due to the pe-

culiar Dirac structure associated with this form factor in (4) there is no quark condensate contribution at the tree level. Therefore, it is important to refine the sum rule analysis of Ref. [19] by including radiative corrections. This requires, in particular, the calculation of the two-loop corrections to the triangle quark-loop diagram. The techniques to handle the corresponding two-loop integrals have been developed in Ref. [22].

II. DERIVATION OF THE SUM RULE

The QCD sum rule analysis of $\xi_3(v \cdot v', \mu)$ proceeds in complete analogy to that of the Isgur-Wise function. For a detailed description of the procedure the reader is referred to Refs. [19, 22]. Here we shall only very briefly sketch the main steps. One considers, in the effective theory, the three-current correlator

$$\int dx \, dx' \, e^{i(k' \cdot x' - k \cdot x)} \langle 0 \, | \, T \left\{ \left[\bar{q} \, \overline{\Gamma}_{M'} h' \right]_{x'}, \left[\bar{h}' \, \Gamma \, i D_\alpha h \right]_0, \left[\bar{h} \, \Gamma_M \, q \right]_x \right\} | \, 0 \rangle = \mathrm{Tr} \left\{ \Xi_\alpha(v, v', k, k') \, \overline{\Gamma}_{M'} \frac{1 + \rlap{/}{v}'}{2} \Gamma \frac{1 + \rlap{/}{v}}{2} \Gamma_M \right\},$$

(12)

where v and v' are the velocities of the heavy quarks, and k and k' are the external off-shell momenta injected into the three-point function. Depending on the choice $\Gamma_M = -\gamma_5$ or $\Gamma_M = \gamma_\mu - v_\mu$, the heavy-light currents interpolate pseudoscalar or vector mesons, respectively. The Dirac structure of the correlator, as shown in the second line, is a consequence of heavy quark symmetry, as reflected in the Feynman rules of HQET. The quantity Ξ_α obeys a decomposition analogous to (4), with coefficient functions $\Xi_\pm$ and Ξ_3 that are analytic in the "off-shell energies" $\omega = 2v \cdot k$ and $\omega' = 2v' \cdot k'$, with discontinuities for positive values of these variables. These functions also depend on the velocity transfer $y = v \cdot v'$. From now on we shall focus on the coefficient $\Xi_3(\omega, \omega', y)$, which is used to construct the sum rule for the subleading form factor $\xi_3(y, \mu)$.

The idea of QCD sum rules is to relate a theoretical approximation to the operator product expansion of Ξ_3 to a hadronic representation of the correlator in terms of physical intermediate states. The lowest-lying states are the ground-state mesons M and M' associated with the heavy-light currents. They lead to a double pole located at $\omega = \omega' = 2\bar{\Lambda}$. The residue is proportional to the function $\xi_3(y, \mu)$. One finds [19]

$$\Xi_3^{\mathrm{pole}}(\omega, \omega', y) = \frac{\bar{\Lambda} \, \xi_3(y, \mu) \, F^2(\mu)}{(\omega - 2\bar{\Lambda} + i\epsilon)(\omega' - 2\bar{\Lambda} + i\epsilon)}, \quad (13)$$

where F corresponds to the scaled meson decay constant $(F \sim f_M \sqrt{m_M})$. Both F and ξ_3 are defined in terms of matrix elements in the effective theory and are scale-dependent quantities. In the deep Euclidean region, the correlator can be calculated perturbatively because of

asymptotic freedom. The idea of Shifman, Vainshtein, and Zakharov was that, at the transition from the perturbative to the nonperturbative regime, confinement effects can be accounted for by including the leading power corrections in an operator product expansion. They are proportional to vacuum expectation values of local quark-gluon operators, the so-called condensates [13]. Following the standard procedure, we write the theoretical expression for Ξ_3 as a double dispersion integral and perform a Borel transformation in ω and ω'. This yields an exponential damping factor in the dispersion integral and eliminates possible subtraction polynomials. Because of the flavor symmetry of HQET, it is natural to set the associated Borel parameters equal: $\tau = \tau' \equiv 2T$. Following Ref. [20], we then introduce new variables $\omega_+ = \frac{1}{2}(\omega + \omega')$ and $\omega_- = \omega - \omega'$, perform the integration over ω_-, and employ quark-hadron duality to equate the remaining integral over ω_+ up to a threshold ω_0 to the Borel transform of the pole contribution in (13). This yields the Borel sum rule

$$\bar{\Lambda} \, \xi_3(y, \mu) \, F^2(\mu) \, e^{-2\bar{\Lambda}/T} = \int\limits_0^{\omega_0} d\omega_+ \, e^{-\omega_+/T} \, \widetilde{\rho}(\omega_+, y)$$

$$\equiv K(T, \omega_0, y) . \quad (14)$$

The effective spectral density $\widetilde{\rho}$ arises after integration of the double spectral density over ω_-. For practical purposes it is useful to notice that the ω_+ dependence of each term in $\widetilde{\rho}(\omega_+, y)$ is known on dimensional grounds [21, 22]. It thus suffices to calculate the Borel transform of Ξ_3, corresponding to the limit $\omega_0 \to \infty$ in (14). The dependence on ω_0 can be reintroduced later.

As pointed out above, the theoretical expression for the right-hand side of the sum rule consists of a perturbative

part and condensate contributions: $K = K_{\mathrm{pert}} + K_{\mathrm{cond}}$. Let us first present the result for the nonperturbative contributions. The leading terms are proportional to the quark condensate (dimension $d = 3$), the gluon condensate ($d = 4$), and the mixed quark-gluon condensate ($d = 5$). For a consistent calculation at order α_s, we calculate the Wilson coefficients of the quark and gluon condensates to one-loop order, and the coefficient of the mixed condensate at the tree level. The truncation of the series of power corrections after the mixed condensate seems safe. The contributions from four-quark operators ($d = 6$) are suppressed relative to the quark condensate by a factor $|\langle \bar{q}q\rangle|/T^3 \sim 1\text{-}5\,\%$. The calculation is most conveniently performed using the coordinate gauge $x \cdot A(x) = 0$ with the origin chosen at the position of the velocity-changing heavy quark current. We find

$$
K_{\mathrm{cond}}(T, \infty, y) = -\frac{2\alpha_s \langle \bar{q}q\rangle\, T}{3\pi} \left[2 + r(y) \right]
$$
$$
+ \frac{\langle \alpha_s GG\rangle}{96\pi}\left(\frac{y-1}{y+1}\right) - \frac{\langle \bar{q}\, g_s \sigma_{\alpha\beta} G^{\alpha\beta} q\rangle}{12T}\,,
$$

$$\tag{15}$$

where

$$
r(y) = \frac{1}{\sqrt{y^2 - 1}}\,\ln\left(y + \sqrt{y^2 - 1}\right).
\tag{16}
$$

Let us now turn to the perturbative contributions to the sum rule. At order α_s, one has to evaluate the bare quark loop as well as the seven two-loop diagrams depicted in Fig. 1. A new feature of the present sum rule, as compared to the sum rule for the Isgur-Wise function considered in Ref. [22], is that there is a diagram (D_7) where a gluon originates from the covariant derivative contained in the current. We denote the Borel transformed contributions of the individual diagrams to the function $K_{\mathrm{pert}}(T, \infty, y)$ by $\hat{D}_i$. Throughout the calculation we use Feynman gauge and dimensional regulariza-
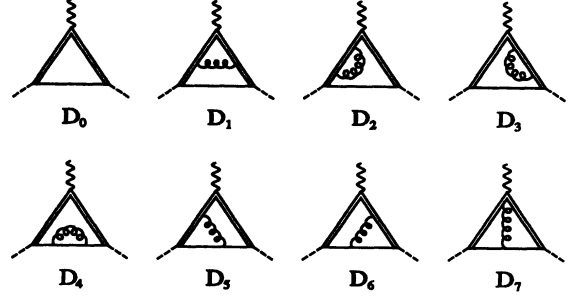

FIG. 1.   One- and two-loop perturbative contributions to the sum rule for the universal form factor $\xi_3(v \cdot v', \mu)$. Heavy quark propagators are drawn as double lines. The wavy line represents the velocity-changing heavy quark current $\bar{h}'\,\Gamma\, iD_\alpha h$.

tion. The bare quark loop is readily calculated and gives [19]

$$
\hat{D}_0 = \frac{2N_c T^D}{(4\pi)^{D/2}}\left(\frac{2}{y+1}\right)^{D/2}\Gamma\!\left(\tfrac{D}{2}\right),
\tag{17}
$$

where D is the dimension of space-time. The evaluation of the two-loop corrections is more complicated. For a detailed and systematic discussion of the techniques used to calculate the two-loop integrals the reader is referred to Ref. [22]. The present calculation proceeds very similar to that in this reference, where the sum rule for the Isgur-Wise function was derived at two-loop order. It is convenient to introduce a constant

$$
A = -\frac{4N_c\, C_F\, g_s^2}{(4\pi)^D}\,\frac{(2T)^{2D-6}}{[2(y+1)]^{D-2}}\,\Gamma\!\left(\tfrac{D}{2}\right)\Gamma\!\left(\tfrac{D}{2} - 1\right),
\tag{18}
$$

where $C_F = (N_c^2 - 1)/2N_c$. Once this quantity is factored out, we find that the contributions of the first four two-loop diagrams in Fig. 1 are the same as the corresponding contributions to the sum rule for the Isgur-Wise function:

$$
\sum_{i=1}^{4} \hat{D}_i = A\left\{ \frac{1}{\epsilon}\left[\frac{3}{2} - y\,r(y)\right] + 2\left[1 - y\,r(y)\right]\ln\left[2(1+y)\right] + 2yh(y) - 4 + \mathcal{O}(\epsilon)\right\}.
\tag{19}
$$

We have expanded the result in $\epsilon = (D - 4)/2$ and introduced the function

$$
h(y) = \frac{1}{\sqrt{y^2 - 1}}\left[L_2(1 - y_-^2) - L_2(1 - y_-)\right] + \frac{3}{4}\sqrt{y^2 - 1}\, r^2(y),
\tag{20}
$$

where $y_- = y - \sqrt{y^2 - 1}$, and $L_2(x)$ is the dilogarithm. The calculation of the remaining three diagrams is more cumbersome. It requires rather elaborate techniques such as Kotikov's method of differential equations [29]. However, remarkable simplifications take place as we add up the various contributions. In particular all dilogarithms, which appear in intermediate steps of the calculation, cancel out. The final result is rather simple:

$$
\sum_{i=5}^{7} \hat{D}_i = A\left\{ \frac{1}{\epsilon} - 2 - \frac{2\pi^2}{3} - (y^2 - 1)\,r^2(y) - (y+1)\left[2 + r(y)\right] + O(\epsilon)\right\}.
\tag{21}
$$

Except for the last term, this is again the same result as for the Isgur-Wise function.

The ultraviolet-divergent terms in the sum of the seven two-loop diagrams match with the anomalous dimensions of the heavy-heavy and heavy-light currents contained in the three-current correlator in (12). Thus the $1/\epsilon$ pole disappears upon renormalization of the currents. In the modified minimal subtraction scheme, the renormalization factors are [3, 10]

$$
Z_{hh} = 1 + \frac{\alpha_s}{2\pi\hat{\epsilon}}\,\gamma(y)\,, \qquad Z_{hl} = 1 - \frac{\alpha_s}{2\pi\hat{\epsilon}}\,,
\tag{22}
$$

where

$$\frac{1}{\hat{\epsilon}} = \frac{1}{\epsilon} + \gamma_E - \ln\frac{4\pi}{\mu^2}, \tag{23}$$

and

$$\gamma(y) = \frac{4}{3}\left[y\,r(y) - 1\right]. \tag{24}$$

Our exact two-loop result for the renormalized perturbative part of the correlator is

$$K_{\text{pert}}(T, \infty, y) = Z_{hh}^{-1}Z_{hl}^{-2}\,\hat{D}_0 + \sum_{i=1}^{7}\hat{D}_i$$

$$= \frac{3T^4}{8\pi^2}\left(\frac{2}{y+1}\right)^2\left\{1 + \frac{\alpha_s}{\pi}\left[\left[2 - \gamma(y)\right]\left(\ln\frac{\mu}{T} + \gamma_E - \frac{11}{6}\right) + \frac{4\pi^2}{9} + \frac{17}{3} + c_{\text{pert}}(y) + \delta c(y)\right]\right\}, \tag{25}$$

where

$$c_{\text{pert}}(y) = \frac{\gamma(y)}{2}\left[4\ln 2 - 3 + \ln\frac{y+1}{2}\right] - \frac{4}{3}\left[y\,h(y) - 1\right] + \ln\frac{y+1}{2} + \frac{2}{3}\left(y^2 - 1\right)r^2(y),$$

$$\delta c(y) = \frac{2}{3} + \frac{2}{3}\left(y + 1\right)\left[2 + r(y)\right] - \frac{4}{9}\left[y\,r(y) - 1\right]. \tag{26}$$

The function $c_{\text{pert}}(y)$ is the same that arises in the calculation of the sum rule for the Isgur-Wise function.

The final expression for the QCD sum rule (14) is obtained when we reintroduce the continuum threshold ω_0 and write the result as a dispersion integral. This gives

$$\bar{\Lambda}\,\xi_3(y, \mu)\,F^2(\mu)\,e^{-2\bar{\Lambda}/T} = \frac{1}{16\pi^2}\left(\frac{2}{y+1}\right)^2\int_0^{\omega_0}d\omega_+\,e^{-\omega_+/T}\,\omega_+^3$$

$$\times\left\{1 + \frac{\alpha_s}{\pi}\left[\left[2 - \gamma(y)\right]\ln\frac{\mu}{\omega_+} + \frac{4\pi^2}{9} + \frac{17}{3} + c_{\text{pert}}(y) + \delta c(y)\right]\right\}$$

$$- \frac{2\alpha_s\langle\bar{q}q\rangle}{3\pi}\left[2 + r(y)\right]\int_0^{\omega_0}d\omega_+\,e^{-\omega_+/T} + \frac{\langle\alpha_s GG\rangle}{96\pi}\left(\frac{y-1}{y+1}\right) - \frac{\langle\bar{q}\,g_s\sigma_{\alpha\beta}G^{\alpha\beta}q\rangle}{12T}. \tag{27}$$

It is instructive to compare this to the sum rule for the product $\bar{\Lambda}\,\xi(y, \mu)$, which can be obtained from the two-loop calculation of Ref. [22]:

$$\bar{\Lambda}\,\xi(y, \mu)\,F^2(\mu)\,e^{-2\bar{\Lambda}/T} = \frac{3}{16\pi^2}\left(\frac{2}{y+1}\right)^2\int_0^{\omega_0}d\omega_+\,e^{-\omega_+/T}\,\omega_+^3\left\{1 + \frac{\alpha_s}{\pi}\left[\left[2 - \gamma(y)\right]\ln\frac{\mu}{\omega_+} + \frac{4\pi^2}{9} + \frac{17}{3} + c_{\text{pert}}(y)\right]\right\}$$

$$- \frac{2\alpha_s\langle\bar{q}q\rangle}{3\pi}\left[y\,r(y) - 1\right]\int_0^{\omega_0}d\omega_+\,e^{-\omega_+/T} - \frac{\langle\alpha_s GG\rangle}{96\pi}\left(\frac{y-1}{y+1}\right) - \frac{(2y+1)}{3}\frac{\langle\bar{q}\,g_s\sigma_{\alpha\beta}G^{\alpha\beta}q\rangle}{4T}. \tag{28}$$

Notice that the μ dependence in (27) and (28) is the same. Thus, our explicit calculation is in accordance with the fact that the function $\eta(y)$ in (6) is renormalization-group invariant. We write

$$\eta(y) = \frac{\xi_3(y, \mu)}{\xi(y, \mu)} = \frac{1}{3} + \Delta(y), \tag{29}$$

and find that $\Delta(y)$ obeys the sum rule

$$\Delta(y)\left[\bar{\Lambda}\,\xi(y)\,F^2\,e^{-2\bar{\Lambda}/T}\right] = \frac{\alpha_s T^4}{12\pi^3}\left(\frac{2}{y+1}\right)^2\left[11 + 6y + (3 + y)\,r(y)\right]\delta_3\!\left(\frac{\omega_0}{T}\right) - \frac{2\alpha_s\langle\bar{q}q\rangle\,T}{9\pi}\left[7 + (3 - y)\,r(y)\right]\delta_0\!\left(\frac{\omega_0}{T}\right)$$

$$+ \frac{\langle\alpha_s GG\rangle}{72\pi}\left(\frac{y-1}{y+1}\right) + \frac{\langle\bar{q}\,g_s\sigma_{\alpha\beta}G^{\alpha\beta}q\rangle}{18T}\left(y - 1\right), \tag{30}$$

where

$$\delta_n(x) = \frac{1}{\Gamma(n+1)}\int_0^x dz\,z^n\,e^{-z}. \tag{31}$$

Since the right-hand side of (30) is of order α_s, in this sum rule one is not sensitive to the running of the quantities $\xi(y)$ and F. Their μ dependence would show up at

order α_s^2. For the analysis of the sum rule it is, therefore, consistent to use

$$\bar{\Lambda}\,\xi(y)\,F^2\,e^{-2\bar{\Lambda}/T} = \frac{9T^4}{8\pi^2}\left(\frac{2}{y+1}\right)^2\delta_3\!\left(\frac{\omega_0}{T}\right)$$

$$- \frac{(2y+1)}{3}\frac{\langle\bar{q}\,g_s\sigma_{\alpha\beta}G^{\alpha\beta}q\rangle}{4T}, \tag{32}$$

which is obtained from (28) by neglecting terms of order α_s. Taking the ratio of (30) and (32) reduces to a minimum the systematic uncertainties in the calculation of $\Delta(y)$.

III. NUMERICAL ANALYSIS AND CONCLUSIONS

In its final form, the sum rule for $\Delta(y)$ very much resembles the sum rules for the other subleading Isgur-Wise functions $\chi_2(y)$ and $\chi_3(y)$, which we derived in Ref. [21]. Accordingly, the numerical analysis proceeds in a similar way. For the QCD parameters we take the standard values

$$\langle \bar{q}q \rangle = -(0.23 \,\text{GeV})^3 \,,$$
$$\langle \alpha_s GG \rangle = 0.04 \,\text{GeV}^4 \,, \tag{33}$$
$$\langle \bar{q} \, g_s \sigma_{\alpha\beta} G^{\alpha\beta} q \rangle = m_0^2 \, \langle \bar{q}q \rangle \,, \quad m_0^2 = 0.8 \,\text{GeV}^2 \,.$$

Our results turn out to be very stable against variations of these numbers within reasonable limits. Furthermore, we use $\alpha_s/\pi = 0.1$ corresponding to a scale $\mu \approx 2\bar{\Lambda} \approx 1$ GeV, which is appropriate for evaluating radiative corrections in the effective theory. Combining (30) and (32), we obtain $\Delta(y)$ and hence $\eta(y)$ as functions of ω_0 and T. These input parameters can be determined from the analysis of a QCD sum rule for the correlator of two heavy-light currents in the effective theory [17, 18]. One finds good stability for $\omega_0 = 2.0 \pm 0.3$ GeV, and the consistency of the theoretical calculation requires that the Borel parameter be in the range $0.6 < T < 1.0$ GeV.

In Fig. 2(a) we show the zero recoil value of the form factor $\eta(y)$ as a function of the Borel parameter, for three different values of the continuum threshold. For $y = 1$ the sum rules simplify considerably. We find

$$\eta(1) = \frac{1}{3} + \frac{14\alpha_s}{9\pi} \frac{\delta_3\left(\dfrac{\omega_0}{T}\right) - \dfrac{8\pi^2}{7} \dfrac{\langle \bar{q}q \rangle}{T^3} \delta_0\left(\dfrac{\omega_0}{T}\right)}{\delta_3\left(\dfrac{\omega_0}{T}\right) - \dfrac{2\pi^2}{9} \dfrac{m_0^2 \langle \bar{q}q \rangle}{T^5}} \,. \tag{34}$$

Neglecting the terms of order α_s, we would recover the result $\eta(1) = 1/3$ derived in Ref. [19]. However, as seen in the figure these contributions are by no means negligible. They enhance the form factor by almost a factor 2. It supports the self-consistency of the sum rule approach that we find stability in essentially the same region of parameter space that leads to stability of the two-current sum rules considered in Refs. [17, 18], and of other three-current sum rules analyzed in Refs. [21, 22].

Over the kinematic range accessible in $\bar{B} \to D^{(*)}\ell\bar{\nu}$ decays, we show in Fig. 2(b) the range of predictions for $\eta(y)$ obtained for $1.7 < \omega_0 < 2.3$ GeV and $0.6 < T < 1.2$ GeV. The numerical analysis confirms our guess that $\eta(y)$ should be a slowly varying function of order unity, which was the motivation for its introduction in the first place. In fact, the sum rule predicts that $\eta(y) \approx 0.6$ essentially independent of y. The main uncertainty comes from the values of α_s and ω_0, which are not very accurately known. However, one should keep in mind that there are systematic uncertainties inherent in QCD sum rules which cannot be estimated by simply varying the input param-
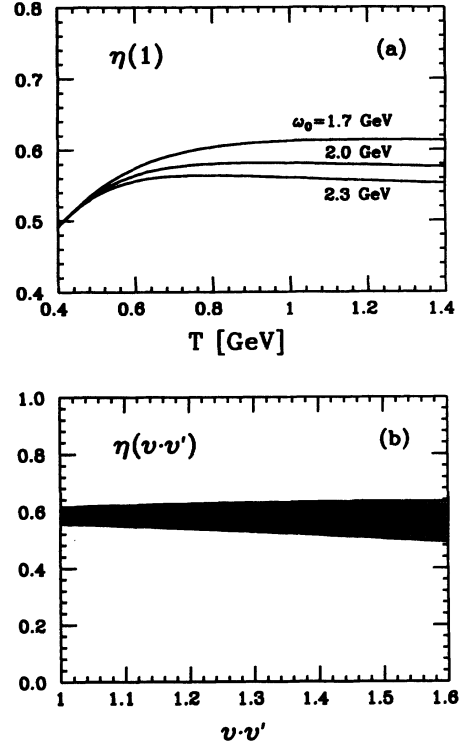

FIG. 2. Numerical evaluation of the sum rule (30): (a) dependence of the zero recoil form factor $\eta(1)$ on the Borel parameter for different values of the continuum threshold; (b) the function $\eta(y)$ for $0.6 < T < 1.2$ GeV and $1.7 < \omega_0 < 2.3$ GeV.

eters and, more generally, are not controlled by any small parameter. (For a detailed discussion of the limits of applicability we refer the reader to Ref. [30].) The only way in which one may estimate the related uncertainties is by comparing QCD sum rule predictions to known quantities. Using such past experience and being conservative, we quote our final result as

$$\eta(y) \approx 0.6 \pm 0.2 \,, \qquad 1.0 < y < 1.6 \,. \tag{35}$$

This result has important implications for the extraction of $|V_{cb}|$ from $\bar{B} \to D\ell\bar{\nu}$ decays. According to (9), the $1/m_Q$ corrections to the decay rate are proportional to $[1.11 - 2.18\,\eta(1)]$, and by a fortunate accident this combination is strongly suppressed for $\eta(1)$ in the range (35). For the symmetry-breaking corrections to the decay rate in (8), we obtain (we use $\bar{\Lambda} = 0.5$ GeV, $m_c = 1.45$ GeV, and $m_b = 4.8$ GeV)

$$SK \approx 1.5 \pm 2.3\% \,, \tag{36}$$

i.e., at most a few percent. This is comparable to the expected size of $1/m_Q^2$ corrections [31]. We conclude that the theoretical uncertainty in the determination of $|V_{cb}|$ from this decay mode is not worse than in $B \to D^*\ell\bar{\nu}$ transitions. Of course, the experimental measurement of $B \to D\ell\bar{\nu}$ near zero recoil is more difficult. The reward of such a measurement, however, would be an independent determination of $|V_{cb}|$ with surprisingly small theoretical uncertainties.

TABLE I. Predictions for the form factor ratio R_2. The zero recoil limit $w = 1$ corresponds to $q^2 = (m_B - m_{D^*})^2$, whereas $w_{max} \approx 1.5$ corresponds to $q^2 = 0$.

	Our results	ISGW [32]	BSW [28]	KS [33]	Sum rules [34]
$R_2(1)$	0.80	0.91	0.85	1.09	0.95
$R_2(w_{max})$	0.84	1.14	1.06	1.00	1.05

Let us finally point out how our sum rule prediction (35) can be tested experimentally, by a measurement of the form factor ratio R_2 in $\bar B \to D^* \ell \bar\nu$ transitions. Using $\eta(y) = 0.6$ in (11) we obtain

$$R_2 \approx 1.0 - 0.2 \left(\frac{2}{v \cdot v' + 1} \right). \tag{37}$$

In Table I we compare this result to the predictions of some popular quark models, as well as to a recent QCD sum rule calculation of the weak decay form factors in the full theory. These models give values for R_2 which are substantially larger than ours. In particular, we note that at $q^2 = 0$, corresponding to the maximal velocity transfer, the models give $R_2 \geq 1$, whereas we find $R_2 \approx 0.84$. This discrepancy should not be too surprising. Since we have worked very hard to understand the origin of the symmetry-breaking corrections, we can hope that our refined sum rule analysis accounts for such effects in a much more detailed way than the naive quark models can.

We end this paper with an interesting speculation. Although there is no reason to believe that it makes any sense to apply the heavy quark expansion to the $D \to K^* \ell \bar\nu$ decay amplitude, we might still believe in a "continuity of signs" and guess that the tendency $R_2 < 1$ should persist, and most likely even become more pronounced, when we imagine changing the heavy quark masses from m_b and m_c to m_c and m_s. This tendency is in fact very consistent with the experimental value of the form factor ratio obtained from an analysis of the joint angular distribution in $D \to K^* \ell \bar\nu$ decays. Taking the weighted average of the results reported by the experiments E691 [35], E653 [36], and E687 [37], we get $R_2^{DK^*}(q^2 = 0) = 0.73 \pm 0.15$. Although we have no right to extrapolate (11) down to the strange quark mass, we take this observation as a confirmation of our prediction that symmetry-breaking corrections suppress R_2.[1]

In conclusion, we have presented the complete next-to-leading order QCD sum rule analysis of the subleading Isgur-Wise functions $\xi_3(v \cdot v', \mu)$ and $\eta(v \cdot v')$, including in particular the two-loop perturbative corrections. We find that effects of order α_s are very important and enhance the form factors. Over the kinematic region accessible in semileptonic decays, the renormalization-group-invariant ratio $\eta(v \cdot v')$ turns out to be essentially constant and equals 0.6 ± 0.2. This leads to an almost complete cancellation of the leading symmetry-breaking corrections to the $\bar B \to D \ell \bar\nu$ decay rate at zero recoil, allowing for a reliable determination of $|V_{cb}|$ from this decay mode.

ACKNOWLEDGMENTS

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[1]The authors of Ref. [38] apply HQET to $D \to K^{(*)} \ell \bar\nu$ decays and obtain $\eta(1) \approx 0.3 \pm 0.4$ (for $\bar\Lambda = 0.4$ GeV) from an overall fit to the data.

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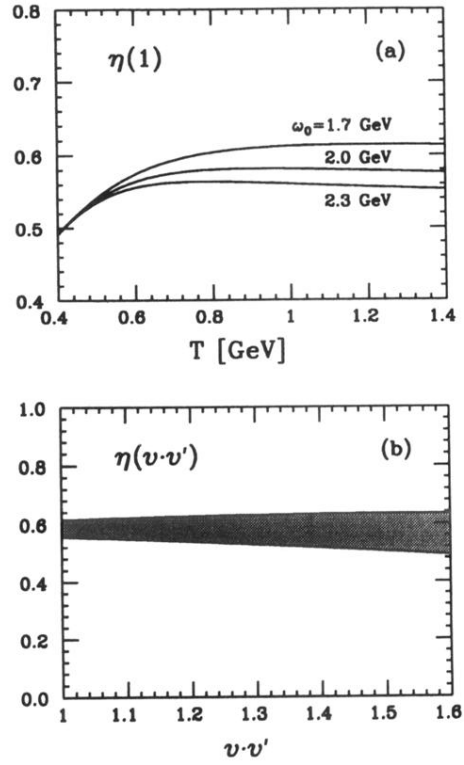
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