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Deep-inelastic scattering from off-shell nucleons

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We derive the general structure of the hadronic tensor required to describe deep-inelastic scattering from an off-shell nucleon within a covariant formalism. Of the large number of possible off-shell structure functions we find that only three contribute in the Bjorken limit. In our approach the usual ambiguities encountered when discussing problems related to off shellness in deep-inelastic scattering are not present. The formulation therefore provides a clear framework within which one can discuss the various approximations and assumptions which have been used in earlier work. As examples, we investigate scattering from the deuteron, nuclear matter, and dressed nucleons. The results of the full calculation are compared with those where various aspects of the off-shell structure are neglected, as well as with those of the convolution model.

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I. INTRODUCTION

The general structure of the hadronic tensor relevant to deep-inelastic scattering (DIS) from an on-mass-shell nucleon ($p^2 = M^2$) which transforms correctly under proper Lorentz and parity transformations, and which is gauge and time-reversal invariant, is well known. In the Bjorken limit the two possible structure functions collapse to one, so that in the case of one flavor electromagnetic deep-inelastic scattering may be expressed in terms of just one quark distribution which is a function of only one variable. (All these statements refer to spin-independent scattering, to which we restrict ourselves throughout this paper.)

The situation is considerably more complex if one is considering, in a covariant formulation, DIS from an off-mass-shell ($p^2 \neq M^2$) hadronic constituent within a composite target. This situation arises, for example, in many calculations relevant to the European Muon Colloboration (EMC) effect, where an off-shell nucleon contained in a nucleus interacts with a high energy probe. Another application of interest is the scattering from a nucleon dressed by a meson cloud. Indeed, because of the added complexity, many calculations ignore the issue completely in the hope that the effects are not large. Typically one neglects not only the possible p^2 dependence in the structure functions, but also assumes no change in the structure of the off-shell hadron tensor. Only in this case can the structure function of the target be written as a one-dimensional convolution between a constituent (nucleon) distribution function within the target and a quark distribution within the constituent [1].

We shall consider scattering from an off-mass-shell nucleon without making these assumptions. The purpose is to develop a theoretical framework which is exact, thus keeping the model-dependent approximations to as late a stage as possible. Of course, in order to make progress we have to restrict our consideration to the interaction with a single off-shell nucleon (impulse approximation). Thus processes where the lepton interacts with quarks in two or more different nucleons (final state interactions) are excluded at this stage, even though these may not be negligible (see Sec. VI B).

It is important to realize that the change in the structure of the off-shell tensor is by no means a trivial matter. There are several distinct differences from the on-shell tensor.

(I) Most obviously, the dependence on the four-momentum squared of the nucleon is no longer trivial, as it is in the case where the target is on shell.

(II) In a covariant formalism the off-shell fermion tensor is a 4×4 matrix in the external fermion legs. This corresponds to the fact that in a relativistic theory it is necessary to consistently incorporate the antiparticle degrees of freedom. Because of this matrix structure, the tensor involves, at least in principle, many more independent functions than in the on-shell case.

(III) Because the incoming particles are off mass shell, the gauge invariance condition for this tensor is not the same as in the on-shell case.

To show this last point, consider the truncated forward virtual Compton amplitude $\widehat{T}_{\mu\nu}(p,q)$, which satisfies the

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well-known generalized Ward identity [2]

$$q^{\mu}\,\widehat{T}_{\mu\nu}(p,q) = -e\left\{S^{-1}(p)S(p+q)\Gamma_{\nu}(p+q,p) - \Gamma_{\nu}(p,p-q)S(p-q)S^{-1}(p)\right\}, \tag{1}$$

where $\Gamma_{\nu}(p+q,p)$ is the γNN vertex function and $S(p)$ is the fermion propagator. For an on-shell nucleon, the full Compton amplitude is

$$T_{\mu\nu}(p,q) = \overline{u}(p)\,\widehat{T}_{\mu\nu}\,u(p), \tag{2}$$

so that inserting Eq. (1) into Eq. (2), and using the Dirac equation, leads to

$$q^{\mu}T_{\mu\nu}(p,q) = 0. \tag{3}$$

Note that the same equation does not hold for the off-shell tensor $\widehat{T}_{\mu\nu}$ [i.e., the right-hand side of Eq. (1) is nonzero], even for the case where the target is a free, pointlike fermion.

Although in calculations of nuclear structure functions the off-shell aspects of the nucleon structure function have usually been ignored, a few partial attempts have been made to try to account for these effects. Unfortunately, these calculations are not without ambiguities [3,4]. Kusno and Moravcsik [5] used the so-called "off-shell-kinematics–on-shell-dynamics" scheme, in which the off-shell nucleon tensor is evaluated at the same energy transfer ν and four-momentum transfer q^2 as the on-shell one, independent of the virtuality of the nucleon. Bodek and Ritchie [6] used a similar scheme; however, they suggested that the off-shell structure functions could be identified with the on-shell ones, evaluated for the same values of q^2 and center of mass energy squared $s = (p+q)^2$, and hence a different value of energy transfer, $\nu \to \nu + (p^2 - M^2)/2$. Dunne and Thomas [7], on the other hand, used an ansatz in which the matrix elements of the hadronic operators in the operator product expansion were assumed to be independent of p^2. The result was a nucleon structure function that was to be evaluated at a shifted value of $q^2[\to \xi(p^2,q^2)q^2$, where ξ is the q^2-rescaling parameter]. This result was mathematically equivalent to the dynamical rescaling model of Close, Roberts, and Ross [8] and Nachtmann and Pirner [9], in which the shift in q^2 was attributed to a change in confinement radius for nucleons bound inside a nucleus.

All of the above treatments use, in one form or other, the familiar convolution formula [1], which amounts to folding the quark momentum distribution in the off-shell constituent with the constituent momentum distribution in the target. In order to derive this formula it is assumed that the form of the off-shell nucleon tensor (i.e., the structure in its Dirac indices) is the same as the on-shell one [10,11]. However, as we show in Secs. II and III, more than one operator contributes in the Bjorken limit, and so there is no *a priori* reason for this to be a valid assumption. The appearance of these other operator structures is closely connected with the antiparticle degrees of freedom arising in any relativistic treatment and constitutes an important part of the off-shell effects. Relativistic calculations have been attempted in the past by Kulagin [12], Nakano [13], and Gross and Liuti [14]; however, their derivations of the convolution model also relied critically on assumptions about the off-shell tensor and the relativistic bound nucleon density matrix, respectively. In fact, to our knowledge, all attempts to derive the simple covariant convolution model have ultimately resorted to some prescription to account for the fact that the bound nucleon has $p^2 \neq M^2$. Without performing a full calculation which self-consistently accounts for the nucleon virtuality, the validity of the various *ad hoc* approximations remains unclear. In short, the naive convolution formula is not a sound starting point for discussing off-shell effects and we make no use of it.

There exist alternative approaches to these just described which do not suffer from off-mass-shell ambiguities. For the nuclear EMC effect, Berger *et al.* [15] used light-front dynamics to calculate the nuclear structure functions. Here all particles are on mass shell, and the transverse momentum and the light-cone variable $p_+ = p_0 + p_L$ are conserved at each vertex, while $p_- = p_0 - p_L$ is not. Alternatively, Johnson and Speth [3] and Heller and Thomas [4] used old-fashioned perturbation theory with the instant form of dynamics, where particles are on mass shell and three-momentum is conserved, but not necessarily energy. Unfortunately, in both of these approaches, the off-mass-shell ambiguities in the definition of the off-shell structure functions are simply replaced by off-energy-shell ambiguities [16]. A review of some of the problems with these approaches may be found in Refs. [17,18].

The advantage of the covariant method in nuclear calculations is that Lorentz invariance is manifest. However, for a consistent treatment within this framework one has to include the antiparticle degrees of freedom, which has not been done up to now. We will set up the formalism in such a way that the structure functions of the physical target are expressed in terms of fully relativistic quark-nucleon and nucleon-target vertex functions. This will enable us to ensure gauge invariance, the Callan-Gross relation, and an unambiguous identification of the scaling variables. All model approximations will be contained entirely in the vertex functions themselves, which, of course, we cannot calculate from first principles.

This paper is organized as follows. In Sec. II we define the general structure of the off-shell tensor in terms of a suitable set of structure functions. In Sec. III we explicitly calculate the scaling properties of these functions. As we shall see, only 3 of 14 possible functions contribute in the Bjorken limit. In Sec. IV we discuss how our formalism can be used to calculate structure functions of composite particles and discuss the limits in which the conventional convolution model may be obtained. In Sec. V we use some simple parametrizations of the relativistic vertex functions to calculate the nucleon valence quark distributions. Using these same vertex functions we then calculate in Sec. VI the structure functions of composite targets containing off-shell nucleons.

II. GENERAL STRUCTURE OF THE OFF-SHELL NUCLEON TENSOR

The process in which we are interested is depicted in Fig. 1, with the photon momentum q and the off-shell nucleon momentum p marked. Because of Hermiticity, covariance, parity, and time-reversal invariance, the corresponding off-shell tensor $\chi_{\mu\nu}$ is a 4×4 matrix depending on p and q, and may in general be written in terms of 14 real functions:

$$\chi_{\mu\nu}(p,q) = \chi^0_{\mu\nu}(p,q) + \not{p}\,\chi^1_{\mu\nu}(p,q) + \not{q}\,\chi^2_{\mu\nu}(p,q), \\ +\gamma_{\{\mu}p_{\nu\}}\,\chi^3(p,q) + \gamma_{\{\mu}q_{\nu\}}\,\chi^4(p,q), \quad (4)$$

where the curly brackets $\{\cdots\}$ around the subscripts indicate the symmetric $\mu\nu$ combination. Here, $\chi^i_{\mu\nu}(p,q)$ are the most general tensors of rank 2 which may be constructed out of q and p:

$$\chi^i_{\mu\nu}(p,q) = P_{T\mu\nu}(p,q)\,\chi^i_T(p,q) + P_{L\mu\nu}(p,q)\,\chi^i_L(p,q) \\ +P_{Q\mu\nu}(p,q)\,\chi^i_Q(p,q) \\ +P_{QL\mu\nu}(p,q)\,\chi^i_{QL}(p,q), \quad i=0,1,2. \quad (5)$$

The functions χ^i on the right-hand side of Eq. (5), as well as χ^3 and χ^4 in Eq. (4), are real scalar functions of q and p. The tensors $P^{\mu\nu}$ are defined by

$$P^{\mu\nu}_T(p,q) = \widetilde{g}^{\mu\nu} + \frac{\widetilde{p}^\mu\widetilde{p}^\nu}{\widetilde{p}^2},$$

$$P^{\mu\nu}_L(p,q) = \frac{\widetilde{p}^\mu\widetilde{p}^\nu}{\widetilde{p}^2},$$

$$P^{\mu\nu}_Q(p,q) = \frac{q^\mu q^\nu}{q^2},$$

$$P^{\mu\nu}_{QL}(p,q) = \frac{1}{\sqrt{-q^2\widetilde{p}^2}}\left(\widetilde{p}^\mu q^\mu + \widetilde{p}^\nu q^\mu\right), \quad (6)$$

where $\widetilde{g}_{\mu\nu} = -g_{\mu\nu} + q_\mu q_\nu/q^2$ and $\widetilde{a}_\mu = a_\mu - q_\mu\, a\cdot q/q^2$, with a_μ being any four-vector.

The above decomposition of the off-shell tensor is of course not unique. It is written in this convenient form because the tensors $P^{\mu\nu}$ turn out to be projection operators [11] and satisfy

$$P^{\mu\nu}_T(p,q)\,P_{T\mu\nu}(p,q) = 2,$$

$$P^{\mu\nu}_L(p,q)\,P_{L\mu\nu}(p,q) = 1,$$

$$P^{\mu\nu}_Q(p,q)\,P_{Q\mu\nu}(p,q) = 1,$$

$$P^{\mu\nu}_{QL}(p,q)\,P_{QL\mu\nu}(p,q) = -2, \quad (7)$$

with all other combinations vanishing. It is important to note that in the Bjorken limit these relations are also true for projectors involving different momenta. That is, the projectors are still orthogonal in this limit and

$$P^{\mu\nu}_T(p_1,q)\,P_{T\mu\nu}(p_2,q) = 2, \text{ etc.} \quad (8)$$

In general, Fig. 1 is a subdiagram of Fig. 2, where $\mathcal{P}$ is the on-shell momentum of the composite target (labeled A). As will be discussed more fully in Secs. IV and V, the hadron tensor for the complete process, $W^A_{\mu\nu}(\mathcal{P},q)$, involves an integral over the nucleon momentum p of the tensor $\chi_{\mu\nu}(p,q)$, traced with another 4×4 matrix originating in the soft, target-constituent part of the diagram. Hence no experiment measures the off-shell tensor by itself, and so it is not possible to measure all the functions χ^i separately. Only combinations thereof give rise to observable experimental quantities. Using the above projectors, we can determine which combinations of off-shell structure functions contribute to the physical ones. In particular, the operators $P^{\mu\nu}_T(\mathcal{P},q)$, $P^{\mu\nu}_L(\mathcal{P},q)$, $P^{\mu\nu}_Q(\mathcal{P},q)$, and $P^{\mu\nu}_{QL}(\mathcal{P},q)$ project from the composite target tensor $W^A_{\mu\nu}(\mathcal{P},q)$ the transverse, longitudinal, and the two possible non-gauge-invariant contributions, respectively, in terms of the scalar functions $\chi^i(p,q)$.

Not all of the functions χ^i will in fact be independent, as the gauge invariance of the theory requires that the latter two contributions vanish. Furthermore, the longitudinal function must also be zero in the Bjorken limit ($\mathcal{P}\cdot q$, $Q^2 \equiv -q^2 \to \infty$, $x = Q^2/2\mathcal{P}\cdot q$ fixed), if the Callan-Gross relation is to be satisfied. That this is indeed the case is shown explicitly in the Appendix. For

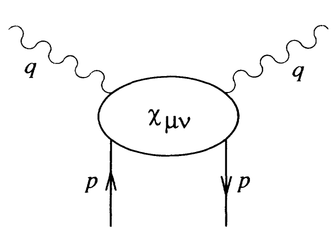

FIG. 1. The truncated nucleon tensor $\chi_{\mu\nu}$.

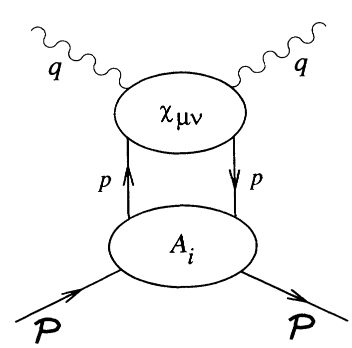

FIG. 2. Scattering from an off-shell nucleon in a composite-target. The functions A_i describe the nucleon–composite-target interaction.

the remaining physical (transverse) contribution we obtain for the coefficients of the χ^i's the following:

$$\frac{1}{2}P_T^{\mu\nu}(\mathcal{P},q)\ \chi_{\mu\nu}(p,q) = \chi_T^0(p,q) + \not{p}\,\chi_T^1(p,q) + \not{q}\,\chi_T^2(p,q)$$
$$+\frac{q^2}{2(p\cdot q)^2}\ (p-y\mathcal{P})^2\left[\chi_L^0(p,q) + \not{p}\,\chi_L^1(p,q) + \not{q}\,\chi_L^2(p,q)\right]$$
$$+\left[-\not{p}+y\not{\mathcal{P}}+\frac{1}{\mathcal{P}\cdot q}\left(\mathcal{P}\cdot p - y\mathcal{P}^2\right)\not{q}\right]\chi^3(p,q), \tag{9}$$

where $y = p\cdot q/\mathcal{P}\cdot q = p_+/\mathcal{P}_+$ is the constituent's light-cone momentum fraction. In the next section we derive the scaling behavior of the functions χ^i using the parton model, by separating the hard, q^2-dependent part of the truncated amplitude $\chi_{\mu\nu}(p,q)$ from the soft, nonperturbative component. We will see that Eq. (9) simplifies considerably, as many terms do not contribute in the Bjorken limit.

A special case of the above formalism is DIS from an on-shell nucleon, described by the tensor which we denote by $W_{\mu\nu}^N(p,q)$. In this case the contribution to the nucleon tensor is given by Eq. (9) traced with $(\not{p}+M)/2$, where $\mathcal{P}$=p and no integration over p is performed:

$$M\ W_{\mu\nu}^N(p,q;p^2=M^2) = \tfrac{1}{2}\mathrm{Tr}\left[(\not{p}+M)\ \widetilde{\chi}_{\mu\nu}(p,q)\right]. \tag{10}$$

This gives the transverse unpolarized on-shell structure function in terms of the on-shell limits of the functions χ^i ($\widetilde{\chi}^i(p,q)\equiv\chi^i(p,q;p^2=M^2)$):

$$\frac{M}{2}W_T^N(p,q) = M\ \widetilde{\chi}_T^0(p,q) + M^2\ \widetilde{\chi}_T^1(p,q)$$
$$+\ p\cdot q\ \widetilde{\chi}_T^2(p,q). \tag{11}$$

Similar expressions can also be found for the other functions (i.e., longitudinal and non-gauge-invariant), but again these vanish in the Bjorken limit.

III. SCALING BEHAVIOR OF THE OFF-SHELL FUNCTIONS χ

In this section we calculate the leading twist contribution to the off-shell structure functions within the covariant quark-parton model. The formal result of the operator product expansion enables us to separate the imaginary part of the forward scattering amplitude, depicted in Fig. 3, into its hard (calculable perturbatively) and soft (nonperturbative) components, denoted by $r_{\mu\nu}$ and $H(k,p)$, respectively. A method similar to this was also discussed in Ref. [19]. This then enables the off-shell tensor $\chi_{\mu\nu}$ to be written as

$$\left[\chi_{\mu\nu}(p,q)\right]_{ab} = \mathcal{I}\left[r_{\mu\nu}(k,q)\right]_{cd}\left[H(k,p)\right]_{dcab}, \tag{12}$$

where $\mathcal{I}$ is the integral operator,

$$\mathcal{I} = \int\frac{d^4k}{(k^2-m^2)^2}\delta([q+k]^2-m^2), \tag{13}$$

and the Dirac matrix structure has been made explicit. (The complete forward scattering amplitude in addition contains the crossed photon diagram, which we do not explicitly take into account. All the formal results of Secs. I–IV remain valid upon inclusion of this diagram. Numerically, it can make a small contribution in the small-x region; however, in the subsequent model calculation in which we consider only two-quark intermediate states there will be no contribution.) In Eq. (13) k is the parton's four-momentum and m its (current) mass. In the following we will drop quark mass terms as the difference between the $m=0$ results and those for $m\sim$ few MeV is negligible. (We shall return to the question of quark masses in Sec. V.) The vector nature of the quark-photon coupling then determines the structure of the tensor $r_{\mu\nu}$ to be

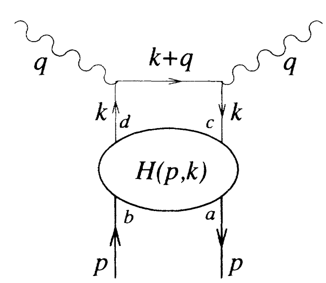

FIG. 3. Leading twist contribution to the off-shell tensor $\chi_{\mu\nu}$. The function $H(p,k)$ describes the soft, nonperturbative physics.

$$r_{\mu\nu}(k,q) = k^2\left[q_\alpha g_{\mu\nu} - (k_\mu + q_\mu)\, g_{\nu\alpha}\right]\gamma^\alpha + \not{k}\left(q^2 g_{\mu\nu} + 4k_\mu k_\nu + 2k_{\{\mu}q_{\nu\}}\right). \tag{14}$$

Following this, the trace over the indices c, d in Eq. (12) may be performed and the results written as

$$\mathrm{Tr}[r_{\mu\nu}[H]_{ab}] = \left\{k^2\left[q_\alpha g_{\mu\nu} - \left(k_\mu + q_{\{\mu}\right) g_{\nu\}\alpha}\right] + k_\alpha\left(q^2 g_{\mu\nu} + 4k_\mu k_\nu + 2k_{\{\mu}q_{\nu\}}\right)\right\}[G^\alpha(p,k)]_{ab}. \tag{15}$$

As G^α is a $4{\times}4$ matrix which transforms like a vector and must be even under parity transformations, its most general form is

$$G^\alpha = I\,(p^\alpha f_1 + k^\alpha f_2) + \not{k}\,(p^\alpha f_3 + k^\alpha f_4) + \not{p}\,(p^\alpha f_5 + k^\alpha f_6) + \gamma^\alpha f_7, \tag{16}$$

where the functions f_i are scalar functions of p and k.

The integrals over k can be done in a standard way. For example, for an integrand containing one free k^α, contracting with p_α and q_α enables us to make the replacement

$$\mathcal{I}k^\alpha \to \mathcal{I}\left\{\rho_1\, p^\alpha + \rho_2\, q^\alpha\right\}. \tag{17}$$

Similarly, for $k^\alpha k^\beta$ terms,

$$\mathcal{I}k^\alpha k^\beta \to \mathcal{I}\left\{\rho_3\, P_T^{\alpha\beta}(p,q) + \tilde{p}^2\rho_1^2\, P_L^{\alpha\beta}(p,q) + \frac{(k\cdot q)^2}{q^2}P_Q^{\alpha\beta}(p,q) - \frac{\tilde{k}\cdot\tilde{p}\;k\cdot q}{\sqrt{-q^2\,\tilde{p}^2}}P_{QL}^{\alpha\beta}(p,q)\right\} \tag{18}$$

and, for $k^\alpha k^\beta\,\not{k}$ terms,

$$\begin{aligned}\mathcal{I}k^\alpha k^\beta\,\not{k} \to \mathcal{I}\Bigg\{&-\rho_3\left(\rho_1 p^{\{\alpha} + \rho_2 q^{\{\alpha}\right)\gamma^{\beta\}} + \rho_3\left(\rho_1\,\not{p} + \rho_2\,\not{q}\right)P_T^{\alpha\beta}(p,q)\\ &+\rho_1\left(\left[2\rho_3 + \tilde{p}^2\rho_1^2\right]\left[\not{p} - \frac{p\cdot q}{q^2}\,\not{q}\right] + \frac{k\cdot q}{q^2}\tilde{p}^2\rho_1\,\not{q}\right)\,P_L^{\alpha\beta}(p,q)\\ &+\frac{(k\cdot q)}{q^2}\left(k\cdot q\rho_1\,\not{p} + \left[k\cdot q\rho_2 + 2\rho_3\right]\not{q}\right)\,P_Q^{\alpha\beta}(p,q)\\ &-\frac{1}{\sqrt{-q^2\,\tilde{p}^2}}\left(\left[k\cdot q\rho_3 + \tilde{p}^2\rho_1^2\right]\not{p} + \left[\tilde{p}^2\rho_1(\rho_3 + k\cdot q\rho_2) - \frac{p\cdot q\;k\cdot q}{q^2}\rho_3\right]\not{q}\right)P_{QL}^{\alpha\beta}(p,q)\Bigg\},\end{aligned} \tag{19}$$

where

$$\rho_1 = \frac{\tilde{k}\cdot\tilde{p}}{\tilde{p}^2},\quad \rho_2 = \frac{k\cdot q}{q^2} - \frac{p\cdot q\;\tilde{k}\cdot\tilde{p}}{q^2\,\tilde{p}^2},\quad \rho_3 = \frac{1}{2}\left(-\tilde{k}^2 + \tilde{p}^2\rho_1^2\right). \tag{20}$$

The χ^i are then completely defined in terms of the functions f_1–f_7, and as all the dependence on the photon momentum q is now explicit, their scaling behavior may be derived in a straightforward manner. We find that χ_T^0 and χ_T^1 are of order 1, while all other χ^i are of order $1/\nu$. Hence we find that deep-inelastic scattering from an off-shell nucleon may be expressed in terms of just three functions:

$$\tfrac{1}{2}P_T^{\mu\nu}(\mathcal{P},q)\;\chi_{\mu\nu}(p,q) = \chi_T^0(p,q) + \not{p}\;\chi_T^1(p,q) + \not{q}\;\chi_T^2(p,q). \tag{21}$$

The complete expressions for the functions χ_T^i are

$$\chi_T^0(p,q) = \mathcal{I}\left\{-\left(k^2 p\cdot q + q^2 k\cdot p\right)f_1(k,p) - \frac{q^2k^2}{2}f_2(k,p)\right\}, \tag{22a}$$

$$\begin{aligned}\chi_T^1(p,q) = \mathcal{I}\Bigg\{&-\left(k^2 p\cdot q + q^2 k\cdot p\right)\left(f_5(k,p) - \frac{q^2}{2p\cdot q}f_3(k,p)\right)\\ &-\frac{q^2k^2}{2}\left(f_6(k,p) - \frac{q^2}{2p\cdot q}f_4(k,p)\right) + \frac{q^4}{2p\cdot q}f_7(k,p)\Bigg\},\end{aligned} \tag{22b}$$

$$\chi_T^2(p,q) = \mathcal{I}\left\{-\left(\frac{p^2q^2}{2p\cdot q} + k\cdot p\right)\left(\left[k^2 + \frac{k\cdot p\;q^2}{p\cdot q}\right]f_3(k,p) + \frac{q^2}{2p\cdot q}\left[k^2\;f_4(k,p) + 2\;f_7(k,p)\right]\right) - k^2\;f_7(k,p)\right\}. \tag{22c}$$

For the other χ it can be easily demonstrated (see the Appendix for details) that for each of the arbitrary functions f_i there are cancellations at leading order in ν in the expressions for $P_L^{\mu\nu}(\mathcal{P},q)\chi_{\mu\nu}(p,q)$, $P_Q^{\mu\nu}(\mathcal{P},q)\chi_{\mu\nu}(p,q)$, and $P_{QL}^{\mu\nu}(\mathcal{P},q)\chi_{\mu\nu}(p,q)$ in the Bjorken limit. Hence the Callan-Gross relation, as well as gauge invariance ($q^\mu W_{\mu\nu}^A = 0$), are assured, independent of the nature of the target-constituent part of the diagram. This result is completely general, so that model-dependent approximations for the vertex functions do not affect these results.

IV. CONVOLUTION MODEL

Before we move on to making model-dependent assumptions for the vertex functions, we need to write down the on-shell tensor $W^A_{\mu\nu}(\mathcal{P},q)$ for the target A in terms of the off-shell tensor $\chi_{\mu\nu}(p,q)$. The full tensor for the composite target is given by

$$M_T\ W^A_{\mu\nu}(\mathcal{P},q) = \int \frac{d^4p}{(2\pi)^4} \frac{2\pi\delta\left([\mathcal{P}-p]^2 - M_R^2\right)}{(p^2-M^2)^2}\ \mathrm{Tr}[\,[IA_0(p,\mathcal{P}) + \gamma_\alpha A_1^\alpha(p,\mathcal{P})]\ \chi_{\mu\nu}(p,q)], \tag{23}$$

where A_0 and A_1 are functions describing the target-constituent part of the complete diagram in Fig. 2, and M_T and M_R are the masses of the target and target recoil systems, respectively. Note that Eq. (23) applies to a target recoil state of definite mass M_R. In general we could also have a sum over all excited recoil states or, equivalently, an integration over the masses M_R weighted by some target recoil spectral function. The transverse structure function of the target is obtained from Eq. (23) by using the transverse projection operator defined in Eq. (6):

$$\begin{aligned} M_T\ W^A_T(\mathcal{P},q) &= \frac{M_T}{2} P_T^{\mu\nu}(\mathcal{P},q)\ W^A_{\mu\nu}(\mathcal{P},q) \\ &= \frac{1}{4\pi^2}\int \frac{dy\ dp^2}{(p^2-M^2)^2}\left\{A_0(p,\mathcal{P})\ \chi^0_T(p,q)\ +\ p\cdot A_1(p,\mathcal{P})\ \chi^1_T(p,q)\ +\ q\cdot A_1(p,\mathcal{P})\ \chi^2_T(p,q)\right\}, \end{aligned} \tag{24}$$

where $p^2 = p_+p_- - \mathbf{p}_T^2$, $p_+ = y\mathcal{P}_+ (= yM_T$ in the target rest frame), and we have used the δ function to fix $p_- = M_T\ +\ (M_R^2 + \mathbf{p}_T^2)/(p_+ - M_T)$.

The convolution model may only be derived from Eq. (24) if we make some additional assumptions. First of all, we need to assume that the target structure function can be written in factorized form, in terms of the nucleon structure function W^N_T and some nucleon distribution function φ:

$$W^A_T(x,Q^2) = \int dy \int dp^2\ \ \varphi(A_0,A_1)\ \ W^N_T(x/y,Q^2,p^2). \tag{25}$$

Furthermore, to obtain the usual one-dimensional convolution formula [1,11] we must assume that W^N_T is independent of p^2:

$$W^A_T(x,Q^2) = \int dy\ \widetilde{\varphi}(y)\ \ W^N_T(x/y,Q^2), \tag{26}$$

where now the integral over p^2 has been absorbed into the definition of $\widetilde{\varphi}$.

There are several ways in which the first assumption might be valid.

Case (a). If all but one of the functions χ^i_T (i=0–2) are zero in the Bjorken limit. Most authors (see, for example, Refs. [1,10,11]) adopt this choice, as this is the case for a pointlike fermion (where only χ^2_T contributes). However, as was shown in Sec. III, all three functions χ^i_T in principle contribute in the Bjorken limit, so that one would require that some of the functions f in Eqs. (22) vanish or cancel. We know of no reason why this should be the case — indeed even the extremely simple quark–nucleon vertex functions which we consider in the next section give rise to more than one nonvanishing χ^i_T.

Case (b). If more than one of the χ^i_T is nonzero, but the nonzero ones are proportional to each other. For example, $f_1 = Mf_5$ and all other f's equal to zero would imply $\chi^0_T = M\chi^1_T$, and so Eq. (25) is obtained. Again, in general there does not seem to be any reason to expect this behavior.

Case (c). If the nonzero nucleon-target functions $A_{0,1}$ multiplying the functions χ^i_T are proportional to each other. An example of this would be if $A_0\ =\ p\cdot A_1/M\ =\ q\cdot A_1\ M/p\cdot q$, which would then give Eq. (25). In general this will not be true unless the $p^2\ =\ M^2$ limit is taken inside the functions $A_{0,1}$.

In short, none of the above conditions are generally satisfied in a self-consistent, fully covariant (relativistic) calculation. Consequently the convolution model interpretation, Eq. (26), of the nuclear structure function in terms of bound nucleon structure functions is inconsistent within this formalism. This difficulty is intrinsically related to the presence of antinucleon degrees of freedom, which are not accounted for in the traditional convolution model. Furthermore, in the absence of the convolution model, the common practice of extracting nucleon structure functions from nuclear DIS data is rather ambiguous. Indeed, the very concept of a structure function of a nucleon bound within a nucleus loses its utility. One is *forced* to consider quark and nuclear degrees of freedom side by side in the calculation of nuclear structure functions.

Using Eq. (24) directly we may compare, for some simple vertex functions, the exact result with those obtained by making the convolution model approximation, Eq. (26). This we will do in the next section. As a final comment, it should be noted that, within the physical assumptions made by the use of the model in the first place (i.e., no final-state interactions), the functions χ^i_T are independent of the physical target, and depend only on the constituent nucleon. By selecting various targets (i.e., by varying $A_{0,1}$) the relative contributions from the functions χ^i_T could in principle be probed, provided, of course, we know the nucleon-target functions sufficiently well. Conversely, once the χ^i_T have been determined for one process, they may be used for all other processes.

V. CALCULATION OF THE NUCLEON STRUCTURE FUNCTION

To calculate the transverse structure function of the complete target requires two sets of functions describing the soft, nonperturbative physics, namely, the quark-nucleon functions f_1–f_7 and the nucleon-target functions A_0, A_1. Here we concentrate on the former set.

We observe that because both the constituent nucleon and struck quark inside the nucleon have spin 1/2, the intermediate spectator state will have either spin 0 or 1. In order to make an overall Lorentz scalar, we therefore need only consider quark-nucleon vertices that transform as a scalar or vector under Lorentz transformations. It is straightforward to identify the form of the vertices that are allowed by Lorentz, parity, and time-reversal invariance; however the specific momentum dependence has to be determined within a model. There will be 15 independent scalar $[\Phi^S_{1-4}(k,p)]$ and vector $[\Phi^V_{1-11}(k,p)]$ vertex functions appearing in the general expression

$$\mathcal{V}^S = I\,\Phi^S_1 + \not{p}\,\Phi^S_2 + \not{k}\,\Phi^S_3 + i\sigma_{\alpha\beta}p^\alpha k^\beta\,\Phi^S_4 \tag{27}$$

for a scalar vertex and

$$\begin{aligned}\mathcal{V}^V_\alpha = {} & \gamma_\alpha\,\Phi^V_1 + p_\alpha\,I\,\Phi^V_2 + k_\alpha\,I\,\Phi^V_3 + i\sigma_{\alpha\beta}p^\beta\,\Phi^V_4 \\ & + i\sigma_{\alpha\beta}k^\beta\,\Phi^V_5 \\ & + p_\alpha\,\not{p}\,\Phi^V_6 + p_\alpha\,\not{k}\,\Phi^V_7 + k_\alpha\,\not{p}\,\Phi^V_8 + k_\alpha\,\not{k}\,\Phi^V_9 \\ & + i\sigma_{\beta\delta}p^\beta k^\delta\,p_\alpha\,\Phi^V_{10} + i\sigma_{\beta\delta}p^\beta k^\delta\,k_\alpha\,\Phi^V_{11}\end{aligned} \tag{28}$$

for a vector vertex.

The functions f_1–f_7 in Eq. (16) can be uniquely determined from these vertex functions. To see this, let us first consider the scalar vertex. The general target-constituent function from Sec. III, $[H(k,p)]_{dcab}$, will be proportional to $(\mathcal{V}^{S\dagger})_{ac}(\mathcal{V}^S)_{db}$. Using the Fierz theorem the Dirac indices can be rearranged into a form that enables the connection with the functions f_i to be explicit:

$$\begin{aligned}f_1 &= 2\,\left[\Phi^S_1\,\Phi^S_2 - k\cdot(p\,\Phi^S_2 + k\,\Phi^S_3)\,\Phi^S_4\right]\delta\,,\\ f_2 &= 2\,\left[\Phi^S_1\,\Phi^S_3 + p\cdot(p\,\Phi^S_2 + k\,\Phi^S_3)\,\Phi^S_4\right]\delta\,,\\ f_3 &= 2\,\left[\Phi^S_2\,\Phi^S_3 + p\cdot k\,(\Phi^S_4)^2 - \Phi^S_1\,\Phi^S_4\right]\delta\,,\\ f_4 &= 2\,\left[(\Phi^S_3)^2 - p^2\,(\Phi^S_4)^2\right]\delta\,,\\ f_5 &= 2\,\left[(\Phi^S_2)^2 - k^2\,(\Phi^S_4)^2\right]\delta\,,\\ f_6 &= 2\,\left[\Phi^S_2\,\Phi^S_3 + p\cdot k\,(\Phi^S_4)^2 + \Phi^S_1\,\Phi^S_4\right]\delta\,,\\ f_7 &= \Big[(\Phi^S_1)^2 - (p\,\Phi^S_2 + k\,\Phi^S_3)^2 \\ &\qquad + \{p^2k^2 - (p\cdot k)^2\}\,(\Phi^S_4)^2\Big]\,\delta,\end{aligned} \tag{29}$$

where $\delta \equiv \delta\left([p-k]^2 - m_S^2\right)$ and m_S is the mass of the scalar spectator system.

Calculating the functions Φ^S_{1-4} from first principles amounts to solving the relativistic, many-body bound state problem. As this is presently not possible one could resort to models such as the MIT bag model. It is not our aim to do this in this paper. Rather, we shall choose a single scalar vertex, say, $\mathcal{V}^S = I\,\Phi^S_1$, and use phenomenological input to constrain its functional form. From Eq. (29) we find that the $I\,\Phi^S_1$ vertex contributes only to f_7:

$$f_7 = (\Phi^S_1)^2\,\delta\left([p-k]^2 - m_S^2\right) \quad \text{[scalar vertex]}. \tag{30}$$

Similarly we choose for the vector vertex a single form, $\mathcal{V}^V_\alpha = \gamma_\alpha\Phi^V_1$, and find that this makes the contributions

$$f_3 = -f_4 = -f_5 = f_6 = -\frac{2\,f_7}{m_V^2} = -\frac{2\,(\Phi^V_1)^2\,\delta\left([p-k]^2 - m_V^2\right)}{m_V^2} \quad \text{[vector vertex]}, \tag{31}$$

where m_V is the mass of the vector spectator state. In writing Eq. (31) we have assumed that the intermediate vector state has a Lorentz structure $-g_{\alpha\beta} + (p_\alpha - k_\alpha)(p_\beta - k_\beta)/m_V^2$. For the sake of simplicity we further assume that only valence quarks are present, so that the scalar or vector spectator may be identified with a diquark. In a more refined calculation one could, for example, integrate over diquark masses using some diquark spectral function.

From Eqs. (30) and (31) we see that even the simplest vertex functions lead to a large number of nonzero functions f_i. This in turn implies that there are scaling contributions to both of the functions χ^1_T and χ^2_T in Eqs. (22), thereby failing to satisfy scenarios (a) and (b) in Sec. IV for the derivation of the convolution model. For more complicated quark-nucleon vertices, even more of the f's will be nonzero.

The k^2 dependence of the functions $\Phi^{S,V}_1$ can be most easily modeled by considering the on-shell nucleon structure function. (We shall approximate the quark's off-shell dependence to be the same in on- and off-shell nucleons — see Sec. VI A for a further discussion on this point.) The large-x limit is known to be dominated by valence u quarks, which implies that the scalar vertex dominates at large x [20]. Now note that, as the spectator state is on mass shell, the quark four-momentum will behave as $k^2 \sim (-m_S^2 - \mathbf{k}_T^2)/(1-x)$ at large x. In order to obtain the correct large-x behavior of the structure function, namely, $W_T^N \sim (1-x)^3$, the k^2 dependence in the scalar vertex function must be $1/k^2$, after we also take into account the two quark propagators, as well as the factor $(1-x)$ arising from the delta function $\delta\left([p-k]^2 - m_R^2\right)$ for the on-shell diquark state of mass m_R ($= m_S$ or m_V) [21].

We fix the large-k^2 behavior of the vector vertex function in a similar way, this time by requiring that we obtain the correct valence d_V/u_V ratio at large x, namely, $\sim (1-x)$. This means that the vertex function for DIS from valence d_V quarks has to go like $(1-x)^4$ for large x, i.e., like $(k^2)^{-5/2}$ [there is an additional $(1-x)^{-2}$ factor arising from the trace for the vector diquark].

It may now seem reasonable to choose a simple monopole form for the scalar vertex function, as was done, for example, in Refs. [21,11], and a corresponding one for the vector vertex. We do not do this, however, for the following reason. The quark propagator $(k^2-m^2)^{-2}$ in Eq. (13) contains a pole. Because the kinematic maximum for k^2 is $(M-m_R)^2$, this pole is in the physical region of k^2 when $m_R+m<M$. The origin of this pole is clear — the model, so far, is not confining and the proton may dissociate into its quark and diquark constituents. One solution would be to make the sum of the quark and diquark masses so large that this cannot occur. However, we do not believe that this is desirable — confinement occurs not because the quark mass is large (it is only a few MeV), but in a dynamical way associated with the nature of the color interaction. The only way that the information about color confinement can enter in this model is through the relativistic quark-nucleon vertex function. A convenient way to ensure that the contribution from a deconfined quark is excluded is to choose a numerator in $\Phi_1^{S,V}$ so that the integrand in the structure function remains finite at the on-shell point, $k^2=m^2$.

For the masses of the scalar and vector diquark, m_S and m_V, the only information available to us is that from low energy models, such as the bag model or the nonrelativistic quark model. There, at a scale (Q^2) of order a few hundred MeV^2, the diquark masses are expected to be somewhere within the range of 600–1100 MeV [20,22]. Furthermore, from the nucleon-$\Delta(1232)$ mass splitting we also anticipate that m_V would be some 200 MeV larger than m_S.

The p^2 dependence of the vertex functions is of course more difficult to obtain, since for this purpose data on nuclear structure functions must be used. In this case the p^2 dependence will not be restricted to the quark-nucleon vertex function alone, but will also be present in the nucleon-nucleus vertices, which introduces an inherent uncertainty in the determination of the former. Nevertheless, the functions $\Phi_1^{S,V}$ do not depend on the nuclear target — that information is contained entirely in the functions $A_{0,1}$. Since for the deuteron the p^2 dependence of the relativistic DNN vertex can be related to known deuteron wave functions [23–25], we may use deuteron DIS data to constrain this universal p^2 dependence of the quark-nucleon vertex functions.

In order to obtain the valence quark distribution for the deuteron, we will use data obtained from muon scattering for $x>0.3$, where valence quarks are known to dominate. Because of isospin symmetry ($u^D=d^D$), only a single experimental quantity for the deuteron (compared with two, $u+d$ and d/u, for the nucleon) is meaningful, namely, $F_{2D}=x(4u^D+d^D)/9=5x(u+d)/9$, where u^D, d^D and u, d are the up and down quark distributions in the deuteron and bound proton, respectively. Hence we cannot differentiate between the p^2 dependence in Φ_1^S and that in Φ_1^V. We therefore choose a simple monopole form and use the same cutoff mass Λ_p in both functions. A detailed comparison between the model and data for $x\lesssim 0.3$ would require separation of the valence and sea components of F_{2D}. Although in principle this could be done by analyzing the $\nu-D$ and $\bar{\nu}-D$ DIS data, in practice those data suffer from poor statistics. Furthermore, typically only the extracted quark distributions in the nucleon are presented [26], and these depend on the theoretical assumptions made to treat binding and Fermi motion corrections.

To summarize, the vertex functions that we use are given by

$$\Phi_1^S(p,k)\propto\frac{(k^2-m^2)}{(k^2-\Lambda_S^2)^2}\frac{(M^2-\Lambda_p^2)}{(p^2-\Lambda_p^2)}, \qquad (32a)$$

$$\Phi_1^V(p,k)\propto\frac{(k^2-m^2)}{(k^2-\Lambda_V^2)^{7/2}}\frac{(M^2-\Lambda_p^2)}{(p^2-\Lambda_p^2)}. \qquad (32b)$$

We find the best fit to the experimental nucleon distributions at $Q^2=4$ GeV^2 (we evolve the curves from $Q_0^2=0.15$ GeV^2 using leading order QCD evolution, with $\Lambda_{\rm QCD}=250$ MeV [27]) for masses $m_S=850$ MeV and $m_V=1050$ MeV, and cutoffs $\Lambda_S=1.2$ GeV and $\Lambda_V=1.0$ GeV, which we fit to the recent parametrizations by Morfin and Tung and Owens [28]. The fits to the u_V+d_V valence quark distribution as well as the valence d_V/u_V ratio are shown in Figs. 4 and 5, respectively. It is remarkable that such simple forms for the vertex functions reproduce the data so well.

Having parameterized the free nucleon vertices, we are now ready to consider the specific cases of DIS from the deuteron, from nuclear matter, and from dressed nucleons. Throughout, we consider the isoscalar valence structure function $xW_T\propto x(u_V+d_V)=3x(q_0+q_1)/2$, where q_0 and q_1 are the quark distributions arising in connection with the scalar and vector diquarks, respectively, normalized so that their first moments are unity [from the spin-flavor wave function of the proton we have $d_V=q_1$ and $u_V=(q_1+3q_0)/2$].

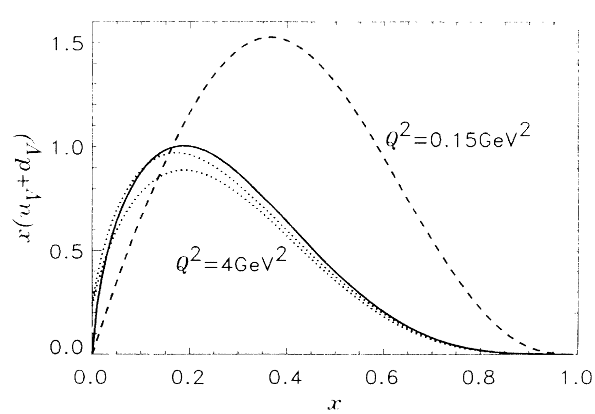

FIG. 4. Valence u_V+d_V quark distribution in the nucleon, evolved from $Q_0^2=0.15$ GeV^2 (dashed curve) to $Q^2=4$ GeV^2 (solid curve), and compared against parametrizations (dotted curves) of world data [28].

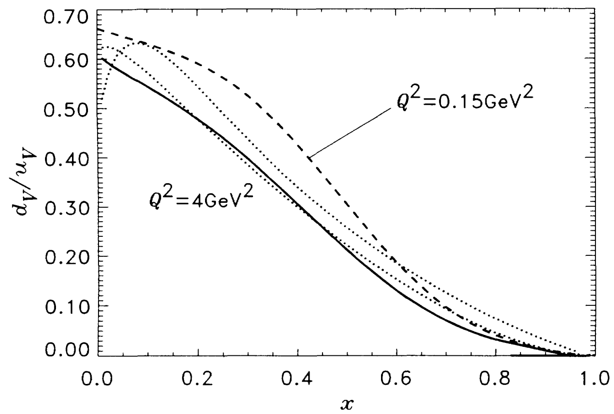

FIG. 5. Valence d_V/u_V ratio, evolved from $Q_0^2 = 0.15$ GeV^2 (dashed curve) to $Q^2 = 4$ GeV^2 (solid curve), and compared against parametrizations (dotted curves) of world data [28].

VI. CALCULATION OF COMPOSITE TARGET STRUCTURE FUNCTIONS

A. DIS From the deuteron

We examine nuclear DIS from a deuteron for several reasons. First, it is critical to know the size of the off-mass-shell corrections to the deuteron structure function if ultimately the nuclear EMC effect data (which usually measures the ratio of nuclear to deuterium structure functions) is to be used to draw conclusions about the differences between quark distributions in free nucleons and those bound in nuclei. Second, in the absence of high-statistics neutrino data, the neutron structure function is often inferred from the deuteron structure function using the naive assumption of additivity of the bound proton and neutron structure functions. Apart from the off-mass-shell effects which we consider here, several other effects spoil this simple assumption. For example, nuclear shadowing is important as $x \to 0$ [29], and of course the deuteron structure function extends beyond $x_N = 1$ $[x_N \equiv (M_D/M)\ x]$ to $x_N = M_D/M$. Hence deviations from additivity occur over much of the range of x. For a reliable extraction of the neutron structure function a systematic computation of these effects is clearly necessary.

The calculation of DIS from the deuteron is more straightforward and reliable than for heavier nuclei, since the relativistic deuteron-nucleon vertex is reasonably well understood. The treatment of the deuteron recoil state is simplified by the fact that most of the time this will be an on-shell nucleon, as this can be expected to dominate the contributions from processes with a recoil Δ or Roper resonance, or a higher mass state.

The structure of the general DNN vertex, with one nucleon on shell, was first derived by Blankenbecler and Cook [30], $\langle N|\psi_N|D\rangle \propto (\not p - M)^{-1}\Gamma^D_\alpha\ \epsilon^\alpha\ \mathcal{C}\bar u^T(\mathcal{P}-p)$, where the DNN vertex function is [31]

$$\Gamma^D_\alpha(p^2) = \gamma_\alpha\ F(p)\ +\ \left(\frac{1}{2}\mathcal{P}_\alpha - p_\alpha\right)\ \frac{G(p)}{M} + \frac{\not p - M}{M}\left[\gamma_\alpha\ H(p)\ +\ \left(\frac{1}{2}\mathcal{P}_\alpha - p_\alpha\right)\ \frac{J(p)}{M}\right], \tag{33}$$

and $\mathcal{C}$ is the charge conjugation operator. The functions F, G, H, and J are related to the $^3S_1$, $^3D_1$, $^1P_1$, and $^3P_1$ deuteron wave functions, u, w, v_s, and v_t, respectively, by

$$F(p) = \pi\sqrt{2M_D}\ (2E_p - M_D)\left(u(\mathbf{p}) - \frac{w(\mathbf{p})}{\sqrt{2}} + \sqrt{\frac{3}{2}}\frac{M}{|\mathbf{p}|}v_t(\mathbf{p})\right), \tag{34a}$$

$$G(p) = \pi\sqrt{2M_D}\ (2E_p - M_D)\left(\frac{M}{E_p+M}u(\mathbf{p}) + \frac{M\ (2E_p+M)}{\mathbf{p}^2}\frac{w(\mathbf{p})}{\sqrt{2}} + \sqrt{\frac{3}{2}}\frac{M}{|\mathbf{p}|}v_t(\mathbf{p})\right), \tag{34b}$$

$$H(p) = \pi\sqrt{2M_D}\ \frac{E_pM}{|\mathbf{p}|}\ \sqrt{\frac{3}{2}}\ v_t(\mathbf{p}), \tag{34c}$$

$$J(p) = -\pi\sqrt{2M_D}\ \frac{M^2}{M_D}\left(\frac{2E_p - M_D}{E_p+M}u(\mathbf{p}) - \frac{(2E_p-M_D)(E_p+2M)}{\mathbf{p}^2}\frac{w(\mathbf{p})}{\sqrt{2}} + \frac{\sqrt{3}M_D}{|\mathbf{p}|}\ v_s(\mathbf{p})\right), \tag{34d}$$

where $E_p = \sqrt{M^2+\mathbf{p}^2}$ and $\mathbf{p}$ is the off-shell nucleon's three-momentum. For the deuteron wave functions we use the model of Buck and Gross [23], with a pseudovector π-exchange interaction.

For the spin-averaged deuteron hadronic tensor we therefore need to evaluate the trace

$$\sum_\lambda \epsilon^{*\alpha}(\lambda,\mathcal{P})\ \epsilon^\beta(\lambda,\mathcal{P})\ \mathrm{Tr}\left[(\not{\mathcal{P}}^T - \not p^T + M)\ \mathcal{C}\overline{\Gamma}^D_\beta(p^2)\ (\not p + M)\ \chi_{\mu\nu}(p,q)\ (\not p + M)\ \Gamma^D_\alpha(p^2)\mathcal{C}\right], \tag{35}$$

where $\epsilon^\alpha(\lambda,\mathcal{P})$ is the polarization vector for a deuteron with helicity λ, and $\overline{\Gamma}^D_\beta = \gamma_0\ \Gamma^{D\dagger}_\beta\ \gamma_0$. This yields the deuteron-nucleon functions

$$A_0^D(p^2) = M \left\{ 4\ F^2 \left[4\ M^2\ +\ 2\ M_D^2\ -\ (p^2 - M^2) \left(-2 + \frac{p^2 - M^2}{M_D^2} \right) \right] \right.$$
$$-8\ F\ G \left[4\ M^2\ -\ M_D^2\ +\ \frac{(p^2 - M^2)}{4M^2} \left(10\ M^2 - M_D^2 + 2\ p^2 + \frac{3\ M^4 - 2\ M^2 p^2 - p^4}{M_D^2} \right) \right]$$
$$+ \frac{G^2}{M^2} \left[\left(4\ M^2 - M_D^2\right)^2 - (p^2 - M^2) \left(4\ M_D^2 - 5\ p^2 - 11\ M^2 + \frac{2p^4 - 2M^4}{M_D^2} \right) \right]$$
$$- \frac{(p^2 - M^2)}{M^2} \left[-12\ H^2\ (p^2 - M^2) + 4\ F\ H\ \left(-5M^2 - 2M_D^2 + p^2 + \frac{(p^2 - M^2)^2}{M_D^2} \right) \right.$$
$$+ \left(p^2 - \frac{(\mathcal{P} \cdot p)^2}{M_D^2} \right) \left(\frac{(p^2 - M^2)}{M^2} \left(-4\ J^2 + 8\ H\ J \right) +\ 16\ F\ J \right.$$
$$\left. \left. \left. + 16\ G\ H\ +\ 8\ G\ J\ \frac{(\mathcal{P} \cdot p - M^2 - p^2)}{M^2} \right) \right] \right\}, \qquad (36a)$$

$$A_{1\alpha}^D(p^2) = 4\ F^2 \left\{ \left(4\ M^2\ +\ 2\ M_D^2\right)\ p_\alpha -\ (p^2 - M^2) \left(\frac{(p^2 - M^2)}{M_D^2} p_\alpha + \left(2 - \frac{(p^2 - M^2)}{M_D^2} \right) \mathcal{P}_\alpha \right] \right\}$$
$$-8\ F\ G \left\{ \left(4\ M^2\ -\ M_D^2\right)\ p_\alpha +\ (p^2 - M^2) \left[\left(1 - \frac{(p^2 - M^2)}{M_D^2} \right) p_\alpha + \frac{\mathcal{P} \cdot p}{M_D^2} \mathcal{P}_\alpha \right] \right\}$$
$$+ \frac{G^2}{M^2} \left\{ \left(4\ M^2 - M_D^2\right)^2\ p_\alpha - (p^2 - M^2) \left[\left(M_D^2 - 4\ M^2\right) \left(2 - \frac{p^2 - M^2}{M_D^2} \right) p_\alpha \right. \right.$$
$$\left. \left. -4 \left(p^2 - \frac{(\mathcal{P} \cdot p)^2}{M_D^2} \right) \mathcal{P}_\alpha \right] \right\}$$
$$- \frac{(p^2 - M^2)}{M^2} \left\{ 4\ H^2\ (p^2 - M^2) \left[p_\alpha - \left(2 - \frac{p^2 - M^2}{M_D^2} \right) \mathcal{P}_\alpha \right] \right.$$
$$-\ 4\ J^2\ \frac{(p^2 - M^2)}{M^2} \left(p^2 - \frac{(\mathcal{P} \cdot p)^2}{M_D^2} \right) \left[p_\alpha - \mathcal{P}_\alpha \right]\ +\ 8\ H\ J\ (p^2 - M^2) \left[p_\alpha - \frac{\mathcal{P} \cdot p}{M_D^2} \mathcal{P}_\alpha \right]$$
$$-\ 8\ F\ H\ M^2 \left[2\ p_\alpha + \left(2 - \frac{p^2 - M^2}{M_D^2} \right) \mathcal{P}_\alpha \right]$$
$$+\ 4\ F\ J \left[\left(3M^2 + p^2 - \frac{(p^2 - M^2)^2}{M_D^2} \right) p_\alpha - \left(p^2 + M^2 - \frac{(p^2 - M^2)^2}{M_D^2} \right) \mathcal{P}_\alpha \right]$$
$$+\ 8\ G\ H \left[\left(M^2 + p^2 - \mathcal{P} \cdot p \right) p_\alpha + \left(p^2 - \frac{p^2 + M^2}{M_D^2} \mathcal{P} \cdot p \right) \mathcal{P}_\alpha \right]$$
$$\left. +\ 8\ G\ J \left(p^2 - \frac{(\mathcal{P} \cdot p)^2}{M_D^2} \right) \left[-2p_\alpha + \mathcal{P}_\alpha \right] \right\}. \qquad (36b)$$

As mentioned in the previous section, we constrain the cutoff parameter Λ_p in the quark- (off-shell) nucleon vertex functions defined in Eqs. (32) by fitting our full, p^2-dependent calculated distribution to the experimental deuteron structure function, using the lepton-deuteron data from the New Muon Collaboration (NMC), BCDMS, and SLAC [32]. However, we still need to fix the normalization constants in Eqs. (32). Naturally, these will be functions of the cutoff Λ_p. Actually, if the exact quark-nucleon vertex functions were known, they would be the same for the off-shell as for the on-shell nucleon. We do not assume this, however, as the vertex functions which we use are only approximations to the exact results. For example, the arguments given in Sec. V, relating to the counting rules which give the k^2 dependence of the vertex functions, are based on quark distributions in an on-shell nucleon. In an off-shell nucleon the connection between x and k^2 is given by the modified expression

$$k^2 = k_+ k_- - k_T^2$$
$$= x M_D \left(p_- - \frac{(\mathbf{p}_T - \mathbf{k}_T)^2 + m_R^2}{M_D (y - x)} \right) - k_T^2, \qquad (37)$$

with p_- now constrained by the δ function for the on-shell nuclear recoil state (see Sec. IV). In principle, the asymptotic k^2 dependence for the quark–off-shell-nucleon vertices expected from counting rules could be determined after integration over the nucleon's momentum. Clearly this is a much more complicated task than was the case for the on-shell nucleon, and we do not believe our simple ansatz for the vertices warrants such a treatment, in which case we shall simply normalize by comparing with the data.

In Fig. 6 we compare the experimental F_{2D} at $Q^2 = 10$ GeV^2 with the calculated total valence quark distribution in the deuteron, $5x(u_V + d_V)/9$, evolved from the same value of $Q_0^2 = 0.15$ GeV^2 (since we use the same diquark masses) as for the free nucleon distributions in Sec. V. The result of the full calculation is almost independent of the value of Λ_p used, after the normalization constants for the vertex functions have been determined by the charge conservation condition. This is because the p_T distribution is strongly peaked at small transverse momenta, $p_T \sim 25$ MeV, so that modification of the large-p_T (or large-$|p^2|$) behavior by altering the form factor cutoff has negligible consequences. Clearly there is very good agreement between the model calculation and the data for $x \gtrsim 0.3$.

From the discussion in Secs. IV and V it should be clear that it is not possible to justify the convolution model for deuteron deep-inelastic scattering. Still, it is of interest to compare our results with those of previous calculations that have made use of convolutionlike formulas. First, we can notice that by taking the on-shell limit ($p^2 \to M^2$) for the kinematic factors in A_0^D and A_1^D in Eqs. (36), we obtain $A_0^D/M = p \cdot A_1^D/M^2 = q \cdot A_1^D/p \cdot q$, thereby satisfying condition (c) in Sec. IV for the convolution model (although this approximation need not be taken in the functions F, G, H, J themselves). Such an approximation is in the spirit of that used in Ref. [14] for the nuclear structure functions. The result of this approximation is shown in the dashed curve of Fig. 6, where we have used the same normalization constants in Eqs. (32) (for $\Lambda_p = \infty$) as those determined in the full calculation. The effect is a reduction in the absolute value of the structure function, without much affect on the shape. By artificially normalizing the new distribution so that the final result conserves baryon number, this curve becomes almost indistinguishable from the full result. However, there is no good reason for using different normalization constants in this approximation, since the $p^2 = M^2$ limit is taken in the nuclear part of the diagram and thus should not in principle affect the quark–off-shell-nucleon vertex.

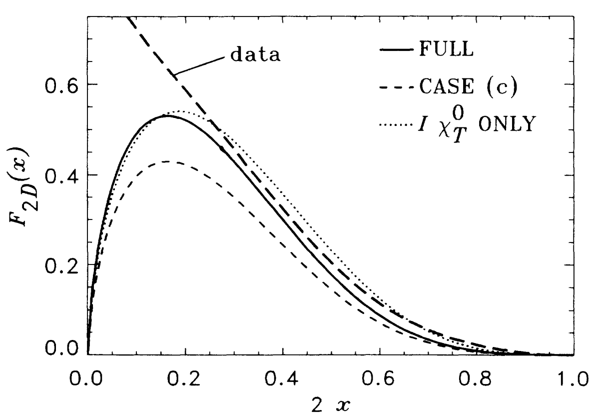

FIG. 6. Valence part of the deuteron structure function: The solid line is the full calculation (with $\Lambda_p = \infty$); the dashed line is with the $p^2 = M^2$ approximation in A_0, A_1 [case (c) in Sec. IV], with the same normalization constants as in the solid curve; the dotted line is the convolution model using only the $\chi_T^0(p,q)$ operator, together with the full nucleon structure function, normalized to baryon number 1. The curves have been evolved from $Q_0^2 = 0.15$ GeV^2 to $Q^2 = 10$ GeV^2 for comparison against the experimental $F_{2D}(x, Q^2 = 10\text{GeV}^2)$ [32].

In other calculations using the convolution model for deuterium the most common prescription has been to drop all terms but $I\chi_T^0$ in the expansion of $\chi_{\mu\nu}$ [in Eq. (35)], and to replace χ_T^0 by the experimental, on-shell structure function of the nucleon [5,33]. In Fig. 6 the dotted curve shows the result after renormalization to ensure baryon number 2 for the deuteron. It is somewhat surprising that the difference in shape between the full result and this ansatz is as small as it is. Still, a discrepancy of $\sim 20\%$ is quite significant in a system as loosely bound as the deuteron.

A numerically significant difference between the convolution approach and the exact calculation is of particular importance if one recalls that the neutron structure function is extracted from structure functions of light nuclei, such as deuterium, using the convolution model. Indeed, in view of the problems which we have just described, it is rather worrying than our knowledge of F_{2n} is based on this. As seen in Fig. 6, depending on the approximation or ansatz taken in calculating F_{2D}, the deviation from the correct, p^2-dependent result, will vary. Still, although unsatisfactory from a theoretical point of view, by artificially renormalizing the deuteron structure function by hand so that it respects baryon number conservation, the differences can be reduced.

A similar situation arises in calculations of the nuclear EMC effect, in which differences between nuclear and deuteron structure functions are explored. Clearly for any accurate description of this effect we need first to have a reliable method of calculating the deuteron structure function. As we have seen, the off-shell effects that are ignored in the deuteron may be compensated for by suitably renormalizing the final result. Whether this can also be done in other, heavier, nuclei is not clear. Certainly in heavy nuclei we would expect off-shell effects to play some role. To date these have not been adequately accounted for, and this is what we turn to next.

B. Nuclear matter

For any nucleus we can easily repeat the above calculation if we know the relativistic nucleon-nucleus vertex functions. Unfortunately, at the present time these are not at all well known for heavy nuclei. A solution to this problem would be to simply parametrize the vertex functions, and to make some assumptions for the nuclear recoil state. Alternatively, if one tried to use nonrelativistic nuclear models as an approximation, it would be difficult to incorporate the off-shell nucleon structure. The best way is to consider first the simpler case of a nucleon embedded in nuclear matter. In this type of calculation the off-shell effects are parametrized in the effective nucleon mass, $M \to M^*$.

Experimentally, the effective nucleon mass at nuclear matter density ($\sim$ 0.15 fm^{-3}) is found to be $\sim 0.7M$ [34]. Theoretically, there is a large number of models for nuclear matter, which predict a wide range of effective nucleon masses. The quantum hadrodynamics model of Serot and Walecka [34], in which pointlike nucleons (in the mean field approximation) are bound by the exchange of scalar (σ) and vector (ω) mesons, predicts rather small effective masses, $M^*/M \simeq$ 0.5–0.6. Somewhat larger masses are obtained when explicit quark degrees of freedom are introduced. For example, in the Guichon model [35], where the σ and ω mesons are allowed to couple directly to quarks inside the nucleons, the value of M^* is typically $\sim 0.9M$. Even larger values are obtained if one includes center-of-mass corrections and self-coupling of the scalar fields [36,37]. Rather than choose a specific nuclear model, we let M^* be a parameter and examine the effect of its variation upon the nucleon structure function, defined in Eq. (11).

Because the quark-nucleon vertex function will now also depend on the effective mass, it would be inappropriate to use the same normalization constants in Eqs. (32) as those determined by normalizing the on-shell nucleon distributions. Therefore the normalization constants in this case must be determined by normalizing the calculated quark distributions in nuclear matter, for $p^2 = M^{*2}$, so that their first moments are unity.

Figure 7 shows the isoscalar valence nucleon structure function $x[u_V(x, p^2 = M^{*2}) + d_V(x, p^2 = M^{*2})]$ for a range of effective masses, $M^*/M \sim$ 0.5–1. There is clearly quite significant softening of the structure function, with the most prominent effects appearing for $0.2 \lesssim x \lesssim 0.6$.

However, it should be remembered that our formalism neglects interactions between the spectator quarks and the surrounding nucleons in the nuclear medium (i.e., it assumes the impulse approximation). This has been found to be quite a poor approximation [36] for nuclear matter. A simple way to estimate the importance of final state interactions is to assume that the strength of the interaction of the spectator diquark with the nuclear medium is 2/3 that of the nucleon interaction, and that it is independent of the mass of the diquark. In that case the diquark mass is modified by $m_R \to m_R^*$, where

$$m_R^* = m_R - \frac{2}{3}(M - M^*) \quad (38)$$

for both scalar and vector diquarks. The effect of this is shown in Fig. 8. As can be seen, interactions of the spectator diquark lead to a hardening of the quark distribution, typically of the same order of magnitude as the nucleon off-shell effects. Combined with the off-shell effects, this gives a structure function which (for $M^*/M \approx 0.7$) is $\sim$ 20–30 % larger than the on-shell result for $x \gtrsim 0.4$. For quantitative comparison against deep-inelastic scattering data on nuclear structure functions it would therefore be very valuable to develop a consistent formalism incorporating both effects.

C. DIS from dressed nucleons

Models of the nucleon which incorporate PCAC (partial conservation of axial-vector current) by including a pion cloud have been used in DIS, among other things, to estimate the size of the πNN form factor [38], and to calculate the flavor symmetry breaking in the proton sea, the possibility of which was recently suggested by the result of the New Muon Collaboration's measurement of the Gottfried sum rule [39]. Previous covariant calculations [40] have all relied upon the same assumptions as for the nuclear calculations, namely, the validity of the convolution formula in the first place, and the lack of any dependence of the bound nucleon structure function on p^2 and $\mathbf{p}_T$. In this section we apply the formalism we have developed for dealing with off-shell effects to the part of this problem where the virtual photon hits the virtual nucleon, with its spectator pion left on mass shell.

In order to calculate this contribution the only additional ingredient which we need is the "sideways" πNN form factor $\Gamma_{\pi NN}(p^2)$, where one nucleon is off mass shell. For this we use the same monopole form that is usually used in the literature [11] (see also [41]):

$$\Gamma_{\pi NN}(p^2) \propto \left(p^2 - \Lambda_{\pi N}^2\right)^{-1}, \quad (39)$$

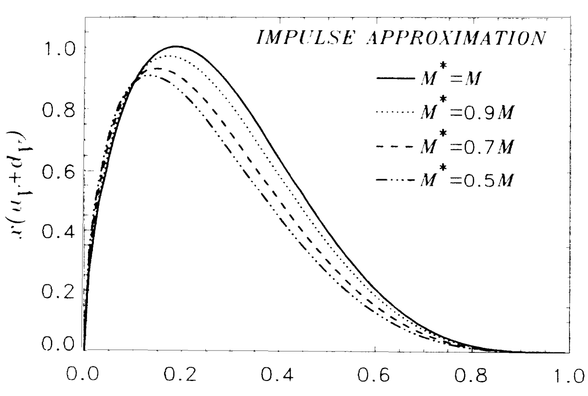

FIG. 7. Nucleon structure function in nuclear matter, in the impulse approximation, for a range of effective nucleon masses, evolved from $Q_0^2 = 0.15$ GeV^2 to $Q^2 = 4$ GeV^2.

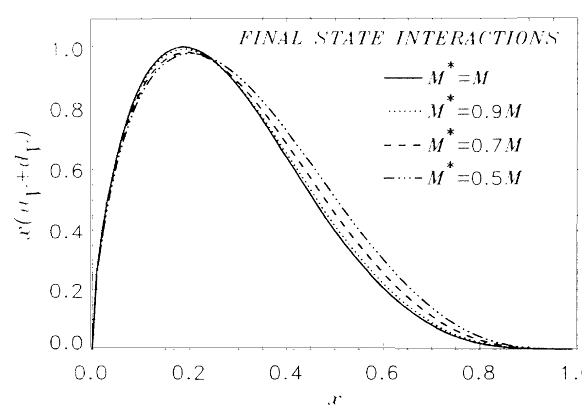

FIG. 8. As in Fig. 7, but including the effects of interaction of the spectator diquark with the nuclear medium.

with $\Lambda_{\pi N} \sim 1.4$ GeV and a pseudoscalar πN coupling. With this, we can rearrange the relevant trace in Eq. (23),

$$\mathrm{Tr}\left[(\not{\mathcal{P}} + M)\ i\gamma_5\Gamma_{\pi NN}(p^2)\ (\not{p} + M)\ \chi_{\mu\nu}(p,q)\ (\not{p} + M)\ i\gamma_5\Gamma_{\pi NN}(p^2)\right], \tag{40}$$

to obtain the nucleon-pion functions $A^{\pi N}_{0,1}$,

$$A^{\pi N}_{0}(p,\mathcal{P}) = \left(-m_\pi^2\ M\right)\Gamma^2_{\pi NN}(p^2), \tag{41a}$$

$$A^{\pi N}_{1\ \alpha}(p,\mathcal{P}) = \left(-m_\pi^2\ p_\alpha\ +\ (p^2 - M^2)(p_\alpha - \mathcal{P}_\alpha)\right)\Gamma^2_{\pi NN}(p^2). \tag{41b}$$

Again, as was the case for the deuteron, by inserting $p^2 \to M^2$ in $A^{\pi N}_{1\alpha}$ we can satisfy the conditions of case (c) in Sec. IV. However, the structure function this time is proportional to $-m_\pi^2$ (i.e., negative), which is clearly unphysical. This illustrates the fact that even though the one-dimensional convolution formula may indeed be obtained from the exact result by certain approximations (e.g., on-shell limit), there is no guarantee that these approximations are physically meaningful.

As was shown in [11], the convolution model may be derived if, among other things, one assumes that the off-shell nucleon structure is the same as that of a pointlike fermion [1], in which case the relevant operator in $\chi_{\mu\nu}$ is $\not{q}\chi^2_T$. As we have seen above, this is only part of the complete expression in the Bjorken limit if one assumes the nucleon quark vertex to be of the form in Sec. V. Nevertheless, the model of [11] can be obtained using these vertices if the following steps are taken: first, the trace in Eq. (40) evaluated with the $\not{q}\chi^2_T$ structure; then to obtain factorization the limits $\mathbf{p}_T = 0$ and $p^2 = M^2$ taken in the "nucleon structure function" (i.e., k-dependent) parts; and finally the full structure of the on-shell nucleon structure function used, as in Eq. (11), rather than just keeping the χ^2_T term. The necessity of the last point is clear, since for the on-shell structure function the *individual* functions χ^i_T are not necessarily positive definite, although the sum of course is positive.

Other authors [42] have implicitly assumed that the relevant operator to be used in the $\chi_{\mu\nu}$ of Eq. (40) is $I\chi^0_T$, similar to what was done in the convolution model calculation for the deuteron discussed in Sec. VI A. However, even with the subsequent replacement of χ^0_T by the full on-shell nucleon structure function in the convolution expression, the result will be proportional to $-m_\pi^2$ since the coefficient of χ^0_T is $A^{\pi N}_0$. Thus it appears that the result of [42] can only be obtained by taking the modulus of a negative structure function.

Clearly, the above procedures are somewhat arbitrary. It is a reflection of the fact that none of the scenarios described in Sec. IV [namely, cases (a)–(c)] for obtaining the convolution model are applicable. As in the deuteron case, the convolution model for dressed nucleons is not derivable from the exact result.

In Fig. 9 we show the result of the convolution model of [11]. This is compared with the result of the calculation including the full p^2 dependence, with the quark-nucleon vertex function evaluated with $\Lambda_p = \infty$, as for the deuteron. For the full calculation we use the same normalization constants for the quark-nucleon vertices as determined from the on-shell nucleon calculation in Sec. V. The results indicate that the full, p^2-dependent calculation gives somewhat smaller results compared with those of the convolution model (although the shapes are quite similar, as can be seen from the dotted curve, where we normalize the scalar and vector vertex functions to give the same first moments as in the convolution model). Such a difference might have been surprising had the convolution expression been a simple approximation to the full result, in which case we may well have expected small off-shell corrections. Unfortunately, this calculation is more difficult to check since there is no clear normalization condition for the structure function. Comparing the first moment of the calculated distributions with the average number of pions in the intermediate state, which can be calculated by considering DIS from the virtual pion, is ambiguous due to the presence of antiparticles in the covariant formulation. [A convolution formula such as Eq. (25) can be written for DIS from virtual pions, since there are no spinor degrees of freedom to spoil this factorization. However, ambiguities in the p^2 dependence of the "off-shell pion structure function" would still remain.] We therefore believe that this fact illustrates the absence of a firm foundation for the covariant convolution model for DIS from dressed nucleons (see [43] for an alternative approach to this calculation).

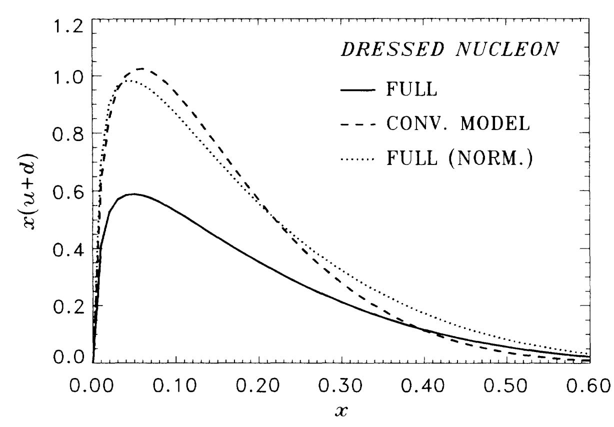

FIG. 9. Contribution to the structure function of a nucleon from DIS off a virtual nucleon with a pion in the final state. The convolution model of Ref. [11] (dashed curve) is compared with the full calculation (for $\Lambda_p = \infty$), using the same normalization for the quark–off-shell-nucleon vertices as for the on-shell vertices (solid), and normalizing the p^2-dependent scalar and vector vertex functions (dotted curve) to give the same first moments as in the convolution model. All curves are evolved from $Q_0^2 = 0.15$ GeV^2 to $Q^2 = 4$ GeV^2.

VII. CONCLUSION

We have investigated within a covariant framework the deep-inelastic scattering from composite particles containing virtual nucleon constituents. The scattering has been treated as a two-step process, in which the off-shell nucleon in the target interacts with the high energy probe. The treatment amounts to neglecting final state interactions. We have constructed the truncated photon-nucleon amplitude from 14 general, independent functions, and used the parton model to show that only 3 of these are relevant in describing the deep-inelastic structure functions in the Bjorken limit. The calculation explicitly ensures current conservation and the Callan-Gross relation.

Within this framework we can unambiguously examine under what conditions the conventional convolution model breaks down. Furthermore, we use some simple models of the relativistic quark-nucleon and nucleon-nucleus vertex functions to investigate this breakdown numerically. While the failure of the convolution model may appear to be an unwelcome complication, it is clear that in any theoretically self-consistent calculation which takes off-mass-shell effects into account it is an inevitable one. Indeed, the "bound nucleon structure function" is an ill-defined quantity within a covariant formulation. This has wide-ranging consequences, as almost all calculations of composite target structure functions (e.g., nuclei, for the EMC effect) have relied upon the validity of the simple convolution model.

We have been able to calculate the deuteron structure function without making any assumptions about the p^2 dependence of the structure functions, and find excellent agreement with the data in the region of x where our model is applicable ($x \gtrsim 0.3$). Making various assumptions for the off-shell nucleons naturally introduces deviations from the exact result. However, by suitably renormalizing the approximated curves by hand to ensure baryon number conservation (as was done in most previous calculations) the differences between the exact results and those of the convolution ansatz are minimized. Although this is most unsatisfactory from a theoretical point of view, phenomenologically the consequences of neglecting the nucleon off-shell effects in the deuteron may not be too great.

To understand the consequences of the off-shell effects in heavy nuclei, we considered a simple model of a nucleon embedded in nuclear matter. We found quite a significant softening of the structure function at intermediate x when the nuclear medium acts to decrease the effective nucleon mass. However, interactions of the spectator diquark state with the surrounding medium tend to make the overall structure function some 20−30% harder at large x ($x \gtrsim 0.4$), for $M^*/M \approx 0.7$, compared with the on-shell result.

The other application which has been examined is DIS from the virtual nucleon component of a physical, or dressed, nucleon, where we also find quite significant differences between the full result and the convolution model. A detailed quantitative understanding of this effect is needed in order to be able to describe the x distributions for all processes where the nucleon's dissociation into a virtual nucleon and meson is expected to be of importance, such as in the measurement of the asymmetry in the light sea quark sector of the proton and neutron, as well as the neutron spin structure function $g_{1n}(x)$.

ACKNOWLEDGMENTS

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APPENDIX: GAUGE INVARIANCE AND THE LONGITUDINAL STRUCTURE FUNCTION

Here we give the full details regarding the vanishing of the longitudinal and non-gauge-invariant structure functions.

Using the projection operators defined in Sec. II we project from the truncated nucleon tensor $\chi_{\mu\nu}$ the contributions to the longitudinal (W_L^A) and non-gauge-invariant (W_Q^A, W_{QL}^A) functions:

$$\begin{aligned} P_L^{\mu\nu}(\mathcal{P},q)\ \chi_{\mu\nu}(p,q) &= \chi_L^0(p,q) + \not p\, \chi_L^1(p,q) + \not q\, \chi_L^2(p,q) - \frac{2\,p\cdot q}{q^2} \not q\, \chi^3(p,q) \\ &\quad + \frac{q^2}{(p\cdot q)^2}\ (p-y\mathcal{P})^2 \left[\chi_T^0(p,q) + \not p\, \chi_T^1(p,q) + \not q\, \chi_T^2(p,q)\ \right], \end{aligned} \tag{A1a}$$

$$\begin{aligned} P_Q^{\mu\nu}(\mathcal{P},q)\ \chi_{\mu\nu}(p,q) &= \chi_Q^0(p,q) + \not p\, \chi_Q^1(p,q) + \not q\, \chi_Q^2(p,q) \\ &\quad + \frac{2\,p\cdot q}{q^2} \not q\, \chi^3(p,q) + 2 \not q\, \chi^4(p,q), \end{aligned} \tag{A1b}$$

$$\begin{aligned} -\frac{1}{2}P_{QL}^{\mu\nu}(\mathcal{P},q)\ \chi_{\mu\nu}(p,q) &= \chi_{QL}^0(p,q) + \not p\, \chi_{QL}^1(p,q) + \not q\, \chi_{QL}^2(p,q) \\ &\quad + \frac{2\,p\cdot q}{q^2} \not q\, \chi^3(p,q) + \not q\, \chi^4(p,q). \end{aligned} \tag{A1c}$$

Following the procedure described in Sec. III we find that all the χ's in Eqs. (A1) are of order $1/\nu$. Hence in the Bjorken limit Eqs. (A1) become

$$P_L^{\mu\nu}(\mathcal{P},q)\chi_{\mu\nu}(p,q), = \not{q}\left(\chi_L^2(p,q) - \frac{2\,p\cdot q}{q^2}\chi^3(p,q)\right), \tag{A2a}$$

$$P_Q^{\mu\nu}(\mathcal{P},q)\;\chi_{\mu\nu}(p,q) = \not{q}\left(\chi_Q^2(p,q) + \frac{2\,p\cdot q}{q^2}\chi^3(p,q) + 2\,\chi^4(p,q)\right), \tag{A2b}$$

$$-\frac{1}{2}P_{QL}^{\mu\nu}(\mathcal{P},q)\;\chi_{\mu\nu}(p,q) = \not{q}\left(\chi_{QL}^2(p,q) + \frac{2\,p\cdot q}{q^2}\chi^3(p,q) + \chi^4(p,q)\right). \tag{A2c}$$

Furthermore, for the functions $\chi(p,q)$ we find that, to leading order in ν,

$$\chi_L^2(p,q) = \frac{2\,p\cdot q}{q^2}\chi^3(p,q), \tag{A3a}$$

$$\chi_Q^2(p,q) = -\frac{2\,p\cdot q}{q^2}\chi^3(p,q) - 2\,\chi^4(p,q), \tag{A3b}$$

$$\chi_{QL}^2(p,q) = \chi_Q^2(p,q) + \chi^4(p,q), \tag{A3c}$$

$$\chi_{L,Q,QL}^{0,1}(p,q) = 0. \tag{A3d}$$

Substituting these expressions into Eqs. (A2) therefore leads to vanishing results for each of the longitudinal and non-gauge-invariant functions. This result is true independent of the production mechanism of the off-shell particle, that is, independent of the functions $A_{0,1}$ as defined in Sec. IV. For the special case of an on-shell nucleon the longitudinal and non-gauge-invariant structure functions are

$$\frac{M}{2}W_L^N(p,q) = M\;\widetilde{\chi}_L^0(p,q) + M^2\;\widetilde{\chi}_L^1(p,q) + p\cdot q\;\widetilde{\chi}_L^2(p,q) - \frac{2\;(p\cdot q)^2}{q^2}\;\widetilde{\chi}^3(p,q) \to 0, \tag{A4a}$$

$$\frac{M}{2}W_Q^N(p,q) = M\;\widetilde{\chi}_Q^0(p,q) + M^2\;\widetilde{\chi}_Q^1(p,q) + p\cdot q\;\widetilde{\chi}_Q^2(p,q) + \frac{2\;(p\cdot q)^2}{q^2}\;\widetilde{\chi}^3(p,q) + 2\,p\cdot q\;\widetilde{\chi}^4(p,q) \to 0, \tag{A4b}$$

$$\frac{M}{2}W_{QL}^N(p,q) = M\;\widetilde{\chi}_{QL}^0(p,q) + M^2\;\widetilde{\chi}_{QL}^1(p,q) + p\cdot q\;\widetilde{\chi}_{QL}^2(p,q) + \frac{2\;(p\cdot q)^2}{q^2}\widetilde{\chi}^3(p,q) + p\cdot q\;\widetilde{\chi}^4(p,q) \to 0, \tag{A4c}$$

where the zero results follow directly from (A3).

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