Chiral-odd and spin-dependent quark fragmentation functions and their applications

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We define a number of quark fragmentation functions for spin-0, -$\frac{1}{2}$, and -1 hadrons, and classify them according to their twist, spin, and chirality. As an example of their applications, we use them to analyze semi-inclusive deep-inelastic scattering on a transversely polarized nucleon.

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I. INTRODUCTION

In high-energy processes, the structure of hadrons is described by parton distributions, or in a broader sense, parton correlations. In previous work [1–5], we have introduced and exploited a number of low-twist parton distributions, with some producing novel spin-dependent and chiral-flip effects in hard scattering processes. These processes in turn allow us to gain access to these distributions experimentally and thereby help us to learn the nonperturbative QCD physics of hadrons. Among the distributions that we have discussed, the quark transversity distribution in the nucleon, which is defined by the following light-cone correlation [1,6],

$$h_1(x)=\frac{1}{2}\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle PS_\perp|\bar{\psi}(0)\not{n}\gamma_5\not{S}_\perp\psi(\lambda n)|PS_\perp\rangle\ , \quad (1)$$

is particularly interesting: it is one of the three distributions which characterize the state of quarks in the nucleon in the leading-order high-energy processes; its unweighted sum rule measures the *tensor charge* of the nucleon, which is identical to the axial charge in nonrelativistic quark models; it is a chiral-odd distribution (containing mixed left- and right-handed quark fields) so it does not appear in many well-known inclusive hard processes such as deep-inelastic scattering. Because of this last feature, we find it cannot be easily measured experimentally.

To see what criteria an underlying physical process has to meet in order to measure the transversity distribution, we consider the so-called "cut diagrams" for the *cross section* of the process, which are obtained by gluing together the Feynman diagrams for the amplitude and its complex conjugate. In a cut diagram, a quark flowing out of a hadron will come back to it after a series of scatterings. For $h_1(x)$ to appear, the chirality of the quark must be flipped when it returns. This occurs if the quark goes through some soft processes during scattering, as shown in Fig. 1(a). The only exception, a hard process which flips chirality, is a mass insertion, shown in Fig. 1(b). For the light (u or d) quarks, the mass insertion is suppressed by $m/\Lambda_{\rm QCD}$ and is ignorable. (Mass insertions might be significant for heavy quarks but they are not the subject of this paper.) Chirality can be flipped in a parton distribution as in the Drell-Yan process shown in Fig. 1(c), where the quark line goes through the interior of another hadron, or in a quark fragmentation process in hadron production shown in Fig. 1(d), where the quark line goes through a fragmentation vertex. To measure the transversity distribution utilizing the second mechanism, we must clarify the structure of fragmentation vertices.

The semi-inclusive hadron production from a quark fragmentation is described by fragmentation functions. As is shown in Ref. [7], parton fragmentation functions in QCD are defined as matrix elements of quark and gluon field operators at light-cone separations. Thus, their twist, spin, and chirality structures shall be as rich as parton distribution functions. In particular, there shall be a corresponding fragmentation function for each parton distribution function defined in Ref. [2]. As we shall show below, there also exist additional fragmentation functions due to hadron final-state interactions. Despite their similarity, fragmentation functions are more difficult to calculate than distribution functions. However, our purpose here is to define them and to study the circumstances under which they contribute to scattering processes.

This paper is organized as follows. In Sec. II, we introduce complete twist-two and -three and a part of twist-

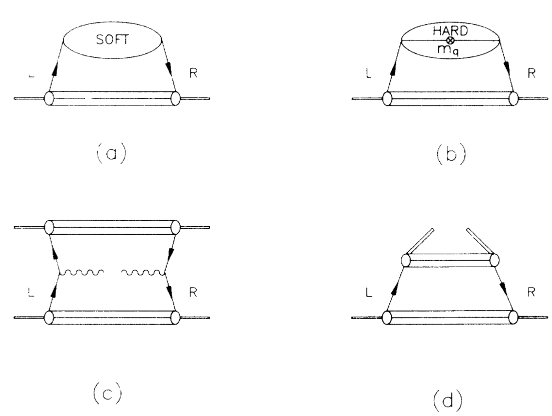

FIG. 1. Processes in which a quark changes its chirality: (a) a generic soft QCD process, (b) mass insertion, (c) the Drell-Yan scattering, and (d) the quark fragmentation.

four quark fragmentation functions for production of spin-0, -$\frac{1}{2}$, and -1 hadrons. In Sec. III, we study measurement of the nucleon's transversity distribution in deep-inelastic scattering using the chiral-odd quark fragmentation functions defined in Sec. II. We specifically consider three hadron-production processes: single-pion production, spin-$\frac{1}{2}$ baryon production, and vector-meson and two-pion production. The first process uses polarized beam and target, and the double spin asymmetry vanishes in the high-energy limit. The second process uses unpolarized lepton beam, but requires measurement of the spin-polarization of the produced baryons. The third process is a single-spin process, which utilizes a fragmentation function arising from hadron final-state interactions. We conclude the paper in Sec. IV.

II. QUARK FRAGMENTATION FUNCTIONS

Quark fragmentation functions were introduced by Feynman to describe hadron production from the underlying hard parton processes [8]. In QCD, it is possible to obtain analytical formulas for these functions in terms of the matrix elements of the quark and gluon fields as for quark distributions in a hadron [7]. In addition to the well-known spin-independent, chiral-even fragmentation function $D(z)$ (we shall call it $\hat{f}_1(z)$] widely discussed in literature, one can introduce various chiral-odd and spin-dependent fragmentation functions, which are capable of producing novel effects in lepton-hadron and hadron-hadron scattering [9]. In this section, we define fragmentation functions involving quark bilinears for production of spin-0, -$\frac{1}{2}$, and -1 hadrons. The discussion here can be easily generalized to gluons and more complicated fragmentation processes.

A. Fragmentation functions for spin-0 meson

Let us consider pion production, or equivalently, production of any hadron whose spin is not observed. In this case, generalizing the procedure in Refs. [2] and [7], we can define three fragmentation functions with quark fields alone,

$$z\int\frac{d\lambda}{2\pi}e^{-i\lambda/z}\langle 0|\gamma^\mu\psi(0)|\pi(P)X\rangle\langle\pi(P)X|\bar{\psi}(\lambda n)|0\rangle = 4[\hat{f}_1(z)p^\mu+\hat{f}_4(z)M^2n^\mu]\ , \quad (2)$$

$$z\int\frac{d\lambda}{2\pi}e^{-i\lambda/z}\langle 0|\psi(0)|\pi(P)X\rangle\langle\pi(P)X|\bar{\psi}(\lambda n)|0\rangle = 4M\hat{e}_1(z)\ , \quad (3)$$

where P is the four-momentum of the pion and p and n are two lightlike vectors such that $p^2=n^2=0$, $p^-=n^+=0$, $p\cdot n=1$, and $P=p+nm_\pi^2/2$. All Dirac indices on quark fields are implicitly contracted. (Our notation for fragmentation functions is analogous to the notation for distribution functions developed in Refs. [2] and [5]; the caret denotes fragmentation.) The mass M is a generic QCD mass scale, and we avoid use of the produced hadron mass because of the singular behavior introduced in the chiral limit [the left-hand side of (3) does not vanish as $m_\pi\to 0$]. The summation over X is implicit and covers all possible states which can be populated by the quark fragmentation. The state $|\pi(P)X\rangle$ is an incoming scattering state between π and X. The renormalization point (μ^2) dependence is suppressed in (2) and (3). QCD radiative corrections induce $\ln\mu^2$ dependence in the fragmentation functions, which is compensated by the $\ln Q^2/\mu^2$ dependence of their coefficients in expressions for observed cross sections. The resulting $\ln Q^2$ dependence, or the Altarelli-Parisi evolution [10], is an important aspect of fragmentation processes which we put aside while we classify their spin and chirality properties. Here we work in $n\cdot A=0$ gauge; otherwise gauge links have to be added to ensure the color gauge invariance. From a simple dimensional analysis, we see that $\hat{f}_1(z)$, $\hat{e}_1(z)$, and $\hat{f}_4(z)$ are twist-two, -three, and -four, respectively; and from their γ-matrix structure, $\hat{f}_1(z)$ and $\hat{f}_4(z)$ are chiral even and $\hat{e}_1(z)$ is chiral odd. Hermiticity guarantees these fragmentation functions are real.

The chiral-odd fragmentation function $\hat{e}_1(z)$ involves both "good" and "bad" components of quark fields on the light cone ($\bar{\psi}\psi=\bar{\psi}_+\psi_-+\bar{\psi}_-\psi_+$, where $\psi_\pm=P_\pm\psi$ with $P_\pm=\frac{1}{2}\gamma^\mp\gamma^\pm$). Using the QCD equation of motion (neglecting the masses for light quarks),

$$i\frac{d}{d\lambda}\psi_-(\lambda n)=-\tfrac{1}{2}\not{n}i\vec{\not{D}}_\perp(\lambda n)\psi_+(\lambda n)\ , \quad (4)$$

where $\vec{D}^\alpha_\perp=\vec{D}^\alpha-\vec{D}\cdot np^\alpha+\vec{D}\cdot pn^\alpha$ and $i\vec{D}^\alpha(\lambda n)=i\vec{\partial}^\alpha-gA^\alpha(\lambda n)$. We rewrite $\hat{e}_1(z)$ in (3) as

$$\hat{e}_1(z)=-\frac{z^2}{8M}\int\frac{d\lambda}{2\pi}e^{-i\lambda/z}[\langle 0|\not{n}i\vec{\not{D}}_\perp(0)\psi_+(0)|\pi(P)X\rangle\langle\pi(P)X|\bar{\psi}_+(\lambda n)|0\rangle + \langle 0|\psi_+(0)|\pi(P)X\rangle\langle\pi(P)X|\bar{\psi}_+(\lambda n)\not{n}i\overleftarrow{\not{D}}_\perp(\lambda n)|0\rangle]\ , \quad (5)$$

where $i\overleftarrow{D}^\alpha(\lambda n)=i\overleftarrow{\partial}^\alpha+gA^\alpha(\lambda n)$. Thus the twist-three fragmentation explicitly involves three parton fields: two quark and one gluon. The appearance of Eq. (5) motivates us to introduce a fragmentation density matrix,

$$\hat{M}^\alpha_{\rho\sigma}(z,z_1)=\int\frac{d\lambda}{2\pi}\frac{d\mu}{2\pi}e^{-i\lambda/z}e^{-i\mu(1/z_1-1/z)}\langle 0|i\vec{D}^\alpha_\perp(\mu n)\psi_\rho(0)|\pi(P)X\rangle\langle\pi(P)X|\bar{\psi}_\sigma(\lambda n)|0\rangle + \int\frac{d\lambda}{2\pi}\frac{d\mu}{2\pi}e^{i\lambda/z}e^{i\mu(1/z_1-1/z)}\langle 0|\psi_\rho(\lambda n)|\pi(P)X\rangle\langle\pi(P)X|\bar{\psi}_\sigma(0)i\overleftarrow{D}^\alpha_\perp(\mu n)|0\rangle\ , \quad (6)$$

where α is restricted to transverse dimensions. It has the following expansion in the Dirac spin space,

$$\hat{M}^\alpha(z,z_1)=M\gamma^\alpha \not{p}\frac{\hat{E}(z,z_1)}{z}+\cdots \tag{7}$$

where $\hat{E}(z,z_1)$ is a real, chiral-odd fragmentation function involving two light-cone fractions and the ellipsis denotes higher-twist contributions. The function $\hat{E}(z,z_1)$ can be isolated from $\hat{M}^\alpha$ through a projection: $\hat{E}(z,z_1)=z/(8M)\mathrm{Tr}\not{n}\gamma_\alpha\hat{M}^\alpha(z,z_1)$. From Eqs. (5)–(7), it is easy to prove,

$$\hat{e}_1(z)=-z\int\hat{E}(z,z_1)d\left[\frac{1}{z_1}\right] . \tag{8}$$

Therefore, $\hat{e}_1(z)$ is just a special moment of $\hat{E}(z,z_1)$. As a consequence, a measurement of $\hat{e}_1(z)$ at one momentum scale is not sufficient to determine its value at other scales, because an Altarelli-Parisi type of evolution equation exists only for $\hat{E}(z,z_1)$, not for a subset of its moments [11]. This property of $\hat{e}_1(z)$ contrasts that of twist-two fragmentation functions, such as $\hat{f}_1(z)$.

B. Fragmentation functions arising from hadron final-state interactions

The quark fragmentations introduced above have a one-to-one correspondence with the quark distributions introduced for a spin-0 meson. In practice, one can define one additional fragmentation function for the pion,

$$z\int\frac{d\lambda}{2\pi}e^{-i\lambda/z}\langle 0|\sigma^{\mu\nu}i\gamma_5\psi(0)|\pi(P)X\rangle\langle\pi(P)X|\bar{\psi}(\lambda n)|0\rangle=4M\epsilon^{\mu\nu\alpha\beta}p_\alpha n_\beta\hat{e}_{\bar{1}}(z) . \tag{9}$$

If there were no final-state interactions between π and X, the state $|\pi(P)X\rangle$ transforms as a free state under time-reversal symmetry and $\hat{e}_{\bar{1}}(z)$ vanishes identically. Thus the magnitude of $\hat{e}_{\bar{1}}(z)$ depends crucially on the effects of hadron final-state interactions.

To illustrate that such fragmentation functions do exist, we consider production of an electron-positron pair from a virtual photon of mass $4m_e^2<q^2<16m_e^2$ in axial-vector quantum electrodynamics. The production cross section is proportional to the vacuum tensor,

$$W^{\mu\nu}=\frac{1}{2\pi}\int d^4x\, e^{iqx}\langle 0|J_5^\mu(x)|e^+(P)e^-\rangle\langle e^+(P)e^-|J_5^\nu(0)|0\rangle . \tag{10}$$

And this, according to Lorentz invariance, has the following decomposition in terms of Lorentz scalars,

$$W^{\mu\nu}=\left[-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right]W_1+\cdots+i(P^\mu q^\nu-P^\nu q^\mu)W^6 . \tag{11}$$

If neglecting the final-state interactions between the electron and positron, one can prove immediately $W_6=0$ due to time-reversal invariance.

However, if taking into account one-photon exchange, one finds,

$$W_6=C\sqrt{1-4m_e^2/q^2}\,\theta(q^2-4m_e^2) , \tag{12}$$

where C is an unimportant numerical constant. The θ function indicates the final-state interaction vanishes if $q^2<4m_e^2$, in particular, if $q^2<0$, $W^{\mu\nu}$ is proportional to the photon-electron scattering cross section, to which we known W_6 does not contribute.

The fragmentation function $\hat{e}_{\bar{1}}(z)$ is chiral-odd and twist-three. It is intimately related to $\hat{e}_1(z)$ introduced in Sec. II A. It is simple to show that it contributes to W_6 type of terms in semi-inclusive production of hadrons in e^+e^- annihilation.

C. Fragmentation functions for spin-$\frac{1}{2}$ baryon

Now we turn to consider the quark fragmentation for a spin-$\frac{1}{2}$ baryon. Eight more fragmentation functions can be introduced through bilinear quark fields besides these in Eqs. (2), (3), and (9). They all depend on the polarization of the baryon: four of them are related to the longitudinal polarization and the other four to the transverse polarization,

$$z\int\frac{d\lambda}{2\pi}e^{-i\lambda/z}\langle 0|\gamma^\mu\gamma_5\psi(0)|B(PS)X\rangle\langle B(PS)X|\bar{\psi}(\lambda n)|0\rangle=4[\hat{g}_1(z)p^\mu(S_\parallel\cdot n)+M\hat{g}_T(z)S_\perp^\mu+M^2\hat{g}_3(z)(S_\parallel\cdot n)n^\mu] , \tag{13}$$

$$z\int\frac{d\lambda}{2\pi}e^{-i\lambda/z}\langle 0|\sigma^{\mu\nu}i\gamma_5\psi(0)|B(PS)X\rangle\langle B(PS)X|\bar{\psi}(\lambda n)|0\rangle=4[\hat{h}_1(z)(S_\perp^\mu p^\nu-S_\perp^\nu p^\mu)+\hat{h}_L(z)M(p^\mu n^\nu-p^\nu n^\mu)(S_\parallel\cdot n)$$
$$+\hat{h}_3(z)M^2(S_\perp^\mu n^\nu-S_\perp^\nu n^\mu)+\cdots] \tag{14}$$

$$z\int\frac{d\lambda}{2\pi}e^{-i\lambda/z}\langle 0|\gamma^\mu\psi(0)|B(PS)X\rangle\langle B(PS)X|\bar\psi(\lambda n)|0\rangle=4[\hat g_{\bar T}(z)MT_\perp^\mu+\cdots] , \quad (15)$$

$$z\int\frac{d\lambda}{2\pi}e^{-i\lambda/z}\langle 0|\gamma_5\psi(0)|B(PS)X\rangle\langle B(PS)X|\bar\psi(\lambda n)|0\rangle=4M\hat h_{\bar L}(z)(S_\parallel\cdot n) , \quad (16)$$

where ellipses denote terms which already appeared in (2), (3), and (9), $B(PS)$ represents the spin-$\frac{1}{2}$ baryon with the four-momentum P and polarization S (we write $S^\mu=S\cdot np^\mu+S\cdot pn^\mu+M_BS_\perp^\mu$ with the baryon mass M_B), and $T^\mu=\epsilon^{\mu\nu\alpha\beta}S_{\perp\nu}p_\alpha n_\beta$ is a transverse vector orthogonal to $S_\perp^\mu$. Again, through dimensional analysis, $\hat g_1(z)$ and $\hat h_1(z)$ are twist-two; $\hat g_T(z)$, $\hat h_L(z)$, $\hat g_{\bar T}(z)$, and $\hat h_{\bar L}(z)$ are twist-three; and $\hat g_3(z)$ and $\hat h_3(z)$ are twist-four. The fragmentation functions $\hat g_{\bar T}(z)$ and $\hat h_{\bar L}(z)$ vanish identically if without final-state interactions.

As was the case for $\hat e_1(z)$, at the level of twist-three, $\hat g_T(z)$ and $\hat h_L(z)$ are not the most general fragmentation functions. Using (4), we derive,

$$\hat g_T(z)=-\frac{z^2}{8M}\int\frac{d\lambda}{2\pi}e^{-i\lambda/z}\langle 0|i\vec D_\perp(0)\cdot S_\perp\not n\gamma_5\psi_+(0)|B(PS_\perp)X\rangle\langle B(PS_\perp)X|\bar\psi_+(\lambda n)|0\rangle$$
$$+\frac{z^2}{8M}\int\frac{d\lambda}{2\pi}e^{-i\lambda/z}\langle 0|\vec D_\perp(0)\cdot T_\perp\not n\psi_+(0)|B(PS_\perp)X\rangle\langle B(PS_\perp)X|\bar\psi_+(\lambda n)|0\rangle+\text{c.c.} , \quad (17)$$

$$\hat h_L(z)=-\frac{z^2}{8M}\int\frac{d\lambda}{2\pi}e^{-i\lambda/z}\langle 0|i\vec{\not D}_\perp(0)\not n\gamma_5\psi_+(0)|B(PS_\parallel)X\rangle\langle B(PS_\parallel)X|\bar\psi_+(\lambda n)|0\rangle+\text{c.c.} , \quad (18)$$

where c.c. stands for complex conjugate. The generalization of Eqs. (17) and (18) to two-light-cone-fraction distributions can be made by defining $\hat M^\alpha(z,z_1)$ for the baryon just as for the pion in Eq. (6). In addition, we need to define a new fragmentation density matrix,

$$\hat N^\alpha_{\rho\sigma}(z,z_1)=-\int\frac{d\lambda}{2\pi}\frac{d\mu}{2\pi}e^{-i\lambda/z}e^{-i\mu(1/z_1-1/z)}\langle 0|i\vec D_\perp^\alpha(\mu n)\psi_\rho(0)|B(PS)X\rangle\langle B(PS)X|\bar\psi_\sigma(\lambda n)|0\rangle$$
$$+\int\frac{d\lambda}{2\pi}\frac{d\mu}{2\pi}e^{i\lambda/z}e^{i\mu(1/z_1-1/z)}\langle 0|\psi_\rho(\lambda n)|B(PS)X\rangle\langle B(PS)X|\bar\psi_\sigma(0)i\overleftarrow{D}_\perp^\alpha(\mu n)|0\rangle , \quad (19)$$

which is the same as $\hat M^\alpha$ except for the minus sign for the first term. Making an expansion in spin space, we have

$$\hat M^\alpha(z,z_1)=M\gamma^\alpha\not p\hat E(z,z_1)/z+iMT_\perp^\alpha\not p\hat G_1(z,z_1)/z+\cdots ,$$
$$\hat N^\alpha(z,z_1)=MS_\perp^\alpha\gamma_5\not p\hat G_2(z,z_1)/z+M\gamma^\alpha\not p\gamma_5\hat H(z,z_1)/z+\cdots . \quad (20)$$

The fragmentation functions can be projected from the density matrices: $G_1=iz/(4M)\mathrm{Tr}\not n\gamma_5T_{\perp\alpha}M^\alpha$, $G_2=-z/(4M)\mathrm{Tr}\not n\gamma_5S_{\perp\alpha}N^\alpha$, and $H=-z/(8M)\mathrm{Tr}\gamma_{\perp\alpha}\not n\gamma_5N^\alpha$. It is easy to prove that

$$\hat g_T(z)=-\frac{z}{2}\int d\left[\frac{1}{z_1}\right][\hat G_1(z,z_1)+\hat G_2(z,z_1)] , \quad (21)$$

$$\hat h_L(z)=-z\int d\left[\frac{1}{z_1}\right]\hat H(z,z_1) . \quad (22)$$

These relations are useful for proving electromagnetic gauge invariance of scattering amplitudes, as an example shows in Sec. III.

D. Fragmentation functions for spin-1 meson

Finally, we consider quark fragmentation functions for vector-meson production. To facilitate counting, let us define the quark-meson forward scattering amplitudes, $A_{hH;h'H'}$, where $h(h')$ and $H(H')$ are quark and meson helicities, respectively. The combination $A_{\frac{1}{2}1;\frac{1}{2}1}+A_{\frac{1}{2}-1;\frac{1}{2}-1}+A_{\frac{1}{2}0;\frac{1}{2}0}$ is independent of the meson polarization, from which we define four fragmentation functions $\hat f_1$, $\hat e_1$ and $\hat e_{\bar 1}$, and $\hat f_4$, depending on what components of quark fields form the amplitude: good-good $(++)$, good-bad $(+-)$, or bad-bad $(--)$. Of course, they are what we have just defined in (2), (3), and (9). Similarly, the combination $A_{\frac{1}{2}1;\frac{1}{2}1}+A_{\frac{1}{2}-1;\frac{1}{2}-1}-2A_{\frac{1}{2}0;\frac{1}{2}0}$ depends on the longitudinal-longitudinal (LL) type of tensor polarization of the meson (see below for definition) and the corresponding four fragmentation functions are $\hat b_1$, $\hat b_2$ and $\hat b_{\bar 2}$, and $\hat b_3$; the combination $A_{\frac{1}{2}1;\frac{1}{2}1}-A_{\frac{1}{2}-1;\frac{1}{2}-1}$ depends on the transverse-transverse (TT) type of vector polarization and the associated fragmentation functions are $\hat g_1$, $\hat h_2$ and $\hat h_{\bar 2}$, and $\hat g_3$; the combination $A_{\frac{1}{2}0;-\frac{1}{2}1}-A_{-\frac{1}{2}1;\frac{1}{2}0}$ is related to the LT type of vector polarization and the associated fragmentation functions are defined as $\hat h_1$, $\hat g_2$ and $\Delta\hat g_{\bar 2}$, and $\hat h_3$; and finally, the combination $A_{\frac{1}{2}0;-\frac{1}{2}1}+A_{-\frac{1}{2}1;\frac{1}{2}0}$ is related to the LT type of tensor polarization and the associate fragmentation functions are defined as $\hat h_{\bar 1}$, $\hat g_{\bar 2}$, $\Delta\hat g_2$, and $\hat h_{\bar 3}$. The spin and twist structures of these twenty fragmentation functions are shown in Table I, and the ones with bar on their subscripts arising from hadron final-state interactions.

Now we relate these fragmentation functions to the matrix elements of the bilinear quark operators. Since the meson polarization vector ϵ^μ appears in bilinear form

TABLE I. Quark fragmentation functions for vector mesons. Note that the functions with bars vanish if there are no final-state interactions.

	Twist-2 $++$	Twist-3 $+-(S)$	Twist-3 $+-(A)$	Twist-4 $--$	Meson polarization
$A_{\frac{1}{2}1\to\frac{1}{2}1}+A_{\frac{1}{2}-1\to\frac{1}{2}-1}+A_{\frac{1}{2}0\to\frac{1}{2}0}$	$\hat{f}_1$	$\hat{e}_1$	$\hat{e}_{\bar{1}}$	$\hat{f}_4$	S
$A_{\frac{1}{2}1\to\frac{1}{2}1}+A_{\frac{1}{2}-1\to\frac{1}{2}-1}-2A_{\frac{1}{2}0\to\frac{1}{2}0}$	$\hat{b}_1$	$\hat{b}_1$	$\hat{b}_{\bar{2}}$	$\hat{b}_3$	$T_{\rm LL}$
$A_{\frac{1}{2}1\to\frac{1}{2}1}-A_{\frac{1}{2}-1\to\frac{1}{2}-1}$	$\hat{g}_1$	$\hat{h}_2$	$\hat{h}_{\bar{2}}$	$\hat{g}_3$	$V_{\rm TT}$
$A_{\frac{1}{2}0\to-\frac{1}{2}1}-A_{-\frac{1}{2}1\to\frac{1}{2}0}$	$\hat{h}_1$	$\hat{g}_2$	$\Delta\hat{g}_{\bar{2}}$	$\hat{h}_3$	$V_{\rm LT}$
$A_{\frac{1}{2}0\to-\frac{1}{2}1}+A_{-\frac{1}{2}1\to\frac{1}{2}0}$	$\hat{h}_{\bar{1}}$	$\hat{g}_{\bar{2}}$	$\Delta\hat{g}_2$	$\hat{h}_{\bar{3}}$	$T_{\rm LT}$

in all the matrix elements, we introduce a rank-two tensor $T^{\mu\nu}=\epsilon^\mu\epsilon^{*\nu}$. Its trace $T^\mu{}_\mu=S$, antisymmetric part $T^{[\mu\nu]}=\epsilon^\mu\epsilon^{*\nu}-\epsilon^\nu\epsilon^{*\mu}$, and traceless-symmetric part $T^{\{\mu\nu\}}=\epsilon^\mu\epsilon^{*\nu}+\epsilon^\nu\epsilon^{*\mu}-(\epsilon\cdot\epsilon^*)g^{\mu\nu}/2$ represent the scalar, vector, and tensor polarization of the meson. Together with p_μ and n_μ, they can be used to build various Lorentz structures to expand the quark matrix elements. The coefficients of the expansion, depending on the polarization and dimension of the associated structures, can be uniquely identified with the fragmentation functions in Table I.

To illustrate this, take the scalar polarization S, from which one can form one scalar S, two vectors Sp^μ and Sn^μ, and one tensor $\epsilon^{\mu\nu\alpha\beta}p_\alpha n_\beta S$, and the coefficients of these structures shall be $\hat{e}_1$, $\hat{f}_1$ and $\hat{f}_4$, and $\hat{e}_{\bar{1}}$, respectively. For the case of tensor polarization, consider the projection of $T^{\{\mu\nu\}}$ in longitudinal directions, $T_{\{\alpha\beta\}}p^\alpha n^\beta(=T)(n^\mu p^\nu+n^\nu p^\mu)$, which characterizes the LL type of tensor polarization. With this one can construct one scalar T, two vectors Tp^μ and Tn^ν, and one tensor $\epsilon^{\mu\nu\alpha\beta}p_\alpha n_\beta T$ and their coefficients are $\hat{b}_2$, $\hat{b}_1$ and $\hat{b}_3$, and $\hat{b}_{\bar{2}}$, respectively. Proceeding in this way, define $A=i\epsilon^{\alpha\beta\gamma\delta}\epsilon_\alpha\epsilon^*_\beta p_\gamma n_\delta$ to characterize the TT type of vector polarization, $S^\mu_\perp=i\epsilon^{\mu\alpha\beta\gamma}p_\alpha n_\beta T_{[\gamma\delta]}n^\delta$ the LT type of vector polarization, and $T^\mu_\perp=\epsilon^{\mu\alpha\beta\gamma}p_\alpha n_\beta T_{\{\gamma\delta\}}n^\delta$ the LT type of tensor polarization, and construct all possible structures with them. The complete expansion of quark matrix elements reads

$$z\int\frac{d\lambda}{2\pi}e^{-i\lambda/z}\langle 0|\gamma^\mu\psi(0)|V(P\epsilon)X\rangle\langle V(P\epsilon)X|\bar{\psi}(\lambda n)|0\rangle=4[\hat{f}_1(z)Sp^\mu+\hat{f}_4(z)M^2Sn^\mu+\Delta\hat{g}_{\bar{2}}(z)MiT_\perp^{[\mu\nu]}n_\nu+\Delta\hat{g}_2(z)MT_\perp^{\{\mu\nu\}}n_\nu+\hat{b}_1(z)Tp^\mu+\hat{b}_4(z)M^2Tn^\mu]\ , \qquad (23)$$

$$z\int\frac{d\lambda}{2\pi}e^{-i\lambda/z}\langle 0|\psi(0)|V(P\epsilon)X\rangle\langle V(P\epsilon)X|\bar{\psi}(\lambda n)|0\rangle=4M[S\hat{e}_1(z)+T\hat{b}_2(z)]\ , \qquad (24)$$

$$z\int\frac{d\lambda}{2\pi}e^{-i\lambda/z}\langle 0|i\gamma_5\psi(0)|V(P\epsilon)X\rangle\langle V(P\epsilon)X|\bar{\psi}(\lambda n)|0\rangle=4MAh_{\bar{2}}(z)\ , \qquad (25)$$

$$z\int\frac{d\lambda}{2\pi}e^{-i\lambda/z}\langle 0|\gamma^\mu\gamma_5\psi(0)|V(P\epsilon)X\rangle\langle V(P\epsilon)X|\bar{\psi}(\lambda n)|0\rangle=4[\hat{g}_1(z)Ap^\mu+M\hat{g}_2(z)S_\perp^\mu+M\hat{g}_{\bar{2}}(z)T_\perp^\mu+M^2\hat{g}_3(z)An^\mu]\ , \qquad (26)$$

$$z\int\frac{d\lambda}{2\pi}e^{-i\lambda/z}\langle 0|\sigma^{\mu\nu}i\gamma_5\psi(0)|V(P\epsilon)X\rangle\langle V(P\epsilon)X|\bar{\psi}(\lambda n)|0\rangle=4[\hat{h}_1(z)(S_\perp^\mu p^\nu-S_\perp^\nu p^\mu)+\hat{h}_{\bar{1}}(z)(T_\perp^\mu p^\nu-T_\perp^\nu p^\mu)+\hat{h}_2(z)MA(p^\mu n^\nu-p^\nu n^\mu)+\hat{h}_3(z)M^2(S_\perp^\mu n^\nu-S_\perp^\nu n^\mu)+\hat{h}_{\bar{3}}(z)M^2(T_\perp^\mu n^\nu-T_\perp^\nu n^\mu)+e_{\bar{1}}(z)M\epsilon^{\mu\nu\alpha\beta}p_\alpha n_\beta S+b_{\bar{2}}(z)M\epsilon^{\mu\nu\alpha\beta}p_\alpha n_\beta T]\ . \qquad (27)$$

Thus, all the twenty fragmentation functions in Table I are expressed in terms of light-cone correlations.

III. MEASURING THE TRANSVERSITY DISTRIBUTION FROM DEEP-INELASTIC SCATTERING

As an example of applying the fragmentation functions defined in the preceding section, we consider measuring the nucleon's transversity distribution $h_1(x)$ through deep-inelastic scattering. Because $h_1(x)$ is chiral odd, it does not appear in *inclusive* deep-inelastic cross section if the current quark masses are neglected. However, $h_1(x)$ does appear in semi-inclusive hadron production if one takes into account the effects of the chiral-odd fragmentation of the struck quark. For pseudoscalar meson production, the leading chiral-odd quark fragmentation function is $\hat{e}_1(z)$; for spin-$\frac{1}{2}$ baryone production, it is $\hat{h}_1(z)$; and for vector-meson production, it is $\hat{h}_{\bar{1}}(z)$ or its generalization to a fragmentation function for two pions. In the following, we discuss these cases separately.

A. Single-pion production

We consider deep-inelastic scattering with longitudinal polarized lepton on transversely polarized nucleon target, focusing on pion production in the current fragmentation region. Since there is no *chiral-odd* twist-two fragmentation function for the pion to couple with the $h_1(x)$ distribution in the nucleon, nor is there a *chiral-even* twist-two transverse-spin-dependent distribution in the nucleon to couple with the fragmentation function $\hat{f}_1(z)$ for the pion, the spin-dependent cross section vanishes at the leading order in Q. At the twist-three level (the order of $1/Q$), $h_1(x)$ contributes through coupling with the chiral-odd fragmentation function $\hat{e}_1(z)$, and so does the chiral-even transverse-spin distribution $g_T(x)$ through the fragmentation function $\hat{f}_1(z)$. Both contributions exist in Fig. 2(a). At the same order, we have to consider also Figs. 2(b) and 2(c), in which one radiative gluon takes part in quark fragmentation, and Figs. 2(d) and 2(e), in which one gluon from the nucleon participates in hard scattering. These processes, representing coherent parton scattering, introduce dependences on the two-light-cone-fraction parton distributions, $G_{1,2}(x,x_1)$, which are the parents of $g_T(x)$ [2], and fragmentation function, $\hat{E}(z,z_1)$, which is the parent of $\hat{e}_1(z)$. However, as we shall show below, they can be eliminated by using QCD equations of motion, and the final result contains only $\hat{e}_1(z)$ and $g_T(x)$.

Let us first consider the contribution from the diagram in Fig. 2(a). Using the definition of the nucleon tensor,

$$W_{\mu\nu}=\frac{1}{4\pi}\int e^{iq\cdot\xi}d^4\xi\langle PS|J_\mu(\xi)J_\nu(0)|PS\rangle\ , \qquad (28)$$

we obtain from this diagram,

$$W^a_{\mu\nu}=\frac{1}{4\pi}\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4P_\pi}{(2\pi)^4}2\pi\delta(P_\pi^2-m_\pi^2)\mathrm{Tr}[M_N(P,S_\perp,k)\gamma_\mu\hat{M}_\pi(k+q,P_\pi)\gamma_\nu]\ , \qquad (29)$$

where m_π is the pion mass,

$$M_N(P,S_\perp,k)_{\alpha\beta}=\int d^4\xi\, e^{i\xi\cdot k}\langle PS_\perp|\bar{\psi}_\beta(0)\psi_\alpha(\xi)|PS_\perp\rangle \qquad (30)$$

is the quark's spin-density matrix for the nucleon and

$$\hat{M}_\pi(k,P_\pi)_{\alpha\beta}=\sum_X\int d^4\xi\, e^{-i\xi\cdot k}\langle 0|\psi_\alpha(0)|\pi(P_\pi)X\rangle\langle\pi(P_\pi)X|\bar{\psi}_\beta(\xi)|0\rangle \qquad (31)$$

is the quark fragmentation density matrix for the pion. Here q is the four-momentum of the virtual photon, and P and $S_\perp$ are the nucleon's four-momentum and polarization vectors, respectively. We choose our coordinate system such that $P=p+nM_N^2/2$, $S_\perp=(0,1,0,0)$, and $q=-x_Bp+\nu n$, where p and n are two lightlike vectors defined in the preceding section, M_N is the mass of the nucleon, and x_B is the Bjorken scaling variable $x_B=Q^2/(2\nu)$.

To perform the k integration in Eq. (29), we make a collinear expansion of quark momentum k *along* p in the fragmentation density matrix,

$$\hat{M}_\pi(k+q,P_\pi)=\hat{M}_\pi(k\cdot np+q,P_\pi)+(k-k\cdot np)^\alpha\frac{\partial\hat{M}_\pi(k\cdot np+q,P_\pi)}{\partial k^\alpha}+\cdots\ . \qquad (32)$$

We temporarily ignore the derivative term, whose contrition will be combined with those from Figs. 2(d) and 2(e) to

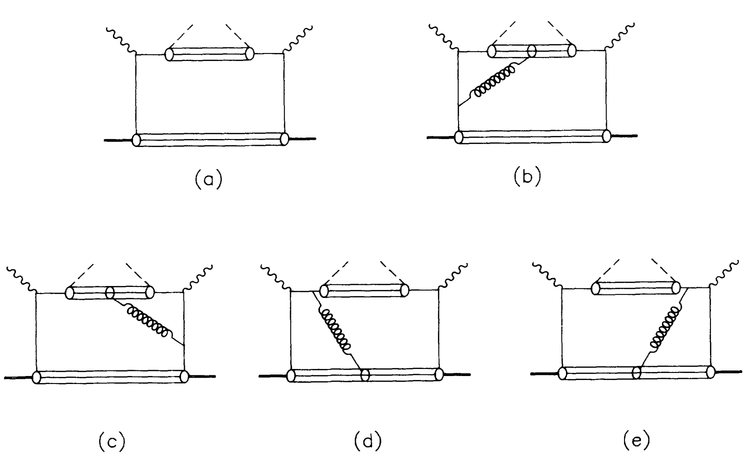

FIG. 2. The twist-two and twist-three cut diagrams for single-pion production in deep-inelastic scattering.

form a gauge-invariant result. The contribution to the k integration from the leading term in Eq. (32) is

$$W^{a}_{\mu\nu}=\frac{1}{4\pi}\int\frac{d^4P_\pi}{(2\pi)^4}2\pi\delta(P_\pi^2-m_\pi^2)\int dx\ \mathrm{Tr}[M_N(xp,S_\perp)\gamma_\mu\hat{M}_\pi(xp+q,P_\pi)\gamma_\nu]\ , \tag{33}$$

where the simplified quark spin-density matrix is

$$M_N(xp,S_\perp)_{\alpha\beta}=\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle PS_\perp|\bar\psi_\beta(0)\psi_\alpha(\lambda n)|PS_\perp\rangle\ , \tag{34}$$

the structure of which has been studied thoroughly in Ref. [1].

To integrate out the transverse components of P_π in Eq. (33), we make a coordinate transformation to a new system in which P_π and q have only longitudinal components. If we label momenta in the new system with a prime, then, to the order of our interest,

$$\begin{aligned}&P'^{-}_\pi=P^-_\pi\ ,\quad P'^{+}_\pi=0\ ,\quad P'^{i}_\pi=0\ ,\\ &q'^{-}=P^-_\pi/z\ ,\quad q'^{+}=q^+\ ,\quad q'^{i}=0\ ,\\ &xp'^{-}=0\ ,\quad xp'^{+}=xp^+\ ,\quad xp'^{i}=-P^i_\pi/z\ .\end{aligned} \tag{35}$$

In the new system, p' has nonvanishing transverse components and as a consequence, the spin and fragmentation density matrices in Eq. (33) are now linked through transverse-momentum integrations. To decouple them, we Taylor expand the spin-density matrix,

$$M_N(xp',S_\perp)=M_N(xp,S_\perp)-\frac{P^i_\pi}{z}\frac{\partial M_N(xp,S_\perp)}{\partial xp^i}+\cdots\ . \tag{36}$$

Here we have ignored the transverse components of n', whose effects are beyond twist-three. The contribution from the derivative term in Eq. (36) will be combined with those from Figs. 2(b) and 2(c) to form a color gauge-invariant expression, as is shown in Eq. (43). And the leading term contribution is

$$W^{a}_{\mu\nu}=\frac{1}{4\pi}\int dz\int dx\ 2\pi\delta((xp+q)^2)\mathrm{Tr}[M_N(xp,S_\perp)\gamma_\mu\hat{M}_\pi(z,p_\pi/z)\gamma_\nu]\ , \tag{37}$$

where the simplified fragmentation density matrix is

$$\hat{M}_\pi(z,p_\pi/z)_{\alpha\beta}=\sum_X\int\frac{d\lambda}{2\pi}e^{-i\lambda/z}\langle 0|\psi_\alpha(0)|\pi(p_\pi)X\rangle\langle\pi(p_\pi)X|\bar\psi_\beta(\lambda n_\pi)|0\rangle\ . \tag{38}$$

Here we have neglected the pion mass and used two additional lightlike vectors p_π and n_π with $p_\pi=z\nu n$ and $p_\pi\cdot n_\pi=1$.

Since spin asymmetry is our main interest, we take the transverse-spin-dependent part of the spin-density matrix from Ref. [5],

$$M_N(x,p,S_\perp)=\tfrac{1}{2}h_1(x)\gamma_5\not{S}_\perp\not{p}+\tfrac{1}{2}g_T(x)M\gamma_5\not{S}_\perp+\cdots\ . \tag{39}$$

From Eqs. (2) and (3), we have the fragmentation density,

$$\hat{M}_\pi(z)=M\frac{\hat{e}_1(z)}{z}+\not{p}_\pi\frac{\hat{f}_1(z)}{z}+\cdots\ . \tag{40}$$

Substituting Eqs. (39) and (40) into Eq. (37) and simplifying the latter, we have

$$W^{a}_{\mu\nu}=\frac{M}{2\nu}\left[\sum_a e_a^2h_1^a(x_B)\int\frac{\hat{e}_1^a(z)}{z}dz\ i\epsilon^{\mu\nu\alpha\beta}p_\alpha S_{\perp\beta}+\sum_a e_a^2g_T^a(x_B)\int\frac{\hat{f}_1^a(z)}{z}dz\ i\epsilon^{\mu\nu\alpha\beta}S_{\perp\alpha}p_{\pi\beta}\right]\ , \tag{41}$$

where the summation runs over different quark flavors and their charge conjugation, and e_a is the electric charge of quarks. As it stands, Eq. (41) does not satisfy electromagnetic gauge invariance, i.e., $W_{\mu\nu}q^\mu\neq0$.

We turn to consider the contribution from Fig. 2(b), which involves an additional transversely polarized gluon. After the collinear expansion and coordinate transformation discussed above, we find

$$W^{b}_{\mu\nu}=\frac{1}{4\pi}\int dx\ dz\ d\left[\frac{1}{z_1}\right]2\pi\delta((q+xp)^2)\mathrm{Tr}\left[M_N(xp,S_\perp)i\gamma_\alpha\frac{i[x\not{p}-(1/z-1/z_1)\not{p}_\pi]}{[xp-(1/z-1/z_1)p_\pi]^2}\gamma_\mu\hat{M}_1^\alpha(z,z_1)\gamma_\nu\right]\ , \tag{42}$$

where the fragmentation density matrix is

$$\hat{M}_1^\alpha(z,z_1)_{\rho\sigma}=\int\frac{d\lambda}{2\pi}\frac{d\mu}{2\pi}e^{-i\lambda/z}e^{-i\mu(1/z_1-1/z)}\langle 0|i\vec{D}_\perp^\alpha(\mu n)\psi_\rho(0)|\pi(P_\pi)X\rangle\langle\pi(P_\pi)X|\bar\psi_\sigma(\lambda n)|0\rangle\ . \tag{43}$$

The partial derivative in $\vec{D}^\alpha_\perp$ comes from the collinear expansion for Fig. 2(a) as explained after Eq. (36). Because the state $|\pi(P_\pi)X\rangle$ is an incoming scattering state which changes to an outgoing scattering state after time reversal, $\hat{\mathcal{M}}_1^a(z,z_1)$ does not have a simple Hermitian conjugation property. As a consequence, if we make an expansion, $\hat{\mathcal{M}}_1^a = M\gamma^\alpha \not{p} \hat{E}_1(z,z_1) + \cdots$, $E_1(z,z_1)$ is not a real quantity. However, its imaginary part, which we are going to ignore, contributes only to single-spin asymmetry. Its real part is just $E(z,z_1)/2$, which was defined in the last section. Inserting the expansion into Eq. (42) and using Eq. (8) to eliminate $\hat{E}(z,z_1)$, we find

$$W_{\mu\nu}^b = \frac{M}{2\nu}\sum_a e_a^2 h_1^a(x)\int dz\, \hat{e}^a(z)\frac{1}{x_B z^2}\frac{1}{p\cdot p_\pi} p_\pi^\mu i\epsilon^{\nu\alpha\beta\gamma} S_{\perp\alpha} p_\beta p_{\pi\gamma} , \tag{44}$$

which is just one of the terms required to make $W_{\mu\nu}$ gauge invariant.

The contribution of Fig. 2(c) can be calculated in the same way and the result is the complex conjugate of Eq. (41) with μ,ν indices interchanged. Combining the $h_1(x)$ term in Eq. (41), and Eq. (44) and its conjugate, we have the chiral-odd part of the spin-dependent nucleon tensor,

$$\begin{aligned} W_{[\mu\nu]}^{a+b+c} &= \frac{M}{2\nu}\sum_a e_a^2 h_1^a(x_B)\int dz \frac{\hat{e}_1^a(z)}{z}\left[i\epsilon^{\mu\nu\alpha\beta} p_\alpha S_{\perp\beta} + \frac{1}{z x_B}\frac{1}{p\cdot p_\pi} i p_\pi^\mu \epsilon^{\nu\alpha\beta\gamma} S_{\perp\alpha} p_\beta p_{\pi\gamma} - \frac{1}{z x_B}\frac{1}{p\cdot p_\pi} i p_\pi^\nu \epsilon^{\mu\alpha\beta\gamma} S_{\perp\alpha} p_\beta p_{\pi\gamma}\right] \\ &= -i\epsilon^{\mu\nu\alpha\beta} q_\alpha S_{\perp\beta}\frac{M}{2\nu}\sum_a e_a^2 \frac{h_1^a(x_B)}{x_B}\int dz \frac{\hat{e}_1^a(z)}{z} , \end{aligned} \tag{45}$$

which is explicitly gauge invariant.

Now we consider the contributions from Figs. 2(d) and 2(e). The calculations here parallel those for Figs. 2(b) and 2(c), and the final result for the chiral-even part of the nucleon tensor, including the contribution from Fig. 2(a), is

$$\begin{aligned} W_{[\mu\nu]}^{a+d+e} &= \frac{M}{2\nu}\sum_a e_a^2 g_T^a(x_B)\int dz \frac{\hat{f}_1^a(z)}{z}\left[i\epsilon^{\mu\nu\alpha\beta} S_{\perp\alpha} p_{\pi\beta} + \frac{x_B z}{p\cdot p_\pi} i p^\mu \epsilon^{\nu\alpha\beta\gamma} S_{\perp\alpha} p_\beta p_{\pi\gamma} - \frac{x_B z}{p\cdot p_\pi} i p^\nu \epsilon^{\mu\alpha\beta\gamma} S_{\perp\alpha} p_\beta p_{\pi\gamma}\right] \\ &= -i\epsilon^{\mu\nu\alpha\beta} q_\alpha S_{\perp\beta}\frac{M}{2\nu}\sum_a e_a^2 g_T^a(x_B)\int dz \hat{f}_1^a(z) . \end{aligned} \tag{46}$$

Adding Eqs. (45) and (46) to the longitudinal-polarization contribution, which is considerably easy to calculate, we have the complete spin-dependent nucleon tensor,

$$W^{\mu\nu} = -i\epsilon^{\mu\nu\alpha\beta}\frac{q_\alpha}{\nu}[(S\cdot n)p_\beta \hat{G}_1(x,z) + S_{\perp\beta}\hat{G}_T(x,z)] . \tag{47}$$

The two structure functions are defined as

$$\hat{G}_1(x,z) = \frac{1}{2}\sum_a e_a^2 g_1^a(x)\hat{f}_1^a(z) , \quad \hat{G}_T(x,z) = \frac{1}{2}\sum_a e_a^2\left[g_T^a(x)\hat{f}_1^a(z) + \frac{h_1^a(x)}{x}\frac{\hat{e}^a(z)}{z}\right] . \tag{48}$$

To isolate the spin-dependent cross section we take the difference of cross sections with left-handed and right-handed leptons,

$$\frac{d^2\Delta\sigma}{dE'd\Omega} = \frac{\alpha_{\rm em}^2}{Q^4}\frac{E'}{EM_N}\Delta l^{\mu\nu} W_{\mu\nu} , \tag{49}$$

where $Q^2 = -q^2$, $k = (E,\mathbf{k})$ and $k' = (E',\mathbf{k}')$ are the incident and outgoing momenta of the lepton, and $\Delta l^{\mu\nu}$ is the spin-dependent part of the lepton tensor, $\Delta l^{\mu\nu} = -\mathrm{Tr}[\gamma^\mu \not{k}' \gamma^\nu \gamma_5 \not{k}] = -4i\epsilon^{\mu\nu\alpha\beta} q_\alpha k_\beta$. It is convenient to express the cross section in terms of scaling variables in a frame where the lepton beam defines the z axis and the x-z plane contains the nucleon polarization vector, which has a polar angle α. In this system, the scattered lepton has polar angles (θ,ϕ) and therefore the momentum transfer $\mathbf{q}$ has polar angles $(\theta,\pi-\phi)$. Defining a conventional dimensionless variable $y = 1 - E'/E$, we can write the cross section as

$$\frac{d^4\Delta\sigma}{dx\, dy\, dz\, d\phi} = \frac{8\alpha_{\rm em}^2}{Q^2}\left\{ \cos\alpha\left[1-\frac{y}{2}\right]\hat{G}_1(x,z) + \cos\phi\,\sin\alpha\sqrt{(\kappa-1)(1-y)}\left[\hat{G}_T(x,z) - \hat{G}_1(x,z)\left[1-\frac{y}{2}\right]\right]\right\} , \tag{50}$$

where $\kappa = 1 + 4x^2M^2/Q^2$ in the second term signals the suppression by a factor of $1/Q$ associated with the structure function $\hat{G}_T$. The existence of $\hat{G}_1$ in the same term is due to a small longitudinal polarization of the nucleon when its spin is perpendicular to the lepton beam.

Equation (50) is one of our main results. As a check, we multiply by z, integrate over it, and sum over all hadron species. Using the well-known momentum sum rule,

$$\sum_{\text{hadrons}} \int dz\, z\hat{f}_1^a(z) = 1 , \tag{51}$$

and the sum rule,

$$\sum_{\text{hadrons}} \int dz\, \hat{e}_1^a(z) = 0 , \tag{52}$$

which is related to the fact that the chiral condensate vanishes in the perturbative QCD vacuum, we get

$$\frac{d^3\Delta\sigma}{dx\,dy\,d\phi} = \frac{4\alpha_{\rm em}^2}{Q^2}\left\{\cos\alpha\left[1-\frac{y}{2}\right]g_1(x) + \cos\phi\,\sin\alpha\sqrt{(\kappa-1)(1-y)}\left[g_T(x) - g_1(x)\left[1-\frac{y}{2}\right]\right]\right\} , \tag{53}$$

where $g_1(x) = \frac{1}{2}\sum_a e_a^2 g_1^a(x)$ and $G_T(x) = g_1(x) + g_2(x) = \frac{1}{2}\sum_a e_a^2[g_1^a(x) + g_2^a(x)]$ are the two conventional spin structure functions. The above result coincides with the same quantity in Ref. [12] if one neglects the terms of order $1/Q^2$ in the latter. The parallelism between the inclusive and semi-inclusive cross sections suggests that both quantities can be extracted from the same set of experiments.

In using Eq. (50) to analyze experimental data, a lower cut on z must be made to ensure the detected particles emerging from the current fragmentation region. To enhance statistics one can integrate z over a region. By varying ϕ, we can separate out the following combinations of structure functions,

$$\int G_1 dz = \frac{1}{2}\sum_a e_a^2 g_1^a(x) N_\pi^a ,$$
$$\int G_T dz = \frac{1}{2}\sum_a e_a^2 \left[g_T^a(x) N_\pi^a + \frac{h_1^a(x)}{x} E_\pi^a\right] , \tag{54}$$

where $N_\pi^a = \int dz f_1^a(z)$ is the pion multiplicity of the quark jet with flavor a and $E_\pi^a = \int dz e^a(z)/z$.

B. Spin-$\frac{1}{2}$ baryon production

In this subsection we study deep-inelastic scattering of unpolarized lepton beam on transversely polarized nucleon target, focusing on spin-$\frac{1}{2}$ baryon production from quark fragmentation. The spin effects in the scattering can be unraveled through measuring the polarization of the produced baryon. This can be done for an unstable hyperon by measuring angular distribution of its decay product. The process was first studied in Ref. [13]. Here we include a formula for the spin-dependent cross section in the lab frame.

The process can be described as in Fig. 2(a), except the produced pion is replaced here by a spin-$\frac{1}{2}$ baryon. From Eq. (14), we find the spin-dependent piece of the fragmentation density matrix,

$$\hat{M}_B(z) = \frac{\hat{h}_1(z)}{z}\gamma_5 \not{S}_{B\perp}\not{p}_B + \cdots , \tag{55}$$

where $p_B = z\nu n$ and S_B are the momentum and polarization of the baryon, respectively. Thus the spin-dependent nucleon tensor is

$$W^{\mu\nu} = -\frac{1}{2\nu}\sum_a e_a^2 h_1^a(x)\frac{\hat{h}_1^a(z)}{z}[(S_\perp^\mu S_{B\perp}^\nu + S_\perp^\nu S_{B\perp}^\mu)p\cdot p_B + (p^\mu p_B^\nu + p^\mu p_B^\nu - g^{\mu\nu}p\cdot p_B)S_\perp\cdot S_{B\perp}] . \tag{56}$$

Contracting it with the unpolarized lepton tensor, $l_{\mu\nu} = \frac{1}{2}{\rm Tr}[\gamma_\mu \not{k}\gamma_\nu \not{k}']$, we have

$$l^{\mu\nu}W_{\mu\nu} = -\frac{4}{\nu}\sum_a e_a^2 h_1^a(x)\hat{h}_1^a(z)[S_\perp\cdot k S_{B\perp}\cdot k p\cdot p_B + k\cdot p k\cdot p_B S_\perp\cdot S_{B\perp}] . \tag{57}$$

Using the lab coordinate system defined in Sec. III A to simplify (57), we find

$$l^{\mu\nu}W_{\mu\nu} = -4Q^2\frac{1-y}{y}\cos(\phi+\phi')\frac{1}{2}\sum_a e_a^2 h_1^a(x)\hat{h}_1^a(z) , \tag{58}$$

where ϕ' is the azimuthal angle between $\mathbf{k}'$ and $\mathbf{S}_B$. This produces the following spin-dependent cross section,

$$\frac{d\Delta\sigma}{dx\,dy\,dz\,d\phi} = -4\frac{\alpha_{\rm em}^2}{Q^2}\frac{1-y}{y}\cos(\phi+\phi')\frac{1}{2}\times\sum_a e_a^2 h_1^a(x)\hat{h}_1^a(z) . \tag{59}$$

This expression reaches maximum if $\mathbf{S}$ and $\mathbf{S}_B$ are the mirror images of each other with respect to the scattering plane.

C. Vector-meson and two-pion production

Here we consider the same setup for deep-inelastic scattering as in Sec. III B, but focusing on vector meson, e.g., ρ, and two-pion production. Our analysis shows that one can define a single-spin asymmetry sensitive to the nucleon's transversity distribution at the leading order in Q; however, its magnitude depends also on the unknown final-state interactions between the detected particle(s) and spectators. Similar ideas have also been proposed in Refs. [14] and [15].

Let us first look at vector-meson production. According to our previous discussion on quark fragmentation for a vector meson, there are two twist-two *chiral-odd* fragmentation functions $\hat{h}_1(z)$ and $\hat{h}_{\bar{1}}(z)$. The former describes the probability of producing vector mesons in vector polarization and the latter in tensor polarization. If one can measure the vector polarization, $\hat{h}_1(z)$ is an ideal choice for coupling with the transversity distribution. However, for the interesting case of ρ meson production, the only way to measure polarization is through its two-pion decay, which registers only tensor polarization. Thus, it appears that $\hat{h}_{\bar{1}}(z)$ is the only choice for coupling with the transversity distribution. However, the size of this fragmentation function depends on unknown final-state interactions.

If one is to measure asymmetry associated with inclusive production of two pions, there are other underlying processes which contribute besides the ρ decay, for instance, the interference production of two pions in their relative s and p waves. The contribution depends on the difference of the phase shifts. To include all the contributions, we directly introduce quark fragmentation functions for two-pion production,

$$\hat{\mathcal{M}}(k,P_{2\pi},l)=\int \frac{d^4\xi}{(2\pi)^4}e^{-ik\cdot\xi}\langle 0|\psi(0)|\pi(l_1)\pi(l_2)X\rangle\langle\pi(l_1)\pi(l_2)X|\bar{\psi}(\xi)|0\rangle \ , \tag{60}$$

where l_1 and l_2 are momenta of two observed pions and $P_{2\pi}=l_1+l_2$ and $l=(l_1-l_2)/2$ are the total and relative momenta, respectively.

The contribution of two-pion fragmentation to the nucleon tensor is

$$W_{\mu\nu}=\frac{1}{4\pi}\int\frac{d^4P_{2\pi}}{(2\pi)^4}\frac{d^4l}{(2\pi)^4}2\pi\delta(l\cdot P_{2\pi}/2)2\pi\delta(4l^2+P_{2\pi}^2)dx\ \mathrm{Tr}[M_N(xp,S_\perp)\gamma_\mu\hat{\mathcal{M}}(xp+q,P_{2\pi},l)\gamma_\nu] \ , \tag{61}$$

where we have neglected the pion mass and made collinear expansion for the initial quark momentum. To proceed further, we make a restriction on the l integrations such that $|l^2|<M^2$, where M is a soft scale on the order of $\Lambda_{\rm QCD}$. Making a collinear expansion for $P_{2\pi}$ and neglecting higher-twist contributions, we have

$$W_{\mu\nu}=\frac{1}{4\pi}\int dx\ 2\pi\delta((xp+q)^2)\int\frac{d^4l}{(2\pi)^4}2\pi\delta(l\cdot p_{2\pi}/2)dx\ \mathrm{Tr}[M_N(xp,S_\perp)\gamma_\mu\hat{\mathcal{M}}(p_{2\pi}/z,l)\gamma_\nu] \ , \tag{62}$$

where $p_{2\pi}=z\nu n$ and the fragmentation density simplifies to

$$\hat{\mathcal{M}}(p_{2\pi}/z,l)=\int\frac{d\lambda}{2\pi}e^{-i\lambda/z}\langle 0|\psi(0)|2\pi(p_{2\pi},l)X\rangle\langle 2\pi(p_{2\pi},l)X|\bar{\psi}(\lambda n_{2\pi})|0\rangle \ , \tag{63}$$

with $n_{2\pi}=p/(z\nu)$. For our purpose, we make the following expansion for the density,

$$\hat{\mathcal{M}}(p_{2\pi}/z,l)=\frac{\hat{H}_1(z,l)}{z}\gamma_5 \not{S}_{2\pi\perp}\not{p}_{2\pi}+\frac{\hat{F}_1(z,l)}{z}\not{p}_{2\pi}+\cdots \ , \tag{64}$$

where $S_{2\pi\perp}^{\alpha}=\epsilon^{\alpha\beta\gamma\delta}p_{2\pi\beta}n_{2\pi\gamma}l_\delta/|1|$ and $S_{2\pi\perp}\cdot p_{2\pi}=0$. The fragmentation function $\hat{H}_1(z,l)$ is real according to Hermiticity and nonvanishing because of the final-state interactions between π's and X.

Substituting Eq. (64) into Eq. (62), we have

$$W^{\mu\nu}=-\frac{1}{2\nu}\sum_a e_a^2h_1^a(x)\int\frac{d^4l}{(2\pi)^4}2\pi\delta(l\cdot p_{2\pi}/2)\frac{\hat{H}_1^a(z,l)}{z}[(S_\perp^\mu S_{2\pi\perp}^\nu+S_\perp^\nu S_{2\pi\perp}^\mu)p\cdot p_{2\pi}+(p^\mu p_{2\pi}^\nu+p^\mu p_{2\pi}^\nu-g^{\mu\nu}p\cdot p_{2\pi})S_\perp\cdot S_{2\pi\perp}] \ . \tag{65}$$

For this, we can calculate the spin-dependent part of the cross section,

$$\frac{d\Delta\sigma}{dx\,dy\,dz\,d\phi}=\frac{4\alpha_{\rm em}^2}{Q^2}\frac{1-y}{y}\frac{1}{2}\sum_a e_a^2h_1^a(x)\int\frac{d^4l}{(2\pi)^2}2\pi\delta(l\cdot p_{2\pi}/2)\hat{H}_1^a(z,l)\sin(\phi+\phi_l) \ , \tag{66}$$

where ϕ_l is the azimuthal angle between $\mathbf{k}'$ and $\mathbf{l}$.

IV. CONCLUSION

In this paper, we define a number of low-twist quark fragmentation functions by analyzing the matrix elements of quark bilinears in light-cone separations and expanding them in terms of various Lorentz structures. Some of these fragmentation functions are chiral odd and polarization dependent, which are not only interesting phenomenologically, but also useful for describing nonperturbative fragmentation processes.

In the examples of using the fragmentation functions, we study measurement of the nucleon's transversity distribution in deep-inelastic scattering, where chirality conservation selects those with odd chirality. Fragmentation functions and parton distributions are frequently coupled in cross sections; thus one can study both in experiments by varying x and z simultaneously. The facilities at CERN, the DESY *ep* collider HERA, and SLAC are particularly useful for learning these nonperturbative hadron observables.

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