Supersymmetry and non-Abelian Chern-Simons systems

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(Received 12 August 1993)

We study SU(n) N=1 supersymmetric Chern-Simons systems in 2+1 dimensions with and without a Maxwell term. Fixing the potential in a specific way, the supersymmetry is extended to N=2 which leads to a system with nontopological soliton solutions. The central charge of the extended supersymmetry is calculated and self-dual equations for the background fields are derived.

PACS number(s): 11.10.Lm, 11.30.Pb

Supersymmetric Chern-Simons systems have recently been considered by a number of authors [1–5]. Attention has in particular been given to a connection between extended supersymmetry and the existence of self-dual soliton solutions in the Abelian Higgs model with a pure Chern-Simons term [2]. The requirement of $N = 2$ supersymmetry was shown to determine a potential with both topological and nontopological soliton solutions. Simultaneously, the central charge of the $N = 2$ extended supersymmetry was identified as the topological charge of the theory. This latter aspect was also addressed in Refs. [6–9]. A non-Abelian SU(n) Higgs model with the corresponding pure Chern-Simons term was studied as well and it was shown that nontopological self-dual solitons exist for a special choice of the potential [10].

In this contribution we consider the supersymmetric generalization of this model with and without a Maxwell term and show that, as in the Abelian case, the requirement of extended supersymmetry leads to a potential with soliton solutions. Here, however, the central charge of the $N = 2$ supersymmetry is the U(1) charge of the system. Subsequently self-dual equations for the background fields are derived.

To construct a non-Abelian supersymmetric Chern-Simons action we require the gauge field connection $\Phi_A = i\Phi^r_A T^r$ ($A = a, \alpha$ with a a Lorentz index and α a spinor index) and the gauge field strength $F_{AB} = iF^r_{AB}T^r$. These quantities satisfy the relations

$$F_{ab} = \partial_a\Phi_b - \partial_b\Phi_a - [\Phi_a, \Phi_b] \; , \tag{1a}$$

$$F_{\alpha b} = D_\alpha\Phi_b - \partial_b\Phi_\alpha - [\Phi_\alpha, \Phi_b] \; , \tag{1b}$$

$$F_{\alpha\beta} = D_\alpha\Phi_\beta + D_\beta\Phi_\alpha - \{\Phi_\alpha, \Phi_\beta\} - 2i(\gamma^a)_{\alpha\beta}\Phi_a . \tag{1c}$$

We use γ matrices $\gamma^0 = \sigma^2, \gamma^1 = i\sigma^1$ and $\gamma^2 = i\sigma^3$ satisfying $\gamma^a\gamma^b = g^{ab} + i\varepsilon^{abc}\gamma_c$, the metric $g_{ab} = \mathrm{diag}(+1, -1, -1)$ and follow the notation of Ref. [11]. The SU(n) Hermitian traceless generators T^r satisfy $[T^s, T^u] = if^{rsu}T^r$. It follows from the Bianchi identities that $F_{ab} = \frac{i}{2}\varepsilon_{abc}\mathcal{D}^\tau(\gamma^c)^\rho_\tau W_\rho$, $F_{\alpha b} = i(\gamma_b)^\rho_\alpha W_\rho$, and $F_{\alpha\beta} = 0$, where the spinor superfield W_ρ satisfies the relation $\mathcal{D}^\rho W_\rho = 0$. Working in the Wess-Zumino gauge we can express the spinor superfields in the form

$$\Phi^r_\alpha = i\theta^\tau(\gamma^a)_{\tau\alpha}A^r_a - 2\theta^2\lambda^r_\alpha \; , \tag{2a}$$

$$\Phi^r_a = A^r_a + i\theta^\tau(\gamma_a)^\rho_\tau\lambda^r_\rho - \tfrac{1}{2}\theta^2\varepsilon_{abc}G^{bc,r} \; , \tag{2b}$$

$$W^r_\alpha = \lambda^r_\alpha - \frac{i}{2}\varepsilon^{abc}\theta^\tau(\gamma_a)_{\tau\alpha}G^r_{bc} - i\theta^2(\gamma^a)^\tau_\alpha(\partial_a\lambda^r_\tau + f^{rsu}A^s_a\lambda^u_\tau) \; , \tag{2c}$$

with $G^r_{ab} = \partial_a A^r_b - \partial_b A^r_a + f^{rsu}A^s_a A^u_b$. From the superfields (2) the supersymmetric generalization of the Chern-Simons action can be written as

$$S_{\mathrm{CS}} = \frac{k}{4}\int d^3x d^2\theta \left(\Phi^{\rho r}W^r_\rho + \frac{i}{6}f^{rsu}\Phi^r_c\Phi^{\alpha,s}(\gamma^c)^\beta_\alpha\Phi^u_\beta \right). \tag{3}$$

We first discuss the supersymmetric generalization of the model introduced in Ref. [10]. As matter fields a scalar superfield Ψ in the SU(n) fundamental representation is introduced,

$$\Psi = \varphi + \theta^\rho\psi_\rho - \theta^2 F \; , \tag{4}$$

in terms of which the total action of the system is

$$S = S_{\mathrm{CS}} + \int d^3x d^2\theta \left[-\tfrac{1}{2}\mathcal{D}^\alpha\Psi^\dagger\mathcal{D}_\alpha\Psi - U(\Psi^\dagger\Psi) \right], \tag{5}$$

where $\mathcal{D}_\alpha\Psi = (D_\alpha - i\Phi^r_\alpha T^r)\Psi$. This action is invariant under SU(n) gauge transformations, under a global U(1) transformation corresponding to a phase transformation of the scalar superfield, and under a supersymmetric transformation

$$\delta\Psi = i\eta^\rho(Q_\rho\Psi) \tag{6}$$

with

$$Q_\rho = -i\left(\frac{\partial}{\partial\theta^\rho} - i\theta^\tau(\gamma^a)_{\tau\rho}\partial_a + i\Phi^r_\rho T^r \right) , \tag{7}$$

$$\delta\Phi^r_\alpha = i\eta^\rho(Q_\rho\Phi^r_\alpha) \; , \tag{8}$$

with

$$Q_\rho\Phi^r_\alpha = -i\left(\frac{\partial}{\partial\theta^\rho} - i\theta^\tau(\gamma^a)_{\tau\rho}\partial_a \right)\Phi^r_\alpha - \frac{i}{2}f^{rsu}\Phi^s_\rho\Phi^u_\alpha. \tag{9}$$

In terms of component fields, the action (5) is

$$S = \int d^3x \left[\tfrac{1}{2}k\varepsilon^{abc}(A^r_a\partial_b A^r_c + \tfrac{1}{3}f^{rsu}A^r_a A^s_b A^u_c) + \tfrac{1}{2}k\lambda^{\rho,r}\lambda^r_\rho + i\psi^{\dagger\alpha}(\gamma^a)^\rho_\alpha D_a\psi_\rho + D^a\varphi^\dagger D_a\varphi \right.$$
$$+ i(\psi^{\dagger\alpha}\lambda^r_\alpha T^r\varphi - \varphi^\dagger T^r\lambda^{\alpha,r}\psi_\alpha) + F^\dagger F - U'(\varphi^\dagger\varphi)(\varphi^\dagger F + F^\dagger\varphi + \psi^{\dagger\alpha}\psi_\alpha)$$
$$\left. - \tfrac{1}{2}U''(\varphi^\dagger\varphi)(\psi^{\dagger\alpha}\varphi\psi^\dagger_\alpha\varphi + \varphi^\dagger\psi^\alpha\varphi^\dagger\psi_\alpha + 2\psi^{\dagger\alpha}\varphi\varphi^\dagger\psi_\alpha) \right] . \tag{10}$$

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Similarly, the transformations (6) and (8) can be rewritten in terms of the component fields as

$$\delta\varphi = \eta^\rho\psi_\rho \,, \quad (11a)$$

$$\delta\psi_\alpha = i\eta^\rho(\gamma^a)_{\rho\alpha}(\partial_a - iA_a^r T^r)\varphi - \eta_\alpha F \,, \quad (11b)$$

$$\delta F = i\eta^\rho(\gamma^a)_{\rho\alpha}(\partial_a - iA_a^r T^r)\psi_\alpha + 2i\eta^\rho\lambda_\rho^r T^r\varphi \,, \quad (11c)$$

$$\delta A_a^r = i\eta^\rho(\gamma_a)_\rho^\alpha\lambda_\alpha^r \,, \quad (11d)$$

$$\delta\lambda_\alpha = -i\tfrac{1}{2}\varepsilon^{abc}\eta^\rho(\gamma_a)_{\rho\alpha}G_{bc}^r \,. \quad (11e)$$

The auxiliary fields F and λ_ρ^r can be eliminated by using their equations of motion

$$F = U'(\varphi^\dagger\varphi)\varphi \,, \quad (12a)$$

$$\lambda_\rho^r = \frac{i}{k}(\varphi^\dagger T^r\psi_\rho - \psi_\rho^\dagger T^r\varphi) \,. \quad (12b)$$

Inserting these relations into Eq. (10) and rearranging some terms using the identity

$$T_{ij}^r T_{kl}^r = \tfrac{1}{2}\delta_{il}\delta_{jk} - \frac{1}{2n}\delta_{ij}\delta_{kl} \,, \quad (13)$$

we obtain the action in the form

$$S = \int d^3x\Big[\tfrac{1}{2}k\varepsilon^{abc}(A_a^r\partial_b A_c^r + \tfrac{1}{3}f^{rsu}A_a^r A_b^s A_c^u) + i\psi^{\dagger\alpha}(\gamma^a)_\alpha^\rho D_a\psi_\rho + D^a\varphi^\dagger D_a\varphi - U'(\varphi^\dagger\varphi)U'(\varphi^\dagger\varphi)\varphi^\dagger\varphi$$
$$-\frac{1}{k}\psi^{\dagger\alpha}T^r\varphi\varphi^\dagger T^r\psi_\alpha - U''(\varphi^\dagger\varphi)\psi^{\dagger\alpha}\varphi\varphi^\dagger\psi_\alpha - U'(\varphi^\dagger\varphi)\psi^{\dagger\alpha}\psi_\alpha + \left(\frac{n-1}{4kn} - \tfrac{1}{2}U''(\varphi^\dagger\varphi)\right)(\psi^{\dagger\alpha}\varphi\psi_\alpha^\dagger\varphi + \varphi^\dagger\psi^\alpha\varphi^\dagger\psi_\alpha)\Big] . \quad (14)$$

Using expressions (12) we find that the supercharge generating the transformations (11a), (11b), and (11d) can be expressed as

$$Q_\alpha^1 = \int d^2x[(\gamma^a)_\alpha^\rho(\gamma^0)_\rho^\tau(D_a\varphi^\dagger\psi_\tau + \psi^\dagger D_a\varphi) + iU'(\varphi^\dagger\varphi)(\gamma^0)_\alpha^\tau(\varphi^\dagger\psi_\tau + \psi_\tau^\dagger\varphi)] \equiv Q_\alpha + Q_\alpha^\dagger \,. \quad (15)$$

It is seen from Eq. (14) that, as in the Abelian case, we are in a position to require that the fermion-number-violating terms disappear from the action. This is achieved if we set

$$U''(\varphi^\dagger\varphi) = \frac{n-1}{2kn} \quad (16)$$

and consequently

$$U'(\varphi^\dagger\varphi) = \frac{n-1}{2kn}(\varphi^\dagger\varphi - v^2) \,. \quad (17)$$

The above condition (16) implies that the resulting action

$$S = \int d^3x\Big[\tfrac{1}{2}k\varepsilon^{abc}(A_a^r\partial_b A_c^r + \tfrac{1}{3}f^{rsu}A_a^r A_b^s A_c^u) + i\psi^{\dagger\alpha}(\gamma^a)_\alpha^\rho D_a\psi_\rho + D^a\varphi^\dagger D_a\varphi$$
$$-\left(\frac{n-1}{2kn}\right)^2(\varphi^\dagger\varphi - v^2)^2\varphi^\dagger\varphi - \frac{n-1}{2kn}(\varphi^\dagger\varphi - v^2)\psi^{\dagger\alpha}\psi_\alpha - \frac{1}{2k}\psi^{\dagger\alpha}\psi_\alpha\varphi^\dagger\varphi - \frac{n-2}{2kn}\psi^{\dagger\alpha}\varphi\varphi^\dagger\psi_\alpha\Big] \quad (18)$$

has an additional symmetry, namely, that of a phase transformation of the whole fermion multiplet or, alternatively, a phase transformation of the whole boson multiplet. The corresponding conserved currents are, respectively,

$$J^a = \psi^{\dagger\rho}(\gamma^a)_\rho^\tau\psi_\tau \,, \quad (19a)$$

$$J^a = i[(D^a\varphi^\dagger)\varphi - \varphi^\dagger D^a\varphi] \,. \quad (19b)$$

It is straightforward to show that these currents are the lowest components of the superfields $\mathcal{D}^\rho\Psi^\dagger(\gamma^a)_\rho^\tau\mathcal{D}_\tau\Psi$ and $i[(\mathcal{D}^a\Psi^\dagger)\Psi - \Psi^\dagger\mathcal{D}^a\Psi]$, respectively. We now consider the first superfield. Taking its time component proportional to θ and integrating over space we find

$$Q_\alpha^2 = i\int d^2x[(\gamma^a)_\alpha^\rho(\gamma^0)_\rho^\tau(D_a\varphi^\dagger\psi_\tau - \psi^\dagger D_a\varphi) + iU'(\varphi^\dagger\varphi)(\gamma^0)_\alpha^\tau(\varphi^\dagger\psi_\tau - \psi_\tau^\dagger\varphi)] \equiv i(Q_\alpha - Q_\alpha^\dagger) \,. \quad (20)$$

This is the second supercharge generating an $N = 2$ supersymmetry of the system. Repeating this for the $-\theta^2$ component of the superfield in question, we arrive, after partial integration and neglecting the surface term, at

$$2T = \int d^2x\Big(2iU'(\varphi^\dagger\varphi)[(D^0\varphi^\dagger)\varphi - \varphi^\dagger D^0\varphi + i\psi^{\dagger\rho}(\gamma^0)_\rho^\tau\psi_\tau] + \frac{1}{k}[\psi^{\dagger\rho}(\gamma^0)_\rho^\tau\psi_\tau\varphi^\dagger\varphi - \psi^{\dagger\rho}\varphi(\gamma^0)_\rho^\tau\varphi^\dagger\psi_\tau] - \varepsilon^{ij}G_{ij}^r\varphi^\dagger T^r\varphi\Big) \,. \quad (21)$$

Using the Gauss law

$$\tfrac{1}{2}\varepsilon^{ij}G_{ij}^r - \frac{1}{k}i(D^0\varphi^\dagger T^r\varphi - \varphi^\dagger T^r D^0\varphi) + \frac{1}{k}\psi^{\dagger\rho}T^r(\gamma^0)_\rho^\tau\psi_\tau = 0 \quad (22)$$

in Eq. (21) we obtain

$$T = -\frac{n-1}{2kn}v^2 i\int d^2x[(D^0\varphi^\dagger)\varphi - \varphi^\dagger D^0\varphi + i\psi^{\dagger\rho}(\gamma^0)_\rho^\tau\psi_\tau] = -\frac{n-1}{2kn}v^2 Q \,, \quad (23)$$

where Q is the global U(1) charge. This is, however, just the central charge of the $N = 2$ supersymmetry as we can check that

$$\{Q_\alpha, Q^{\dagger\beta}\} = (\gamma^a)_\alpha^\beta P_a - \delta_\alpha^\beta T . \quad (24)$$

We can similarly investigate the second supercurrent, namely the one of which the lowest component is given by expression (19b). In this case, however, we get exactly the same results as for the first supercurrent after using equations of motion and partial integration. It is now straightforward to derive the self-dual equations for the background fields. As in the Abelian case [2] we multiply (24) by $\frac{1}{2}(1 \pm \gamma^0)$ and take the trace to obtain

$$P_0 = \pm T + \{Q_{\pm\alpha}, (Q_{\pm\alpha})^\dagger\} \quad (25)$$

with

$$\begin{aligned} Q_{\pm\alpha} &= \tfrac{1}{2}(1 \pm \gamma^0)_\alpha^\beta Q_\beta \\ &= \int d^2x \{ \tfrac{1}{2}(1 \pm \gamma^0)_\alpha^\beta \psi_\beta [D_0\varphi^\dagger \pm i\varphi^\dagger U'(\varphi^\dagger\varphi)] \\ &\quad \mp \tfrac{1}{2}(\gamma^1 \mp i\gamma^2)_\alpha^\beta \psi_\beta (D_1\varphi^\dagger \pm iD_2\varphi^\dagger)\}. \end{aligned} \quad (26)$$

As Eq. (25) contains an anticommutator of adjoint operators we get a lower bound for expectation values of P_0:

$$P_0 \geq \frac{n-1}{2kn} v^2 |Q| , \quad (27)$$

which is saturated for the states annihilated by $Q_{\pm\alpha}$. For a state built around classical background fields this is fulfilled if the background fields obey the Gauss law (22), with the fermion fields set to zero, as well as

$$D_1\varphi \mp iD_2\varphi = 0 , \quad (28a)$$

$$D_0\varphi \mp iU'(\varphi^\dagger\varphi)\varphi = 0 . \quad (28b)$$

The upper (lower) sign is for negative (positive) charge Q. Equations (28) are just the self-dual equations derived in Ref. [10]. As to the particle spectrum of the supersymmetric model discussed, we can deduce from action (18) that there are n complex scalar fields and n fermions with mass $m = \frac{n-1}{2kn}v^2$, carrying a unit of the global U(1) charge in the symmetric phase. In the asymmetric phase there are two neutral bosons and two neutral fermions with mass $m = 2(\frac{n-1}{2kn}v^2)$, together with $n-1$ complex vector bosons and fermions of mass $\frac{v^2}{2k}$ carrying $\frac{n}{n-1}$ units of U(1) charge.

A model related to the one studied above is a supersymmetric non-Abelian generalization of the model discussed in Ref. [12]. It includes a Maxwell term in the action and an additional scalar field $\mathcal{N}$ in the adjoint representation of SU(n):

$$\mathcal{N}^r = N^r + \theta^\rho \chi_\rho^r - \theta^2 \mathcal{F}^r . \quad (29)$$

The action of this model reads

$$\begin{aligned} S = S_{\text{CS}} + \int d^3x d^2\theta \big[&\tfrac{1}{4} W^{\alpha,r} W_\alpha^r - \tfrac{1}{4}\mathcal{D}^\alpha \mathcal{N}^r \mathcal{D}_\alpha \mathcal{N}^r \\ &- \tfrac{1}{2}\mathcal{D}^\alpha \Psi^\dagger \mathcal{D}_\alpha \Psi - U(\Psi, \mathcal{N}) \big] , \end{aligned} \quad (30)$$

where the potential is taken in the form

$$U(\Psi, \mathcal{N}) = c_1 \Psi^\dagger \mathcal{N}^r T^r \Psi + \tfrac{1}{2} c_2 \mathcal{N}^r \mathcal{N}^r + c_3 \Psi^\dagger \Psi. \quad (31)$$

The supersymmetric transformations of the compenent fields are given by Eqs. (11) together with

$$\delta N^r = \eta^\rho \chi_\rho^r , \quad (32a)$$

$$\delta\chi_\alpha^r = i\eta^\rho (\gamma^a)_{\rho\alpha} (\partial_a N^r + f^{rsu} A_a^s N^u) - \eta_\alpha \mathcal{F}^r, \quad (32b)$$

which is generated by

$$\begin{aligned} Q_\alpha^1 = \int d^2x \big[&(\gamma^a)_\alpha^\rho (\gamma^0)_\rho^\tau (D_a\varphi^\dagger \psi_\tau + \psi_\tau^\dagger D_a\varphi \\ &+ \chi_\tau^r D_a N^r - \tfrac{1}{2}\lambda_\tau^r \varepsilon_{abc} G^{bc,r}) \\ &+ i(\gamma^0)_\alpha^\tau (F^\dagger \psi_\tau + \psi_\tau^\dagger F + \chi_\tau^r \mathcal{F}^r) \big] \\ \equiv Q_\alpha + Q_\alpha^\dagger , \end{aligned} \quad (33)$$

with F and $\mathcal{F}$ substituted by their equations of motion.

The supersymmetry can be extended to $N = 2$ by adjusting the potential parameters to $c_1 = -1$ and $c_2 = -k$. The remaining parameter we set at $c_3 = -\frac{n-1}{2kn}v^2$ and consequently express the auxiliary fields as

$$F = -\left(N^r T^r + \frac{n-1}{2kn} v^2 \right) \varphi , \quad (34a)$$

$$\mathcal{F}^r = -\varphi^\dagger T^r \varphi - kN^r . \quad (34b)$$

Using the above relations we obtain the action expressed in terms of component fields:

$$\begin{aligned} S = \int d^3x \Big[&-\tfrac{1}{4} G^{ab,r} G_{ab}^r + \tfrac{1}{2} k \varepsilon^{abc} (A_a^r \partial_b A_c^r + \tfrac{1}{3} f^{rsu} A_a^r A_b^s A_c^u) + i\psi^{\dagger\alpha} (\gamma^a)_\alpha^\rho D_a \psi_\rho + D^a \varphi^\dagger D_a \varphi \\ &+ \frac{i}{2} \lambda^\alpha (\gamma^a)_\alpha^\rho D_a \lambda_\rho + \frac{i}{2} \chi^\alpha (\gamma^a)_\alpha^\rho D_a \chi_\rho + \frac{k}{2} \lambda^{\alpha,r} \lambda_\alpha^r + \frac{k}{2} \chi^{\alpha,r} \chi_\alpha^r + \tfrac{1}{2} D^a N^r D_a N^r - f^{rsu} \chi^{\alpha,r} \lambda_\alpha^s N^u \\ &+ i(\psi^{\dagger\alpha} \lambda_\alpha^r T^r \varphi - \varphi^\dagger T^r \lambda^{\alpha,r} \psi_\alpha) + \psi^{\dagger\alpha} \chi_\alpha^r T^r \varphi + \varphi^\dagger T^r \chi^{\alpha,r} \psi_\alpha + \frac{n-1}{2kn} v^2 \psi^{\dagger\alpha} \psi_\alpha \\ &+ \psi^{\dagger\alpha} N^r T^r \psi_\alpha - \varphi^\dagger \left(N^s T^s + \frac{n-1}{2kn} v^2 \right) \left(N^r T^r + \frac{n-1}{2kn} v^2 \right) \varphi - \tfrac{1}{2} (\varphi^\dagger T^r \varphi + kN^r)(\varphi^\dagger T^r \varphi + kN^r) \Big] . \end{aligned} \quad (35)$$

To find the generators of the extended supersymmetry we can make use of the fact that the special choice of the parameters c_1 and c_2 leads to an additional U(1) symmetry. Namely, we can transform the whole ψ multiplet as $\psi' = e^{i\varepsilon}\psi$ and simultaneously transform the fields

$$\chi^{r'} = \chi^r \cos\varepsilon + \lambda^r \sin\varepsilon , \quad (36a)$$

$$\lambda^{r'} = -\chi^r \sin\varepsilon + \lambda^r \cos\varepsilon . \quad (36b)$$

This symmetry generates the conserved current $J^a = \psi^{\dagger\rho}(\gamma^a)_\rho^\tau \psi_\tau + i\chi^{\rho,r}(\gamma^a)_\rho^\tau \lambda_\tau^r$ which is the $\theta = 0$ part of a conserved supercurrent

$$\mathcal{D}^\rho \Psi^\dagger (\gamma^a)_\rho^\tau \mathcal{D}_\tau \Psi + i\mathcal{D}^\rho \mathcal{N}^r (\gamma^a)_\rho^\tau W_\tau^r . \quad (37)$$

The second generator of the $N = 2$ supersymmetry is obtained by space integration of the time component (37) proportional to θ:

$$Q_\alpha^2 = i\int d^2x\Big[(\gamma^a)_\alpha^\rho(\gamma^0)_\rho^\tau\Big(D_a\varphi^\dagger\psi_\tau - \psi_\tau^\dagger D_a\varphi + i\lambda_\tau^r D_a N^r + \frac{i}{2}\chi_\tau^r\varepsilon_{abc}G^{bc,r}\Big) + i(\gamma^0)_\alpha^\tau(F^\dagger\psi_\tau - \psi_\tau^\dagger F + i\mathcal{F}^r\lambda_\tau^r)\Big] \equiv i(Q_\alpha - Q_\alpha^\dagger)\ . \tag{38}$$

The space integration of the time component (37) proportional to $-\theta^2$ gives the central charge. After using the equations of motion and partial integration we find the central charge in the same form as in expression (23). As Eqs. (24) and (25) hold, it is straightforward to find the self-dual equations for the background fields which saturate the condition (27). In the same way as inferred from Eq. (26) we get

$$D_1\varphi \mp iD_2\varphi = 0\ , \tag{39a}$$

$$D_0\varphi \pm i\Big(N^rT^r + \frac{n-1}{2kn}v^2\Big)\varphi = 0\ , \tag{39b}$$

$$D_iN^r \mp G_{i0}^r = 0\ , \tag{39c}$$

$$D_0N^r = 0\ , \tag{39d}$$

$$G_{12}^r \mp (\varphi^\dagger T^r\varphi + kN^r) = 0\ . \tag{39e}$$

The upper (lower) sign is for negative (positive) charge Q. Configurations saturating condition (27) must in addition to Eqs. (39) satisfy the Gauss law following from Eq. (35) by variation of A_0^r:

$$D_iG^{i0,r} + \tfrac{1}{2}k\varepsilon^{ij}G_{ij}^r - i(D^0\varphi^\dagger T^r\varphi - \varphi^\dagger T^rD^0\varphi) + \psi^{\dagger\rho}T^r(\gamma^0)_\rho^\tau\psi_\tau + f^{rsu}N^sD^0N^u - \frac{i}{2}f^{rsu}[\lambda^{\rho,s}(\gamma^0)_\rho^\tau\lambda_\tau^u + \chi^{\rho,s}(\gamma^0)_\rho^\tau\chi_\tau^u] = 0\ , \tag{40}$$

with the fermion fields set to zero. The self-dual equations (39) are similar to those derived in Ref. [12] for the Abelian version of the model. To obtain the equations of Ref. [12], one has to start from the supersymmetric potential $\mathcal{N}(\Psi^\dagger\Psi - v^2) + \frac{k}{2}\mathcal{N}^2$. This potential, however, cannot be generalized to the non-Abelian case, which is why we chose to work with the potential (31).

A specific solution of the above equations may now be investigated for the case $\varphi = (0,\ldots,0,f)$, $N = (0,\ldots,0,N^D)$, $A_a = (0,\ldots,0,A_a^D)$. Introducing $\bar{A}_a = A_a^D\sqrt{\frac{n-1}{2n}} + \partial_a\arg f$ we get $\partial_0N^D = 0$, $\partial_0|f| = 0$, $\partial_0\bar{A}_i = 0$, and $\sqrt{\frac{n-1}{2n}}N^D = \bar{A}_0 + \frac{n-1}{2kn}v^2$. Consequently, Eqs. (39a), (39e), and (40) reduce to

$$\bar{A}_i \mp \varepsilon_{ij}\partial_j\ln|f| = 0\ , \tag{41a}$$

$$\partial_i\partial_i\ln|f|^2 + 2\Big(|f|^2\sqrt{\frac{n-1}{2n}} - kN^D\Big) = 0\ , \tag{41b}$$

$$-\partial_i\partial_iN^D + k\Big(kN^D - \sqrt{\frac{n-1}{2n}}|f|^2\Big) + \frac{n-1}{n}|f|^2\Big(N^D - \frac{v^2}{k}\sqrt{\frac{n-1}{2n}}\Big) = 0\ . \tag{41c}$$

These equations can be compared to those derived and solved in Ref. [12] for the Abelian model.

The particle spectrum of this model is as follows. In the symmetric phase, there are n complex scalar fields φ and n fermion fields ψ with the mass $\frac{n-1}{2kn}v^2$ carrying a unit of global U(1) charge, n^2-1 gauge bosons A_a^r, and scalar bosons N^r, and the same number of fermions χ^r, λ^r, all of which have the mass k and zero U(1) charge. In the asymmetric phase the vacuum expectation values of the scalar fields are $\langle\varphi\rangle = (0,\ldots,v)$ and $\langle N\rangle = (0,\ldots,\frac{v^2}{k}\sqrt{\frac{n-1}{2n}})$. There are two scalar degrees of freedom with mass $\frac{1}{2}\sqrt{k^2 + 4\frac{n-1}{n}v^2} \pm \frac{1}{2}k$ and a gauge boson with two propagating modes of the same mass and with zero U(1) charge. Then there are $2(n-1)$ gauge bosons with two propagating modes of masses $\frac{v^2}{2k}$ and $k + \frac{v^2}{2k}$. Moreover, there are $2(n-1)$ boson degrees of freedom of mass $k + \frac{v^2}{2k}$, both the former and the latter carrying $\frac{n}{n-1}$ units of the global U(1) charge. Finally, there are $(n-1)^2 - 1$ gauge bosons and the same number of scalars, all with the mass k and zero U(1) charge. Obviously, the number of fermion degrees of freedom and their mass spectrum as well as the U(1) charges are exactly the same as that of the bosons.

We have discussed above the connection between extended supersymmetry and the existence of soliton solutions in non-Abelian SU(n) Chern-Simons systems. The self-dual equations and the central charge have been derived for models without and with a Maxwell term. In the former case the potential is determined completely from the extended supersymmetry requirement whereas in the latter the $\Psi^\dagger\Psi$ term of the potential is not fixed. Unlike in the Abelian case, the central charge is here not the topological charge of the system, but the global U(1) charge and the soliton solutions are nontopological.

[1] Z. Hlousek and D. Spector, Nucl. Phys. **B344**, 763 (1990).
[2] Ch. Lee, K. Lee, and E. J. Weinberg, Phys. Lett. B **243**, 105 (1990).
[3] P. P. Srivastatva and K. Tanaka, Phys. Lett. B **256**, 427 (1991).
[4] E. A. Ivanov, Phys. Lett. B **268**, 203 (1991).
[5] S. J. Gates and H. Nishino, Phys. Lett. B **281**, 72 (1992).
[6] P. Di Vecchia and S. Ferrara, Nucl. Phys. **B130**, 93 (1977).
[7] E. Witten and D. Olive, Phys. Lett. **78B**, 97 (1978).
[8] Z. Hlousek and D. Spector, Nucl. Phys. **B370**, 143 (1992).
[9] Z. Hlousek and D. Spector, Nucl. Phys. **B397**, 173 (1993).
[10] K. Lee, Phys. Rev. Lett. **66**, 553 (1991).
[11] S. J. Gates, M. T. Grisaru, M. Roček, and W. Siegel, *Superspace, or One Thousand and One Lessons in Supersymmetry* (Benjamin-Cummings, Reading, MA, 1983).
[12] Ch. Lee, K. Lee, and H. Min, Phys. Lett. B **252**, 79 (1990).