Charged global monopoles

J. R. Morris
Department of Chemistry/Physics/Astronomy, Indiana University Northwest, 3400 Broadway, Gary, Indiana 46408
(Received 6 July 1993)

A model containing a global SO(3) scalar isovector coupled to a local U(1) complex scalar field is investigated. It is argued that there exists a parameter range for which the complex scalar is coaxed out of its vacuum state in the core of a global monopole and gives rise to a charged global monopole. The estimated maximum charge for a quantum mechanically stable configuration has the potential for being quite large, and the Abelian Coulombic force can rival the strength of the Goldstone force within a finite range outside the core of the monopole. The possibility is explored that a global antimonopole and a global monopole with like charges can form a "meson" bound state. It is found, however, that owing to pair creation of light charged particles the monopoles will lose charge (unless there exists a parameter fine tuning), thus causing the mesons to decay, if they form at all.

PACS number(s): 11.15.Ex, 11.30.Qc, 14.80.Hv

I. INTRODUCTION

The speculation that phase transitions may have occurred in the early Universe gives rise to the intriguing possibility that various types of topological defects, such as domain walls, cosmic strings [1], superconducting cosmic strings [2], magnetic monopoles [3], and dyons [4] may have been created. Such objects can have important implications for particle physics, astrophysics, and cosmology. Recently there has also been interest in global monopoles [5–7] which can arise from a spontaneous symmetry breaking of a global group such as SO(3). For instance, global monopoles formed at the grand unification scale may serve as seeds for galaxy and large-scale structure formation [7]. Ordinary global monopoles, however, would be detectable only through a direct coupling to global scalar fields or by gravitational effects [6]. Although interactions associated with Goldstone boson fields are long-range interactions, they may be difficult to detect at energy scales much lower than the symmetry breaking scale associated with the formation of the global monopole, due to an effective decoupling at low energies [8]. Questions regarding whether a global monopole could possess an electric charge and what consequences would ensue therefore become relevant. It is interesting therefore to consider a possible model of charged global monopoles having an associated long-range Abelian field through which electromagnetic interactions can exist, with the possibility of enhancing detectability. In addition, there is the prospect that an electric charge may help to stabilize a global monopole-antimonopole pair against collapse [9] and allow meson-like bound states to form.

Presented here is a simple model which can describe a global monopole possessing a charge arising from a local U(1) gauge group. A type of approach similar in spirit to that of Witten's [2] is presented to argue that there exists a parameter range for which a vanishing complex scalar field in the presence of a global monopole [which emerges from the spontaneously broken global SO(3) group] is unstable, thus coaxing the complex scalar field out of its vacuum to form a condensate in the core of the monopole. The scalar condensate can possess an Abelian charge residing in or near the monopole core. The possibility is examined that a global monopole-antimonopole pair with electric charges of the same sign can bind together to form a charged "meson."

The model is presented in Sec. II. In Sec. III the stability of a charged global monopole against loss of charge due to the pair production of light charged particles, such as electrons, is examined. It is found that, unless an extreme fine tuning of one of the coupling constants exists, the charged global monopoles are unstable against a loss of charge so that the charge "evaporates" from the monopole, which implies that meson states are not stable against decay, if they form at all. A summary and discussion forms Sec. IV.

II. THE MODEL

Specifically, the model consists of a scalar isovector field χ coupled to a complex scalar field ϕ and is described by the Lagrangian

$$L=\tfrac{1}{2}\partial^\mu\chi\cdot\partial_\mu\chi+(D^\mu\phi)^*(D_\mu\phi)-V-\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}\ , \tag{1}$$

where the potential V is given by

$$V=\tfrac{1}{4}\lambda(\chi\cdot\chi-\eta^2)^2+f(\chi\cdot\chi-\eta^2)\phi^*\phi+m^2\phi^*\phi+g(\phi^*\phi)^2\ , \tag{2}$$

and

$$D_\mu=\nabla_\mu+ieA_\mu,\quad F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu\ . \tag{3}$$

The coupling constants λ, f, and g, as well as the ϕ particle mass m, are assumed to be positive, real quantities. The stable vacuum state of the theory is located by $\chi=|\chi|=\eta$, $\phi=0$, and the χ particle mass is $m_\chi=(2\lambda)^{1/2}\eta$.

For the case whether the fields ϕ and A_μ vanish identically, the model admits a topological stable, spherically symmetric, global monopole solution [5] $\boldsymbol{\chi}=\chi(r)\hat{\mathbf{r}}$, where near the origin, in the monopole core, $\chi\approx ar$ and asymptotically, at large distances outside the monopole core, $\chi\rightarrow\eta$. The monopole core is taken to have a radius $r_0\approx 1/m_\chi\approx(\lambda^{1/2}\eta)^{-1}$.

Following a similar line of reasoning that Witten [2] has used to argue the existence of superconducting cosmic strings, it can be argued that for the present model there exists a parameter range for which the complex scalar field given by $\phi=0$, with $A_\mu=0$, is unstable in the background field of the global monopole, and must therefore relax to a lower energy state for which $\phi\neq 0$ in the monopole core. Therefore, consider a global monopole background field given by $\boldsymbol{\chi}=\chi(r)\hat{\mathbf{r}}$ with $A_\mu=0$. Upon looking at small fluctuations of ϕ about $\phi=0$, we have, approximately,

$$\nabla_\mu\partial^\mu\phi+[f(\chi^2-\eta^2)+m^2]\phi=0\ . \tag{4}$$

Let us take $\phi(\mathbf{r},t)=\phi_0(\mathbf{r})e^{-i\Omega t}$ and use $\chi\approx ar$ near the center of the monopole core, so that (4) becomes

$$-\nabla^2\phi_0+(fa^2r^2)\phi_0=E\phi_0,\quad E=\Omega^2+(f\eta^2-m^2)\ . \tag{5}$$

Equation (5) is just the Schrödinger equation for a particle of mass $\mu=1/2$ in a simple harmonic oscillator potential with an associated "spring constant" $k=2fa^2$. The (normalizable) ground-state "wave function" has an energy

$$E=\frac{3}{2}\omega=\frac{3}{2}\left[\frac{k}{\mu}\right]^{1/2}=3a\sqrt{f} \tag{6}$$

which implies that, for this state, $\Omega^2=3a\sqrt{f}-(f\eta^2-m^2)$. Therefore the solution $\phi=0$ is unstable when $\Omega^2<0$, i.e., for a set of parameters satisfying the approximate inequality

$$3a\sqrt{f}<(f\eta^2-m^2)\ , \tag{7}$$

in which case the scalar field ϕ is coaxed out of its vacuum state, with $\phi\neq 0$ in the core of the global monopole. Furthermore, we expect, by continuity, that solutions corresponding to excitations exist for $A_\mu\neq 0$. In particular (since there exists a conserved current $j_\mu=i[\phi^*(D_\mu\phi)-\phi(D_\mu\phi)^*]$) solutions which describe a global monopole with an Abelian "electric" charge are assumed to exist for a set of parameters restricted by (7).

Adopting the ansatz

$$\phi(r,t)=\frac{1}{\sqrt{2}}F(r)e^{i\Psi(t)},\quad \Psi(t)=\omega t\ , \tag{8}$$

$$\boldsymbol{\chi}(\mathbf{r})=\chi(r)\hat{\mathbf{r}},\quad \hat{\mathbf{r}}=(\sin\theta\cos\varphi,\sin\theta\sin\varphi,\cos\theta)\ , \tag{9}$$

$$A_\mu(r)=\frac{1}{e}[P(r)-1]\partial_\mu\Psi=\frac{\omega}{e}[P(r)-1]\delta_\mu^0\ , \tag{10}$$

the equations of motion can be written as

$$\frac{1}{r^2}\partial_r(r^2\partial_r\chi)-\frac{2}{r^2}\chi-[\lambda(\chi^2-\eta^2)+fF^2]\chi=0\ , \tag{11}$$

$$\frac{1}{r^2}\partial_r(r^2\partial_rF)+\omega^2P^2F-[f(\chi^2-\eta^2)+m^2+gF^2]F=0\ , \tag{12}$$

$$\frac{1}{r^2}\partial_r(r^2\partial_rP)-e^2F^2P=0\ , \tag{13}$$

and the current density becomes

$$j_\mu=i[\phi^*(D_\mu\phi)-\phi(D_\mu\phi)^*]=-\omega F^2P\delta_\mu^0\ . \tag{14}$$

The equations of motion are subjected to the boundary conditions

$$\chi\rightarrow 0,\ F\rightarrow F_0\equiv[(f\eta^2-m^2)/g]^{1/2},\quad P\rightarrow P_0=\text{const, as } r\rightarrow 0\ , \tag{15a}$$

$$\chi\rightarrow\eta,\ F\rightarrow 0,\ P\rightarrow\text{const, as } r\rightarrow\infty\ . \tag{15b}$$

Using (11)–(15) and working to $O(r^2)$, the solutions near the origin are approximately given by

$$\chi\approx ar\ , \tag{16a}$$

$$F\approx F_0-\tfrac{1}{6}\omega^2P_0^2r^2\ , \tag{16b}$$

$$P\approx P_0+\tfrac{1}{6}e^2F_0^2P_0r^2\ , \tag{16c}$$

where the constants a and P_0 are to be determined numerically.

As $r\rightarrow\infty$, (11)–(15) yield the asymptotic solutions

$$\chi\approx\eta\left[1-\frac{1}{\lambda\eta^2r^2}\right]\approx\eta\left[1-\frac{\delta^2}{r^2}\right]\ , \tag{17a}$$

$$F\approx(\text{const})\exp[-(m^2-\omega^2B^2)^{1/2}r],\quad m^2>\omega^2B^2\ , \tag{17b}$$

$$P\approx-\frac{A}{r}+B\ , \tag{17c}$$

where A and B are positive constants, and $\delta\approx(\lambda^{1/2}\eta)^{-1}$ is the core radius.

By (3) and (10) the "ordinary," or "physical" radial component of the electric field is $E_r=F_{0r}=-(\omega/e)\partial_rP$, which by (17c) asymptotically becomes $E_r\approx-(\omega A/er^2)$. Consequently, the total electric charge of the global monopole is

$$q\equiv Ne=\lim_{r\rightarrow\infty}\int E_rr^2d\Omega=-4\pi\omega A/e\ , \tag{18}$$

where N is the number of unbalanced charge carriers in the monopole, and at asymptotic distances,

$$E_r\approx\frac{q}{4\pi r^2}=\frac{Ne}{4\pi r^2}\ . \tag{19}$$

The energy density of the charged global monopole configuration is

$$T_{00}=\frac{1}{2}(\partial_r\chi)^2+\frac{\chi^2}{r^2}+\frac{1}{2}(\partial_rF)^2+\frac{1}{2}\omega^2F^2P^2+\frac{1}{2}\frac{\omega^2}{e^2}(\partial_rP)^2+V(\chi,F)\ . \tag{20}$$

By (14) and (17) it is seen that the charge density vanishes rapidly at asymptotic distances, so that the monopole's charge is localized in or near the monopole core. For approximation purposes let us take

$$F\approx\begin{cases}F_0, & r\leq\delta\ ,\\ 0, & r>\delta\ ,\end{cases}\quad P\approx\begin{cases}P_0, & r\leq\delta\ ,\\ B, & r>\delta\ ,\end{cases}\quad \chi\approx\begin{cases}0, & r\leq\delta\ ,\\ \eta, & r>\delta\ ,\end{cases} \tag{21}$$

so that the energy density in the monopole core is roughly approximated by

$$T_{00,\text{core}}\approx\tfrac{1}{2}\omega^2F_0^2P_0^2+V(0,F_0)\ . \tag{22}$$

Then by (14) and (21)

$$N=\int j_0 d^3x\approx-\omega F_0^2P_0\delta^3 \tag{23}$$

which implies that $\omega P_0\approx -N/(F_0^2\delta^3)$. Therefore (22) becomes, for the charge state given by $q=Ne$ (N = integer),

$$T^{(N)}_{00,\text{core}}\approx\frac{N^2}{2F_0^2\delta^6}+V(0,F_0)\ , \tag{24}$$

so that the mass of the charged monopole core is roughly

$$E_N\approx T^{(N)}_{00\text{core}}\delta^3\approx\frac{N^2}{2F_0^2\delta^3}+V(0,F_0)\delta^3\ . \tag{25}$$

For a monopole with a charge Ne to be quantum mechanically stable against spontaneous decay into a monopole with a charge $(N-1)e$ plus a free ϕ boson, we require that

$$\Delta E\equiv E_N-E_{N-1}\approx\frac{2N-1}{2F_0^2\delta^3}<m\ . \tag{26}$$

Therefore, for $\Delta E\approx m$, by (25) and (26) the maximum charge number, $N_{\max}$, that a stable monopole can possess is roughly estimated to be given by

$$N_{\max}\approx\frac{1}{2}+mF_0^2\delta^3\approx\frac{1}{2}+\frac{m(f\eta^2-m^2)}{g\lambda^{3/2}\eta^3}\ . \tag{27}$$

Thus, for $N_{\max}<1$ we may infer that no quantum mechanically stable charged global monopole exists for this model, while, on the other hand, sufficiently small values of $g\lambda^{3/2}$ and η/m could give rise to the existence of highly charged monopole states. A good approximation of $N_{\max}$ probably requires numerical studies.

By (19) the Coulombic force between a monopole with charge state N_1 and a monopole with charge state N_2 separated by a distance $r>\delta$ is roughly

$$F_c\approx\frac{\alpha N_1N_2}{r^2},\quad \alpha=\frac{e^2}{4\pi}\ . \tag{28}$$

By (20), (21), and (25) the mass of an ordinary uncharged SO(3) global monopole is roughly

$$M(r)\approx E_0+\int_\delta^r\frac{\eta^2}{r'^2}4\pi r'^2dr'\approx E_0+4\pi\eta^2(r-\delta)\ , \tag{29}$$

so that the attractive Goldstone force [5] between a monopole and an antimonopole is roughly

$$F_G\approx\frac{\partial M(r)}{\partial r}\approx 4\pi\eta^2\ . \tag{30}$$

From (28) and (30) therefore, the magnitude of the Coulomb force becomes comparable to that of the Goldstone force for a separation distance of roughly

$$r_c\approx\left[\frac{\alpha N_1N_2}{4\pi}\right]^{1/2}\frac{1}{\eta}\approx\left[\frac{\alpha\lambda N_1N_2}{4\pi}\right]^{1/2}\delta\ . \tag{31}$$

For $r>r_c$, of course, the constant Goldstone force dominates.

The competing effects of an attractive Goldstone force and a repulsive Coulomb force between a monopole and an antimonopole with like charges allows for the possibility of charged global monopole bound states representing charged "mesons." However, it will be seen that (assuming the absence of a parameter fine tuning) such meson states, if they form at all, are unstable against decay.

III. INSTABILITY AGAINST LOSS OF CHARGE

Although it has been argued that for a sufficiently large ϕ boson mass m and a sufficiently small value of $g\lambda^{3/2}$ a charged global monopole can be quantum mechanically stable against a loss of charge due to ϕ boson emission, we must investigate the possibility that an instability may develop due to the pair creation of light charged particles, such as electrons and positrons, in the strong electric field near the monopole core [10]. If a particle-antiparticle pair is likely to be created from vacuum, then one of the particles will be absorbed by the monopole and hence act to neutralize the charge of the monopole, while the antiparticle is repelled away from the monopole. The net effect is an "evaporation" of the monopole charge due to pair production.

In the absence of a magnetic field the critical electric field strength for which particle-antiparticle pairs are likely to be produced is roughly $E_c\approx m^2/e$, where here m refers to the mass of the particle and antiparticle being produced and e is the magnitude of the particle's electric charge. Let us consider a charged global monopole as a spherical shell of charge with a charge Ne and a radius $r\approx\delta$. Pair production is likely to occur in a region where (roughly) $\delta\leq r\leq r_0$, where r_0 is the critical radius, i.e., $E(r_0)=E_c\approx m^2/e$, which by (19) gives

$$r_0\approx\left[\frac{N}{4\pi}\right]^{1/2}\frac{e}{m}\ , \tag{32}$$

which implies that $r_0/\delta\approx(\lambda N/4\pi)^{1/2}e\eta/m$, so that for $\lambda^{1/2}\eta/m\gg 1$ pair production is likely to take place within a radius $r_0\gg\delta$.

Let us consider the possibility of enhancing stability against charge evaporation by introducing a magnetic field $\mathbf{B}$ generated by a circulation of the charged ϕ condensate in the core of the monopole. In regions where $E^2-B^2\geq E_c^2$ pairs are copiously produced, while for $E^2-B^2\leq 0$ there is no pair production, and for $0<E^2-B^2\ll E_c^2$ pair production is suppressed. For approximation purposes consider, for simplicity, a ring of current in the xy plane with charge Ne and ring radius $r\approx\delta$. The magnetic moment of the current is

$$\mu\approx i\pi\delta^2\approx\frac{Ne}{2}\delta^2\omega\ , \tag{33}$$

where ω is the angular velocity associated with the current. The magnitude of the magnetic field is $B(r) \approx \mu / r^3$. Setting $B(r_0) = E_c \approx m^2/e$ it is determined that

$$\omega \approx 2 \left[\frac{N}{64\pi^3} \right]^{1/2} \frac{e}{m\delta^2} \approx 2 \left[\frac{N}{64\pi^3} \right]^{1/2} \frac{e\lambda\eta^2}{m} . \tag{34}$$

But the linear velocity for the current is $v \approx \omega\delta \leq 1$, so that approximately

$$\omega \leq \delta^{-1} \approx \lambda^{1/2}\eta . \tag{35}$$

Equations (34) and (35) can be satisfied simultaneously only for the case $\lambda \leq \lambda_c \approx (64\pi^3/N)(m/2e\eta)^2$, which implies that for the case $\lambda > \lambda_c$ there does not exist a magnetic field strong enough to suppress pair production. Thus, in this latter case, the charged global monopole is expected to be charge neutralized by electrons (or positrons), ending as a neutral global monopole with nonzero lepton number. Thus (for $\lambda > \lambda_c$), mesons, if they form at all, will be unstable against decay through neutralized monopole-antimonopole annihilations. Since the symmetry breaking energy scale is assumed to be extremely large compared to the electron mass, the situation for which $\lambda < \lambda_c$ would require an extreme fine tuning of the coupling constant λ.

IV. DISCUSSION

In summary, a simple model consisting of a broken symmetric SO(3) scalar isovector coupled to a complex scalar has been investigated, and it has been argued that there exists a parameter range for which the model admits charged global monopoles as solutions. The electric charge is localized within or near the monopole core and its maximum value is determined by the model parameters. The possibility exists that the maximum charge can be quite large, and the Coulomb force can be comparable in strength to the Goldstone force at a distance determined by the charge states of the interacting monopoles. The situation wherein an antimonopole and a monopole with like charges might bind together to form a multiply charged meson state has been examined. However, for a symmetry breaking mass scale that is much larger than the electron mass, it is found that the Coulomb interaction can provide no more than a temporary stability against global monopole-antimonopole annihilation, at best (unless there is an extreme fine tuning for the coupling constant λ); a global monopole loses electric charge so that the meson states are unstable against decay, if they form at all. The charge evaporation by pair production is due to the small size of the monopole core, to which the monopole charge is confined. Thus, if such objects were physically realized subsequent to a phase transition in the early Universe, then stable multiply charged global monopole-antimonopole meson states would not be expected to exist.

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[8] See, for eample, R. N. Mohapatra, *Unification and Supersymmetry*, 2nd ed. (Springer, Berlin, 1992), and references therein.

[9] A somewhat related situation is involved in the case of the embedded superconducting cosmic string [R. Holman, S. Hsu, T. Vachaspati, and R. Watkins, Phys. Rev. D **46**, 5352 (1992)] where it has been pointed out that the electromagnetic effects associated with the existence of a charged scalar condensate in the string core can provide some stability against decay (the decay being due to the longitudinal contraction of the string with a subsequent annihilation of the magnetic monpole and antimonopole at either end of the string) by providing a current in the string which can be reflected off of the monopoles. See also T. Vachaspati and R. Watkins, Phys. Lett. B (to be published), where it is shown that the electroweak Z string can be stabilized by the presence of charged boson bound states.

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