BRST quantization of the chiral Schwinger model in the extended field-antifield space

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It is shown that the quantization of the chiral Schwinger model in the Batalin-Vilkovisky framework can be carried out in an extended space of fields and antifields, where the master equation has a local solution. The Wess-Zumino term is generated in this way, avoiding the use of nonlocal expressions. The nilpotency of the new BRST charge is proven explicitly.

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The Batalin-Vilkovisky (BV) Lagrangian BRST quantization [1] is a powerful method of quantization of field theories. It is very useful for the treatment of a wide range of theories, including those gauge theories whose constraints do not close an algebra. However, when anomalous gauge theories are concerned, the pathological problems of this kind of theories emerge troubling the construction of a gauge-independent generating functional. As was shown recently by Troost, van Nieuwenhuizen, and Van Proeyen, the presence of anomalies corresponds to the nonexistence of local solutions to the master equation [2]. This fact is the BRST reflection of those original perturbative calculations of Feynman diagrams that gave rise to the so-called anomalous Ward identities and that, from the current algebra point of view, appear as a failure of the chiral generators in closing an algebra in perturbation theory [3,4].

The chiral Schwinger model (CSM) is a two-dimensional theory that is very useful for understanding several features of anomalous models. Jackiw and Rajaraman [5] showed that a unitary and consistent effective theory can be constructed for this model, in spite of losing the gauge invariance. On the other hand, following the idea of Faddeev and Shatashvili [6] of introducing additional degrees of freedom through the Wess-Zumino term, in Ref. [7] the Faddeev-Popov procedure was applied to obtain a gauge-independent vacuum functional. In Ref. [8] we showed that a gauge-independent generating functional for the CSM can be built up using the BV procedure. In this case a nonlocal solution for the master equation was considered. Afterwards, the introduction of an auxiliary field (the so called Wess-Zumino field) makes it possible to write out a local generating functional. The same procedure is worthless in the non-Abelian case, where one is not able to build a nonlocal solution for the master equation. The naive application of the BV procedure to this model would not work, in the sense that the equation that defines the method, the so-called master equation, has no solution.

In the same spirit of the Faddeev-Shatashvili works, recently we proposed in Ref. [9] the introduction of extra degrees of freedom in the BV formalism. We showed that an enlargement of the field-antifield space of the chiral two-dimensional QCD (QCD_2) model makes possible the construction of local solutions for the master equation. There, by the introduction of a field-antifield pair associated with the gauge symmetry group we got a gauge-independent generating functional for chiral QCD_2, obtaining the Wess-Zumino-Witten action coupled to the gauge field as a solution for the master equation at first order in $\hbar$. More recently, a generalization of this procedure to treat in a generic way anomalous gauge theories with a closed, irreducible classical gauge algebra was proposed by Gomis and Paris [10].

The aim of the present work is first to show that the quantization of the chiral Schwinger model (CSM) also can be held in an extended space where the master equation has a local solution. Then we will build up the BRST generator and prove, in the canonical quantization framework, that the inclusion of the new field-antifield pair associated with the gauge group leads to the nilpotency of this generator.

Let us consider the classical action for the CSM:

$$S_0=\int d^2x\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\bar{\psi}\not{D}\frac{1-\gamma_5}{2}\psi\right] . \qquad (1)$$

As usual, the quantum action W in the BV scheme must satisfy the master equation

$$\tfrac{1}{2}(W,W)=i\hbar\Delta W , \qquad (2)$$

where

$$(X,Y)=\frac{\partial_r X}{\partial\phi}\frac{\partial_l Y}{\partial\phi^*}-\frac{\partial_r X}{\partial\phi^*}\frac{\partial_l Y}{\partial\phi} ,$$
$$\Delta\equiv\frac{\partial_r}{\partial\phi^A}\frac{\partial_l}{\partial\phi^*_A} . \qquad (3)$$

W can be expanded in powers of $\hbar$:

$$W=S+\sum_{j=1}^{\infty}\hbar^j M_j . \qquad (4)$$

The standard zero order term corresponds to S_0 plus the field-antifield terms, which become the gauge-fixing terms when the antifields are restricted to the gauge surface $\phi^*=\partial\Psi/\partial\phi$, Ψ being the gauge fermion. Thus, one gets

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$$S=S_0+\int d^2x\{A_\mu^*\partial^\mu c+i\psi^*\psi c-i\bar{\psi}\bar{\psi}^*c\} . \quad (5)$$

The master Eq. (2) at first order in $\hbar$ is

$$(M_1,S)=i\Delta S, \quad (6)$$

where $e^{\Delta S}$ is the BRST Jacobian, thus bearing the anomalous properties of the path integral measure. The direct action of the operator

$$\Delta\equiv\frac{\partial_r}{\partial\phi^A}\frac{\partial_l}{\partial\phi_A^*}$$

on the action (5) would lead to ΔS being proportional to $\delta(0)$, because of the two functional derivatives. This ill-defined expression makes evident the need of a regularization scheme [2]. In the present case, the only effective contribution to ΔS comes from the fermion-antifermion terms of the gauge-fixing action $S_{\rm GF}$:

$$\int d^2x(\psi^*c\psi+\bar{\psi}c\bar{\psi}^*) .$$

As explained in Ref. [8], we temporarily transform it in a nonlocal expression by doing a point splitting using a kernel $K(x-y)$. By means of this kernel we introduce the Fujikawa regularization method [11], which leads to the regularized expression

$$\Delta S=\frac{i}{4\pi}\int d^2xc[(1-a)\partial_\mu A^\mu-\epsilon^{\mu\nu}\partial_\mu A_\nu] . \quad (7)$$

As already mentioned, the master Eq. (6) with the action (5) will not admit local solutions.

In order to remove this obstacle, we propose to enlarge the field-antifield configuration space including, in addition to the fields that are present at the classical level, also the field θ associated with the Lie algebra of gauge group U(1) together with the corresponding antifield θ^*. The θ field will become dynamical only at the quantum level. Since the classical action S_0 is independent of θ, the model is invariant under arbitrary variations of θ,

$$\theta\to\theta+\lambda , \quad (8)$$

besides the usual gauge symmetry of this model. Now, the generator of this extra symmetry must be included in the Hessian matrix of the extended action solution of the master equation at the classical level. This is attained simply adding to the action (5) the corresponding term

$$\bar{S}=S_0+\int d^2x\{A_\mu^*\partial^\mu c+i\psi^*\psi c-i\bar{\psi}\bar{\psi}^*c+\theta^*c\} . \quad (9)$$

Observe that now the action $\bar{S}$ depends on θ^* through the inclusion of the term θ^*c, and this extra term plays a fundamental role modifying the master Eq. (6), in such a way that one can construct a local solution, depending also on the field θ.

The modified master equation at order $\hbar$ is

$$(M_1,\bar{S})=\int\left[\frac{\partial_r M_1}{\partial A^\mu}\frac{\partial_l\bar{S}}{\partial A_\mu^*}+\frac{\partial_r M_1}{\partial\theta}\frac{\partial_l\bar{S}}{\partial\theta^*}\right]=i\Delta\bar{S} . \quad (10)$$

It is worth remarking the inclusion of θ field leaves $\Delta S=\Delta\bar{S}$, as can easily be seen from (9). Now, in this framework, the master equation at first order in $\hbar$ admits local solutions. It is easy to verify that the usual Wess-Zumino term for the chiral Schwinger model,

$$M_1=\frac{1}{4\pi}\int d^2x\left\{\frac{a-1}{2}\partial_\mu\theta\partial^\mu\theta-\partial_\mu\theta[(a-1)A^\mu+\epsilon^{\mu\nu}A_\nu]\right\} , \quad (11)$$

satisfies the master equation (10). The higher order contributions to the master equation (2) vanish; so the quantum action is just

$$W=\bar{S}+\hbar M_1 . \quad (12)$$

This result is the same as that of Ref. [6], thus showing that it is possible to build up an action that leads to a gauge-independent vacuum functional for the chiral Schwinger model, without making use of the Faddeev-Popov trick as in Ref. [7] or using nonlocal expressions as in [3].

Let us now study the above procedure from the canonical point of view. It is well known that the presence of anomalies breaks down the nilpotency of Q. We will now investigate the operator Q^2 to understand the effect of the enlargement in the configuration space of fields and antifields. What we are really going to calculate is the anticommutator $[Q,Q]_+=2Q^2$.

Both actions (5) and (12) represent, from the Hamiltonian point of view, constrained systems. Their quantization can be obtained, when there is no operator ordering problem, calculating the Dirac (anti)brackets for the classical theory, and then associating them with the (anti)commutators of the quantum fields as usual: $\{A,B\}_{\rm DB}\to(-i/\hbar)[A,B]_\pm$. There is a pair of constraints that is especially important in our analysis of the effect of the inclusion of the θ field in the canonical algebra: the primary constraint associated with the momentum conjugate to A_0 and the secondary constraint that comes from the time evolution of the former, namely the Gauss law.

So, for action (5), where θ and θ^* are not present, the classical Hamiltonian analysis leads to the constraints

$$\bar{\Omega}_0\equiv\Pi_0\approx 0 , \qquad \bar{\Omega}_1\equiv\{\Omega_0,\mathcal{H}\}_{\rm DB}=\Pi_1'+i\bar{\psi}\gamma_0\frac{1-\gamma_5}{2}\psi\approx 0 , \quad (13)$$

where $\mathcal{H}$ is the Hamiltonian density corresponding to the action S, Eq. (5), and the prime means ∂_1.

In order to construct the canonical generator of the BRST transformations, we now study the transformation of the fields and antifields, which are defined by

$$\delta\Phi=(\Phi,W)|_\Sigma\,\rho . \quad (14)$$

Here, W is the quantum action given in Eq. (4) and ρ is the parameter of the transformation. So, before adding the M_1 term to action (5), the BRST transformation of the fields and antifields are given by

$$\delta\Phi=(\Phi,S)|_\Sigma\,\rho .$$

As the action S depends linearly on the antifields, the BRST transformation are independent of the gauge condition. Thus,

$$\begin{aligned}
&\delta_0 A^\mu=\partial^\mu c\rho ,\\
&\delta_0\psi=i\psi c\rho ,\\
&\delta_0\bar\psi=-i\bar\psi c\rho ,\\
&\delta_0 c=0 ,\\
&\delta_0 A^*_\mu=\partial^\nu F_{\mu\nu}\rho-\bar\psi\gamma_\mu\frac{1-\gamma_5}{2}\psi ,\\
&\delta_0\psi^*=\tfrac{1}{2}\bar\psi\,\overleftarrow{\not D}(1-\gamma_5)\rho-i\psi^* c\rho ,\\
&\delta_0\bar\psi^*_0=-\tfrac{1}{2}\not D(1-\gamma_5)\psi\rho+i\bar\psi^* c\rho ,\\
&\delta_0 c^*=A^*_\mu\partial^\mu\rho+\psi^*\psi\rho-\overline{\psi\psi}{}^*\rho .
\end{aligned}\tag{15}$$

The Noether's current associated with these transformations is easily calculated:

$$J_\mu=-\partial^\nu F_{\mu\nu}c+\bar\psi\gamma_\mu\frac{1-\gamma_5}{2}\psi c .\tag{16}$$

So, the generator Q_0 of transformations (15) is thus

$$Q_0=\int dx_1\left[\Pi_1'+\bar\psi\gamma_0\frac{1-\gamma_5}{2}\psi\right]c .\tag{17}$$

Observe that Q_0 is exactly the Gauss law times the ghost, as one hopes from the canonical structure of the model.

The only contribution to $[Q_0,Q_0]_+$ comes from the fermionic current term

$$[Q_0,Q_0]_+=\int dx_1\int dy_1 c(x)\left[\bar\psi(x)\gamma_0\frac{1-\gamma_5}{2}\psi(x),\bar\psi(y)\gamma_0\frac{1-\gamma_5}{2}\psi(y)\right]_- c(y)\Bigg|_{x_0=y_0} .\tag{18}$$

So, Q_0^2 is essentially proportional to the equal time commutator of the chiral current:

$$J_0^5=\bar\psi(x)\gamma_0\frac{1-\gamma_5}{2}\psi(x) .$$

The calculation of this current algebra requires a careful management at the quantum level as it involves the product of operators at the same point. To overcome this problem a regularization scheme is needed. As is well known, the calculation of this commutator generates the Schwinger term, transforming the current algebra into a Kac-Moody one. The Bjorken-Johnson-Low limit [12] enables one to relate the vacuum expectation value of this current commutator to the second functional derivative of the effective action, obtained from the fermionic integration [4,13–15]. In fact, following Ref. [15], defining

$$G_{\mu\nu}(x,y)=\frac{\delta^2 S_{\rm eff}[A]}{\delta A^\mu(x)\delta A^\nu(y)} ,$$

the vacuum expectation value of the current commutator is

$$\langle[J_\mu^5(x),J_\nu^5(y)]_-\rangle=\lim_{\epsilon\to 0}[G_{\mu\nu}(x_1,x_0;y_1,y_0+\epsilon)-G_{\mu\nu}(x_1,x_0;y_1,y_0-\epsilon)] .$$

As is well known, the effective action ($S_{\rm eff}$) is equivalent to the logarithm of the determinant of the fermionic chiral operator $[\not\partial+\not A(1-\gamma_5)/2]$. This result was first obtained by Jackiw [16,5] from the previously Schwinger's calculation [17] for the standard Schwinger model. Alternatively, by means of a decoupling gauge transformation, one can relate this determinant to the corresponding Jacobian [18]. The regularization of this Jacobian is analogous to that previously used in the calculation of ΔS. So, it can be performed following Fujikawa's prescription also, finally, getting, for the current commutator,

$$i[J_0^5(x),J_0^5(y)]_-=\frac{\hbar^2}{\pi}\partial_1\delta(x_1-y_1) .$$

The appearance of the Schwinger term means the breakdown of the gauge symmetry which also leads, as we shall see, to the breakdown of the Q_0 nilpotency. Introducing the above commutator in Eq. (18), we obtain

$$[Q_0,Q_0]_+=\frac{i}{\pi}\int dx_1 c(x)\partial_1 c(x) .\tag{19}$$

Now, we will analyze in the same way what happens with the extended action (12). The addition of the Wess-Zumino term M_1, Eq. (11), obtained as a solution of the $\hbar$ order master equation, comes mainly to modify the constraints (13), leaving

$$\begin{aligned}
&\bar\Omega_0\equiv\Pi_0\approx 0 ,\\
&\bar\Omega_1\equiv\{\Omega_0,\mathcal{H}+\mathcal{H}_1\}\\
&\quad=\Pi_1'+i\bar\psi\gamma_0\frac{1-\gamma_5}{2}\psi-\frac{\hbar}{4\pi}[(a-1)A_0+A_1]\\
&\quad-\left[\Pi_\theta-\frac{\hbar}{4\pi}\theta'\right]\approx 0 .
\end{aligned}\tag{20}$$

$\mathcal{H}_1$ is the Hamiltonian density corresponding to the $\hbar M_1$ term and Π_θ is the momentum conjugate to the field θ. We can see that, as a consequence of the introduction of the M_1 term, the chiral constraint $[\Pi_\theta-(h/4\pi)\theta']$ appears in the modified Gauss law. Now, at quantum level, the commutator of the modified Gauss law with itself is zero because the additional terms came to cancel the Schwinger term arising from the fermionic chiral current.

Let us now see how does the inclusion of the extra pair θ,θ^* and the M_1 term contribute to the BRST transformations. According to Eq. (14),

$$\delta\Phi=(\Phi,\bar S)|_\Sigma\,\rho+\hbar(\Phi,M_1)|_\Sigma\,\rho .$$

For the field θ, the only contribution comes from the $\theta^* c$

term in the action $\bar{S}$, then

$$\delta\theta = c\rho \ . \tag{21}$$

As the M_1 given in (11) does not depend on the antifields, it does not modify the transformations corresponding to the fields, given in Eq. (15). However, it gives rise to additional terms in transformation of the antifields:

$$\begin{aligned} \delta A^*_\mu &= \delta_0 A^*_\mu + \frac{\hbar}{4\pi}[(1-a)\partial_\mu\theta + \epsilon^{\mu\nu}\partial_\nu\theta]\rho \ , \\ \delta c^* &= \delta_0 c^* + \theta^*\rho \ , \\ \delta\theta^* &= \frac{\hbar}{4\pi}\{(1-a)\partial_\mu\partial^\mu\theta - [(1-a)\partial_\mu A^\mu + \epsilon^{\mu\nu}\partial^\mu A_\nu]\} \ . \end{aligned} \tag{22}$$

It is worth remarking that the BRST variation of the antifields are proportional to the equations of motion; thus, from the canonical point of view, they can be considered as being zero. So, the generator of these transformations is $Q = Q_0 + Q'$ with

$$Q' = -\int dx_1 c \left[\frac{\hbar}{4\pi}[(a-1)A_0 + A_1] + \left[\Pi_\theta - \frac{\hbar}{4\pi}\theta' \right] \right] . \tag{23}$$

The canonical commutation relations for the model described by the action (12) can be calculated taking into account that now the modified Gauss law is a first class constraint at the quantum level. The nonvanishing equal time commutators that will contribute to the anticommutator $[Q',Q']_+$ are

$$\begin{aligned} [\theta'(x), \Pi_\theta(y)]_- &= i\hbar\partial_1\delta(x_1 - y_1) \ , \\ [\Pi_1'(x), A_1(y)]_- &= -i\hbar\partial_1\delta(x_1 - y_1) \ . \end{aligned} \tag{24}$$

These Schwinger-like terms come to cancel the analogous contribution of Q_0^2. Thus,

$$[Q',Q']_+ = 2[Q_0,Q']_+ = -\frac{i}{2\pi}\int dx_1 c(x)\partial_1 c(x) \ . \tag{25}$$

Finally, the BRST generator Q is nilpotent:

$$Q^2 = \tfrac{1}{2}[Q,Q]_+ = 0 \ . \tag{26}$$

Thus, the physical states of the theory are defined by the cohomology class of the generator of BRST transformations:

$$Q = \int dx_1 c \left[-\partial^\nu F_{0\nu} + \bar{\psi}\gamma_0 \frac{1-\gamma_5}{2}\psi - \frac{\hbar}{4\pi}[(a-1)A_0 + A_1] + \left[\Pi_\theta - \frac{\hbar}{4\pi}\theta' \right] \right] . \tag{27}$$

Observe that since the physical state must be annihilated by this BRST operator, the new term added in order to restore the nilpotency impose the chiral constraint $\Pi_\theta = (\hbar/4\pi)\theta'$, on the enlarged Hilbert space.

Concluding, we saw that it is possible to generalize the Batalin-Vilkovisky Lagrangian method, including a field associated with the gauge group element (the Wess-Zumino field). Thus, a gauge-independent formulation for an anomalous theory can be built up without using any nonlocal expressions. As can be easily realized, for nonanomalous theories, this extension in the space of fields and antifields will not affect the theory, as the Wess-Zumino field would not become dynamical, since $M_1 = 0$ is a solution of the master equation in this case. From the canonical point of view, it was shown that the BRST charge has its nilpotency restored in the extended space. Also, a relevant fact is the presence of the chiral constraint in the new BRST generator. This means that the physical sector of the extended Hilbert space includes only one chiral sector of the extra bosonic field. Its nongauge invariance comes to compensate the anomalous behavior of the fermionic measure.

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