Renormalization of gauge-invariant composite operators in the light-cone gauge

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(Received 19 July 1993)

We generalize to composite operators concepts and techniques which have been successful in proving renormalization of the effective action in the light-cone gauge. Gauge-invariant operators can be grouped into classes, closed under a matrixwise renormalization. In spite of the presence of nonlocal counterterms, an "effective" dimensional hierarchy still guarantees that any class is endowed with a finite number of elements. The main result we find is that gauge-invariant operators under renormalization mix only among themselves, thanks to the very simple structure of Lee-Ward identities in this gauge, contrary to their behavior in covariant gauges.

PACS number(s): 11.10.Gh, 11.15.Bt

I. INTRODUCTION

Composite operators often occur in calculations of physical cross sections. A celebrated example is deep-inelastic scattering where short-distance products of currents are expressed in terms of local operators by means of a Wilson expansion [1]. But, strictly speaking, the Lagrangian density itself is an instance of composite operator.

As quantum fields are distribution-valued operators, one can easily realize that, taking products at the same space-time point, gives rise to singularities. Hence we see the need of first considering a procedure of regularization and then performing the necessary subtractions in a consistent way to operatively define their finite parts [2], at least in a perturbative context.

The peculiar phenomenon occurring in composite operator renormalization is their *mixing*. Locality and polynomiality in the masses of counterterms guarantee the presence of a *dimensional hierarchy*: counterterms can only have canonical dimensions less than or equal to the ones of the operators we are considering. Therefore, the number of counterterms which mix is finite [2].

All those concepts and techniques naturally apply to gauge theories with the proviso that they have to comply with Ward identities taking care of redundant degrees of freedom. In covariant gauges (typically in generalized Feynman gauges) the relevant Slavnov-Taylor identities involve unphysical operators (Faddeev-Popov ghosts). As a consequence, a deep thorough analysis [3] has shown that gauge-invariant operators *do mix* with unphysical ones under renormalization.

The situation radically changes in the so-called *physical* gauges $n_\mu A^\mu = 0$, n_μ being a constant vector, where there is no need of Faddeev-Popov fields and Lee-Ward identities are straightforward [4]. This is the reason why such gauges have been largely adopted in the past for phenomenological applications [5].

Only recently however has a systematic approach been developed with a sound basis on the axioms of canonical quantum field theory. Effective action renormalization has been proven, so far, at any order in the loop expansion, only in the the light-cone (LC) gauge ($n^2 = 0$) [6]. Essential to this goal is to endow the "spurious" singularity, occurring in the vector propagator, with a causal prescription [Mandelstam-Leibbrandt (ML) prescription [7,8]], as suggested by a careful canonical quantization [9]. This prescription in turn is the source of a potentially serious difficulty: nonlocal counterterms are needed, already at the one loop level, to make one particle irreducible vertices finite [8].

It is clear that nonlocality could in principle destroy dimensional hierarchy. Should the mixing involve an infinite number of independent counterterms, even for a single insertion, the very program of composite operator renormalization would be in jeopardy.

Happily this is not the case. Generalizing concepts and techniques which have been successful in proving renormalization of the effective action, in the next sections we show that a new kind of "effective" dimensional hierarchy can be established which is enough to prove renormalization at any order in the loop expansion, at least for gauge-invariant composite operators, which are the ones directly involved in phenomenological applications [10]. Actually the very simple structure of Lee-Ward identities, which survives renormalization in this case, will allow us to reach a rather strong result: in the LC gauge, under renormalization gauge invariant operators mix only among themselves, in classes with finite numbers of elements.

The problems one encounters when treating more general operators will be briefly discussed in the Conclusions.

In Sec. II we introduce our notation, we define the generating functionals with composite operators insertions, and derive the Lee-Ward identities they have to satisfy. Section III is devoted to generalize the Bogoliubov-Parasiuk-Hepp-Zimmermann (BPHZ) subtraction method [11–13] to our problem and prove the gauge invariance of renormalized composite operators. In Sec. IV we discuss power counting in the LC gauge and the need of introducing a more general criterion of superficial degree of divergence, in relation to Weinberg's theorem [14]. In Sec. V we explore all the constraints the counterterms have to satisfy and in Sec. VI we prove that in the mixing of gauge-invariant operators a unique independent nonlocal structure can appear with a mass dimension equal to *one*, the same one encounters when renormalizing the effective action. Concrete examples of mixing are presented in Sec. VII, while remarks and comments concerning further developments are contained in the Conclusions.

II. THE GENERATING FUNCTIONALS WITH COMPOSITE OPERATORS

We start by defining our Lagrangian and our notation,

$$\mathcal{L}_{\rm GI} = -\tfrac{1}{2}{\rm Tr}\,(F_{\mu\nu}F^{\mu\nu}) + \bar\psi\,(i\not{\mathcal{D}} - m)\,\psi \ , \tag{1}$$

where $F_{\mu\nu}$ is the usual field tensor in the adjoint representation of the algebra su(N):

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]\,, \tag{2}$$

$$A_\mu = A^a_\mu \tau^a, \quad a = 1, \ldots, N^2 - 1\,, \tag{3}$$

$$\mathrm{Tr}(\tau^a\tau^b) = \tfrac{1}{2}\delta^{ab}\,, \tag{4}$$

$$\left[\tau^a, \tau^b\right] = if^{abc}\tau^c\,, \tag{5}$$

f^{acb} being the structure constants of the group which are completely antisymmetric in this basis. $\mathcal{D}_\mu$ is the covariant derivative acting on the fundamental representation:

$$\mathcal{D}_\mu = \partial_\mu - igA_\mu\,. \tag{6}$$

The Lagrangian density in Eq. (1) is invariant under gauge transformations, as is well known. Their infinitesimal form is

$$\delta^{[\omega]}\psi(x) = ig\,\omega(x)\psi(x)\,, \tag{7a}$$

$$\delta^{[\omega]}A_\mu(x) = D_\mu\omega(x)\,, \tag{7b}$$

where D_μ is the covariant derivative acting on the adjoint representation

$$D_\mu = \partial_\mu - ig[A_\mu,\,\cdot\,] \tag{8}$$

and $\omega(x)$ are the infinitesimal parameters of the transformation. In order to quantize the theory, we introduce the light-cone gauge fixing

$$\mathcal{L}_{\rm GF} = -\lambda(x)\cdot n_\mu A^\mu(x)\,, \tag{9}$$

$\lambda(x)$ being Lagrange multipliers and n_μ a fixed lightlike four-vector $n^2 = 0$.

In the following, dimensional regularization will be understood in the framework of perturbation theory. In 2ω dimensions the coupling constant g will be replaced with $g\mu^{2-\omega}$ where μ is a mass scale.

From Eqs. (1) and (9) we can construct the usual functional W which generates the Green's functions of the theory

$$W[J,K,\eta,\bar\eta] = \mathcal{N}\int[dA][d\lambda][d\psi][d\bar\psi]\exp\left[i\int d^4x\,(\mathcal{L}_{\rm GI} + \mathcal{L}_{\rm GF} + \mathcal{L}_S)\right]\,, \tag{10}$$

where

$$\mathcal{L}_S = J_\mu\cdot A^\mu + K\cdot\lambda + \bar\eta\psi - \bar\psi\eta. \tag{11}$$

Then we can define in the usual way the functional $Z = \frac{1}{i}\ln W$ which generates the connected Green's functions; from Z we get the "classical " fields $\mathcal{A}_\mu$, Λ, Ψ, $\bar\Psi$, and, eventually, the functional Γ which generates the proper vertices of the theory:

$$\Gamma[\mathcal{A},\Lambda,\Psi,\bar\Psi] = Z[J,K,\eta,\bar\eta] - \int d^4x\left(J\cdot\mathcal{A} + K\cdot\Lambda + \bar\Psi\eta - \bar\eta\Psi\right). \tag{12}$$

The derivatives with respect to Grassmann variables are understood as left derivatives; with our conventions we get, in particular,

$$\bar\Psi \equiv \frac{\delta Z}{\delta\eta} \Rightarrow \eta = \frac{\delta\Gamma}{\delta\bar\Psi}\,, \qquad \Psi \equiv \frac{\delta Z}{\delta\bar\eta} \Rightarrow \bar\eta = \frac{\delta\Gamma}{\delta\Psi}. \tag{13}$$

We also notice that invariance under a shift in λ of the path integral entails the condition

$$n\mathcal{A} = K, \tag{14}$$

which in turn guarantees that any Green's function containing nA, but no λ, vanishes.

In this section we are mainly concerned with the generalization of such generating functionals to the case in which composite operators are considered. Such a generalization is presented, for instance, in Refs. [15,16]. We denote by $X = X[A,\psi,\bar\psi]$ a polynomial built from the original fields and their derivatives taken at the same space-time point; Green's functions with insertions of such operators usually exhibit further singularities. The technique one uses to take those insertions into account is to introduce, in the definition of W, a source term related to X. In the general case we shall consider a set of operators X_i each associated to a source Σ_i, by adding the Lagrangian density

$$\mathcal{L}_X = \sum_i \Sigma_i\cdot X_i\,. \tag{15}$$

In the following we shall not be concerned with composite operators involving Lagrange multipliers as they would affect the equations of motion of the field λ that enter in the derivation of the Lee identities. Moreover we shall limit ourselves to gauge-invariant composite operators, but in the final section where we shall briefly dwell on possible generalizations.

A crucial point to remark is that the functional Γ with insertions is defined by means of a Legendre transformation involving only the classical fields we have already considered:

$$\Gamma[\mathcal{A},\Lambda,\Psi,\bar{\Psi},\Sigma_i] = Z[J,K,\eta,\bar{\eta},\Sigma_i] - \int d^4x\,(J\cdot\mathcal{A} + K\cdot\Lambda + \bar{\Psi}\eta - \bar{\eta}\Psi)\,. \tag{16}$$

As a consequence, one can prove the equality

$$\left.\frac{\delta\Gamma}{\delta\Sigma_i}\right|_{\mathcal{A},\Lambda,\Psi,\bar{\Psi},\Sigma_{j\neq i}} = \left.\frac{\delta Z}{\delta\Sigma_i}\right|_{J,K,\eta,\bar{\eta},\Sigma_{j\neq i}}, \tag{17}$$

where on the left-hand side (right-hand side) "classical" fields (original sources) are kept fixed beside the sources $\Sigma_{j\neq i}$.

By solving the equations of motion of the Lagrange multiplier it is possible to make explicit the dependence of Γ on Λ,

$$\Gamma[\mathcal{A},\Lambda,\Psi,\bar{\Psi},\Sigma_i] = \widetilde{\Gamma}[\mathcal{A},\Psi,\bar{\Psi},\Sigma_i] - \int d^4x\,\Lambda\cdot n\mathcal{A} \tag{18}$$

and to convince oneself that the gauge-fixing term does not renormalize. $\widetilde{\Gamma}$ is customarily called the "reduced generating functional."

As we are concerned with gauge-invariant operators, Lee-Ward identities, which have a very simple form in light-cone gauge, will not entail further difficulties in presence of insertions. In order to derive the Ward identities, we follow the standard technique of performing a change of variable in the path integral corresponding to an infinitesimal gauge transformation. The related functional determinant in this gauge is independent of the fields, as is well known. As we are here considering gauge-invariant insertions, they cannot affect the form of the Ward identity

$$D^{ab}_\mu\left[\frac{\delta}{i\delta J}\right]\left\{\frac{\delta}{i\delta K^b}n^\mu - J^{\mu b}\right\}W + ig\mu^{2-\omega}\left(\bar{\eta}\tau^a\frac{\delta W}{i\delta\bar{\eta}} + \frac{\delta W}{i\delta\eta}\tau^a\eta\right) = 0\,, \tag{19}$$

where W depends also on sources Σ_i related to composite operators. We can get rid of the term with a second-order functional derivative in Eq. (19), using the equations of motion for the Lagrange multiplier. Then we derive from Eq. (19) the following Lee identity for the reduced functional $\widetilde{\Gamma}$:

$$D^{ab}_\mu[\mathcal{A}]\frac{\delta\widetilde{\Gamma}}{\delta\mathcal{A}^b_\mu} + ig\mu^{2-\omega}\left(\frac{\delta\widetilde{\Gamma}}{\delta\Psi}\tau^a\Psi + \bar{\Psi}\tau^a\frac{\delta\widetilde{\Gamma}}{\delta\bar{\Psi}}\right) \equiv \Delta^a\widetilde{\Gamma} = 0\,, \tag{20}$$

Δ^a being the functional differential operator which describes an infinitesimal gauge transformation of the "classical" fields. We shall use the same symbol also for the analogous operator acting on functionals of elementary fields. Equation (20) means that $\widetilde{\Gamma}$ is gauge invariant. We stress the fact that $\widetilde{\Gamma}$ depends also on possible sources related to gauge-invariant composite operators.

III. GAUGE INVARIANCE OF RENORMALIZED OPERATORS

In order to renormalize either the action or a composite operator, we adopt the graph-by-graph subtraction method (or BPHZ method) summarized by Bogoliubov's $\overline{R}$ operator on Feynman graphs [11–13]. We just stress the fact here that in the presence of diagrams with operator insertions the definition of one-particle irreducible (1PI) diagrams remains the same if the operator vertices are treated just like ordinary interaction vertices.

In the following we shall work in the minimal subtraction scheme (MS) on dimensionally regularized diagrams: we denote by $\mathcal{K}G$ the singular part of the Laurent expansion of the graph G in the neighborhood of $\omega = 2$. The renormalized graph RG is obtained by subtracting the singular part from the subdivergence-free diagram $\overline{R}G$:

$$RG = (1-\mathcal{K})\overline{R}G\,. \tag{21}$$

We shall also use the notation

$$CG = -\mathcal{K}\overline{R}G \tag{22}$$

to indicate the specific counterterm necessary to renormalize the graph G. CG is different from zero if and only if G is 1PI and superficially divergent. In this case a specific counterterm chosen to produce CG as an additional Feynman rule has to be added to the Lagrangian. If G involves composite operator vertices it contributes to the renormalization of the Σ-dependent terms; otherwise it renormalizes the original Lagrangian.

By performing this procedure for every 1PI diagram up to the order of l loop, one builds an action $S^{\{l\}}$ renormalized to this order. A synthetic and completely equivalent description of this method is given through the generating functional Γ. We define

$$S^{\{0\}}[A,\psi,\bar{\psi},\Sigma_i] = \int d^4x\,(\mathcal{L}_{\rm GI}[A,\psi,\bar{\psi}] + \mathcal{L}_X[A,\psi,\bar{\psi},\Sigma_i]), \tag{23}$$

as the unrenormalized action. In this definition the gauge-fixing term is excluded as it does not renormalize; hence, $S^{\{0\}}$ is gauge invariant. We denote by $\widetilde{\Gamma}^{\{l\}}$ the reduced generating functional obtained from the action $S^{\{l\}}$ and perform the loopwise expansion

$$\widetilde{\Gamma}^{\{l\}} = \sum_{m=0}^{\infty}\widetilde{\Gamma}^{\{l\}}_m\,; \tag{24}$$

$\widetilde{\Gamma}^{\{l\}}_m$ represents the m-loop contribution to $\widetilde{\Gamma}^{\{l\}}$. Now we are able to define iteratively the renormalized action

$$S^{\{l\}}[A,\psi,\bar{\psi},\Sigma_i] = S^{\{l-1\}}[A,\psi,\bar{\psi},\Sigma_i] - \mathcal{K}\widetilde{\Gamma}_l^{\{l-1\}}[A,\psi,\bar{\psi},\Sigma_i]\,, \qquad (25)$$

where $\mathcal{K}$ picks up just the singular part at $\omega = 2$ of the regularized expression $\widetilde{\Gamma}_l^{\{l-1\}}[A,\psi,\bar{\psi},\Sigma_i]$; in this functional, the fields A, ψ, and $\bar{\psi}$ take the place of the corresponding classical fields.

In general, even in a covariant theory, if the dimension of X_i is ≥ 4, an infinite number of counterterms of arbitrarily high degree in Σ_i are introduced in the renormalized action by Eq. (25). The "renormalized operator" $[X_k]^{\{l\}}$ is defined by

$$[X_k]^{\{l\}}(x) \equiv \left.\frac{\delta S^{\{l\}}[A,\psi,\bar{\psi},\Sigma_i]}{\delta\Sigma_k(x)}\right|_{\Sigma_i=0\,\forall i}\,; \qquad (26)$$

the operator is renormalized only in the sense that Green's functions with at most one single insertion of $[X_k]^{\{l\}}$ are finite in the renormalized theory. If finite Green's functions with more operator insertions are needed one has to consider the whole renormalized action $S^{\{l\}}$ whose functional $W^{\{l\}}[J,K,\eta,\bar{\eta},\Sigma_i]$ is finite up to order l at any degree in Σ_i.

A "weak" form of renormalization will also be considered in which only counterterms at most linear in the sources Σ_i are introduced. To this purpose we define

$$\widetilde{\Gamma}^L \equiv \widetilde{\Gamma}\Big|_{\Sigma_i=0} + \int d^4x \sum_j \left(\left.\frac{\delta\widetilde{\Gamma}}{\delta\Sigma_j(x)}\right|_{\Sigma_i=0}\Sigma_j(x)\right)\,, \qquad (27)$$

the part of $\widetilde{\Gamma}$ linear in the sources Σ_i. Then we define recursively as weakly renormalized action

$$S_w^{\{l\}}[A,\psi,\bar{\psi},\Sigma_i] = S_w^{\{l-1\}}[A,\psi,\bar{\psi},\Sigma_i] - \mathcal{K}\widetilde{\Gamma}_l^{L\{l-1\}}[A,\psi,\bar{\psi},\Sigma_i]\,. \qquad (28)$$

Of course one gets

$$S_w^{\{l\}}[A,\psi,\bar{\psi},\Sigma_i] = S^{\{l\}}[A,\psi,\bar{\psi}] + \sum_i \Sigma_i[X_i]^{\{l\}}\,, \qquad (29)$$

where the first term on the right-hand side (RHS) is the renormalized action one would obtain if operator insertions were absent. Only the linear parts in the Σ_i's of the generating functionals obtained from S_w are finite. We shall now prove the following proposition.

Proposition 1. Let $S^{\{0\}}$ be the action with insertions of gauge invariant operators X_i, defined by Eq. (23) and $S^{\{l\}}$ the action renormalized up to l loops according to Eq. (25); then (1) $S^{\{l\}}$ is gauge invariant $\forall l$,

$$\Delta^a S^{\{l\}}[A,\psi,\bar{\psi}] = 0, \qquad (30)$$

and (2) the renormalized operators $[X_i]^{\{l\}}$ are gauge invariant $\forall l$,

$$\Delta^a [X_i]^{\{l\}}[A,\psi,\bar{\psi}] = 0. \qquad (31)$$

Proof (2) follows directly from (1) and Eq. (26). Let us show (1) by induction on l; as obviously the thesis holds if $l = 0$, we have just to prove the inductive step. Let us start from Eq. (30). The form of Lee identities is not affected by renormalization:

$$\Delta^a\widetilde{\Gamma}^{\{l\}}[\mathcal{A},\Psi,\bar{\Psi}] = 0. \qquad (32)$$

The same equation must hold for the singular part of the Laurent expansion in $\omega = 2$ and for each contribution in the loopwise expansion:

$$\Delta^a\mathcal{K}\widetilde{\Gamma}_m^{\{l\}}[\mathcal{A},\Psi,\bar{\Psi}] = 0. \qquad (33)$$

From Eq. (25) we finally obtain the desired result

$$\Delta^a S^{\{l+1\}}[A,\psi,\bar{\psi}] = 0. \;\square \qquad (34)$$

Of course, from Eq. (29) it also follows that the weakly renormalized action $S_w^{\{l\}}$ is gauge invariant.

IV. POWER COUNTING IN THE LIGHT-CONE GAUGE

The main feature of Feynman graphs in the light-cone gauge is the presence of spurious poles introduced by the particular form of the free gauge field propagator:

$$\langle 0|TA_\mu^a(x)A_\nu^b(y)|0\rangle_{g=0} = \int \frac{d^{2\omega}k}{(2\pi)^4}e^{ik(x-y)}\frac{-i\delta^{ab}}{k^2+i\epsilon}\left[g^{\mu\nu} - \frac{n^\mu k^\nu + n^\nu k^\mu}{[[nk]]}\right]. \qquad (35)$$

The prescription of the spurious pole is the so-called Mandelstam-Leibbrandt (ML) prescription [7–9]

$$\frac{1}{[[nk]]} \overset{\text{Man}}{\equiv} \frac{1}{nk + i\epsilon\sigma(n^*k)} \overset{\text{Lei}}{\equiv} \frac{n^*k}{(nk)(n^*k)+i\epsilon}\,, \qquad (36)$$

$\sigma(\cdot)$ being the sign function and n_μ^* a new four-vector on the light-cone independent from n_μ. The choice of n_μ^* represents a further violation of Lorentz covariance. We choose $n_\mu = \frac{1}{\sqrt{2}}(1,0,0,1)$ and $n_\mu^* = \frac{1}{\sqrt{2}}(1,0,0,-1)$ in a particular Lorentz frame. Therefore

$$\frac{1}{[[nk]]} = \frac{1}{k^+ + i\epsilon\sigma(k^-)}\,, \qquad (37)$$

where we have introduced the light-cone coordinates (LCC's)

$$k^\pm = k_\mp = \frac{k^0 \pm k^3}{\sqrt{2}}\,. \qquad (38)$$

One can prove that the derivative of the ML distribution with respect to p^μ is given by[1]

[1]We take here the opportunity of correcting Eq. (A6.2) of reference [17].

$$\frac{\partial}{\partial p^\mu}\frac{1}{[[np]]} = -\frac{n_\mu}{[[np]]^2} - 2\pi i n^*_\mu \delta(np)\delta(n^*p) . \quad (39)$$

This result will be useful when discussing the form of nonpolynomial counterterms in Sec. V.

The ML distribution has also correct homogeneity properties with respect to both n_μ and n^*_μ; this can be seen observing that

$$n^\mu \frac{\partial}{\partial n^\mu}\frac{1}{[[np]]} = -\frac{1}{[[np]]} , \quad (40)$$

and that Eq. (36) is manifestly invariant under dilation of the vector n^*_μ. As a consequence, the homogeneity degrees of a composite operator with respect to both gauge vectors are preserved under renormalization.

One can show [6,17] that using the ML prescription, the Euclidean UV power counting is a good convergence criterion for the corresponding minkowskian integrals. On the other hand, the spurious poles behave as convergence factors only for the "longitudinal" variables k^0 and k^3 and not for the "transverse" ones k^1 and k^2. It follows that in light-cone gauge a diagram may have a divergence associated to certain proper subsets of the integration variables yet involving all integration momenta. From an analytical point of view, i.e., as for Weinberg's theorem, these divergences are subdivergences, but from a graphical point of view they are to be considered as overall divergences because they are not related to subdiagrams and therefore they are not removed by counterterms in the graph-by-graph subtraction method. Hence we shall call "superficially divergent" a graph G if it exhibits positive power counting on some subset (proper or not) of its integration variables not limited to a proper subdiagram of G. In the following we introduce an appropriate superficial degree of divergence consistent with this definition.

First we consider a one-loop diagram $G^{(1)}$. We denote by $\delta_\forall(G^{(1)})$ the usual degree of divergence one obtains by a dilation of all the variables $\{k^\forall\} = \{k^0, k^1, k^2, k^3\}$ and we define the analogous quantity $\delta_\perp(G^{(1)})$ obtained considering just the transverse variables $\{k^\perp\} = \{k^1, k^2\}$. $\delta_\perp$ differs from $\delta_\forall$ on differentials,

$$\delta_\perp(d^4k) = 2 , \quad (41)$$

or in the cases

$$\delta_\perp(nk) = \delta_\perp(n^*k) = \delta_\perp\left(\frac{1}{[[nk]]}\right) = 0 , \quad (42)$$

while we shall keep, for a single component,

$$\delta_\perp(k^\mu) = \delta_\forall(k^\mu) = 1 , \quad (43)$$

since the result of integrals will always be written in four-vector notation.

The "superficial degree of divergence" of $G^{(1)}$ is then defined as

$$\delta(G^{(1)}) = \max\left\{\delta_\forall(G^{(1)}), \delta_\perp(G^{(1)})\right\} . \quad (44)$$

It is easy to show that $\delta(G^{(1)})$ is the maximum degree among the ones related to all possible subsets of integration variables.

Now we consider a graph G with l integration momenta $k_1, \ldots, k_l$. We still define

$$\{k_i^\forall\} = \{k_i^0, k_i^1, k_i^2, k_i^3\} , \quad (45)$$
$$\{k_i^\perp\} = \{k_i^1, k_i^2\}, \quad i = 1, \ldots, l, ,$$

and denote by

$$\delta_{v_1,\ldots,v_l}(G), \quad v_i \in \{\forall, \perp\} , \quad (46)$$

the degree of divergence of G related to the variables

$$\{k_1^{v_1}\} \cup \{k_2^{v_2}\} \cup \cdots \cup \{k_l^{v_l}\} . \quad (47)$$

The superficial degree of divergence of G is now defined as

$$\delta(G) = \max_{v_i \in \{\forall,\perp\}} \{\delta_{v_1,\ldots,v_l}(G)\} . \quad (48)$$

It is easy to realize that this definition leads to a sufficient condition for convergence. To show that it is not too cautious, we look at the following two-loop example: for the integral

$$I = \int d^{2\omega}k_1\, d^{2\omega}k_2 \frac{k_1^\mu\, k_1^\nu\, k_2^\rho}{(k_1-q)^2(k_1-k_2)^4 k_2^2\, [[n(k_1+k_2+s)]]\,[[n(k_1+p)]]\,[[nk_1]]^2} , \quad (49)$$

one finds

$$\delta(I) = \max\left\{\begin{array}{rcr} \delta_{\forall\forall} & = & -1 \\ \delta_{\perp\forall} & = & 0 \\ \delta_{\forall\perp} & = & -3 \\ \delta_{\perp\perp} & = & -1 \end{array}\right\} = 0, \quad (50)$$

and hence I may diverge in the variables $\{k_1^\perp\} \cup \{k_2^\forall\}$. We remark that I has negative mass dimension and no subdivergences.

We say that a diagram G is superficially convergent (divergent) if $\delta(G) < 0$ [$\delta(G) \geq 0$]. We say that G has a subdivergence if it has a superficially divergent 1PI proper subdiagram.

It is crucial to notice that, while in covariant gauges the usual degree $\delta(G)$ has a dimensional meaning because it equals the dimension of the 1PI momentum-space graph, in the light-cone gauge $\delta(G)$ depends on the particular topological structure of the graph. As a consequence, different graphs contributing to the same proper vertex have in general a different degree and any proper vertex, whatever its dimension, can have superficially divergent graphs. Therefore, in the light-cone gauge power counting arguments do not limit the type of counterterms entering the Lagrangian or a composite operator under renormalization. In particular, as even diagrams with negative mass dimension may be superficially divergent, nonlocal, i.e., nonpolynomial in external momenta, counterterms are generally expected.

V. GENERAL FORM OF NONPOLYNOMIAL COUNTERTERMS

In a covariant field theory and in particular in Yang-Mills theories with covariant gauges, the so-called BPH theorem holds. The counterterm CG of a 1PI graph G is polynomial in the external momenta and thereby the locality of the Lagrangian or a composite operator is preserved under renormalization. The theorem does not hold in the light-cone gauge: the lacking argument in the proof is that in a covariant theory the action of the derivative with respect to an external momentum on a graph G lowers its degree of divergence. In the light-cone gauge this is not true. Consider for instance a graph G with l integration momenta $k_1,\ldots,k_l$. Suppose that G has a spurious pole of the form

$$\frac{1}{[[n(k_1+r+\text{other momenta})]]}, \qquad (51)$$

r being an external momentum. The degree $\delta(G)$ is not necessarily lowered by a differentiation with respect to r^μ, as the degrees $\delta_{\perp,v_2,\ldots,v_l}$ are not. As a consequence, CG is not in general a polynomial in r^μ. However it is easy to see that a suitable number of derivatives,

$$\frac{\partial}{\partial p^\alpha}, \quad \alpha\in\{-,1,2\}, \qquad (52)$$

acting on a graph G, does indeed make it converge [see Eq. (39)]. Therefore the BPH theorem is modified as follows.

Proposition 2. Let G be a 1PI diagram in light-cone gauge and p an external momentum. If $\delta(G)\geq 0$, then CG is a polynomial in the components p^α, $\alpha\in\{-,1,2\}$.

Possible nonlocalities of counterterms are therefore limited to nonpolynomial functions of $p_i^+ = np_i$. We shall see that the nonlocalities can only appear as spurious poles $[[np]]^{-1}$ in the external momenta.

By the same arguments one can show that in light-cone gauge, as in covariant gauges, for a 1PI graph G, the counterterm CG is polynomial in the fermionic masses.

Some results about the most general form of nonlocal counterterms prove to be very important in renormalization theory. The following proposition states that the only possible nonlocalities of a counterterm are spurious poles $\frac{1}{np}$ in the external momenta. In the proof, the following "splitting formula" holding for the ML distribution is used:

$$\frac{1}{[[n(p_1+k)]]}\frac{1}{[[n(p_2+k)]]} = \frac{1}{[[n(p_2-p_1)]]}\left(\frac{1}{[[n(p_1+k)]]}-\frac{1}{[[n(p_2+k)]]}\right). \qquad (53)$$

Proposition 3. Let G be a 1PI graph in light-cone gauge. Without loss of generality we can consider

$$G=\int\frac{d^{2\omega}k_1\ldots d^{2\omega}k_l \;\; f(p,k,n,n^*,g_{\mu\nu})}{\prod_{j=1}^{\alpha}\left(t_j^2(k,p)-m_j^2\right)\prod_{k=1}^{\beta}[[ns_k(k,\widetilde{p})]]}, \qquad (54)$$

where p_i are the external momenta, $\widetilde{p}_j$ $(j=1,\ldots,\beta)$ are linear combination of the p_i, $f(\hat{f})$ is a polynomial in its arguments, $t_j(\hat{t}_j)$ are linear combinations of the p_i and k_i, $s_k(\hat{s}_k)$ are linear combinations of the k_i and $\widetilde{p}_i$, and $m_j(\hat{m}_j)$ are possible fermionic masses. Then G can be expressed as a sum

$$G=\sum_k I_k, \qquad (55)$$

where each I_k is of the form

$$\begin{aligned} I &= \prod_{r=1}^{\gamma}\frac{1}{n\hat{s}_r(\widetilde{p})}\hat{I} \\ &= \prod_{r=1}^{\gamma}\frac{1}{n\hat{s}_r(\widetilde{p})} \\ &\quad\times\int\frac{d^{2\omega}k_1\cdots d^{2\omega}k_l \;\; \hat{f}(p,k,n,n^*,g_{\mu\nu})}{\prod_{j=1}^{\alpha}\left(\hat{t}_j^2(k,p)-\hat{m}_j^2\right)\prod_{m=1}^{l}[[nk_m]]^{\beta_m}}, \end{aligned} \qquad (56)$$

with $\beta_m\geq 0$ $\forall m$ and

$$\sum_{m=1}^{l}\beta_m+\gamma=\beta \quad\Rightarrow\quad \gamma\leq\beta. \qquad (57)$$

Corollary 4. CG is a meromorphic function in the variables $p_i^+ = np_i$ with poles at most of order β.

Proof: (corollary) it follows directly from the form of the integrals $\hat{I}$ observing that $C\hat{I}$ is polynomial in the external momenta as the spurious poles are p_i independent.

□

Proof: (proposition) by induction on l. We first show the thesis for $l=1$. If no spurious poles depend on k_1, then G is already of the form I with $\beta_1=0$ and $\gamma=\beta$. Otherwise, by using formula (53), one can factor out of the integrand all spurious poles but one $[[n(\widetilde{p}+k_1)]]^{-\beta_1}$ that can possibly be of higher order $(\beta_1>1)$ if multiple poles were originally present in G. By shifting k_1 these poles become $[[nk_1]]^{-\beta_1}$ and therefore G is decomposed as in Eq. (55).

Let us now assume that the thesis holds for $l-1$ loops; we can apply to the integral in $d^{2\omega}k_l$ the same procedure above considering as "external momenta" also the variables k_i $i=1,\ldots,l-1$.

G is therefore reduced to a sum of terms of the form

$$\int\frac{d^{2\omega}k_1\cdots d^{2\omega}k_{l-1}}{\prod_{r=1}^{\beta-\beta_l}[[n\hat{s}_r(\widetilde{p},k_1,\ldots,k_{l-1})]]}\int\frac{d^{2\omega}k_l\,\hat{f}(p,k,n,g_{\mu\nu})}{\prod_{j=1}^{\alpha}\left(\hat{t}_j^2(k,p)-\hat{m}_j^2\right)\,[[nk_l]]^{\beta_l}}$$

$$=\int\frac{d^{2\omega}k_l}{[[nk_l]]^{\beta_l}}\int\frac{d^{2\omega}k_1\cdots d^{2\omega}k_{l-1}\;\;\hat{f}(p,k,n,g_{\mu\nu})}{\prod_{j=1}^{\alpha}\left(\hat{t}_j^2(k,p)-\hat{m}_j^2\right)\prod_{r=1}^{\beta-\beta_l}[[n\hat{s}_r(\widetilde{p},k_1,\ldots,k_{l-1})]]}.$$

We can now apply the inductive hypothesis to the multiple integral on the RHS considering p_i and k_l as external momenta but remembering that the variables $\widetilde{p}_i$ do not depend on k_l. As a consequence, the spurious poles extracted from the multiple integral can be factorized out of the integral in $d^{2\omega}k_l$ giving only terms of the form (56). □

We now discuss a feature of Feynman integrals in the light-cone gauge that is fundamental in selecting the possible structures involved in operator renormalization. To this purpose let us consider the following peculiar property of the ML prescription under the algebraic splitting:

$$\frac{1}{nk+i\epsilon\sigma(n^*k)}\frac{1}{n(k-s)+i\epsilon\sigma[n^*(k-s)]}=\frac{1}{ns+i\epsilon\sigma(n^*s)}\left[\frac{1}{n(k-s)+i\epsilon\sigma[n^*(k-s)]}-\frac{1}{nk+i\epsilon\sigma(n^*k)}\right]. \quad (58)$$

In the limit $s_\mu \to 0$, no singularity is present in either term of the equality; in particular at the left-hand side we have a double pole at $nk = 0$ with causal prescription and at the right-hand side the pole at $ns = 0$ is canceled by a corresponding zero of the quantity in square brackets. However, would we consider the limit $ns \to 0$ with $n^*s \neq 0$, a singularity at $ns = 0$ would persist owing to the dependence on n^*.

This is at variance with the behavior of a prescription involving only one gauge vector, after the disposal of Poincaré-Bertrand terms [17], and is at the root of the nonlocal behavior of some counterterms in light-cone gauge with the ML prescription.

It is however clear that, should we restrict the spurious denominators to the subregions $n^*s_i = -ns_i$, s_i being any generic four-vector, we would get

$$\frac{1}{ns+i\epsilon\sigma(n^*s)}\to\frac{ns}{(ns)^2-i\epsilon}=\mathrm{CPV}\frac{1}{ns}, \quad (59)$$

and would recover locality by the very same argument which is used in the proof in the one-vector spacelike case [17]. We stress that this restriction must be understood in the sense of the theory of distributions. In particular multiple poles should always be interpreted as derivatives; otherwise one immediately runs into powers of CPV prescription, i.e., meaningless quantities.

Having the above discussed property in mind, we now require that all acceptable nonlocal structures in counterterms have to become local when $n^*\partial$ is replaced by $-n\partial$.

Actually a replacement of the kind $n^*\partial \to \kappa\, n\partial$, with any constant κ, would do the job. Our previous choice is reminiscent of the condition $n^* \to -n$, advocated in Ref. [6] to recover locality in analogy with the spacelike planar gauge. We stress however that the present condition is imposed in a form of a phase-space restriction while standing on the light cone; in this sense it is closer to the spirit of the discussion in Ref. [18].

We shall show in the next section that this criterion is extremely efficient in selecting among *a priori* possible nonlocal structures the only acceptable one: Ω, the same already present when renormalizing the effective action [6].

VI. CONSTRUCTION OF RENORMALIZATION CLASSES

In covariant Yang-Mills theories, the renormalization of gauge invariant composite operators is governed by the BPH theorem; the locality of counterterms guarantees that a composite operator can only mix with operators of lower or equal canonical dimension. This *dimensional hierarchy* automatically limits the number of renormalization constants needed by a single renormalized operator. Nevertheless, in covariant gauges, the renormalization of composite operators is a very complicated matter because of the presence of nonphysical degrees of freedom that contribute nontrivially to renormalized operators. For this reason the gauge invariance of a composite operator is generally lost under renormalization [3].

In light cone gauge the situation is opposite. Renormalization preserves gauge invariance of the operator, but the presence of nonlocal counterterms could allow in principle an infinite number of independent structures to appear.

Our aim is to show that on the contrary the renormalization of a gauge invariant composite operator involves only a finite number of renormalization constants and that nonlocal terms do not affect physical quantities. From now on we will focus on weak renormalization (a single insertion) because, in the more general case, an infinite number of counterterms is expected on general grounds also in covariant theories if the operator has dimension ≥ 4.

Let us consider a local gauge invariant operator X being a Lorentz tensor of rank i (i free Lorentz indices), with homogeneity degrees O_n and O_{n^*} with respect to the gauge vectors and mass dimension d_m. The most general form of the renormalized operator $[X]$ is a structure having the same characteristics of X mentioned above but locality since poles of the form $n\partial^{-1}$ may be present; however the structure has to become local if the substitution $n^*\partial \to n\partial$ is performed. Such structures will be called *quasilocal.* We observe that the *canonical* dimension of the field A_μ cannot be defined in light-cone gauge as the UV behavior of its propagator does depend on the gauge vector; the only well-defined dimension of operators in light-cone gauge is mass dimension. As a consequence in the expression of the renormalized operator $[X]$ we shall consider mass parameters as part of mixed operators and shall work with dimensionless renormalization constants. Hence we can state the following.

Proposition 5. Local or quasilocal gauge invariant composite operators with the same mass dimension d_m, the same homogeneity degrees O_n and O_{n^*}, and the same tensorial rank i form a class that is closed under renormalization.

We want to show that each of these *renormalization classes* contain a finite number of independent operators.

In the construction of quasilocal gauge invariant structures, in addition to the usual covariant tensors, spinors and derivatives, the following covariant nonlocal integral operator can be used as a building block:

$$[nD^{-1}]^{ab} = \delta^{ab}\frac{1}{[[n\partial]]} - g\, f^{acd}\frac{1}{[[n\partial]]}\left\{nA^{c}[nD^{-1}]^{db}\right\} ; \tag{60}$$

the formula has to be understood recursively and can easily be expanded in powers of g. Because of the negative mass dimension of nD^{-1}, for any given composite operator, an infinite set of possible independent counterterms can be obtained still satisfying the requirements of correct mass dimension, homogeneity, and tensorial structure. On the contrary, only very few structures containing nD^{-1} become local when $n^*\partial \to n\partial$; by explicit construction one realizes that the only acceptable ones are those in which nD^{-1} is carried by the following combination trasforming in the adjoint representation:

$$\Omega^a = \frac{n^\mu n^{*\nu}}{n^* n}[nD^{-1}]^{ab}F^b_{\mu\nu}. \tag{61}$$

This structure is peculiar since $n_\mu n^*_\nu F^{\mu\nu}$ develops a factor nD in the numerator when $n^*\partial \to n\partial$:

$$\begin{aligned}\Omega|_{n^*\partial\to n\partial} &= \frac{1}{n^*n}\,\frac{1}{nD}\,(n\partial\, n^*A - n\partial\, nA - ig[nA, n^*A]) \\ &= \frac{1}{n^*n}\,\frac{1}{nD}\,(nD\, n^*A - nD\, nA) \\ &= \frac{1}{n^*n}(n^*A - nA).\end{aligned} \tag{62}$$

Such a condition is indeed stronger than the one considered in Ref. [6] as structures like

$$n_\mu n^*_\nu F^{\mu\nu} \times (\text{nonlocal}), \tag{63}$$

or

$$\frac{1}{nD}\, n^*D \times (\text{anything}), \tag{64}$$

become local if one replaces $n^* \to n$ but not if $n^*\partial \to n\partial$.

The crucial point here to observe is that Ω has positive mass dimension; hence, for any given operator the number of possible independent gauge invariant counterterms built from local covariant objects and Ω, is automatically limited by dimensionality arguments. Equivalently, each renormalization class is finite. Moreover, by directly inspecting the expansion of Ω,

$$\Omega^a = \Omega^a_0 + \textstyle\sum_{k=1}^{\infty} g^k \Omega^a_k, \quad \Omega^a_0 = n^*A^a - \frac{n^*\partial}{n\partial} nA^a,$$
$$\Omega^a_k = (-1)^{k+1} f^{ab_k h_{k-1}} f^{h_{k-1} b_{k-1} h_{k-2}} \ldots f^{h_2 b_2 h_1} f^{h_1 b_1 c} \tfrac{1}{n\partial}\left\{nA^{b_k} \tfrac{1}{n\partial}\left\{nA^{b_{k-1}} \tfrac{1}{n\partial}\left\{\cdots \tfrac{1}{n\partial}\left\{nA^{b_1} \tfrac{n^*\partial}{n\partial} nA^c\right\}\cdots\right\}\right\}\right\}, \tag{65}$$

one learns that all nonlocal terms appearing in the renormalized operator $[X]$ will be proportional to the field nA; therefore only the local part of the renormalized operator $[X]$ will contribute to λ-independent Green's functions [see Eq. (14)].

What we have said before can be summarized as follows

Proposition 6. Let X be a local or quasilocal gauge invariant composite operator; then (1) $[X]$ involves a finite number of renormalization constants and (2) possible nonlocal terms of $[X]$ are proportional to nA and therefore do not contribute to physical quantities.

Let us see how to build a basis of independent operators for a given renormalization class characterized by the mass dimension d_m, the homogeneity degrees O_n and O_{n^*}, and the tensor rank i of its gauge invariant operators. In the following table we list the "blocks" that can be used to build a local or quasilocal operator:

$$\begin{aligned} f &= \text{No.}[\bar\psi\cdots\psi]\,, \\ q &= \text{No.}[F_{\mu\nu} = F^a_{\mu\nu}\tau^a]\,, \\ p &= \text{No.}[D^{ab}_\mu \text{ or } \mathcal{D}_\mu]\,, \\ \omega &= \text{No.}[\Omega = \Omega^a\tau^a]\,, \\ j &= \text{No.}[n_\mu]\,, \\ k &= \text{No.}[n^*_\mu]\,, \\ l &= \text{No.}[(n^*n)^{-1}]\,, \\ g &= \text{No.}[\gamma^\mu]\,, \\ r &= \text{No.}[g^{\mu\nu}]\,, \\ m &= \text{No.}[\text{masses } m \text{ or derivatives } \partial_\mu]\,, \end{aligned} \tag{66}$$

where of course $g^{\mu\nu}$ is understood only with free indices. The positive integer variables $f, q, \ldots, r$ denote the multiplicity of a single factor inside a given operator. Of course, as already anticipated in (66), derivatives acting on gauge invariant quantities are also allowed, each one entailing a unit dimension in mass. The values the variables can assume are subject to the costraints due to mass dimension and homogeneity with respect to n and n^*:

$$3f + 2q + p + \omega + m = d_m\,, \tag{67a}$$

$$j - \omega - l = O_n, \tag{67b}$$

$$k - l = O_{n^*}. \tag{67c}$$

Eq. (67a) gives an upper limit to all the variables on the LHS; the remaining variables are always limited by imposing the correct rank i of the operators and their independence.

Of course more operators can correspond to the same combination of variables. Starting from the table above, one has first to build all possible combinations, whose number is however *finite* for a given composite operator, and then to check their independence. Some examples are discussed in the next section.

VII. EXAMPLES

In this section we discuss two simple examples of mixing.

The first gauge invariant composite operator we consider is $\bar\psi\psi$. One can easily realize that the only allowed counterterms are

$$[\bar\psi\psi] = \zeta_1\bar\psi\psi + \zeta_2\bar\psi\frac{\not n \not n^*}{2nn^*}\psi + \zeta_3 m^3. \tag{68}$$

In this case renormalization involves only local gauge invariant fermionic bilinears, in addition to the mass term. The renormalization of this operator is related to the one of the mass term in the Lagrangian [6]; the ensuing constraints on ζ_1 and ζ_2 are

$$\begin{aligned}\zeta_1 &= Z_2\left(1-\frac{\partial}{\partial m}\delta m\right),\\ \zeta_2 &= 0,\end{aligned} \tag{69}$$

while no restriction is imposed on ζ_3. An explicit one-loop calculation confirms this prediction and gives

$$\begin{aligned}\zeta_1 &= 1 - \frac{g^2}{8\pi^2}\frac{N^2-1}{2N}\frac{1}{2-\omega} + O(g^4),\\ \zeta_2 &= 0 + O(g^4),\\ \zeta_3 &= \frac{N}{4\pi^2}\frac{1}{2-\omega} + O(g^2).\end{aligned} \tag{70}$$

As a second example we consider the fermionic U(1) conserved current $\bar\psi\gamma_\mu\psi$. There are several independent gauge invariant structures with mass dimension 3 with which it can *a priori* mix

$$\begin{aligned}[\bar\psi\gamma_\mu\psi] &= \zeta_1\,\bar\psi\gamma_\mu\psi + \zeta_2\,\frac{n_\mu}{nn^*}\,\bar\psi\not n^*\psi + \zeta_3\,\frac{n^*_\mu}{nn^*}\,\bar\psi\not n\psi\\ &\quad + \zeta_4\,\bar\psi\frac{\not n\gamma_\mu\not n^*}{nn^*}\psi + \zeta_5\,n^\nu F^a_{\nu\mu}\Omega^a\\ &\quad + \zeta_6\,n_\mu\frac{n_\nu n^*_\sigma}{nn^*}F^{\nu\sigma,a}\Omega^a\,.\end{aligned} \tag{71}$$

Some of them are nonlocal, i.e., involve Ω. However, this current is related to the fermion propagator by the U(1) Ward identity which sets constraints between ζ_i's and the wave function renormalization constants Z_2 and $\tilde Z_2$ [6]:

$$\begin{aligned}\zeta_1 &= Z_2,\\ \zeta_2 &= -\zeta_3 = Z_2\tilde Z_2(1-\tilde Z_2^{-1}),\\ \zeta_4 &= \zeta_5 = \zeta_6 = 0.\end{aligned} \tag{72}$$

These relations agree with the general result [2] that the renormalized current $[\bar\psi\gamma^\mu\psi]^{\{l\}}$ coincides with the Noether current derived from the renormalized action $S^{\{l\}}$. An explicit one-loop calculation fully confirms Eqs. (72) and reproduces the values [6]

$$\begin{aligned}\zeta_1 &= 1 + \frac{g^2}{16\pi^2}\frac{N^2-1}{2N}\frac{1}{2-\omega} + O(g^4),\\ \zeta_2 &= -\zeta_3 = -\frac{g^2}{8\pi^2}\frac{N^2-1}{2N}\frac{1}{2-\omega} + O(g^4),\\ \zeta_4 &= \zeta_5 = \zeta_6 = 0 + O(g^4).\end{aligned} \tag{73}$$

Finally a last example, definitely requiring nonlocal counterterms, is the Lagrangian density itself, as discussed in [6]; of course further structures involving total derivatives must be considered in this case.

VIII. CONCLUSIONS

In this paper we have solved the problem of renormalizing at any order in the loop expansion gauge invariant composite operators in Yang-Mills theories quantized in light-cone gauge with the correct causal ML prescription in vector propagator. We have here generalized the treatment developed in Refs. [6,17], concerning effective action.

The main results are the proof that renormalization preserves the gauge invariance of composite operators (Sec. III) and the full characterization of admissible nonlocal structures in counterterms, which can be only carried by the quantity Ω, and therefore cannot contribute to physical quantities; hence we get the proof that the renormalization of a composite operator always entails a finite number of renormalization constants (Sec. VI).

Specific examples of mixing under renormalization are presented in Sec. VII; in particular we have found quite instructive the behavior of the U(1) conserved fermionic current, which is endowed with a direct physical interest.

Generalizations to gauge-dependent operators are *a priori* possible; however, one is immediately faced with a basic difficulty concerning Lee-Ward identities, which are no longer form-invariant under renormalization. In addition, physical applications are usually concerned with gauge invariant composite operators.

A few preliminary results have already appeared in the literature [19], while a pedagogical review on the whole subject will be reported elsewhere [20].

[1] K.G. Wilson, Phys. Rev. **179**, 1699 (1969).
[2] See, e.g., J. Collins, *Renormalization* (Cambridge University Press, Cambridge, England, 1984), and references therein.
[3] S. Joglekar and B.W. Lee, Ann. Phys. (N.Y.) **97**, 160 (1976).
[4] W. Kummer, Acta Phys. Austr. **41**, 315 (1975).
[5] See, e.g., Yu.L. Dokshitzer, D.I. D'yakonov, and S.I. Troyan, Phys. Rep. **58**, 269 (1980).
[6] A. Bassetto, M. Dalbosco, and R. Soldati, Phys. Rev. D **36**, 3138 (1987).
[7] S. Mandelstam, Nucl. Phys. **B213**, 149 (1983).

[8] G. Leibbrandt, Phys. Rev. D **29**, 1699 (1984).
[9] A. Bassetto, M. Dalbosco, I. Lazzizzera, and R. Soldati, Phys. Rev. D **31**, 2012 (1985).
[10] See, e.g., A.P. Bukhvostov, G.V. Frolov, L.N. Lipatov, and E.A. Kuraev, Nucl. Phys. **B258**, 601 (1985).
[11] N.N. Bogoliubov and O. Parasiuk, Acta Math. **97**, 227 (1957).
[12] K. Hepp, Commun. Math. Phys. **2**, 301 (1966).
[13] W. Zimmermann, Commun. Math. Phys. **15**, 208 (1969).
[14] S. Weinberg, Phys. Rev. **118**, 838 (1960).
[15] J. Zinn-Justin, *Quantum Field Theory and Critical Phenomena* (Oxford University Press, Oxford, England, 1989).
[16] M. Le Bellac, *Quantum and Statistical Field Theory* (Oxford University Press, Oxford, England, 1991).
[17] A. Bassetto, G. Nardelli, and R. Soldati, *Yang-Mills Theories in Algebraic Non-Covariant Gauges* (World Scientific, Singapore, 1991).
[18] A. Bassetto, in *Physical and Nonstandard Gauges*, edited by P. Gaigg, W. Kummer, and M. Schweda (Springer-Verlag, New York, 1990).
[19] A. Bassetto, Phys. Rev. D **47**, 727 (1993).
[20] C. Acerbi, Tesi di Laurea, Università di Padora, 1993.