Effective field theory of an internal string

Y. M. Cho
Institute for Advanced Study, Princeton, New Jersey 08540
*and Center for Theoretical Physics, Seoul National University, Seoul 151-742, Korea**

(Received 3 March 1993)

We obtain an effective field theory of an internal string by constructing a gauge theory of the Virasoro-Kac-Moody symmetry associated with $P\otimes G$, where P is the Poincaré group and G is the grand unification group. The theory automatically breaks the Virasoro-Kac-Moody group down to $P\otimes H\otimes \mathrm{U}(1)$, where H is a subgroup of G and U(1) is the Cartan subgroup of the Virasoro group. After the spontaneous symmetry breaking the mass spectrum of the theory is characterized by two completely different mass scales: the Planck scale which is responsible for the harmonic oscillator spectrum of the string and the elementary particle scale which is responsible for the fine structure of the mass spectrum. The theory allows a straightforward supersymmetric generalization.

PACS number(s): 11.25.−w, 11.10.Lm, 11.15.Ex, 11.25.Mj

Recently the string theory [1,2] has become a leading candidate of the theory of everything, the ultimate theory which unifies all the interactions in nature, and has been studied extensively. In spite of the extensive studies, however, an effective field of theory of string which could describe the dynamics of a propagating colored string in the four-dimensional curved space-time has not been available so far. The purpose of this paper is to present a gauge theory of the Virasoro-Kac-Moody symmetry associated with the Poincaré group P and the grand unification group G, which could provide a field-theoretic description of a colored string. After an inevitable spontaneous symmetry breaking the particle spectrum of the theory at the pre-confinement level consists of an infinite tower of massive spin-two and spin-one particles and a finite number of the massless particles made of the graviton as well as the gauge fields of the unbroken subgroup $H\otimes \mathrm{U}(1)$, where H is a subgroup of G and U(1) is the Cartan subgroup of the Virasoro group. In addition, the theory contains light scalar particles, the dilaton, and an adjacent multiplet of H, as the pseudo Goldstone particles of the symmetry breaking.

In the limit where the size of the string can be neglected, an effective field theory of string must satisfy the following minimal requirements. First of all, such a theory must have the Virasoro invariance before an inevitable spontaneous symmetry breaking [3,4]. This is so because in this limit the Virasoro group becomes an internal symmetry of the string theory which assures the reparametrization invariance of the string. Second, it must have the general invariance to accommodate the gravitational interaction. In fact, it is more likely that the theory must have the Virasoro-Kac-Moody symmetry associated with the Poincaré group P, if it can successfully describe the propagation of the string in a four-dimensional curved space-time [4,5]. Finally it must have the Kac-Moody symmetry associated with the color SU(3) to accommodate the strong interaction [6,7]. Indeed it must have the Kac-Moody symmetry associated with the grand unification group G to achieve the full unification of all the interactions. So what we need for an effective field theory of string (in the limit that the string can be approximated to a point) is a gauge theory of the Virasoro-Kac-Moody symmetry associated with the Poincaré group and the grand unification group.

So far nobody has proposed to consider the gauge theory of the Virasoro-Kac-Moody group as a possible string field theory. The reason probably is that there are two serious objections that one can raise to this type of theory. First, this type of theory must be nonunitary, since the gauge fields of the Virasoro group (which should form an adjoint representation) do not form a unitary representation. Second, it should be nonrenormalizable, because it should include nonrenormalizable interactions (in particular, the gravitational interaction). It, therefore, seems useless to consider such a theory. Nevertheless, we find that these objections are not insurmountable. First, this type of theory could easily be made unitary with a spontaneous symmetry breaking, although this has not been so well known to the physics community. In fact, we have recently shown that the gauge theory of the Virasoro group does become unitary after the gauge group is broken down to the Cartan subgroup [3]. Similarly, we will show in the following that all the physical fields of the gauge theory of the Virasoro-Kac-Moody group become explicitly unitary after the desired spontaneous symmetry breaking, although they certainly do not form unitary representations under the full symmetry group. This will resolve the nonunitarity problem completely.

The second objection looks more serious, since the theory does not seem to satisfy the usual conditions of the renormalizability. But notice that the gauge theory of the Virasoro-Kac-Moody group is expected to become the string theory in the point limit of the string, although it probably does not describe the full string theory itself.

*Permanent address.

0556-2821/94/49(2)/1007(5)/$06.00

On the other hand, the full string theory is supposed to be finite. This implies that, with the help of the higher-order corrections coming from the full string theory, one could make the theory renormalizable or even finite. With this reservation one could regard the gauge theory of the Virasoro-Kac-Moody group as an acceptable candidate of string field theory.

To construct such a theory we need a general representation of the Virasoro-Kac-Moody symmetry associated with an arbitrary non-Abelian group G. We first present a representation, which we call *the product representation* [6]. The Virasoro-Kac-Moody group is the semidirect product group of the Virasoro group and the Kac-Moody group whose algebra is given by

$$\begin{aligned}
&[L_m, L_n] = f^k_{mn} L_k = (m-n) L_{m+n} ,\\
&[L_m, T_{an}] = f^{ck}_{man} T_{ck} = -n T_{am+n} ,\\
&[T_{am}, T_{bn}] = f^{ck}_{ambn} T_{ck} = f^c_{ab} T_{cm+n} ,
\end{aligned} \tag{1}$$

where k, m, and n are the indices of the Virasoro group and a, b, and c are the indices of the associated group G. Notice that here we do not consider the central extension for simplicity. The product representation is given by

$$\begin{aligned}
&(L_m)^k_n = [(\alpha+1)m + \beta - k]\delta^k_{m+n} ,\\
&(T_{am})^{\alpha k}_{\beta n} = (T_a)^\alpha_\beta (T_m)^k_n = (T_a)^\alpha_\beta \delta^k_{m+n} .
\end{aligned} \tag{2}$$

Notice that L_m forms an arbitrary (α,β) representation of the Virasoro group [3,8] and T_{am} forms a representation of the Kac-Moody group made of two matrices T_a and T_m, where T_a is an arbitrary matrix representation of G which acts only on the indices of the associated group and T_m is an infinite-dimensional matrix which acts only on the indices of the Virasoro group. When T_a forms an adjoint representation, we obtain the following product representation ϕ^{ak}:

$$\begin{aligned}
&\delta_\xi \phi^{ck} = i\xi^m (L_m)^k_n \phi^{cn} = i[(\alpha+1)m + \beta - k]\xi^m \phi^{ck-m} ,\\
&\delta_\alpha \phi^{ck} = \alpha^{am} (T_{am})^{ck}_{bn} \phi^{bn} = f^c_{ab} \alpha^{am} \phi^{bk-m} ,
\end{aligned} \tag{3}$$

where ξ^m and α^{am} are the infinitesimal parameters of the Virasoro-Kac-Moody group.

With this preliminary we now show how to construct a gauge theory of the Virasoro-Kac-Moody symmetry associated with the Poincaré group and the grand unification group. The simplest way to construct such a theory is to start from the Einstein-Yang-Mills theory based on the principal fiber bundle $P(M_5, G)$, whose fiber is the grand unification group G and the base manifold is a five-dimensional manifold M_5 which itself forms a fiber bundle $B(M_4, S^1)$ made of the closed string S^1 and the four-dimensional space-time M_4. In a coordinate basis $(\partial_\mu \oplus \partial_5)$ the most general metric on M_5 can be written as [4,9]

$$\gamma_{AB} = \begin{pmatrix} \gamma_{\mu\nu} + e^2\kappa^2\phi^2 A_\mu A_\nu & e\kappa A_\mu \phi^2 \\ e\kappa\phi^2 A_\nu & \phi^2 \end{pmatrix} , \tag{4}$$

where e is the coupling constant of the Virasoro group and κ is a scale parameter which characterizes the size of the string. But in the block-diagonal basis $(\hat\partial_\mu \oplus \partial_5)$ where [4,9]

$$\begin{aligned}
&\hat\partial_\mu = \partial_\mu - e\kappa A_\mu \partial_5 ,\\
&[\hat\partial_\mu, \hat\partial_\nu] = -e\kappa F_{\mu\nu}\partial_5, \quad [\hat\partial_\mu, \partial_5] = e(\partial_\theta A_\mu)\partial_5 ,\\
&F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - e(A_\mu \partial_\theta A_\nu - A_\nu \partial_\theta A_\mu) ,
\end{aligned} \tag{5}$$

we have

$$\gamma_{AB} = \begin{pmatrix} \gamma_{\mu\nu} & 0 \\ 0 & \phi^2 \end{pmatrix} . \tag{6}$$

Notice that here we have identified x^5 with $\kappa\theta$, where θ is the dimensionless string coordinate of S^1.

Let the gauge potentials of G in the coordinate basis $(\partial_\mu \oplus \partial_5)$ be A^a_μ and ϕ^a. Clearly the field strengths of the potentials are given by

$$\begin{aligned}
F^c_{\mu\nu} &= \partial_\mu A^c_\nu - \partial_\nu A^c_\mu + g f^c_{ab} A^a_\mu A^b_\nu ,\\
F^c_{\mu 5} &= \partial_\mu \phi^c + g f^c_{ab} A^a_\mu \phi^b - \frac{1}{\kappa}\partial_\theta A^c_\mu \\
&= \hat D_\mu \phi^c .
\end{aligned} \tag{7}$$

In the basis $(\hat\partial_\mu \oplus \partial_5)$, however, we have [10]

$$\begin{aligned}
&\hat F^c_{\mu\nu} = \hat\partial_\mu \hat A^c_\nu - \hat\partial_\nu \hat A^c_\mu + g f^c_{ab} \hat A^a_\mu \hat A^b_\nu + e\kappa F_{\mu\nu}\phi^c ,\\
&\hat F^c_{\mu 5} = \hat\partial_\mu \phi^c + g f^c_{ab} \hat A^a_\mu \phi^b - \frac{1}{\kappa}\partial_\theta \hat A^c_\mu - e(\partial_\theta A_\mu)\phi^c ,
\end{aligned} \tag{8}$$

where $\hat A^c_\mu = A^c_\mu - e\kappa A_\mu \phi^c$ is the gauge potential in the basis $\hat\partial_\mu$. Notice that

$$\begin{aligned}
&\hat F^c_{\mu\nu} = F^c_{\mu\nu} + e\kappa A_\mu (\hat D_\nu \phi^c) - e\kappa A_\nu (\hat D_\mu \phi^c) ,\\
&\hat F^c_{\mu 5} = F^c_{\mu 5} .
\end{aligned} \tag{9}$$

Obviously, the Einstein-Yang-Mills theory on M_5 must be invariant under the (infinitesimal) general coordinate transformation

$$\begin{aligned}
&x^\mu \to x^{\mu'} = x^\mu + \xi^\mu(x,\theta) ,\\
&\theta \to \theta' = \theta + \xi(x,\theta) .
\end{aligned} \tag{10}$$

The coordinate transformation generated by ξ^μ describes the gauge transformation of the Kac-Moody symmetry associated with the Poincaré group (or more precisely the group of the four-dimensional general coordinate transformation), and the one generated by ξ describes the gauge transformation of the Virasoro group [4,11]. Under the Virasoro transformation we have

$$\begin{aligned}
&\delta_\xi \gamma_{\mu\nu} = -\xi \partial_\theta \gamma_{\mu\nu} ,\\
&\delta_\xi A_\mu = -\frac{1}{e}\partial_\mu \xi + (\partial_\theta \xi) A_\mu - \xi \partial_\theta A_\mu ,\\
&\delta_\xi \phi = -(\partial_\theta \xi)\phi - \xi \partial_\theta \phi ,\\
&\delta_\xi \hat A^c_\mu = -\xi \partial_\theta \hat A^c_\mu ,\\
&\delta_\xi \phi^c = -(\partial_\theta \xi)\phi^c - \xi \partial_\theta \phi^c ,
\end{aligned} \tag{11}$$

from which we obtain

$$\delta_\xi F_{\mu\nu}=(\partial_\theta\xi)F_{\mu\nu}-\xi\partial_\theta F_{\mu\nu}\ ,$$
$$\delta_\xi \hat{F}^c_{\mu\nu}=-\xi\partial_\theta\hat{F}^c_{\mu\nu}\ , \tag{12}$$
$$\delta_\xi(\hat{D}_\mu\phi^c)=-(\partial_\theta\xi)\hat{D}_\mu\phi^c-\xi\partial_\theta(\hat{D}_\mu\phi^c)\ .$$

Upon the Fourier decomposition (11) can be written as [4,6]

$$\begin{aligned}
&\delta_\xi\gamma^k_{\mu\nu}=i(m-k)\xi^m\gamma^{k-m}_{\mu\nu}\ ,\\
&\delta_\xi A^k_\mu=-\frac{1}{e}\partial_\mu\xi^k+i(2m-k)\xi^m A^{k-m}_\mu\ ,\\
&\delta_\xi\phi^k=-ik\xi^m\phi^{k-m}\ ,\\
&\delta_\xi\hat{A}^{ck}_\mu=i(m-k)\xi^m\hat{A}^{ck-m}_\mu\ ,\\
&\delta_\xi\phi^{ck}=-ik\xi^m\phi^{ck-m}\ .
\end{aligned} \tag{13}$$

The results shows that $\gamma^k_{\mu\nu}$, $\hat{A}^{ck}_\mu$, ϕ^k, and ϕ^{ck} form (0,0), (0,0), $(-1,0)$, and $(-1,0)$ representations of the Virasoro group, respectively. More importantly it shows that A^k_μ transforms exactly as the gauge potential of the Virasoro group. From (13) we have

$$\begin{aligned}
&\delta_\xi F^k_{\mu\nu}=i(2m-k)\xi^m F^{k-m}_{\mu\nu}\ ,\\
&\delta_\xi\hat{F}^{ck}_{\mu\nu}=i(m-k)\xi^m\hat{F}^{ck-m}_{\mu\nu}\ ,\\
&\delta_\xi(\hat{D}_\mu\phi^{ck})=-ik\xi^m(\hat{D}_\mu\phi^{ck-m})\ ,
\end{aligned} \tag{14}$$

which shows that $F^k_{\mu\nu}$, $\hat{F}^{ck}_{\mu\nu}$, and $\hat{D}_\mu\phi^{ck}$ form (1,0), (0,0), and $(-1,0)$ representations of the Virasoro group, respectively.

Under the infinitesimal gauge transformation of G generated by $\boldsymbol{\alpha}(x,\theta)$, we have

$$\begin{aligned}
&\delta_\alpha\gamma_{\mu\nu}=0,\quad \delta_\alpha A_\mu=0,\quad \delta_\alpha\phi=0\ ,\\
&\delta_\alpha\hat{A}^c_\mu=-\frac{1}{g}\partial_\mu\alpha^c+f^c_{ab}\alpha^a\hat{A}^b_\mu\ ,\\
&\delta_\alpha\phi^c=-\frac{1}{g\kappa}\partial_\theta\alpha^c+f^c_{ab}\alpha^a\phi^b\ .
\end{aligned} \tag{15}$$

This can be written as

$$\begin{aligned}
&\delta_\alpha\gamma^k_{\mu\nu}=0,\quad \delta_\alpha A^k_\mu=0,\quad \delta_\alpha\phi^k=0\ ,\\
&\delta_\alpha\hat{A}^{ck}_\mu=-\frac{1}{g}\partial_\mu\alpha^{ck}+i\frac{e}{g}m\alpha^{cm}A^{k-m}_\mu\\
&\qquad\qquad+f^c_{ab}\alpha^{am}\hat{A}^{bk-m}_\mu\ ,\\
&\delta_\alpha\phi^{ck}=-i\frac{k}{g\kappa}\alpha^{ck}+f^c_{ab}\alpha^{am}\phi^{bk-m}\ ,
\end{aligned} \tag{16}$$

from which we have

$$\begin{aligned}
&\delta_\alpha F^k_{\mu\nu}=0\ ,\\
&\delta_\alpha F^{ck}_{\mu\nu}=f^c_{ab}\alpha^{am}\hat{F}^{bk-m}_{\mu\nu}\ ,\\
&\delta_\alpha(\hat{D}_\mu\phi^{ck})=f^c_{ab}\alpha^{am}(\hat{D}_\mu\phi^{bk-m})\ .
\end{aligned} \tag{17}$$

Obviously, the gauge transformation corresponds to the Kac-Moody symmetry associated with G. But there are two points to be emphasized here. First (16) tells us that *the gauge transformation is nonlinearly realized.* The scalar multiplet ϕ^{ck} transforms nonlinearly, but nevertheless admits the unique covariant derivative which transforms linearly under the Kac-Moody transformation. The nonlinear realization has a very important physical implication as we will see in the following. Second (16) tells us that, up to the normalization factor $i(e/g)$ which can easily be absorbed to A^k_μ, A^k_μ and $\hat{A}^{ck}_\mu$ form the gauge potentials of the Virasoro-Kac-Moody symmetry associated with G. The corresponding field strengths of the Virasoro-Kac-Moody group are given by $F^k_{\mu\nu}$ and $\tilde{F}^{ck}_{\mu\nu}$, where $\tilde{F}^{ck}_{\mu\nu}$ is defined by

$$\begin{aligned}
\tilde{F}^{ck}_{\mu\nu}&=\partial_\mu\hat{A}^{ck}_\nu-\partial_\nu\hat{A}^{ck}_\mu\\
&\quad+g[i(e/g)(f^{ck}_{man}A^m_\mu\hat{A}^{an}_\nu+f^{ck}_{amn}\hat{A}^{am}_\mu A^n_\nu)\\
&\qquad+f^{ck}_{ambn}\hat{A}^{am}_\mu\hat{A}^{bn}_\nu]\\
&=\hat{F}^{ck}_{\mu\nu}-e\kappa F^m_{\mu\nu}\phi^{ck-m}\ .
\end{aligned} \tag{18}$$

Notice that the above definition of $\tilde{F}^{ck}_{\mu\nu}$ explicitly incorporates the semidirect product structure of the Virasoro-Kac-Moody group. From (18) we have

$$\delta_\alpha\tilde{F}^{ck}_{\mu\nu}=i(e/g)m\alpha^{cm}\tilde{F}^{k-m}_{\mu\nu}+f^c_{ab}\alpha^{am}F^{bk-m}_{\mu\nu}\ , \tag{19}$$

which indeed confirms the fact that, again up to the normalization factor $i(e/g)$, $F^k_{\mu\nu}$ and $\tilde{F}^{ck}_{\mu\nu}$ form the field strengths of the Virasoro-Kac-Moody group.

Now we are ready to write down the gauge theory of the Virasoro-Kac-Moody group. After the dimensional reduction of the five-dimensional Einstein-Yang-Mills Lagrangian $\mathcal{L}_5$ down to M_4, we obtain

$$\mathcal{L}=\int\mathcal{L}_5\frac{d\theta}{2\pi}=\mathcal{L}_1+\mathcal{L}_2\ ,$$

$$\mathcal{L}_1=\frac{(\sqrt{\gamma})^{-k}}{16\pi G}\left[\phi^{k-l}R^l+\frac{e^2\kappa^2}{4}(\phi^3)^{k-m-n-p-q}\gamma^{\mu\nu m}\gamma^{\alpha\beta n}F^p_{\mu\alpha}F^q_{\nu\beta}-\frac{pq}{4\kappa^2}(\phi^{-1})^{k-m-n-p-q}\gamma^{\mu\nu m}\gamma^{\alpha\beta n}(\gamma^p_{\mu\alpha}\gamma^q_{\nu\beta}-\gamma^p_{\mu\nu}\gamma^q_{\alpha\beta})\right]\ ,$$

$$\mathcal{L}_2=(\sqrt{\gamma})^{-k}[-\tfrac{1}{4}\phi^{k-m-n-p-q}\gamma^{\mu\nu m}\gamma^{\alpha\beta n}\hat{F}^{ap}_{\mu\alpha}\hat{F}^{aq}_{\nu\beta}-\tfrac{1}{2}(\phi^{-1})^{k-l-m-n}\gamma^{\mu\nu l}(\hat{D}_\mu\phi^{am})(\hat{D}_\nu\phi^{an})]\ , \tag{20}$$

where R^k is the Fourier components of the four-dimensional scalar curvature R in the basis $\hat{\partial}_\mu$. Notice that $\mathcal{L}_1$ describes a gauge theory of the Virasoro-Kac-Moody symmetry associated with P [4,11], and $\mathcal{L}_2$ describes a gauge theory of the Kac-Moody symmetry associated with G [6]. Remarkably *the Lagrangian does not, in fact cannot, contain any Higgs-type potential for the scalar multiplets ϕ^k and ϕ^{ak}.* The gauge invariance simply forbids any polynomial potential for the scalar multiplets from the Lagrangian.

To discuss the physical content of the theory notice that $\mathcal{L}_5$ has the unique vacuum

$$\langle \gamma_{\mu\nu}^k \rangle = \eta_{\mu\nu}\delta_0^k, \quad \langle \phi^k \rangle = \delta_0^k, \quad \langle \phi^{ak} \rangle = 0 \ . \tag{21}$$

The vacuum breaks the Virasoro-Kac-Moody group down to $P \otimes G \otimes \mathrm{U}(1)$, where U(1) is the Cartan subgroup of the Virasoro group. This must be clear from (13) and (16). Expanding the fields around the vacuum and setting

$$\gamma_{AB} \simeq \begin{bmatrix} \eta_{\mu\nu} + e\kappa h_{\mu\nu} & e\kappa A_\mu \\ e\kappa A_\nu & 1 + 2e\kappa\varphi \end{bmatrix} \tag{22}$$

we obtain, with $e^2\kappa^2 = 16\pi G$, [4]

$$\mathcal{L}_1 = -\tfrac{1}{4}[(\partial_\mu \rho_{\alpha\beta}^{-n})(\partial_\mu \rho_{\alpha\beta}^{n}) - (\partial_\mu \rho_{\alpha\alpha}^{-n})(\partial_\mu \rho_{\beta\beta}^{n}) + 2(\partial_\mu \rho_{\mu\nu}^{-n})(\partial_\nu \rho_{\alpha\alpha}^{n}) - 2(\partial_\mu \rho_{\mu\alpha}^{-n})(\partial_\nu \rho_{\nu\alpha}^{n}) - (n^2/\kappa^2)(\rho_{\alpha\beta}^{-n}\rho_{\alpha\beta}^{n} - \rho_{\alpha\alpha}^{-n}\rho_{\beta\beta}^{-n})]$$
$$-\tfrac{3}{2}(\partial_\mu \varphi^0)^2 - \tfrac{1}{4}(\partial_\mu A_\nu^0 - \partial_\nu A_\mu^0)^2 + \text{interactions} \ , \tag{23}$$

where all the four-dimensional contractions are made with the flat metric $\eta_{\mu\nu}$ and

$$\rho_{\mu\nu}^0 = h_{\mu\nu}^0 + \eta_{\mu\nu}\varphi^0 \ ,$$
$$\rho_{\mu\nu}^n = h_{\mu\nu}^n + i\frac{\kappa}{n}(\partial_\mu A_\nu^n + \partial_\nu A_\mu^n) - \frac{2\kappa^2}{n^2}\partial_\mu\partial_\nu\varphi^n \quad (n \neq 0) \ . \tag{24}$$

So, after the symmetry breaking, $\mathcal{L}_1$ contains an infinite tower of massive spin-two particles $\rho_{\mu\nu}^n$ $(n \neq 0)$, the massless graviton $\rho_{\mu\nu}^0$, the massless gauge fields A_μ^0 of U(1), and the massless dilaton φ^0. Notice that *it is not $h_{\mu\nu}^0$, but $\rho_{\mu\nu}^0$, which describes the massless spin-two graviton* [4,12]. Now, expanding $\mathcal{L}_2$ around the vacuum (21) we obtain [6]

$$\mathcal{L}_2 = -\tfrac{1}{4}H_{\mu\nu}^{a-n}H_{\mu\nu}^{an} - \tfrac{1}{2}(n/\kappa)^2 C_\mu^{a-n}C_\mu^{an}$$
$$-\tfrac{1}{2}(D_\mu \phi^{a0})^2 + \text{interactions} \ , \tag{25}$$

where $H_{\mu\nu}^{ak}$ is the field strength of C_μ^{ak} given by

$$C_\mu^{a0} = A_\mu^{a0} \ ,$$
$$C_\mu^{an} = A_\mu^{an} + i\frac{\kappa}{n}\partial_\mu \phi^{an} \quad (n \neq 0) \ . \tag{26}$$

So $\mathcal{L}_2$ contains an infinite tower of the massive spin-one fields C_μ^{an} $(n \neq 0)$, the massless gauge fields C_μ^{a0} of G, and the massless scalar multiplet ϕ^{a0} which forms an adjoint representation of G. This shows that *the mass spectrum of the Lagrangian (20) becomes that of a harmonic oscillator whose mass scale is determined by the Planck scale.* Furthermore, all the massive modes are doubly degenerate. The degeneracy follows from the Hermiticity of the Lagrangian (20), which guarantees the charge conjugation invariance of the theory under the unbroken subgroup U(1) of the Virasoro group. These are precisely what one expects from a string theory.

Certainly the vacuum (21) is unique from the 5-dimensional point of view. After the dimensional reduction, however, there is nothing which can forbid ϕ^{ak} to acquire a nonvanishing expectation value. So the Lagrangian (20) in general has the vacuum

$$\langle \gamma_{\mu\nu}^k \rangle = \eta_{\mu\nu}\delta_0^k, \quad \langle \phi^k \rangle = \delta_0^k, \quad \langle \phi^{ak} \rangle = \mu^a \delta_0^k \ . \tag{27}$$

Remarkably, this vacuum breaks the Kac-Moody group down to H, where H is the subgroup of G which forms the little group of μ^a. More significantly, the symmetry breaking generates the following mass matrix for the gauge fields A_μ^{ak} [6]:

$$M_{am,bn}^2 = \left[\left[\frac{m}{\kappa} \right]^2 \delta_{ab} - 2ig\frac{m}{\kappa} f_{ab}^c \mu^c + g^2 f_{ac}^e f_{bd}^e \mu^c \mu^d \right] \delta_{m+n}^0 \ . \tag{28}$$

So the mass spectrum of the vector fields is characterized by two completely different mass scales: the Planck scale which is responsible for the harmonic-oscillator spectrum of the string and the elementary particle scale which is responsible for the fine structure of the mass spectrum. The mass spectrum of the spin-two particles remains unchanged and is characterized by the Planck scale.

In summary we have succeeded in constructing a prototype gauge theory of the Virasoro-Kac-Moody symmetry associated with the Poincaré group and the grand unification group, which is capable of describing the dynamics of a propagating colored string in the four-dimensional curved space-time. We conclude with the following remarks.

(1) A most remarkable characteristics of the theory is that *it describes a theory which is not based on the unitary representations of the underlying symmetry group.* Indeed none of the fields which appear in the Lagrangian (20) form a unitary representation under the Virasoro group [3,4]. Nevertheless, the Hamiltonian of the theory becomes explicitly positive-definite as we have demonstrated. This is because after the symmetry breaking all the physical fields become explicitly unitary under the unbroken subgroup. This guarantees the unitary of the theory.

(2) Another remarkable characteristics of the theory is that *it is a nonlinearly realized gauge theory.* Notice that the nonlinear realization described by (16) is of a novel

type, totally different from the standard Callan-Coleman-Wess-Zumino nonlinear realization [13]. Furthermore *the nonlinear realization makes the spontaneous symmetry-breaking geometric* [14]. In spite of the fact that the nonlinear gauge invariance precludes any Higgs-type potential from the Lagrangian, the symmetry breaking must take place. The theory simply does not admit a vacuum which remains invariant under the gauge transformation.

(3) The spontaneous symmetry breaking must leave the gauge fields of the unbroken subgroup massless. But the theory also has the massless scalar fields which can be interpreted as the Nambu-Goldstone fields of the symmetry breaking. The interpretation follows from the fact that the vacuum of the theory is degenerate, because there is no potential for the scalar fields. Notice, however, that after the symmetry breaking there is nothing which can guarantee the scalar fields to remain massless. So *the scalar fields become the pseudo Goldstone fields of the symmetry breaking.*

(4) Notice that *when the five-dimensional metric (4) becomes flat, the Lagrangian (20) describes a gauge theory of the Kac-Moody group associated with G*, which is nonlinearly realized. The gauge theory of the Kac-Moody group is very interesting in its own right, and could be made to be renormalizable as we have suggested at the beginning of this paper. Assuming the renormalizability one may be able to calculate the masses of the pseudo Goldstone particles through the quantum correction.

(5) The theory may be capable of generating a second-step symmetry breaking through the higher-order quantum correction. Indeed it is quite possible that one may start from the vacuum (21) but end up with the vacuum (27) through the quantum correction which induces a Coleman-Weinberg-type effective potential. According to this scenario the quantum correction can generate the elementary particle scale which provides the fine structure to the mass spectrum.

(6) As we have emphasized, the physical metric of the unified theory which describes the massless graviton is $\rho^0_{\mu\nu}$ defined by (24). Furthermore the dilaton ϕ^0 becomes a pseudo Goldstone particle which should acquire a small mass after the quantum correction. These facts should have important physical implications in the cosmology and the fifth force [12,15].

(7) So far we have not considered the central extension of the theory. One might need the central extension if the theory develops an anomaly which necessitates the central extension. At this moment, however, we find no compelling reason for the central extension of the theory.

(8) Obviously, the theory allows a straightforward supersymmetric generalization. With the supersymmetric generalization the theory becomes an effective field theory of an internal superstring.

A detailed discussion of the subject with a supersymmetric generalization will be published elsewhere [16].

It is a great pleasure to thank S. O. Ahn for encouragement. This work was supported in part by the Ministry of Education and by the Korea Science and Engineering Foundation.

[1] Y. Nambu, Lecture at the Copenhagen Summer Symposium, 1970 (unpublished); T. Goto, Prog. Theor. Phys. **46**, 1560 (1971).

[2] M. B. Green, J. H. Schwarz, and E. Witten, *Superstring Theory* (Cambridge University Press, Cambridge, England, 1987); M. Kaku, *Introduction to Superstrings* (Springer-Verlag, New York, 1988).

[3] Y. M. Cho, Phys. Rev. Lett. **67**, 3469 (1991); Y. M. Cho, in *Proceedings of the XXVth International Conference on High Energy Physics,* Singapore, 1990, edited by K. K. Phua and Y. Yamaguchi (World Scientific, Singapore, 1991).

[4] Y. M. Cho and S. W. Zoh, Phys. Rev. D **46**, R2290 (1992); **46**, 3483 (1992); Y. M. Cho, in *Proceedings of XXIst International Conference on Differential Geometric Methods in Theoretical Physics,* Tianjin, 1992, edited by M. L. Ge and C. N. Yang (World Scientific, Singapore, 1992).

[5] I. Bars and S. Yankielowicz, Phys. Rev. D **35**, 3878 (1987); Phys. Lett. B **196**, 329 (1987); I. Bars, Nucl. Phys. **B317**, 395 (1988); M. Kaku, Phys. Rev. D **38**, 3052 (1988).

[6] Y. M. Cho and S. W. Zoh, Phys. Lett. B **292**, 55 (1992); Y. M. Cho, in *Proceedings of Vth Asia Pacific Physics Conference,* edited by S. P. Chia and K. S. Low (World Scientific, Singapore, 1992).

[7] A. C. Cadavid and R. J. Finkelstein, Lett. Math. Phys. **16**, 279 (1988); **17**, 159 (1989).

[8] I. Kaplansky, Commun. Math. Phys. **86**, 49 (1982); B. L. Feigin and D. B. Fuks, Funct. Anal. Appl. **16**, 47 (1982).

[9] Y. M. Cho, J. Math. Phys. **16**, 2029 (1975); Y. M. Cho and P. G. O. Freund, Phys. Rev. D **12**, 1711 (1975); Y. M. Cho and P. S. Jang, *ibid.* **12**, 3789 (1975).

[10] Y. M. Cho, Phys. Rev. D **35**, R2628 (1987); Phys. Lett. B **186**, 38 (1987); **199**, 358 (1987); Y. M. Cho and D. S. Kimm, J. Math. Phys. **30**, 1570 (1989).

[11] L. Dolan and M. J. Duff, Phys. Rev. Lett. **52**, 14 (1984); C. S. Aulakh and D. Sahdev, Phys. Lett. **164B**, 293 (1985).

[12] Y. M. Cho, Phys. Rev. Lett. **68**, 3133 (1992).

[13] S. Coleman, J. Wess, and B. Zumino, Phys. Rev. **177**, 2239 (1969); C. G. Callan, S. Coleman, J. Wess, and B. Zumino, *ibid.* **177**, 2247 (1969); Y. M. Cho, Phys. Rev. D **13**, 2810 (1978).

[14] Y. M. Cho, Phys. Rev. Lett. **55**, 2932 (1985); Y. M. Cho and P. Y. Pac, Mod. Phys. Lett. A **3**, 1473 (1988); Y. M. Cho and J. H. Yoon, Phys. Rev. D **47**, 3465 (1993).

[15] Y. M. Cho, Phys. Rev. D **41**, 2462 (1990); Y. M. Cho and D. H. Park, Gen. Relativ. Gravit. **23**, 741 (1991).

[16] Y. M. Cho and S. W. Zoh, Report SNUTP 92-63 (unpublished).