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Wormhole created from vacuum fluctuation

Liao Liu
Department of Physics, Beijing Normal University, Beijing 100875, China
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Recently a number of wormhole solutions to general relativity plus matter have been found, but none of them is connected to the vacuum matter fluctuation. In this paper, we point out that vacuum fluctuation in the one-loop approximation in Einstein's theory can create Lorentzian wormholes, which act as a throat connecting two de Sitter universes.

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I. INTRODUCTION

As we know from quantum field theory in curved spacetime, when the curvature of spacetime is comparable to the Planck curvature $c^3G^{-1}\hbar^{-1}\sim 10^{66}$ cm^{-2}, the spacetime fluctuation induced by vacuum matter fluctuations cannot be neglected [1]. Such geometric fluctuations might cause the spacetime to have a quantum-foam-like structure with all possible topologies. Thus a simple connected smooth spacetime on a large scale might be really a multiply connected spacetime with handles or wormholes everywhere on a small scale of the Planck length. However, as far as we know no one has given a concrete realization of this interesting conjecture until now. We, in this paper, try to show that such a concrete realization does exist.

II. ONE-LOOP CONTRIBUTION TO EFFECTIVE ACTION

By using conformal time instead of cosmic time, we may write the Robertson-Walker metric of the Universe as

$$\begin{aligned} ds^2&=a^2(\eta)[d\eta^2-d\chi^2-\sin^2\chi\, d\Omega_2^2], \quad k=1\ ,\\ ds^2&=a^2(\eta)[d\eta^2-dx^2-dy^2-dz^2], \quad k=0\ ,\\ ds^2&=a^2(\eta)[d\eta^2-d\chi^2-\sinh^2\chi d\Omega_2^2], \quad k=-1\ . \end{aligned} \tag{1}$$

Assume the dimension of $a(\eta)$ is a length in the Planck system of unit $c=\hbar=G=1$; then all the coordinates in (1) are dimensionless.

As is known, though the classical conformal invariant matter fields are traceless, the quantum matter fields in the one-loop approximation do have a trace anomaly, which is given by

$$\langle T\rangle=\alpha\Box R+\beta(R_{\mu\nu}R^{\mu\nu}-\tfrac{1}{3}R^2)+\gamma C_{\mu\nu\lambda\tau}C^{\mu\nu\lambda\tau}\ . \tag{2}$$

The constants appearing in Eq. (2) are

$$\alpha=\frac{1}{2880\pi^2}[N_0+3N_{1/2}-6N_1] \tag{3}$$

$$\beta=\frac{1}{2880\pi^2}[-N_0-\tfrac{11}{2}N_{1/2}-64N_1]\ , \tag{4}$$

where N_0 is the number of scalar fields, $N_{1/2}$ is the number of Weyl fields, and N_1 is the number of gauge fields. For conformal flat spacetime, the term containing γ in Eq. (2) is zero and this is the case we will consider here.

For the metric (1), the trace anomaly is [3]

$$\begin{aligned}\langle T\rangle=&(6\alpha/a^5)[a\ddddot{a}-4\dot{a}\dddot{a}+(6/a)\dot{a}^2\ddot{a}-3\ddot{a}^2+2(\dot{a}^2-a\ddot{a})K]\\ &-(12\beta/a^5)[(1/a)\dot{a}^2\ddot{a}-(1/a^2)\dot{a}^4-(\dot{a}^2-a\ddot{a})K]\ ,\end{aligned} \tag{5}$$

where

$$\begin{aligned} &R_c^0=-(3/a^4)(a\ddot{a}-\dot{a}^2)\ ,\\ &R_1^1=R_2^2=R_3^3=-(1/a^4)(a\ddot{a}+\dot{a}^2+2a^2K)\ ,\\ &R=(6/a^3)(\ddot{a}+aK)\ ,\\ &C_{\mu\nu\lambda\tau}=0\ . \end{aligned} \tag{6}$$

Let Γ_1 be the one-loop contribution to the effective action. From the relation

$$\langle T\rangle=(2/\sqrt{-g}\,)g_{\mu\nu}\delta\Gamma_{1/\delta}g_{\mu\nu}=(1/a^3)\delta\Gamma_1/\delta a \tag{7}$$

we can get [3]

$$\Gamma_1=V\int d\eta[3\alpha(\ddot{a}/a)^2+6\alpha(\dot{a}/a)^2+\beta(\dot{a}/a)^4+6\beta(\dot{a}/a)^2]\ . \tag{8}$$

In (8), we have taken $K=1$ and

$$V=\int_0^\pi d\chi\int_0^\pi d\theta\int_0^{2\pi}d\varphi\sin^2\chi\sin\theta=2\pi^2\ . \tag{9}$$

This is just the case for the wormhole solution we want to have.

The total effective action is

$$\Gamma=\Gamma_0+\Gamma_1\ , \tag{10}$$

where the classical Einstein Γ_0 is

$$\Gamma_0=\int\sqrt{-g}\,d^4x\,\phi R=V\int d\eta[6\phi a(\ddot{a}+a)]\ , \tag{11}$$

so

$$\begin{aligned}\Gamma=V\int d\eta[&6\phi a(\ddot{a}+a)+3\alpha(\ddot{a}/a)^2+6\alpha(\dot{a}/a)^2\\ &+\beta(\dot{a}/a)^4+6\beta(\dot{a}/a)^2]\ ,\end{aligned} \tag{12}$$

where

$\phi=1/(16\pi G)$.

For simplicity, let us consider conformal invariant scalar fields, where

$$\beta=-\alpha\ .$$

So (12) can be rewritten as

$$\Gamma=V\int d\eta[6\phi a(\ddot{a}+a)+3\alpha(\ddot{a}/a)^2-\alpha(\dot{a}/a)^4]\ . \quad (13)$$

III. WORMHOLE SOLUTION

From (13), the Lagrangian is

$$L=[6\phi a(\ddot{a}+a)+3\alpha(\ddot{a}/a)^2-\alpha(\dot{a}/a)^4]\ . \quad (14)$$

The a equation in conformal time η is then

$$4\phi\ddot{a}+4\phi a-4\alpha a^{-5}\dot{a}^4+16\alpha a^{-4}\dot{a}^2\ddot{a}-6\alpha a^{-3}\ddot{a}^2-8\alpha a^{-3}\dot{a}\dddot{a}+2\alpha a^{-2}\ddddot{a}=0\ . \quad (15)$$

Note the relation between cosmic time t and conformal time η is

$$d/d\eta=a(d/dt)\ . \quad (16)$$

Then the a equation in Euclidean cosmic time t is

$$-2\phi a^2\ddot{a}-2\phi a\dot{a}^2+2\phi a+3\alpha a\dot{a}\dddot{a}-7\alpha\dot{a}^2\ddot{a}+\alpha a\ddot{a}^2+\alpha a^2\ddddot{a}=0\ . \quad (17)$$

This is a nonlinear ordinary differential equation of $a(t)$ in fourth order; it seems hopeless to find an exact solution of it, but fortunately we find that

$$a(t)=A\cosh(t) \quad (18)$$

is an exact solution of it, if $\alpha=2\phi$, $A=i$.

Proof. Put (18) into (17); we have

$$-\phi A^2\cosh^2 t-\phi A^2\sinh^2 t+\alpha A^2\cosh^2 t+\phi=0 \quad (19)$$

or

$$A^2\cosh^2 t(-2\phi+\alpha)+\phi(A^2+1)=0\ .$$

Evidently, $a(t)=A\cosh(t)$ is an exact solution of (17), if

$$\alpha=2\phi \text{ or } N_0=(360\pi)/G\simeq 1131\ ,$$

and (20)

$$A=i(=il_P),\quad l_P=(G\hbar c^{-3})^{1/2}\ .$$

Thus the proof is completed. At first sight, it seems there is no meaning for a solution with an imaginary scale factor; however, under the transformation

$$t\to it\ ,$$

$$A\to iA\ ,$$

the signature in (1) changes from $(+---)$ to $(-+++)$, but t and $a(t)$ remain real, so we have really found a Lorentzian wormhole of throat radius

$$a(t)|_{t=0}=l_P$$

which connects two de Sitter universes.

IV. CONCLUSIONS

To conclude, though recently a number of wormhole solutions to general relativity plus matter have been found, none of them is connected to the vacuum matter quantum fluctuation [4]. In this paper we succeed in finding a Lorentzian wormhole solution in Einstein's gravity theory without matter. Such wormholes are created from pure vacuum fluctuations of a certain number N of conformal invariant quantum scalar matter fields in the one-loop approximation. Our result is a realization of Wheeler's quantum foam conjecture proposed more than thirty years ago.

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